Proceedings of a Symposium in honor of Robert J. Myers, edited by Yung-Ping Chen and George F. Rohrlich, *Checks and Balances in Social Security*, pp. 357, published by the University Press of America, Lanham, Md. 20706, 1986; \$17.50 (paperback), \$32.25 (hardcover).

This volume contains twenty-seven papers which were presented in a Symposium held to honor Robert J. Myers. The basic organization of the book is in five parts, plus a foreword by U.S. Senator John Heinz, a foreword by U.S. Senator Daniel Patrick Moynihan, two appendices, and an addendum paper.

Robert Myers began his career in Social Security in 1934 as a junior actuary for the Committee on Economic Security and served as Chief Actuary, Social Security Administration, from 1947 to 1970, as Deputy Commissioner of Social Security from 1981 to 1982, and as Executive Director, National Commission on Social Security Reform during 1982–83. He has gained worldwide recognition for his knowledge and wisdom regarding social insurance, and this Symposium Proceedings is a fitting tribute.

Part One is entitled "Perspectives on the Origin and on the Legislative and Administrative Evolution of Social Security in the United States." The opening paper by George Rohrlich, a co-editor of the volume, is concerned with historical perspectives on Social Security. The second paper, "Social Security as a Floor of Protection," was written by Yung-Ping Chen. Dr. Chen convened and directed the Symposium, and served as coeditor. The third and fourth papers, by James W. Kelley and Jack S. Futterman, discuss significant differences that have occurred in recent years in the way Social Security legislation is enacted, and the need for continuity and expertise of top management in the person of the Commissioner of Social Security.

Part Two consists of five papers whose theme is "Financing, Budgetary, Taxation, and Actuarial Aspects." The first paper, by Joseph Humphreys (staff member of Committee on Finance, U.S. Senate), discusses the many consequences of separating the general budget of the federal government from the Social Security budget. C. L. Trowbridge, a former Social Security Chief Actuary, offers a mathematical formulation for determining the actuarial balance and the financial impact of the various economic and demographic factors. The third paper, presented by Richard S. Foster, Acting Deputy Chief Actuary for Social Security, describes the short-range financial status of Social Security as seen in October 1982, the time of the conference. Alicia Munnell, Vice-President and Economist, Federal Reserve Bank of Boston, discusses taxation of Social Security benefits in the fourth paper. The last paper of Part Two proposes a joint public and private review committee composed of actuaries to examine the financial viability of the system. It was written by Barnet Berin, managing director and chief actuary at William M. Mercer-Meidinger-Hansen Inc.

Part Three is entitled "The Future Role of Social Security as Seen by the Insurance Industry, Pension Planners, Organized Labor, and the General Public." The first three papers set forth positions taken by the insurance industry. Robert Beck, Chairman and Chief Executive Officer, Prudential Insurance Company, and a member of the National Commission on Social Security Reform, describes five aspects of the problems as of October 1982, and offers short- and long-run solutions. James Swenson, Vice-President

and Associate Actuary of Prudential Insurance Company, emphasizes the need for legislation to create a safe base for Social Security's future. James Douds and Michael Kerley, National Association of Life Underwriters, describe the affirmative position toward Social Security taken by their organization and insurance sales force over the years. L. Edwin Wang used a questionnaire to obtain and present the views of the Social Security program expressed by a group of pension planners representing 80 percent of the large church employees' benefits plans in the U.S. One of the former Social Security Chief Actuaries, Dwight Bartlett, explores the proposition that the job of Social Security would be made easier if a private pension scheme would take over part of its task. Burt Seidman, Director of Social Security of the AFL-CIO, discusses some principles which he feels should guide Social Security. The final paper of Part Three is by Milton Gwirtzman, who discusses the highlights from some two dozen public polls concerned with public confidence in Social Security and related matters.

