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SOME APPLICATIONS OF CREDIBILITY THEORY TO GROUP INSURANCE

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ABSTRACT

The rating and underwriting of group insurance differs greatly from that of other forms of insurance. Presented herein are some credibility formulas that are designed to be specifically applicable to group insurance. These include credibility formulas that are based on the size of the group, are adjusted for group member changes, are nonlinear to lessen the effect of large claims, and are for high deductible plans, for competitive situations and for varying time periods. The practical motivation and use of these formulas are discussed. The method used here is compared to some other credibility models. The formulas then are applied to some actual insurance data.

I. INTRODUCTION

Outline of the Paper

The first two sections are introductory. Section III presents a number of formulas for the credibility of experience by size of group. In each of these formulas, the credibility increases with size of group. The differences between the formulas reflect both some simplifying assumptions and two different ways of expressing the size of the group: by number of coverage units and by manual premium volume.

The formulas in Section III assume that no coverage units leave or enter the group. Section IV presents a method to modify the credibility when risk units leave the group, assuming that all risk units leaving the group do so at the end of year 1 and are immediately replaced by new units. Other scenarios are more realistic but beyond the scope of this paper. Section IV presents only the number of units formula (not premium volume), but the reader should be able to derive other formulas analogous to those in Section III.

In Section V a method for estimating appropriate "pooling" points is presented. An alternate method using a quadratic formula also is included. Section VI suggests a similar methodology for high deductible plans. Section VII shows how competitive pressures might affect the credibility.

This paper does not discuss estimation techniques. Nevertheless, the credibility values are useful only if they can be estimated by using the assumption that they do not vary over time. Section VIII evaluates this assumption. Section IX includes a brief history of credibility theory and compares the methodology used in this paper to some widely known work in this area. Section X gives a simplified method of varying the credibility for various lengths of the experience period. Section XI describes the data that are used to illustrate the formulas of the paper.

The Group Insurance Environment

Most of the discussion in this paper refers to group medical insurance, but might apply equally to other group insurance coverages. Group medical insurance here refers to broad comprehensive major medical coverage that reimburses the insured for practically all necessary health-related services, subject only to a very high payment limit and modest deductible and coinsurance amounts. Some common features of group medical insurance are listed below.

1. The insurance coverage is written with rates guaranteed for a period of time that is almost always one year.
2. Groups usually consist of all the employees of a single company, a governmental unit, or members of a union. In general, group membership is based on the employer-employee relationship, and eligible dependents of the employees are covered.
3. The coverage is normally written without individual health underwriting, down to a minimum size that is often ten employees.
4. The rates are expressed per employee and per dependent unit. Although the insurer may base the rates on the characteristics of the employees, such as their ages at the beginning of the year, the rates themselves do not vary during the year. Thus, the insurer bears the risk of changes in the employee group.
5. There may be provision for retrospective experience rating, particularly on groups of more than 100 employees.
6. Insurers have a manual or tabular rating system that includes many factors. A few of those commonly used are geographic location, age, gender, industry, occupation, size of group, and the benefit plan offered.
7. Most insurers use the group's own claim experience in setting the rates. The method of using this experience varies widely and depends on the size of the group. For groups in the 100- to 300-member range, the method usually involves blending the group's own experience rate with the manual rate. In general, this analysis is performed by group underwriters who have had little education in statistics. They usually look to the actuary to supply a credibility table for the purpose of blending

of rates. The actuary must supply a table or formula for the credibility that does not vary by too many factors and is relatively easy to explain to the group underwriter.

8. The purchaser of the insurance is knowledgeable about his benefits and premium rates. If he feels unfairly treated, he is likely to move the coverage to another insurer. The market is relatively competitive, and the customer can usually find another insurer that will write the same benefits at an attractive price. Furthermore, the customer is usually advised by a consultant or insurance broker who has little loyalty to the insurer but may have a strong influence on the customer's choice. It is often necessary for the insurer to explain and justify its rating methods to the brokers. The marketing department of the insurance company is frequently involved in these communications and rating decisions. Thus, no matter how correct the actuary's theoretical evaluation of rating methods, the final rates are often the result of negotiation and compromise. Satisfying the group brokers who influence the client may be the only way the insurer can continue to stay in the market.
9. Although most insurers will use a group's own claim and premium experience in setting rates on even the smaller groups, they may not report the experience to the group in any form that is useful to a competing insurer.
10. Insurance company data bases are often designed with only the needs of the claim department in mind. The actuary or underwriter may not be able to obtain good historical data. Very often the claims data base and the coverage or premium data bases will be inconsistent. This may make it difficult or impossible to match a group member's claim history with his dates of beginning or ending coverage. Sometimes the insurance company does not even maintain the membership records.

