TRANSACTIONS OF SOCIETY OF ACTUARIES
1988 VOL. 40 PT 1

RELATIONSHIPS BETWEEN STATUTORY AND GENERALLY ACCEPTED ACCOUNTING PRINCIPLES (GAAP)

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ABSTRACT

In Volume XXXIV of the *Transactions*, Mr. Donald R. Sondergeld published an article entitled "Profitability As a Return on Total Capital [2]." This article demonstrated how returns on GAAP capital are related to the internal rate of return, which is calculated on a statutory basis. The purpose of this paper is to further develop this relationship. In particular, if statutory losses are viewed as an investment of corporate surplus and the subsequent statutory gains are viewed as a repayment of this investment with interest, then GAAP earnings represent the interest on this investment and GAAP surplus represents the principal amount of the investment outstanding at a particular point in time.

I. INTRODUCTION

Internal rate of return and return on total capital are two widely used profitability measures. Underlying each of these measures is the notion of an investor who is investing capital in a project. Internal rate of return measures the return over the life of the project and return on total capital measures the return during each accounting period.

What is often overlooked is how the capital is being repaid over the life of the project. In other words, as the project returns funds to the investor, what portion of these funds represents interest on the investment and what portion represents a repayment of capital? The repayment schedule influences not only the duration of the investment, but also the return that should be expected.

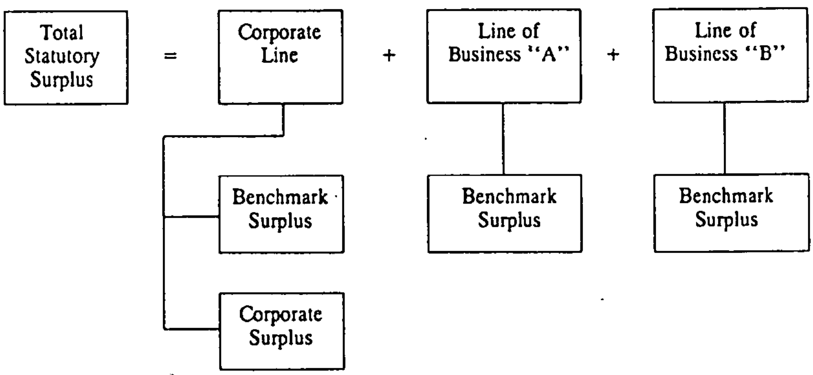
This paper develops a calculation for the portion of statutory earnings representing interest on the investment and for the portion representing repayment of capital. In particular, Section II presents a structure for surplus and the notion of surplus transfers. Section III discusses statutory earnings and GAAP earnings. Section IV, which is the heart of this paper, demonstrates the relationship between statutory earnings and GAAP earnings. Finally, Section V discusses the relationship between internal rate of return and return on total capital.

II. STRUCTURE OF SURPLUS

A. Benchmark Surplus

Statutory surplus can be divided into two accounts: benchmark surplus and corporate surplus. Benchmark surplus is an allocation of statutory surplus "to provide for plausible deviations from expected experience."[1]

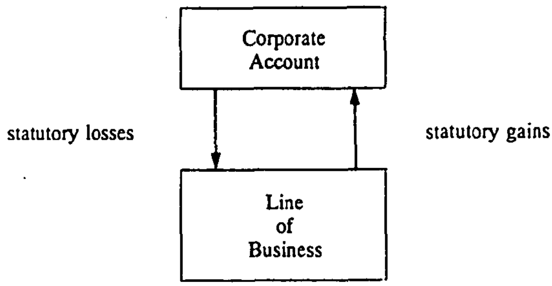
Each line of business has a benchmark surplus account. The balance of statutory surplus is corporate surplus and is held in a corporate account. Pictorially, the structure of statutory surplus is as follows:



B. Surplus Transfers

In each accounting period that an operating line of business incurs a statutory loss, statutory surplus would be transferred from the corporate surplus account to that line of business. Conversely, in each accounting period that an operating line of business incurs a statutory gain, the gain would be transferred from the line of business to the corporate surplus account. Pictorially, the transfer occurs as shown on the next page.

[1]The April 10, 1986 "Exposure Draft on Life Insurance Company Valuation Principles" by the Committee on Life Insurance Company Valuation Principles defined two surplus accounts: designated surplus and vitality surplus. These two accounts are equivalent to benchmark surplus and corporate surplus, respectively.



In addition, statutory surplus is transferred between the corporate surplus account and the benchmark surplus account to maintain benchmark surplus at a target level. Thus, at the end of each accounting period, the operating line of business' assets equal its statutory liabilities plus benchmark surplus.