Part Four consists of eight papers devoted to the theme "The Quest for a Steady Future Despite Continuing Sources of Uncertainty." In the first paper, James Dillman discusses the findings and recommendations of the National Commission on Social Security (1979-81). David Koitz looks at Social Security's financial problems as a specialist in social legislation, and also as a citizen-taxpayer. The third paper, by Carolyn Weaver (staff member of Senate Finance Committee), analyzes features of the Social Security program as it has evolved over fifty years. She discusses inequities which she foresees Congress will grapple with for decades to come. Elizabeth Duskin, Vice-Chairman of the National Commission on Social Security, uses that experience in discussing the role of advisory committees or commissions and the successful process of compromise pursued by the National Commission on Social Security Reform. In the fifth paper, Lawrence Atkins points out the various escape routes from Social Security coverage and the ensuing erosion of the Social Security tax base. Janice Gregory, staff member of the Social Security Subcommittee of the House Ways and Means Committee, examines the interplay between demographic and political factors that impact on Social Security. The next paper, "A Bond Plan to Restore Public Confidence in Social Security," by Yung-Ping Chen, proposes that when Social Security trust funds are low, beneficiaries will receive, except for lowincome persons, their benefits in cash and deferred benefits (bonds) to be payable when the trust funds rise above a predetermined level. The last paper of Part Four is by Robert J. Myers. It details changes in Social Security following the 1983 Amendments. Mr. Myers points out where Congress followed the recommendations of the Commission on Social Security Reform and where it went beyond or departed from the Commission's recomendations.

Part Five's three papers are concerned with Social Security programs in other countries. In the first paper, William Yoffee shows the impact of Social Security standards as set forth in International Labor Conventions on developed and developing nations. Vladimir Rys reflects on past, present, and future developments in Social Security as seen by a sociologist. Giovanni Tamburi writes about future cost patterns as viewed by an actuary.

Biographic and professional data, as well as selected publications of Robert J. Myers, are contained in an appendix. It is a real service to scholars and friends for this to appear.

An addendum paper, "Components of Trends in Social Security Costs," by Francisco R. Bayo, Milton P. Glanz, and Charles L. Trowbridge, expands and complements the sixth paper with empirical work. This paper also appears in *TSA* XXXVIII, 1986.

The co-editors Yung-Ping Chen and George Rohrlich prepared a lengthy introduction to the Proceedings, and their "guide through the 27 chapters" was very helpful in the preparation of this review.

I found the book to be fascinating reading and feel that it should be beneficial to all actuaries. It contains an absorbing description of the genesis of the Social Security program, insiders' thoughts on the administration of the program in recent years, and a description of how presidential commissions on Social Security are appointed, their tasks, their accomplishments, and their concerns for the future. The book captures the intense public debate about the long-term role of Social Security at a point in time when public confidence in its future was at its lowest point. The restoration of that confidence is occurring, partially because of the 1983 Amendments which were helped greatly by the long experience, expertise, and service of Mr. Myers as Executive Director, National Commission on Social Security Reform.

As the Introduction says (p. xii), these proceedings "express our and all the conference participants' love, respect and admiration for Bob and for what he has wrought in this half century of zealous and sustained effort on behalf of Social Security in this country and abroad." This reviewer would agree completely in that expression and is thankful that these Proceedings are available.

JOHN A. BEEKMAN

Dick London, Graduation: The Revision of Estimates, pp. 183, published by ACTEX Publications, Abington, Conn. 06230, 1985.

London's Graduation: The Revision of Estimates supersedes the graduation monograph of Morton D. Miller and the study note of T.N.E. Greville on the Society of Actuaries syllabus for graduation. For those whose only exposure to graduation of data came from Miller and the Study Notes, there is much in Mr. London's book which is familiar. There are also several new topics and different presentations of old topics. The text covers graduation using moving-weighted-average formulas, Whittaker's method, the Bayesian process, parametric functions, and two-dimensional techniques. Additionally, statistical considerations and smooth-junction interpolation are covered.

The most significant change from prior texts is the emphasis of the Bayesian process in graduation. Bayesian graduation involves using a multi-variate version of Bayes' Theorem for conditional probabilities and requires a formal expression of "prior opinion" regarding the "true values" being estimated. The prior opinion takes the form of a probability distribution for the true values. Whittaker's method is shown to be a formal application of Bayesian statistics to graduation. The Kimeldorf-Jones method is examined in detail in the chapter on Bayesian graduation.

While Bayesian graduation has not yet achieved great use in practice, its inclusion in the text offers an interesting addition to the traditional graduation techniques and brings to mind the fact that graduation of data is not a static field.

From an educational viewpoint, the text admirably presents material and reinforces it in the exercises at the end of each chapter. Many of the exercises are derivations of the theory presented in the text and are important to the understanding of the material. At times the text becomes a bit "chatty" and the consistency of notation fails. Overall, the

text covers an amount of material appropriate for a one semester course, and the student should be able to follow the material as long as he or she takes the time to work through the exercises. Though the text does not provide an exhaustive discussion of the subject, an extensive bibliography is provided to assist in an in-depth study of a particular topic.