II. PRELIMINARIES AND NOTATION

Let X_1 through X_{n+1} be random variables. We want to approximate the conditional expectation $E(X_{n+1}|X_1 . . . X_n)$ with the linear expression

$$C + \sum_{i=1}^{n} Z_i \cdot X_i,$$

where C and the Z_i's are constants. We want to pick these constants so that the expected squared error,

$$E\left[C + \sum_{i=1}^{n} Z_i \cdot X_i - E(X_{n+1}|X_1 . . . X_n)\right]^2,$$

is minimized. The solution is the n simultaneous linear equations:

$$\sum_{i=1}^{n} Z_i \cdot \text{cov}(X_i,X_j) = \text{cov}(X_{n+1}, X_j) \text{ for } j=1,n \tag{2.1}$$

and

$$C = E(X_{n+1}) - \sum_{i=1}^{n} Z_i \cdot E(X_i),$$

where $\text{cov}(X,Y)$ is the covariance of X and Y or $E\{[X - E(X)]\,[Y - E(Y)]\}$. To show this, set the $n+1$ partial derivatives with respect to the Z_i's and C equal to zero.

In particular, if $n = 1$, then:

$$Z_1 = Z = \text{cov}(X_1,X_2)/\text{var}(X_1) \tag{2.2}$$

and

$$E(X_2|X_1) \approx ZX_1 + E(X_2) - ZE(X_1) = ZX_1 + \mu_2 - Z\mu_1,$$

where $\mu_i = E(X_i)$. Z is called the credibility. More simply, if $\mu_1 = \mu_2 = \mu$,

$$E(X_2|X_1) \approx ZX_1 + (1 - Z)\mu. \tag{2.3}$$

This result is valid as long as the moments exist and there is a unique set of Z_i's that satisfy (2.1).

Assume that there are m risks that taken together form a group. The risks could be, for example, the employees and dependent units of a group insurance customer. The total expected claims for the group for some insurance coverage is to be estimated for an insuring period of one year. The experience of the ith risk in insurance year t will be written X_{it}, with $1 \le i \le m$ and $t = 1,2,\ldots$. The X_{it} values can be manual premium loss ratios. That is, $X_{it} = Y_{it}/P_i$, where Y_{it} is the actual claims incurred for risk i in year t and P_i is the manual premium for risk i.

Let:

$$X_{\cdot t} = \frac{\sum_{i=1}^{m} X_{it}P_i}{\sum_{i=1}^{m} P_i} = \frac{\sum_{i=1}^{m} Y_{it}}{\sum_{i=1}^{m} P_i}. \tag{2.4}$$

Then $X_{\cdot t}$ is the average claim experience for the group in year t. Often it will be convenient to treat the case that $P_i = P$ for all i, and therefore:

$$X_{\cdot t} = \frac{1}{m}\sum_{i=1}^{m} X_{it} \tag{2.5}$$

III. CREDIBILITY BY GROUP SIZE

The method used here has been called a layered or hierarchical model. This has been discussed in Jewell [6] and others. Some alternative credibility models are summarized by Venter [9].

Approximate $E(X_{\cdot 2}|X_{\cdot 1}) \approx ZX_{\cdot 1} + C$ by using (2.3) and (2.2) for the case (2.5). Then:

$$Z = \frac{\operatorname{cov}(X_{\cdot 1},X_{\cdot 2})}{\operatorname{var}(X_{\cdot 1})} = \frac{\operatorname{cov}\left[\frac{1}{m}\sum_{i=1}^{m} X_{i1}, \frac{1}{m}\sum_{i=1}^{m} X_{i2}\right]}{\operatorname{var}\left[\frac{1}{m}\sum_{i=1}^{m} X_{i1}\right]} \tag{3.1}$$

and $C = (1 - Z)\mu$. Now assume that the variances and covariances do not depend on the individual risks, so that:

$$a_{11} = \operatorname{var}(X_{i1}), \tag{3.2}$$

$$a_{12} = \operatorname{cov}(X_{i1}, X_{i2}), \tag{3.2}$$

$$b_{11} = \operatorname{cov}(X_{i1}, X_{j1}), \tag{3.2}$$

and

$$b_{12} = \operatorname{cov}(X_{i1}, X_{j2}), \tag{3.2}$$

for all $1 \leq i \leq m$, $1 \leq j \leq m$, and $i \neq j$. Then (3.1) becomes:

$$Z = \frac{ma_{12} + m(m-1)b_{12}}{ma_{11} + m(m-1)b_{11}} = \frac{a_{12} + (m-1)b_{12}}{a_{11} + (m-1)b_{11}} \tag{3.3}$$

or

$$Z = \frac{k_1 + (m-1)k_2}{1 + (m-1)k_3} \tag{3.4}$$

where $k_1 = a_{12}/a_{11}$, $k_2 = b_{12}/a_{11}$, and $k_3 = b_{11}/a_{11}$.

Formula (3.4) is useful and simple. Furthermore, it can be explained intuitively in that the credibility is equal to k_1 if $m = 1$. As m increases, more information is available for each risk, so that the credibility increases. The highest credibility is k_2/k_3, which should be close to 1.