For example, assume a line of business is expected to incur statutory earnings and expects to hold benchmark surplus as follows:

Year	Total Statutory Earnings	Benchmark Surplus	
		Change	Balance (E)
1	−1,000	100	100
2	100	150	250
3	200	250	500
4	400	−200	300
5	600	−300	0

(E) denotes end of year

Then the following surplus transfers would be made between the corporate surplus account and the line of business:

Year	Total Statutory Earnings	Change in Benchmark Surplus	Total Surplus Transfers
1	−1,000	100	−1,100
2	100	150	−50
3	200	250	−50
4	400	−200	600
5	600	−300	900
Total	300	0	300

Thus, over this five-year period corporate surplus would appear as follows:

Year	Corporate Surplus (B)	Total Surplus Transfers	Corporate Surplus (E)
1	2,000	−1,100	900
2	900	− 50	850
3	850	− 50	800
4	800	600	1,400
5	1,400	900	2,300

(B) denotes beginning of year
(E) denotes end of year

Algebraically,

$$ {}_t^SX = {}^SE_t, \tag{1}$$

$$ {}_t^bX = {}^bE_t - \Delta_t^bS, \tag{2}$$

and

$$ {}_t^{Sb}X = {}_t^SX + {}_t^bX \tag{3}$$

where

${}_t^SX$ = surplus transferred from the corporate surplus account to the line of business (excludes the effect of benchmark surplus);

${}^SE_t$ = statutory earnings;

${}_t^bX$ = surplus transferred from the corporate surplus account to the line of business to maintain benchmark surplus at a target level;

${}^bE_t$ = earnings on benchmark surplus;

Δ_t^bS = change in benchmark surplus; and

${}_t^{Sb}X$ = total surplus that is transferred from the corporate surplus account to the line of business.

C. GAAP Surplus

The total GAAP surplus for an operating line of business is the sum of GAAP adjustments for the line plus benchmark surplus. The major GAAP adjustments are the deferred acquisition expense asset, the difference between the statutory reserves and the GAAP benefit reserves, and deferred taxes. Algebraically,

$$ {}_t^GS = {}_t^GVE + ({}_t^SV - {}_t^GVB) - {}_t^GV' \tag{4}$$

and

$$ {}^{Gb}_{t}S = {}^{G}_{t}S + {}^{b}_{t}S \tag{5} $$

where

${}^{G}_{t}S$ = GAAP surplus;

${}^{G}_{t}VE$ = deferred acquisition expense asset;

${}^{S}_{t}V$ = statutory reserve;

${}^{G}_{t}VB$ = GAAP benefit reserve;

${}^{G}_{t}V'$ = deferred tax liability;

${}^{Gb}_{t}S$ = total GAAP surplus; and

${}^{b}_{t}S$ = benchmark surplus.

III. EARNINGS

A. Statutory Insurance Cash Flow

For ease of presentation, statutory insurance cash flow incurred within a line of business during an accounting period will be divided into two parts: cash flow incurred at the beginning of the period and cash flow incurred at the end of the period. The cash flow incurred at the beginning of the period will consist of the gross premium minus commissions and expenses. The cash flow incurred at the end of the period will consist of investment income minus benefits and federal income taxes. Investment income will include the interest on the statutory reserve and benchmark surplus, but it will exclude the interest on corporate surplus. In the case of a mutual company, federal income taxes would include the surplus tax on benchmark surplus, but it would exclude the surplus tax on corporate surplus. Both the interest and tax on corporate surplus would be included in the corporate segment. Algebraically,

$$ CF^B_t = GP_t - C_t - EA_t - EM_t \tag{6} $$

$$ CF^E_t = {}^{S}I_t + {}^{b}I_t - B_t - {}^{S}TX_t - {}^{b}TX_t, \tag{7} $$

and

$$ CF_t = CF^B_t + CF^E_t \tag{8} $$

where

CF_t^B = cash flow incurred at the beginning of the period;
GP_t = gross premium;
C_t = commissions;
EA_t = acquisition expenses;
EM_t = maintenance expenses;
CF_t^E = cash flow incurred at the end of the period;
${}^SI_t$ = investment income (excluding interest on benchmark surplus);
${}^bI_t$ = interest on benchmark surplus;
B_t = benefits;
${}^STX_t$ = federal income taxes (excluding tax on benchmark surplus);
${}^bTX_t$ = tax on benchmark surplus; and
CF_t = total cash flow.

Table I presents a numeric example of cash flow for a hypothetical block of business. This example will be built on in later sections of this paper.

TABLE I

t	GP_t	$-$	C_t	$-$	EA_t	$-$	EM_t	$+$	${}^SI_t$	$+$	${}^bI_t$	$-$	B_t	$-$	${}^STX_t$	$-$	${}^bTX_t$	$=$	CF_t
0	1,000.00		550.00		700.00		50.00		−30.00		0.00		250.00		−430.50		0.00		−149.50
1	900.00		0.00		0.00		60.00		149.00		5.85		350.00		155.70		0.22		488.93
2	800.00		0.00		0.00		70.00		158.00		7.65		500.00		103.47		0.30		291.88
3	700.00		0.00		0.00		80.00		157.00		8.55		650.00		82.44		0.33		52.78
4	600.00		0.00		0.00		90.00		136.00		7.65		750.00		263.78		0.30		−360.43
	4,000.00		550.00		700.00		350.00		570.00		29.70		2,500.00		174.89		1.15		323.66

B. Statutory Earnings

Statutory earnings at the end of the accounting period will be equal to the cash flow incurred during the period minus the change in the statutory reserve. The cash flow would exclude the interest and tax on benchmark surplus. Similarly, the earnings on benchmark surplus will be equal to the interest on benchmark surplus minus the tax on benchmark surplus.