From a practical viewpoint, some of the graduation methods are highly computer oriented and lend themselves well to application in actual situations encountered by a practicing actuary. For example, while the calculations involved in a two-dimensional graduation or a regression method are extremely onerous when done manually, the computer greatly simplifies the calculations and allows the theory to be applied. In the final chapter of the text, each method of graduation is analyzed and critiqued. This chapter is very important for the practitioner.

Overall, the text is needed because of the computer revolution and the extensive amount of research that has taken place in this field since the publication of prior texts. The text presents a modernized view of the subject of graduation over that contained in prior texts.

ROBERT M. BEUERLEIN AND WILLIAM B. FRYE

Nathan Keyfitz and John A. Beekman, Demography Through Problems, pp. 136, published by Springer-Verlag, Berlin, distributed by Springer-Verlag New York, Inc., New York, N.Y. 10010, 1984.

Readers should have no need for a lengthy review of the depth of knowledge these two distinguished authors have in the field of Demography. In total, they possess over a century of writing, research, and teaching experience in this field.

The topics covered in this book include: Populations That Are Not Age-Dependent; The Life Table (much of which is a review to someone having studied Life Contingencies); Use of Stable Theory; Births and Deaths under Stability; Projection and Forecasting; Stochastic Population Models.

In the preface the authors point out that "The book . . . is an experiment in the teaching of population theory and analysis." Of the 136 pages, only 18 are given over to narrative. The rest of the text consists of a "sequence of problems (338) where each is a self-contained puzzle, and the successful solution of each puts the students in a position to tackle the next, as a means of securing the active participation of the learner and so the mastery of a technical subject."

Whether the book is completely successful in this regard may be open to debate. We have used the text at the University of Waterloo for the last two years and have been totally delighted with it. It is brief but clear and almost without error (I have found only one!).

However, a text that is suitable in a lecture/tutor environment does not necessarily translate perfectly into the correspondence mode of the Society.

And more's the pity. Since the totally justified removal of *Introduction to Demography* by Spiegelman, there has been virtually no truly Demographic material on the Associateship syllabus and little in total. Demography is important enough to all actuaries (and vital to many) that it at least warrants elective status in the new flexible syllabus.

This book was plainly written specifically for that reason. It obviously satisfies the page-count paranoia of the Society, but only by introducing some pedagogical weak-nesses. Without at least an accompanying study note, this text may prove difficult for the correspondent student.

However, for those with special interest in Demography, I highly recommend this carefully presented text and sincerely hope that it will someday find its place on the Society's syllabus.

ROBERT L. BROWN

Burton T. Beam, Jr. and John J. McFadden, *Employee Benefits*, pp. 486, published by Richard D. Irwin, Inc., Homewood, Ill. 60430, 1985.

The book has one introductory chapter, then two chapters which cover the mechanics of OASDHI, unemployment insurance, workers' compensation and other government programs. The next eleven chapters describe the mechanics of group insurance. Following are ten chapters on qualified retirement plans, two chapters on all other employee benefits from executive compensation to free parking, and a final chapter on employee benefit planning. The emphasis is on the means of providing employee benefits, rather than a philosophy of why employee benefits are provided.

The treatment is thorough, with an orderly, logical, straight-forward style. Each chapter gives an overview, then a broad outline, then the details of the outline. One senses that much of the information comes from secondary sources rather than from the author's experience in the field. For example, on page 216 under the discussion of "Premium-Delay Arrangements," the authors states that, "the insurance company still has a statutory obligation to maintain the claim reserve, and therefore, it must use assets other than the employer's premiums for this purpose. In most cases these assets come from the insurance company's surplus." One experienced in the language of life insurance accounting would state the concept some other way. What the authors mean is that since liabilities remain constant and assets decline, surplus is reduced.

The style is refreshingly free of insurance company jargon. One need not be steeped in insurance lingo to understand any of the book. However, one is startled to read the phrase *insurer's financial experts* used to describe what those of us in the business call *actuaries*.

There are implicit philosophical assumptions behind some of the authors' conclusions. For example, in discussing the question "Should the employer have a qualified plan?" the authors (probably unconsciously) introduce a socialist bias in referring to the effect on a company's income taxes: "In other words, a qualified plan represents a form of compensation, part of which is *paid by the federal government* rather than by the employer [emphasis added]" (page 265).