If instead $X_{\cdot t} = \sum_{i=1}^{m} X_{it} P_i / \sum_{i=1}^{m} P_i$ as in (2.4), then

$$Z = \frac{a_{12} + \left[m \frac{P.}{P'_.} - 1\right] b_{12}}{a_{11} + \left[m \frac{P.}{P'_.} - 1\right] b_{11}} = \frac{k_1 + \left[m \frac{P.}{P'_.} - 1\right] k_2}{1 + \left[m \frac{P.}{P'_.} - 1\right] k_3} \tag{3.6}$$

where $P. = \frac{1}{m} \sum_{i=1}^{m} P_i$ and $P'_. = \sum_{i=1}^{m} P_i^2 / \sum_{i=1}^{m} P_i$.

Formula (3.6) is the same as (3.4), except that m is adjusted by the factor $P./P'_.$. The reciprocal of this factor,

$$\frac{P'_.}{P.} = 1 + \frac{\frac{1}{m} \sum_{i=1}^{m} (P_i - P.)^2}{P.^2},$$

is a measure of the relative variance of the P_i values. If all the premiums were of equal size, then the most information would be available from one risk to another.

Other assumptions can be made concerning the covariance structure. If we assume that $b_{11} = b_{12}$, then (3.4) becomes:

$$Z = \frac{k_1 + (m - 1)k_2}{1 + (m - 1)k_2} \tag{3.7}$$

and (3.6) becomes:

$$Z = \frac{k_1 + \left[m \frac{P.}{P'_.} - 1\right] k_2}{1 + \left[m \frac{P.}{P'_.} - 1\right] k_2}. \tag{3.8}$$

This formula has the nice property that $\lim_{m \to \infty} Z = 1$. I believe that (3.7) or (3.8) will be the formula that many group insurance companies will want to use for their credibility table.

Venter [9] has made some good arguments that the variances are inversely proportional to the premium. These arguments lead to two possible sets of assumptions:

(1) $b_{11} = b_{12} = c_1$, $a_{12} = c_1 + c_2$, and $\text{var}(X_{i1}) = c_1 + c_2 + c_3/P_i$, which yields:

$$Z = \frac{c_2 + m \dfrac{P.}{P!} c_1}{\dfrac{c_3}{P!} + c_2 + m \dfrac{P.}{P!} c_1},$$

or

(2) $\text{cov}(X_{i1},X_{j1}) = \text{cov}(X_{i1},X_{j2}) = c_1/(P_iP_j)$ for all $1 \leq i \leq m$, $1 \leq j \leq m$, $i \neq j$; $a_{12} = (c_1+c_2)/P_i^2$; and $a_{11} = (c_1+c_2+c_3)/P_i^2$; which yields:

$$Z = \frac{c_2 + mc_1}{c_3P. + c_2 + mc_1}.$$

IV. CREDIBILITY ADJUSTED FOR GROUP MEMBER CHANGES

Assume that a set of the risks in the group leaves and is replaced by other risks. Suppose p is the proportion of risks that persist in the group ($0 < p < 1$). That is, exactly $(1 - p)m$ risks leave and pm stay. Let

$$X_{pt} = \frac{1}{pm} \sum_{i=1}^{pm} X_{it}$$

where $\{1 \ldots pm\}$ refers to those risks that stay, and

$$X_{qt} = \frac{1}{qm} \sum_{i=pm+1}^{m} X_{it}$$

where $\{pm+1 \ldots m\}$ refers to those risks that leave and are replaced ($q = 1 - p$). Then,

$$X._t = pX_{pt} + qX_{qt},$$

and using the linear approximation as before:

$$E(X_{p2}|X._1) \approx Z_pX._1 + (1 - Z_p)\mu$$

and

$$E(X_{q2}|X._1) = Z_qX._1 + (1 - Z_q)\mu.$$

Solving these equations as in Section III yields:

$$Z_p = \frac{a_{12} + (m-1)b_{12}}{a_{11} + (m-1)b_{11}}, \qquad Z_q = \frac{mb_{12}}{a_{11} + (m-1)b_{11}} \tag{4.1}$$

and

$$\begin{aligned} E(X_{\cdot 2}|X_{\cdot 1}) &= E(pX_{pt} + qX_{qt}|X_{\cdot 1}) = pE(X_{pt}|X_{\cdot 1}) + qE(X_{qt}|X_{\cdot 1}) \\ &\approx p[Z_pX_{\cdot 1} + (1 - Z_p)\mu] + q[Z_qX_{\cdot 1} + (1 - Z_q)\mu] \\ &= ZX_{\cdot 1} + (1 - Z)\mu \end{aligned}$$

where

$$\begin{aligned} Z &= pZ_p + qZ_q \\ &= \frac{p[a_{12} + (m - 1)b_{12}] + qmb_{12}}{a_{11} + (m - 1)b_{11}} \\ &= \frac{pa_{12} + (m - p)b_{12}}{a_{11} + (m - 1)b_{11}}. \end{aligned} \tag{4.2}$$

This last formula can be derived in a slightly different way: Assume that possibly up to m new risks, $m + 1$ through $2m$, may take the place of the initial m risks, each with probability $1 - p$. Define the random variables p_i $(1 \le i \le m)$, where $pr\{p_i = 1\} = p$ and $pr\{p_i = 0\} = 1 - p$. Assume the p_i values are independent of each other and of the X_{it} $(1 \le i \le 2m)$.