Algebraically,

$$ {}^SE_t = (CF_t - {}^bI_t + {}^bTX_t) - \Delta_t^S V, \tag{9}$$

$$ {}^bE_t = {}^bI_t - {}^bTX_t, \tag{10}$$

and

$$^{Sb}E_t \;=\; ^{S}E_t + {}^{b}E_t \tag{11}$$

where

$^{S}E_t$ = statutory earnings (excluding interest and tax on benchmark surplus);

$\Delta_t^S V$ = change in statutory reserve; and

$^{Sb}E_t$ = total statutory earnings.

For example, assume a closed block of business incurs the statutory insurance cash flows appearing in Table I. The statutory reserve and benchmark surplus for this block over this period is the following:

TABLE II

t	^S_tV	^b_tS
0	0.00	0.00
1	650.00	65.00
2	850.00	85.00
3	950.00	95.00
4	850.00	85.00
5	0.00	0.00

Then statutory earnings would be as follows:

TABLE III

t	CF_t	$-$	$\Delta_t^S V$	$=$	$^{Sb}E_t$
0	−149.50		650.00		−799.50
1	488.93		200.00		288.93
2	291.88		100.00		191.88
3	52.78		−100.00		152.78
4	−360.43		−850.00		489.57
Total	323.66		0.00		323.66

C. Surplus Transfers Refined

In order to honor some algebraic subtleties in the formulas that will be presented later, the formulas for surplus transfers that were presented in Section II.B need to be refined to reflect transfers that occur at the beginning of the accounting period and transfers that occur at the end of the accounting period. In particular, in the first accounting period

$$^{S}_{0}X^{B} = CF^{B}_{0} \tag{12}$$

and

$$ {}^{S}_{0}X^{E} = (CF^{E}_{0} - {}^{b}I_{0} + {}^{b}TX_{0}) - \Delta^{S}_{0}V \tag{13} $$

where

${}^{S}_{0}X^{B}$ = surplus transferred from the corporate surplus account to the line of business at the beginning of the first accounting period (excludes the effect of benchmark surplus); and

${}^{S}_{0}X^{E}$ = surplus transferred from the corporate surplus account to the line of business at the end of the first accounting period (excludes the effect of benchmark surplus).

After the first accounting period (or when cash flow turns positive),

$$ {}^{S}_{t}X^{B} = 0 \tag{12a} $$

and

$$ {}^{S}_{t}X^{E} = {}^{S}E_{t} \tag{13a} $$

where

${}^{S}_{t}X^{B}$ = surplus transferred from the corporate surplus account to the line of business at the beginning of the period (excludes the effect of benchmark surplus); and

${}^{S}_{t}X^{E}$ = surplus transferred from the corporate surplus account to the line of business at the end of the period (excludes the effect of benchmark surplus).

Since it is assumed that investment income and taxes are incurred at the end of each accounting period, the formula for benchmark surplus would be refined as follows:

$$ {}^{b}_{t}X^{B} = 0 $$

and

$$ {}^{b}_{t}X^{E} = {}^{b}I_{t} - {}^{b}TX_{t} - \Delta^{b}_{t}S $$

where

${}^{b}_{t}X^{B}$ = surplus transferred from the corporate surplus account to the line of business at the beginning of the period to maintain benchmark surplus at a target level; and

${}^{b}_{t}X^{E}$ = surplus transferred from the corporate surplus account to the line of business at the end of the period to maintain benchmark surplus at a target level.

Finally, the refinement of the total surplus that is transferred from the corporate surplus account to the line of business is as follows:

$$ {}^{Sb}_{t}X^{B} = {}^{S}_{t}X^{B} + {}^{b}_{t}X^{B} \tag{14} $$

and

$$ {}^{Sb}_{t}X^{E} = {}^{S}_{t}X^{E} + {}^{b}_{t}X^{E} \tag{15} $$

where

${}^{Sb}_{t}X^{B}$ = total surplus that is transferred from the corporate surplus account to the line of business at the beginning of the period; and

${}^{Sb}_{t}X^{E}$ = total surplus that is transferred from the corporate surplus account to the line of business at the end of the period.

For example, the total surplus transfers for the closed block of business that was presented earlier would appear as follows:

TABLE IV

t	${}^{Sb}_{t}X^{B}$	+	${}^{Sb}_{t}X^{E}$	=	${}^{Sb}_{t}X$
0	−300.00		−564.50		−864.50
1	0.00		268.93		268.93
2	0.00		181.88		181.88
3	0.00		162.78		162.78
4	0.00		574.57		574.57
Total	−300.00		623.66		323.66

D. GAAP Earnings

GAAP earnings at the end of the accounting period will be equal to the statutory earnings at the end of the period plus the change in the GAAP adjustments. Equivalently, GAAP earnings equal the cash flow incurred during the period plus the change in the deferred acquisition expense asset, minus the change in the GAAP benefit reserve and the change in deferred tax liability. Algebraically,

$$ {}^{G}E_{t} = (CF_{t} - {}^{b}I_{t} + {}^{b}TX_{t}) + \Delta^{G}_{t}VE - \Delta^{G}_{t}VB - \Delta^{G}_{t}V' \tag{16} $$

and

$$^{Gb}E_t = {}^{G}E_t + {}^{b}E_t \tag{17}$$

where

$^{G}E_t$ = GAAP earning (excluding interest and tax on benchmark surplus);
$\Delta_t^G VE$ = change in deferred policy acquisition cost asset;
$\Delta_t^G VB$ = change in GAAP benefit reserve;
$\Delta_t^G V'$ = change in GAAP deferred tax liability; and
$^{Gb}E_t$ = total GAAP earnings.