In discussing separate accounts, the authors state, "Separate accounts funding was developed to avoid comingling pension assets with all of the general assets of the insurance company, since an insurance company's general assets have traditionally been invested in long term, low return investment vehicles" (page 331). This sentence indicates less understanding of the investment process than the authors probably have; I

know of no insurance company whose investment philosophy dictates low return investment vehicles.

In discussing pension plan funding, the authors make the usual layman's definition of an annuity, "The annuity purchase rate is based on the plan's retirement age, the life expectancy of the retiree, and the assumed investment return in the post-retirement period" (page 335). One would expect better things from professionals in the employee benefit field. Even if they are not actuaries themselves, they could have had sections such as this edited by an actuary. It is, as all actuaries have had drummed into their heads, not life expectancy that determines the annuity purchase rate, but mortality rates.

Relatively minor lapses such as this one prevent this book from being a more valuable source book for actuarial students. It may be, nevertheless, a useful reference book.

In commenting on wellness programs the authors state, "Recent studies have shown that the cost of establishing and maintaining many of these programs are more than offset by the lower amounts paid for medical expense, disability, and death benefits" (page 453). The authors do not cite the programs or the companies in which these programs have offset the costs of medical expenses. One is skeptical of such thinly supported broad general conclusions.

The final chapter, on employee benefit planning, offers sound advice: "Too often the proper design of an employee benefit plan is viewed as a one-time decision rather than as an evolving process. As times and organizations change, employers' answers to the questions raised in this chapter may also have to change. For this reason, these issues must be frequently restudied to determine whether a group benefit plan is continuing to meet its desired purpose" (page 461).

The authors review old and new ways to determine what types of benefits should be provided:

the growing consensus seems to be that the traditional methods of determining the types and levels of benefits to offer have lost much of their effectiveness. These include basing benefits on the following factors:

The employer's perception of the employees' needs. . . .

What competitors are doing . . .

Tax Laws and Regulations. . . .

In the last few years employers have increasingly taken a marketing research approach to employee benefit planning. The employees' preferences for benefits are determined similar to the way that consumers' demands for products are determined.

Marketing research techniques must be used with caution. They can have a negative effect on employee morale unless the employer is committed to using their results in benefit decision making. Therefore this approach should not be undertaken unless the employer intends to base expenditures for benefits on satisfying what employees perceive as their needs. In addition, employees must be made aware that changes in an overall benefit program will be subject to financial constraints and possibly tradeoffs among benefits.

The book gives thorough treatment of the mechanics of group insurance, qualified retirement plans, social security benefits, and other employee benefits including executive benefits, vacation, holidays, etc., but the final chapter on planning may give the most

valuable insight. The book would not, however, adequately replace *Employee Benefit Planning* by Rosenbloom and Holliman and the current Part 6 syllabus.

WILLIS B. HOWARD

Michael J. Boskin, Too Many Promises: The Uncertain Future of Social Security, pp. 196, Dow Jones-Irwin, Homewood, III. 60430, 1986.

(The opinions expressed herein are those of the reviewer and do not necessarily reflect those of the Social Security Administration.)

Most books written about Social Security fall into one of two categories: either textbook-style or crisis-style. This book falls into the second category. Boskin writes as an economist, telling what he thinks is wrong with Social Security and what he believes should be done to correct the problems.

He says that Social Security is a big program that has grown in small increments without much thought given to the overall result. It affects the retirement planning and savings rates of most Americans. It distributes wealth in a sometimes haphazard way. The Social Security payroll tax affects the number of jobs, and the pay scales, in the U.S. economy, in ways that were unforeseen at the beginning of the program.

Boskin presents a number of interesting points. I agree with much of what he has to say. He presents a generally clear argument for scaling back Social Security in favor of more self-sufficiency for retirees (with an expanded welfare program to act as a safety net). Unfortunately, he insists on seeing everything in economic terms. Virtually all issues concerning Social Security have political, social, and other facets besides their economic implications; Boskin presents incomplete or one-sided arguments when he disregards these other factors.

Boskin views Social Security as basically an old-age retirement program; all his discussion of the issues focuses on that part of Social Security, although his cost estimates usually include survivors and disability insurance. When he wants to show large costs, he also includes hospital insurance. It is not always clear exactly which parts of Social Security are included in his figures.