Define $X'_{i2} = p_iX_{i2} + (1 - p_i)X_{i'2}$ *for* $i' = i + m$ and $X'_{\cdot 2} = \frac{1}{m}\sum_{i=1}^{m} X'_{i2}$.

Then $E(X'_{i2}|X_{\cdot 1}) \approx ZX_{\cdot 1} + (1 - Z)\mu$.

Since

$$\begin{aligned} \text{cov}(X'_{i2},X_{i1}) &= p\text{cov}(X_{i2},X_{i1}) + (1 - p)\,\text{cov}(X_{i'2}, X_{i1}) \\ &= pa_{12} + (1 - p)b_{12}, \end{aligned}$$

$$\begin{aligned} Z &= \frac{pa_{12} + (1 - p)b_{12} + (m - 1)b_{12}}{a_{11} + (m - 1)b_{11}} \\ &= \frac{pa_{12} + (m - p)b_{12}}{a_{11} + (m - 1)b_{11}}. \end{aligned}$$

In this section the claims in year 2 have been expressed as a linear function of the total claims in year 1. Of course, a better method would be to separate the claims in year 1 between the two sets of risks and write the approximation formula as a linear function of both. I leave it to the reader to derive the formulas for the two credibilities. As mentioned above, such split claim data may not be available.

V. NONLINEAR CREDIBILITY FORMULAS

A nonlinear formula is often suggested in an attempt to make the rating less sensitive to the large claim. Jewell and Schnieper [7] discuss a method in which the credibility depends on the experience. In the experience rating process, many insurers place an upper limit on the amount incurred by one group member. This limit is called the pooling level. The credibility model above lets us calculate an optimum pooling level based on the least-squares criterion:

Let $X_{i1}^{\nu} = \min\{X_{i1},\nu\}$, where ν is the pooling point yet unknown and $X_{\cdot 1}^{\nu} = \frac{1}{m}\sum_{i=1}^{m} X_{i1}^{\nu}$. Approximate $E(X_{\cdot 2}|X_{\cdot 1}^{\nu}) \approx C_{\nu} + Z_{\nu}X_{\cdot 1}^{\nu}$. This will be a better estimate than those in Section III if:

$$E[E(X_{\cdot 2}|X_{\cdot 1}) - (C_{\nu} + Z_{\nu}X_{\cdot 1}^{\nu})]^2 < E[E(X_{\cdot 2}|X_{\cdot 1}) - (C + ZX_{\cdot 1})]^2.$$

It is easy to show that the best estimate will be found by maximizing the quantity:

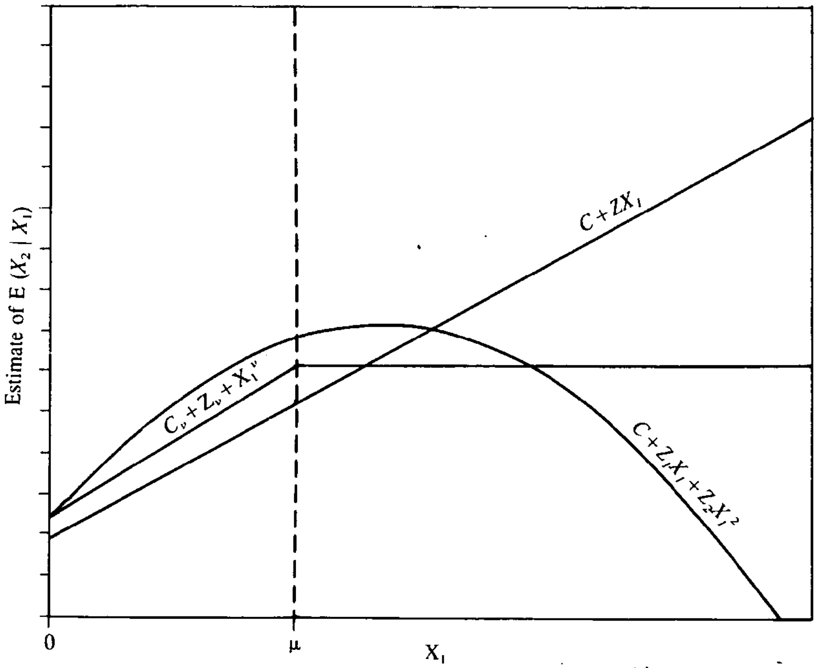
$$R_{\nu} = \frac{[\mathrm{cov}(X_{\cdot 1}^{\nu},X_{\cdot 2})]^2}{\mathrm{var}(X_{\cdot 1}^{\nu})} = \frac{[a_{12}^{\nu} + (m-1)b_{12}^{\nu}]^2}{a_{11}^{\nu} + (m-1)b_{11}^{\nu}}$$