Continuing with the previous example, the GAAP benefit reserve, the deferred policy acquisition cost asset, the deferred tax liability and GAAP surplus would be as follows:

TABLE V

t	${}_t^G VB$	${}_t^G VE$	${}_t^G V'$	${}_t^{Gb}S$
0	0.00	0.00	0.00	0.00
1	366.84	972.37	439.44	881.09
2	592.19	707.25	337.77	712.29
3	611.88	455.87	277.90	611.09
4	405.37	219.63	232.49	516.77

Based on these values, GAAP earnings would be as follows:

TABLE VI

t	CF_t	−	$\Delta_t^G VB$	+	$\Delta_t^G VE$	−	$\Delta_t^G V'$	=	$^{Gb}E_t$
0	−149.50		366.84		972.37		439.44		16.59
1	488.93		225.34		−265.13		−101.66		100.12
2	291.88		19.69		−251.38		− 59.87		80.68
3	52.78		−206.51		−236.24		− 45.42		68.47
4	−360.43		−405.37		−219.63		−232.49		57.80
Total	323.66		0.00		0.00		0.00		323.66

IV. RELATIONSHIP BETWEEN STATUTORY AND GENERALLY ACCEPTED ACCOUNTING PRINCIPLES

A. Relationship

In the first couple of policy years, a life insurance policy usually incurs statutory losses. These losses are then followed by statutory gains. Statutory losses can be viewed as an investment of corporate surplus, and the subsequent statutory gains can be viewed as a repayment of this investment with interest. From this point of view, GAAP earnings represents the interest on these surplus investments, the change in GAAP surplus represents the portion of the investment principal which is repaid during the year, and GAAP surplus represents the amount of the investment principal outstanding at a particular point in time. In particular, this section will show that statutory earnings equals GAAP earnings plus the decrease in GAAP surplus. Algebraically,

$$^{Sb}E_t - \Delta_t^b S = {}^{Gb}E_t - \Delta_t^{Gb} S \tag{18}$$

where

$\Delta_t^{Gb} S$ = change in total GAAP surplus.

B. Demonstration

The relationship shown in Equation (18) can be demonstrated algebraically as follows. Section III.D presented the following equation for total GAAP earnings:

$$^{Gb}E_t = CF_t + \Delta_t^G VE - \Delta_t^G VB - \Delta_t^G V'. \tag{17a}$$

Equations (4) and (5) in Section II.C lead to the following equation:

$$\Delta_t^{Gb} S = \Delta_t^G VE + (\Delta_t^S V - \Delta_t^G VB) - \Delta_t^G V' + \Delta_t^b S. \tag{19}$$

Now, Equations (17a) and (19) lead to the following:

$$\begin{aligned} ^{Gb}E_t - \Delta_t^{Gb} S &= [CF_t + \Delta_t^G VE - \Delta_t^G VB - \Delta_t^G V'] - [\Delta_t^G VE \\ &\quad + (\Delta_t^S V - \Delta_t^G VB - \Delta_t^G V' + \Delta_t^b S] \\ &= CF_t - \Delta_t^S V - \Delta_t^b S \\ &= {}^{Sb}E_t - \Delta_t^b S, \end{aligned}$$

which demonstrates the relationship stated in equation (18).

Table VII, which is based on Tables I–VI, is a numerical demonstration of the relationship between statutory earnings and GAAP earnings.

TABLE VII

t	$^{Sb}_{t}X$	=	$^{Sb}E_t$	−	$\Delta_t^b S$	=	$^{Gb}E_t$	−	$\Delta_t^{Gb} S$
0	−864.50		−799.50		65.00		16.59		881.09
1	268.93		288.93		20.00		100.12		−168.81
2	181.88		191.88		10.00		80.68		−101.20
3	162.78		152.78		−10.00		68.47		− 94.32
4	574.57		489.57		−85.00		57.80		−516.77

V. RELATIONSHIP BETWEEN INTERNAL RATE OF RETURN AND RETURN ON TOTAL CAPITAL

A. Definitions

The internal rate of return (IRR) is the interest rate at which the present value of the surplus transfers between the corporate account and the line of business is equal to zero. Algebraically, the internal rate of return is the root of the following polynomial:

$$0 = {}^{Sb}_{0}X^{B} + \sum_{t=1}^{n} ({}^{Sb}_{t-1}X^{E} + {}^{Sb}_{t}X^{B})(1 + \text{IRR})^{-t} + {}^{Sb}_{N}X^{E}\,(1 + \text{IRR})^{-N}. \quad (20)$$

The return on total capital for an accounting period is defined as the total GAAP earnings for that period divided by the total GAAP surplus at the beginning of that period plus the surplus that is transferred at the beginning of the period:

$$\text{ROTC}_t = \frac{{}^{Gb}E_t}{{}^{Gb}_{t}S - {}^{Sb}_{t}X^{B}} \quad (21)$$

where

ROTC_t = return on total capital.