Boskin's basic approach is to view Social Security in terms of "equity." He never defines this term, but it becomes apparent that he means simply that each contributor to Social Security should get exactly the same rate of return on his contributions. Implicit in his writing is the assumption that everyone could agree on one definition of equity and, in particular, on his definition. However, his simplistic economic definition of equity would produce much controversy in the political arena. One lesson learned in the Social Security area is that equity is in the eye of the beholder; each person tends to define equity slightly differently, favoring him- or herself or particular groups to which he or she belongs or favors. The writing of the Social Security law (or any other law) is a continual battle among various groups to have their version of equity pushed forward. By assuming that battle away, Boskin sidesteps many of these ongoing controversies.

A closely related term appearing often in the book is "actuarially fair," although again it is never defined. Boskin says that everyone should get an "actuarially fair" deal out of Social Security. In his models, everyone evidently has the same health status and the same mortality curve, so that he never has to deal with messy real-life details. For instance, some groups live longer than others. Do you give the longer-lived group a smaller monthly benefit so that their total lifetime benefits will be "actuarially fair"? Some groups (such as AIDS sufferers) live a very short time. How do you define "actuarially fair" for them? These are issues that actuaries will recognize as having been around and debated for a long time.

Boskin sometimes has to bend the facts to fit his way of thinking. He mentions numerous times that Social Security benefits are a "function of previous Social Security contributions." He makes this statement to lay the groundwork for his criticism of the relationship of benefits to contributions, but it is a misstatement. In fact, benefits are a function of previous earnings, not of previous contributions. There was never intended to be more than a loose connection between benefits and contributions.

Boskin says that there is a "myth that Social Security is providing identical treatment to all groups in the population." This is a convenient statement for him to make, since he would propose to have such "identical treatment." But in fact I know of no such myth. I think it has been obvious from the beginning that Social Security pays back relatively more to low-wage earners, wives, widows, children, and (as Boskin points out) workers retiring before tax rates and maximum wage bases have reached their ultimate levels.

Boskin also selects his facts and figures in a onc-sided manner, at times, to promote his argument. In Table 4.2, titled "Who Gets How Good a Deal from Social Security— Transfers by Income Class," he uses workers currently at or near retirement to illustrate his argument that high-wage earners get a good deal (even better than low-wage earners). In Table 4.3, "Who Gets How Good a Deal from Social Security—Marital Status and Sex," which follows immediately afterward, he uses workers who will retire twentyfive years in the future in his illustrations. If he had used the workers from Table 4.3 in Table 4.2, he would have come to just the opposite conclusion in that prior table.

It becomes apparent at many points that this book must have been written prior to the 1983 Social Security Amendments, and then hastily updated to include the amendments' effects (even though the book was not published until 1986). The updating was incomplete. The most glaring example of this is Chapter 3, "Retirement Patterns and Policies." Boskin goes through an entire chapter on retirement, mentioning a retirement age of 65 numerous times, without once mentioning that the 1983 Amendments increased the retirement age to 67. In that same chapter, he describes the effects of the Social Security earnings test without mentioning how those effects will be influenced by the change in the earnings test scheduled for 1990. Because of the 1983 Amendments, the benefit withholding rate will be cut back in that year from 1-for-2 to 1-for-3; this would have an important bearing on his argument.

Boskin's bottom line is that Social Security is too big and unfair. His solution is to cut it back by eliminating dependents' benefits, and by providing a strictly contributionrelated, nonweighted, retirement benefit. He does not provide a completely specified proposal; he puts it in general terms, i.e. "tying Social Security benefits directly to contributions." He sketches out a plan to accumulate contributions at interest and pay out an "actuarially fair" benefit at retirement. He would provide survivors and disability benefits but does not explain how or at what level. In the last chapter, he describes his proposal and provides cost estimates and a table of winners and losers for five general

proposals (retirement age increased to 68 immediately, reduced benefits for high-wage earners, etc.). He describes problems with each of the five proposals. I was expecting and hoping to see similar cost estimates for his proposal, but they were not there! Thus a comparison of the costs of his proposal with the others listed, or with the present program, is not possible. My conclusion is that Boskin has an incomplete proposal which is too ill-defined to make cost estimates possible. After a few minutes' reflection, I thought of a half-dozen serious problems with his proposal. Those problems could probably be overcome, but only by changing a seven-word proposal into one with many messy details which could not possibly provide the desired equity. In other words, it would be a lot more like what we have now.