with respect to ν, where again the four parameters are defined as in (3.2): $a_{11}^{\nu} = \mathrm{var}(X_{i1}^{\nu})$, $a_{12}^{\nu} = \mathrm{cov}(X_{i1}^{\nu},X_{i2})$, $b_{11}^{\nu} = \mathrm{cov}(X_{i1}^{\nu},X_{j1}^{\nu})$, and $b_{12}^{\nu} = \mathrm{cov}(X_{i1}^{\nu},X_{j2})$ for $1 \le i \le m$, $1 \le j \le m$, and $i \ne j$. Presumably as m increases, ν will also increase. Figure 1 shows how a graph of the formula might look for $m = 1$.

Another method of accomplishing a result similar to that of the pooling formula would be to use a quadratic instead of a linear formula. For $m = 1$, approximate $E(X_{12}|X_{11}) \approx C + Z_1X_{11} + Z_2X_{11}^2$. Define some higher-order

FIGURE 1

GRAPHS OF CREDIBILITY FORMULAS ACHIEVED WITH LINEAR, POOLING AND QUADRATIC FORMULAS



moments as: $\sigma_{ij} = E[(X_{11} - \mu)^i(X_{12} - \mu)^j]$ with $1 \leq i \leq 4$ and $1 \leq j \leq 2$. Then solve by using the following equations:

$$Z_2 = \frac{\sigma_{20}\sigma_{21} - \sigma_{30}\sigma_{11}}{\sigma_{40}\sigma_{20} - \sigma_{30}^2 - \sigma_{20}^3}$$

and

$$Z_1 = \frac{\sigma_{11}}{\sigma_{20}} - \left(\frac{\sigma_{11}}{\sigma_{20}} + 2\mu\right) \cdot Z_2. \qquad (5.1)$$

The evaluation of $E(X._2|X._1) \approx C + Z_1X._1 + Z_2X^2._1$ needs sixteen moments:

$$
\begin{aligned}
\alpha_1 &= E[(X_{i1} - \mu_1)(X_{i2} - \mu_2)] = a_{12}.\\
\alpha_2 &= E[(X_{i1} - \mu)(X_{j2} - \mu_2)] = b_{12}.\\
\alpha_3 &= E[(X_{i1} - \mu_1)^2] = a_{11}.\\
\alpha_4 &= E[(X_{i1} - \mu)(X_{j1} - \mu)] = b_{11}.\\
\alpha_5 &= E[(X_{i1} - \mu_1)^3].\\
\alpha_6 &= E[(X_{i1} - \mu_1)^2 (X_{j1} - \mu_1)].\\
\alpha_7 &= E[(X_{i1} - \mu_1)(X_{j1} - \mu_1)(X_{k1} - \mu_1)].\\
\alpha_8 &= E[(X_{i1} - \mu_1)^2 (X_{i2} - \mu_1)].\\
\alpha_9 &= E[(X_{i1} - \mu_1)^2 (X_{j2} - \mu_2)].\\
\alpha_{10} &= E[(X_{i1} - \mu_1)(X_{i2} - \mu_1)(X_{j1} - \mu_1)].\\
\alpha_{11} &= E[(X_{i1} - \mu_1)(X_{j1} - \mu_1)(X_{k2} - \mu_2)].\\
\alpha_{12} &= E[(X_{i1} - \mu_1)^4].\\
\alpha_{13} &= E[(X_{i1} - \mu_1)^3 (X_{j1} - \mu_1)].\\
\alpha_{14} &= E[(X_{i1} - \mu_1)^2 (X_{j1} - \mu_1)^2].\\
\alpha_{15} &= E[(X_{i1} - \mu_1)^2 (X_{j1} - \mu_1)(X_{k1} - \mu_1)].\\
\alpha_{16} &= E[(X_{i1} - \mu_1)(X_{j1} - \mu)(X_{k1} - \mu_1)(X_{l1} - \mu_1)].
\end{aligned}
$$

with $1 \le i \le m$, $1 \le j \le m$, $1 \le k \le m$, $1 \le l \le m$, and $i \ne j \ne k \ne l$; and in (5.1):

$$
\begin{aligned}
\sigma_{11} &= [\alpha_1 + (m-1)\alpha_2]/m.\\
\sigma_{20} &= [\alpha_3 + (m-1)\alpha_4]/m.\\
\sigma_{30} &= [\alpha_5 + 3(m-1)\alpha_6 + (m-1)(m-2)\alpha_7]/m^2.\\
\sigma_{21} &= [\alpha_8 + (m-1)\alpha_9 + 2(m-1)\alpha_{10} + (m-1)(m-2)\alpha_{11}]/m^2.\\
\sigma_{40} &= [\alpha_{12} + 4(m-1)\alpha_{13} + 3(m-1)\alpha_{14} + 6(m-1)(m-2)\alpha_{15}\\
&\quad + (m-1)(m-2)(m-3)\alpha_{16}]/m^3.
\end{aligned}
$$

Figure 1 also shows how a graph of the quadratic function might look.