Based on the example that has been developed throughout this paper, the return on total capital is:

TABLE VIII

t	${}^{Gb}E_t$	÷ [${}^{Gb}_{t}S$	− ${}^{Sb}_{t}X^{B}$]	= ROTC_t
0	16.59	0.00	−300.00	5.53%
1	100.12	881.09	0.00	11.36%
2	80.68	712.29	0.00	11.33%
3	68.47	611.09	0.00	11.20%
4	57.80	516.77	0.00	11.19%

The internal rate of return is 10.59 percent.

B. Relationship between IRR and ROTC

What Mr. Sondergeld demonstrated in his 1982 paper, and is the basis of this paper, is the following equation:

$$0 = {}^{Sb}_{0}X^{B} + \sum_{t=1}^{n} ({}^{Sb}_{t-1}X^{E} + {}^{Sb}_{t}X^{B}) \times \prod_{s=0}^{t-1} (1 + \text{ROTC}_s)^{-1} \quad (22)$$

In that paper [2] he also makes that statement that ". . .if the actuary tells management that a plan of insurance will produce a [10.59] percent statutory IRR, he should also tell management that the GAAP ROTC that is expected to appear, which is equivalent to the [10.59] percent IRR, is [5.53] percent in the first year, [11.36] percent in the second year, [11.36] percent in the third year, [11.20] percent in the fourth year, and [11.19] percent in the fifth year." (Note that the numbers were changed to reflect the example that is presented in this paper.)

The actuary could enhance this statement by presenting the following tables:

Period	Statutory Earnings	=	GAAP Earnings	+	Repayment of Capital
0	−799.50		16.59		−816.09
1	283.30		94.49		188.81
2	184.53		73.33		111.20
3	144.56		60.25		84.31
4	482.22		50.45		431.77

Period	GAAP Earnings	÷	Outstanding Investment (B)	=	ROTC
0	16.59		300.00		5.53%
1	94.49		816.09		11.58
2	73.33		627.28		11.69
3	60.25		516.08		11.67
4	50.45		431.77		11.68

This presentation, which excludes the effect of benchmark surplus, would not only show each period's expected return but also how much capital is expected to be invested in a line of business and how this capital is expected to be repaid.

REFERENCES

1. PROMISLOW, S.D. "A New Approach to the Theory of Interest," *TSA* XXXII (1980):53–118.
2. SONDERGELD, D.R. "Profitability As a Return on Total Capital," *TSA* XXXIV (1982):415–33.
3. TEICHROEW, D., ROBICHEK, A.A. AND MONTALBANO, M. "An Analysis of Criteria for Investment and Financing Decisions under Certainty," *Management Science* XII (November 1965):151–79.

DISCUSSION OF PRECEDING PAPER

FRANK S. IRISH:

In this paper, Mr. Lombardi presents a very helpful analysis of how the operation of a corporate account interacts with the calculation of rates of return. Essentially, the corporate account is used as a reservoir from which money is drawn (or to which money is returned) in order to maintain the capital base within each line of business at the proper level for determining rates of return.

I would raise a minor question about the way in which he expresses the transfers to and from the account. The variable called X (with its many subscripts) is described as "surplus transferred from the corporate surplus account." The equations make it clear, however, that X is negative when large surplus strains occur and positive when earnings are high. Thus, it would appear that Mr. Lombardi has his signs reversed and that in reality large transfers *from* the corporate account must occur when there are large surplus strains and that transfers *to* the corporate account occur when earnings are high. Although this might be a mere semantic quibble, I think that it could easily be a source of confusion and could obscure the concept that a corporate account is a reservoir that funds the capital needs of the lines of business and that benefits from their excess earnings.

A more important issue is the timing of the transfers. In Mr. Sondergeld's paper,* on which Mr. Lombardi drew heavily, as well as in this paper, there is an awkwardness in having the transfers occur mainly at the beginning of the year for the first year, but at the end of the year in subsequent years. Furthermore, the base for calculating the rate of return in the first year is the excess of expenses over premium, which ignores the very important elements of first-year benchmark surplus and first-year statutory reserve strain (as a matter of fact, it leaves statutory reserves out of consideration altogether in the calculation of the first-year capital base).

Both Mr. Sondergeld and Mr. Lombardi have conceptualized their product line as an asset share. This leads them to set their time periods in a policy-year mode (for example, the assumption that expenses flow at the beginning of the year). However, accounting is carried on in a calendar-year atmosphere, and their equations simply do not work well for calendar years. In order to fix this, one should assume that cash flows occur continuously over the year and also that changes in reserves and benchmark surplus occur

*SONDERGELD, D.R. "Profitability As a Return on Total Capital," TSA XXXIV (1982): 415–33.

continuously over the year. In addition to putting rate of return calculations more in accord with accounting realities, this would also cure the beginning/end-of-year awkwardness.