VI. HIGH DEDUCTIBLE CREDIBILITY

Often group medical coverage is written with a very high deductible amount, frequently to protect the employer's self-insured medical plan from the fluctuation due to large claims. This coverage is called specific stop loss insurance, and the deductible amount is called the specific attachment point. Currently, insurers rate this product in one of two different ways. The first method uses tabular rates that take into account age, sex, industry, specific level, etc. In this method the group's own experience is ignored. The second method applies a percentage, which varies only by the specific level, to the group's estimated total claims. These two methods can lead to wildly different results, and there is some controversy as to which is superior. The credibility model above produces an answer that is a mixture of both methods.

Let $X^s_{i2} = \max \{0, X_{i2} - s)$ where s is the desired specific attachment point. Define: $X^s_{\cdot 2} = \frac{1}{m}\sum_{i=1}^{m} X^s_{i2}$, $E(X^s_{i2}) = \mu_s$, and again the moments $a^s_{12} = \mathrm{cov}(X_{i1}, X^s_{i2})$ and $b^s_{12} = \mathrm{cov}(X_{i1}, X^s_{j2})$ with $1 \le i \le m$, $1 \le j \le m$, and $i \neq j$. Then approximate: $E(X^s_{\cdot 2}|X_{\cdot 1}) \approx C + ZX_{\cdot 1}$, so that $C = \mu_s - Z\mu$, and:

$$Z = \frac{a^s_{12} + (m - 1)b^s_{12}}{a_{11} + (m - 1)b_{11}}$$

by formula (3.3).

VII. CREDIBILITY UNDER COMPETITION

Taylor [8] treated a case of credibility under competitive pricing. In that model it was assumed that the purchasing decision would be based more heavily on experience than on the correct credibility. The result was that the insurer, in order to minimize losses, would need to raise the credibility. Here I treat the case in which the competition uses lower credibility because, as is often the case in group insurance sales, the competition does not have access to the experience of the prospective group.

Assume that a group has an unknown maximum premium that it will pay. Assume that this maximum is normally distributed with mean $\mu + c$ and variance s^2, where $\mu = E(X_{\cdot 2})$ and c is a constant that is related to the cost of changing insurers. Let the charged premium be P_c. The gain F in year 2 is equal to $P_c - X_{\cdot 2}$ if the group renews, and 0 if it does not. If the maximum that the group will pay is independent of its claims experience, then:

$$E(F) = [P_c - E(X_{\cdot 2}|X_{\cdot 1})] \cdot \left\{1 - \Phi\left[\frac{P_c - (\mu + c)}{s}\right]\right\}$$

$$\approx [P_c - ZX_{\cdot 1} - (1 - Z)\mu] \cdot \left\{1 - \Phi\left[\frac{P_c - (\mu + c)}{s}\right]\right\}$$

where $\Phi(x) = \int_{-\infty}^{x} e^{-t^2/2}\, dt$.

This can be approximately solved for the P_c that maximizes $E(F)$. The insurer could use renewal experience to estimate c and s, which might depend on the size of the group. Note that if s is very small, then:

$$P_c = \max\{E(X_{\cdot 2}|X_{\cdot 1}), \mu + c\} \approx \max\{ZX_{\cdot 1} + (1 - Z)\mu, \mu + c\}.$$

This is the equivalent of not giving any credibility to low claim experience and full credibility to high claim experience. This is probably a good strategy to maximize profits, but it is possible that it would not be acceptable to the insurance brokers.

VIII. COVARIANCE STRUCTURE MODEL

Because the intent of this credibility method is to use prior data of variances and covariances to estimate the credibility, it is worth investigating the implicit assumptions regarding the covariance structure. Assume we have a claim series X_t with constant expectation $\mu = E(X_t)$ for all t and the set of possible approximations:

$$E[X_{t+n}|X_t \ldots X_{t+n-1}] \approx C_n^t + \sum_{i=1}^{n} Z_{in}^t X_{t+i-1}.$$

Then we want $Z_{in}^t = Z_{in}$ independently of t. By using (2.1) and induction on n, it is easy to show that

$$\frac{\operatorname{cov}(X_{t+i}, X_{t+j})}{\operatorname{var}(X_{t+i})} = r_{j-i} \quad ,$$

also independent of t, and that therefore,

$$\operatorname{var}(X_j) = \left(\frac{r_1}{r_{-1}}\right)^{j-i} \cdot \operatorname{var}(X_i)$$

for all i and j. Note that the series is not stationary but has constant normalized autocorrelations. Compare this to the structure in Gerber and Jones [4]. In fact, the series is stationary in the wide sense up to a geometrically changing scale parameter. Presumably we have for $j > i > 0$: $r_j < r_i$, $r_1^2 < r_2 < r_1$, and $0 < Z_{in} < Z_{jn}$. Also, $r_1 > r_{-1}$, which implies $\operatorname{var}(X_j) > \operatorname{var}(X_i)$.

IX. RELATIONSHIPS WITH OTHER CREDIBILITY MODELS

In this paper, I have treated the credibility formula as the best approximation of the conditional expectation based on the least-squares criterion. This method has an advantage in that no probability distribution is assumed.

It is also simple to explain. It requires the existence of a few second moments. Gerber [5] presents a simple explanation of this method.

The need to use an average between a class mean and a global mean has been recognized for a long time in insurance rate-making (see, for example, Bailey [1]). Later approaches used Bayesian statistical methods. In these it was assumed that an underlying risk parameter's prior distribution is modified by experience to a posterior distribution. The mean of the posterior distribution is used as the estimate for the rates. The prior distribution is set subjectively. If the prior distribution and the likelihood are related in a certain way, the posterior mean will be exactly a linear function of the prior mean and the experience mean.

Later (see Bühlmann and Straub [2]), a least-squares approach is used with no explicit distribution assumption. Bühlmann [3] gives a good explanation of this model.

Although risk parameters are used in Bühlmann's model but not in the method used in this paper, the two are essentially equivalent. To see this, first note that for any parameter ϑ and random variables X_t:

$$\mathrm{var}(X_t) = E[\mathrm{var}\,(X_t|\vartheta)] + \mathrm{var}[E(X_t|\vartheta)] \tag{9.1}$$

and for $r \neq t$:

$$\mathrm{cov}(X_t,X_r) = \mathrm{cov}[E(X_t|\vartheta),\, E(X_r|\vartheta)] + E[\mathrm{cov}(X_t,X_r|\vartheta)]. \tag{9.2}$$

Bühlmann then assumes that the X_t values are independent and identically distributed, given ϑ. In which case:

$$\begin{aligned} E(X_t|\vartheta) &= \mathrm{E}(X_r|\vartheta), \\ \mathrm{var}(X_t|\vartheta) &= \mathrm{var}(X_r|\vartheta), \\ \mathrm{cov}(X_t,X_r|\vartheta) &= 0, \end{aligned} \tag{9.3}$$

and

$$\mathrm{cov}[E(X_t|\vartheta),\, E(X_r|\vartheta)] = \mathrm{var}[E(X_t|\vartheta)]. \tag{9.4}$$

If (9.3) and (9.4) are substituted into (9.2), it becomes:

$$\mathrm{cov}(X_t,X_r) = \mathrm{var}[E(X_t|\vartheta]. \tag{9.5}$$

If (9.5) and (9.1) are substituted into formula (2.2) above, it becomes:

$$Z = \frac{\text{cov}(X_1,X_2)}{\text{var}(X_1)} = \frac{\text{var}[E(X_t|\vartheta)]}{E[\text{var}(X_t|\vartheta)] + \text{var}[E(X_t|\vartheta)]}$$

$$= \frac{1}{1 + k}$$

with $k = E[\text{var}(X_t|\vartheta)]/\text{var}[E(X_t|\vartheta)]$. This is the formula that appears in Bühlmann [3, page 123], with $n = 1$.

X. MULTIYEAR CREDIBILITY

The formulas of Section III can be extended to multiple years of experience. Assume that $\text{var}(X_{it}) = a_{11}$, $\text{cov}(X_{it},X_{ir}) = a_{12}$, $\text{cov}(X_{it},X_{jt}) = b_{11}$, and $\text{cov}(X_{it},X_{jr}) = b_{12}$ for all $1 \leq t \leq n+1$, $1 \leq r \leq n+1$, $t \neq r$, and $1 \leq i \leq m$, $1 \leq j \leq m$, $i \neq j$. Then approximating:

$$E(X_{\cdot n+1}|X_{\cdot 1} \ldots X_{\cdot n}) \approx C + \sum_{t=1}^{n} Z_t \cdot X_{\cdot t}, \tag{10.1}$$

it is easy to prove using (2.1) that in this case $Z_i = Z$ is constant.

Let $X_{\cdot\cdot} = \frac{1}{n} \sum_{t=1}^{n} X_{\cdot t}$ and $Z_T = nZ$. Then (10.1) becomes:

$$E(X_{\cdot n+1}|X_{\cdot\cdot}) \approx Z_T X_{\cdot\cdot} + (1 - Z_T)\mu. \tag{10.2}$$

A short calculation yields:

$$Z_T = \frac{n[a_{12} + (m-1)b_{12}]}{a_{11} + (m-1)b_{11} + (n-1)[a_{12} + (m-1)b_{12}]}$$

$$= \frac{nZ_{11}}{1 + (n-1)Z_{11}} \tag{10.3}$$

where $Z_{11} = \dfrac{a_{12} + (m-1)b_{12}}{a_{11} + (m-1)b_{11}}$.