In order to carry this out, it is not necessary to make surplus transfers continuously. Typically, transfers will be annual. It is necessary to make interest adjustments to the extent that surplus carried in the line departs from benchmark surplus between transfers. The equation for rate of return then becomes (using simplified notation):

$$E' = \Delta S - i(\bar{S} - \bar{S}')$$

$$\text{ROTC} = \frac{E'}{\bar{S}' - 1/2\bar{E}'}$$

This is an approximate solution to the equation:

$$(d/dt)S = \delta(r)\, S' + \delta(i)(S - S')$$

where

S = GAAP surplus (before transfers)
S' = benchmark surplus plus GAAP adjustments
E' = earnings adjusted for interest on excess surplus
i = yield on assets
$\delta(r)$ = force of interest related to ROTC
$\delta(i)$ = force of interest related to yield on assets.

If transfers are carried out annually, then $S = S'$ at the beginning of the calendar year, and the interest adjustment is small but not always negligible.

DONALD R. SONDERGELD:

The purpose of my discussion is to comment on the author's interpretation of GAAP earnings. The author chose to interpret GAAP earnings entirely as interest on invested surplus, where the interest rate is the Return on Total Capital (ROTC). This is not unreasonable. Another way is to consider GAAP earnings to be partially repayment of principal and partially interest. The interest rate can be the Internal Rate of Return (IRR), or some other basis can be used to allocate GAAP earnings between principal and interest.

Interpretation of Earnings

Statutory earnings can be thought of as a repayment of principal and interest, where the interest rate is the IRR. As statutory and GAAP earnings are equal over time, statutory and GAAP earnings can both be thought of as consisting of principal and interest.

My paper showed the relationship between the statutory IRR and the unlevel GAAP ROTCs. It also pointed out that management should be concerned when a product with a satisfactory IRR must be GAAP'ed in such a way as to produce an unsatisfactory pattern of GAAP ROTCs. The analysis in my paper also was based upon the assumption that surplus didn't change when a policy was written, if all acquisition expenses are capitalized. When all expenses are not capitalized, a portion of GAAP earnings must represent a payment of the noncapitalized investment.

In Section IV-A Mr. Lombardi states, "GAAP earnings represents the interest on these surplus investments, the change in GAAP surplus represents the portion of investment principal which is repaid during the year, and GAAP surplus represents the amount of the investment principal outstanding at a particular point in time."

I subscribe to the last two thoughts, but would like to offer another interpretation of what GAAP earnings represent. If the expected ROTC in any year is greater (or less) than the expected IRR, this implies that additional principal is invested (repaid).

Statutory and GAAP Examples

The following is an example of a product that has a 10 percent expected IRR and a 7.19 percent actual IRR.

STATUTORY EARNINGS

t	Expected	Actual
0	−100	−100
1	70	68
2	44	42
	14	10

On an *expected* basis, an actuary might tell management that the 70 can be thought of as 10 interest on the 100 investment, plus 60 repayment of principal; also, that the 44 can be thought of as 4 interest on the 40 investment, plus 40 repayment of principal. This allocation is based on the IRR.

After two years, an actuary might tell management that the *actual* IRR was only 7.19 percent. Thus, the 68 in year one can be thought of as 7.19 interest on the 100 investment plus 60.81 repayment of principal. Also, the

42 in the second year can be thought of as 2.81 interest on the 39.19 outstanding investment plus 39.19 repayment of principal.

Let us now look at some GAAP examples on the same product, using various GAAP assumptions:

t	A	B	C	D
	Expected GAAP Earnings			Actual GAAP Earnings
1	10	12	8	9
2	4	2	6	1
	14	14	14	10
	Expected GAAP Surplus			Actual GAAP Surplus
0	100	100	100	100
1	100 + 10 − 70 = 40	100 + 12 − 70 = 42	100 + 8 − 70 = 38	100 + 9 − 68 = 41
2	40 + 4 − 44 = 0	42 + 2 − 44 = 0	38 + 6 − 44 = 0	41 + 1 − 42 = 0
	Expected ROTC			Actual ROTC
1	10%	12%	8%	9%
2	10%	4.8%	15.8%	2.4%

Under statutory reporting, an actuary can tell management how much of the actual earnings that were reported represents repayment of principal and how much represents repayment of interest. Under GAAP the same is true. It is a matter of opinion as to how principal and interest are allocated.

Under GAAP B, the GAAP surplus changes are -58 and -42 in years one and two, and the earnings are 12 and 2 in year one and two. The author will interpret the -58 and -42 to be repayment of principal and the 12 and 2 GAAP earnings entirely as interest. Under the interpretation of using an interest rate equal to the IRR, we can say that 10 of the 12 earnings in year one represents interest and $+2$ is a reinvestment of principal. Similarly, in year two, an interpretation is that 4 of the 2 of earnings represents interest and -2 represents a repayment of principal. Of course, other allocations of interest and repayment of principal are possible.

On an expected basis, we can interpret 10 of GAAP earnings as interest in year one for GAAP A, B, and C, with the excess of 10 over expected earnings being interpreted as a *repayment* of principal in year one. A similar comment applies to year two. Conversely, the excess of expected earnings over 10 is a *reinvestment* of principal. Note that this is the same result we had on a statutory basis.

There is more than one way to interpret what GAAP earnings represents and the same is true of statutory earnings. Formula (40) in my paper showed that the present value of the *statutory transfers*, discounted at either the IRR or at the ROTCs, equals zero. Refer to the author's formulas (20) and (22);

this means that another way of looking at the statutory earnings stream is using the ROTC yield rates instead of the IRR. This can be visualized by simply changing the headings in the above table from "Expected (or Actual) GAAP Surplus" to "Expected (or Actual) Outstanding Statutory Investment."

Under GAAP A, the change in GAAP surplus is $40 - 100 = -60$ in year one and $0 - 40 = -40$ in year two. Mr. Lombardi will say that these changes can be thought of as repayment of principal and that the GAAP earnings of 10 in year one and 4 in year two represents interest. As the ROTCs and IRR are equal, let me agree.

Let us now look at the actual statutory results from the earlier example (of -100, 68, and 42), but let's assume only 70 can be capitalized on GAAP basis E instead of the 100 that was capitalized on basis D. We then have the following results:

	Actual GAAP Earnings		Actual GAAP Surplus		Actual ROTC	
t	D	E	D	E	D	E
0	—	−30	100	100	—	—
1	9	29	100+9−68= 41	100−30+29−68= 31	9%	−1%
2	1	11	41+1−42= 0	31+11−42= −30	2.4%	35.48%
	10	10				

Under GAAP D, we might say that the total GAAP earnings of 10 represents interest, we might say that the 9 and the 1 can be interpreted entirely as interest. Alternatively, we can think of 7.19 of the 9 as interest and 2.81 of the 1 as interest.

Under GAAP E, we might say that of the 10 total GAAP earnings, 10 represents interest. Then how is the interest distributed between year one and two? If you say that it is -1 and 11, then the 11 of interest in year two happens to be greater than the total of 10. However, it could be defined as 9 and 1, 7.19 and 2.81, or some other allocation. Note that the GAAP earnings for year 1 are $-30 + 29 = -1$. I allocated 100 of statutory and GAAP surplus to the product and then immediately wrote down 30 of GAAP surplus an instant after issue in the first policy year. Note that the present value of the statutory transfers equals zero when discounted at the statutory IRR of 7.19% or at the ROTCs of -1% and 35.48%:

$$-100 + \frac{68}{(1.0719)} + \frac{42}{(1.0719)^2} = 0 = -100 + \frac{68}{(0.99)} + \frac{42}{(0.99)(1.3548)}$$

Multiple Yield Rates

Formula (20) can be used to develop an IRR. However, my paper also referred to the technique to use, outlined in Teichroew and Montalbano's paper,* when there are multiple roots. An example of this might be helpful.

t	Statutory Earnings	Outstanding Balance at 12.2%
0	−100	−100
1	170	57.8
2	−163	−98.1
3	110	0
	17	

If we solve the cubic equation

$$-100 + 170v - 163v^2 + 100v^3 = 0,$$

we find there is one real solution of $i = 12.2\%$ and two imaginary solutions.

However, using the solution $i = 12.2\%$ means that the company is earning 12.2 percent on its investment and is also paying 12.2 percent when the sign changes and it is in a borrowing mode.

If the company is willing to borrow only at 5 percent, it can solve for i during the period it is in an investing mode. This produces $i = 10\%$. Then we have:

t	Outstanding Balance
0	−100, accumulates at 10%
1	60, accumulates at 5%
2	−100, accumulates at 10%
3	0

Let's look at a possible set of GAAP statistics:

t	GAAP Earnings	GAAP Surplus
0		= 100
1	8.0	100 − 170 + 8 = −62
2	6.2	−62 + 163 + 6.2 = 107.2
3	2.8	107.2 − 110 + 2.8 = 0
	17.0	

*Teichroew, D., Robichek, A.A. and Montalbano, M. "An Analysis of Criteria for Investment and Financing Decisions under Certainty," *Management Science* XII (November 1965):151–79.

If this were the case, the GAAP ROTCs would be: 8%, -10%, and 2.6%, which satisfy Equation (22).

$$\frac{100}{1} + \frac{170}{(1.08)} - \frac{163}{(1.08)(0.90)} + \frac{110}{(1.08)(0.90)(1.026)} = 0$$

$$-100 + 157.4 - 167.7 + 110.3 = 0$$

The purpose of this discussion was to suggest other methods of interpreting statutory and GAAP earnings.

(AUTHOR'S REVIEW OF DISCUSSION)

LOUIS J. LOMBARDI:

I thank Mr. Irish and Mr. Sondergeld for taking the time to comment on my paper.

A more rigorous presentation of this paper would not have used such a simplified cash flow model with transfers occurring either at the beginning or the end of the year. Instead, the formulas could have been developed to more accurately reflect the incidence of cash flow and the actual accounting cycle (for example, monthly, quarterly). I chose not to do this because of the additional complexity that it introduces. I received a number of informal comments on this paper, and the most common criticism was there was too much algebra. I hope that this simplification does not hamper the basic concepts that this paper is trying to present and that more refined formulas can be developed without too much difficulty.