Although there is little justification for the assumption that $\text{cov}(X_{it},X_{ir})$ does not depend on t and r, (10.3) can be used as a reasonable adjustment for experience periods that are not exactly one year. For example, for nine months of experience, calculate the formula for $n = 0.75$. Of course, if the insurer regularly renews groups by using experience periods of less than a year, then the covariances should be estimated by using data for similar periods. This is often the case, because, in general, groups must be given at least 30 days' notice of rate adjustments, and it might take a number of months to gather data, calculate the rate increase, and notify all the renewing groups. Still, the formula might then be used for a consistent adjustment for the group that has experience for an unusual period.

XI. THE DATA

I applied some of the above formulas to group medical insurance data obtained from Health Care Service Corporation, Blue Cross/Blue Shield of Illinois. The data consist of claims from 1985 and 1984 for groups that were continuously in force during the full two years. The size of the groups selected ranged from 10 to about 100 employees.

Table 1 summarizes the data. The claims were adjusted relative to the company's tabular premium basis. Line 1 shows the total adjusted claims and line 2 the mean adjusted claims. Line 3 is the sum of the squares of the adjusted claims. Line 6 is the sum of the squares of the group totals. The sum of the products of claims from different risk units in the same group in the same year is on line 7 and is equal to line 6 less line 3. Thus the $\hat{b}_{11}$ covariance is calculated on line 9. Line 10 shows the sum of the product of the two years of the adjusted claims. Analogously to $\hat{b}_{11}$, $\hat{b}_{12}$ is calculated on lines 13 to 16. The factors $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_3$ are calculated according to (3.4). The three columns on the right are the calculation of $\hat{a}_{12}$ and $\hat{b}_{12}$ for three specific levels as in Section VI.

Table 2 shows the credibility calculated for various values of m (employees plus dependent units), s (the specific level), and p (the individual persistency in Section IV).

Table 3 shows the results for the first method in Section V, in which ν is the pooling point by size of group.

TABLE 1

ESTIMATION OF PARAMETERS*

	1985	1984	1985		
Deductible	\$0	\$0	\$10,000	\$25,000	\$50,000
1. Adjusted Claims	5,758,924	5,810,928	1,529,298	818,657	564,746
2. Mean of Adjusted Claims	515.71	520.37	134.42	71.96	49.64
3. Sum of Squares		$4.38E+10$			
4. Mean of Squares		3.926,302			
5. $\hat{a}_{11}$		3,655,521			
6. Sum of Squares of Groups		$2.143E+11$			
7. Net Squares		$1.704E+11$			
8. Mean		346,228			
9. $\hat{b}_{11}$		75,447			

	1985–1984			
Deductible (1985)	\$0	\$10,000	\$25,000	\$50,000
10. Sum of Products	$1.29E+10$	$6.94E+09$	$4.18E+09$	$3.01E+09$
11. Mean of Products	1,158,637	609,778	367,403	264,340
12. $\hat{a}_{12}$	890,280	539,830	329,959	238,509
13. Sum of Products of Groups	$1.816E+11$	$5.90E+10$	$3.67E+10$	$2.64E+10$
14. Net Products	$1.686E+11$	$5.21E+10$	$3.25E+10$	$2.34E+10$
15. Mean of Products	342,522	105,801	65,965	47,613
16. $\hat{b}_{12}$	74,164	35,853	28,521	21,782
$\hat{k}_1 = \hat{a}_{12}/\hat{a}_{11}$	24.35%	14.8%	9.0%	6.5%
$\hat{k}_2 = \hat{b}_{12}/\hat{a}_{11}$	2.03%	1.0%	0.8%	0.6%
$\hat{k}_3 = \hat{b}_{11}/\hat{a}_{11}$	2.06%	2.1%	2.1%	2.1%

*1984 is year 1 and 1985 is year 2.

TABLE 2

CREDIBILITY

	p			
m	100%	90%	80%	70%
1	24.4%	22.1%	19.9%	17.7%
25	48.8	47.4	45.9	44.4
50	61.5	60.4	59.3	58.2
75	69.0	68.2	67.3	66.4
100	74.0	73.3	72.5	71.8
150	80.2	79.6	79.1	78.5
200	83.8	83.4	82.9	82.5
250	86.3	85.9	85.5	85.2
500	91.8	91.6	91.4	91.2
1,000	94.9	94.8	94.7	94.6
2,500	96.9	96.9	96.8	96.8
5,000	97.6	97.6	97.5	97.5
10,000	97.9	97.9	97.9	97.9
50,000	98.2	98.2	98.2	98.2
100,000	98.3	98.3	98.3	98.3
∞	98.3	98.3	98.3	98.3

TABLE 3

POOLING POINTS

m	v
1	\$25,000
10	7,000
15	7,000
20	8,000
25	9,000
50	9,000
75	14,000
100	20,000
150	25,000
200	25,000
250	25,000
300	25,000
500	100,000
1,000	100,000
∞	100,000

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