With regard to the development of more refined formulas, in addition to the continuous formulas that Mr. Irish suggests, I would like to suggest two additional approaches. The first approach would be an adaptation of the formulas presented in Huffman's paper "Asset Share Mathematics" [1]. Another approach would be to develop monthly formulas, as opposed to yearly formulas. I have found this second approach very useful. It is easier to understand, and it is very easy to switch between policy year and calendar year.

When Mr. Sondergeld makes the statement that "another way is to consider GAAP earnings to be partially repayment of principal and partially interest," he is essentially criticizing some of the perceived shortcomings of current GAAP practices and offering an alternative accounting system for measuring earnings that has come to be known as Level Return on Equity Method.

Though this method is very sound and consistent with the pricing process when a target internal rate of return is the primary pricing objective, it does contradict some of the basic principles on which GAAP is currently based, in particular, the principle that GAAP earnings accurately measure the results of current operations. The accounting profession would have a very difficult time accepting the statement that current earnings include a partial repayment of principal. Another contradiction appears when a company is subject to an alternate minimum tax that is based on GAAP earnings. If one accepts the statement that GAAP earnings include a partial repayment of principal, the company would be paying tax on this repayment.

VI. RETURN FROM OPERATIONS VERSUS RETURN FROM NET EARNINGS ON SURPLUS

It is revealing to split return on total capital into two parts. The first part of this return is due to the net earnings (net investment income and capital gains after taxes) on surplus and represents the aftertax earnings on the assets backing surplus. The second part is the return on total capital due to net operating income, which is total GAAP earnings (or net income) less the net earnings on surplus. This part represents the additional return a company is able to earn from the products and services it sells. A primary objective of surplus management is to invest surplus in the various lines of business so as to provide a return higher than that earned on the assets backing surplus. The return due to net operating income is a measure of this additional return.

Algebraically,

$$ROTC_t^S = \frac{I_t - TX_t}{{}^{Gb}_{t}S - {}^{Sb}_{t}X^B}$$

and

$$ROTC_t^O = \frac{{}^{Gb}E_t - I_t + TX_t}{{}^{Gb}_{t}S - {}^{Sb}_{t}X^B}$$

where

$ROTC_t^S$ = return on total capital due to net earnings on surplus;

I_t = interest on GAAP surplus (including interest on benchmark surplus);

TX_t = tax on GAAP surplus (including tax on benchmark surplus); and

$ROTC_t^O$ = return on total capital due to net operating income.

Using the example that was built on throughout this paper, return on total capital split in this manner would appear as follows.

TABLE IX

t	$[I_t$	$- \; TX_t]$	$\div \; [$	$-$	$] \; =$	$+$	$= \; ROTC_t$
0	0.00	0.00	0.00	−300.00	0.00	5.53%	5.53%
1	87.46	28.78	881.09	0.00	6.66%	4.70	11.58
2	70.38	22.26	712.29	0.00	6.76	4.57	11.69
3	60.16	18.39	611.09	0.00	6.84	4.36	11.67
4	50.83	15.41	516.77	0.00	6.85	4.34	11.68

VII. MARKET VALUE RETURN

One drawback of internal rate of return is that, for most life insurance products, it takes a very long time for this profit measure to be calculated by using the actual earnings of a block of business (assuming that the information is available). For a mature block of business, return on total capital is a good approximation for internal rate of return if the yearly returns are fairly level. Another useful profit measure might be a market value return.

This measure is calculated in the same manner as internal rate of return, but instead of doing this calculation over the life of a block of business, it assumes that the block is sold at some point before all contracts mature. Some percentage of the total GAAP surplus could be used as the market value. Algebraically, the market value return is the root of the following polynomial.

$$0 = {}^{Sb}_{0}X^{B} + \sum_{t=1}^{m} \left[{}^{Sb}_{t-1}X^{E} + {}^{Sb}_{t}X^{B}\right] \times (1 + MVR_m)^{-t} + R_m \times {}^{Gb}_{t}S \times (1 + MVR_m)^{-m}$$

where

MVR_m = market value return; and

R_m = ratio of market value to book value, at time m.

Table X is a comparison of return on total capital, market value return (assuming market value equals book value), and internal rate of return.

TABLE X

t	$ROTC_t$	MVR_t	IRR
0	5.53%	5.53%	10.59%
1	11.58	9.78	10.59
2	11.69	10.31	10.59
3	11.67	10.50	10.59
4	11.68	10.59	10.59

XI. CONCLUSION

When Mr. Sondergeld's paper "Profitability As a Return on Total Capital" [2] was first published, the concepts that were presented were at least several years ahead of the industry. I view Mr. Sondergeld's paper, together with Horn's paper "Life Insurance Earnings and the Release from Risk Policy Reserve System" [3] and Anderson's paper "Gross Premium Calculations and Profit Measurement for Nonparticipating Insurance" [4], as three of the most insightful papers on this subject matter.

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