STEVEN F. MCKAY

A Ramble Through the Actuarial Countryside: The Collected Papers, Essays and Speeches of Frank Mitchell Redington, F.I.A., edited with commentary by Gary Chamberlin, pp. 549, published by the Staple Inn Actuarial Society, Attn.: R. C. Wilkinson, Reliance Mutual Insurance, Mount Ephrain, Tunbridge Wells, Kent TN4 8BL, England, 1986, U.S. \$35 or Can \$45.

Frank Redington (1906–1984) was indeed an inspired and inspiring actuary. To have his thoughts neatly classified into book format with editorial explanations of topic, time and circumstances is a favor of which actuaries interested in our profession's philosophy would do well to take advantage. This volume is unreservedly recommended.

Mr. Redington was a mental wrestler with actuarial problems, whether of principle or technique, studied often over years or decades. He had the rare gift of clarity as well as good humor in conveying the essence of each problem, the matters that puzzled him and the conclusions he reached. He was Chief Actuary of the mighty Prudential Assurance Company of London from 1951 to 1968; he was President of the Institute of Actuaries from 1958 to 1960; and he was awarded the Institute's Gold Medal in 1968 "for work of pre-eminent importance in the actuarial field."

Here is a list of the volume's nine sections. I have not in all cases stated the section headings but have expressed them in terms designed to convey the theme of each section to an actuary who does not have the book before him.

Section 1. Ideas and statistical enquiries in Redington's early actuarial years.

- Section 2. Balance sheet principles, including pioneer thoughts on immunization.
- Section 3. Speeches as president of the Institute.
- Section 4. Policy dividend scales on the British bonus system.
- Section 5. Patterns in life table mortality rates.
- Section 6. Probability, economics and mortality.
- Section 7. A general appraisal late in Redington's long career.
- Section 8. Last reflections. Reminiscences.
- Section 9. Redington's poem, "Mathematics," written in his fiftieth year.

The book closes with a bibliography, a chronology, and selected quotations among which this reviewer's favorite is, "The briefest glance at the past tells us one fundamental actuarial lesson, that our strength lies in no way at all in the infallibility of our forecasts; it lies in our power to measure and deal with our own fallibility, to face and to assess our own uncertainty."

E. J. MOORHEAD

Hans U. Gerber, Lebensversicherungsmathematik, pp. 120, published for the Vereinigung Schweizerischer Versicherungsmathematiker by Springer-Verlag, Berlin, distributed by Springer-Verlag New York, Inc., New York, N.Y. 10010, 1986.

This well-written and attractively published book has been influenced by computer developments and by the younger generation's knowledge of probability theory. These factors have led to a probabilistic approach to actuarial models and formulas and to the relegation of commutation functions to a brief appendix. In 120 pages the book covers many of the main concepts presented in the Society's textbook, *Actuarial Mathematics*, and in addition devotes a chapter each to interest theory and to the estimation of basic probabilities. The book is directed to younger readers who take pleasure in applied mathematics and who wish an introduction to life insurance mathematics. A well-organized and elegant introduction awaits their reading.

The text is written in German but with its many formulas in the international language of mathematics, and with some dictionary assistance, it is not difficult to follow. To actuarial students, with lively curiosity, it can be both a supplement and an aid to *Actuarial Mathematics*; to practitioners, it can be a useful reference for following up some points they may come up against in applying actuarial mathematics; to educators it can provide a modern introduction to basic actuarial mathematics.

Chapter 1 deftly covers the main concepts of compound interest mathematics and ends with an algorithm for the rate of return from an investment of p repaid by payments r_k at times τ_k , k = 1, 2, ..., n. This is offered as being either more efficient or simpler than two standards methods for cases where a unique solution exists.

Chapter 2 discusses the basic random variables T, future lifetime of a life aged x; K, the curtate future lifetime; and S, the final fractional year of life, so that T = K + S. Probability distributions for K, T and S are defined. Under uniform distribution of deaths in the years of age, K and S are independent random variables which subsequently lead to simple formulas connecting continuous and discrete functions, for example, $\tilde{A}_x = (i/\delta)A_x$. Many of the standard life table concepts are explored here.

Chapter 3 on life insurance considers the random variables

$$Z = c_{K+1} v^{K+1}$$

and

$$Z = c(T)v^T$$

where c_{K+1} and c(T) denote amounts of insurance for policy year K+1 and at time point T, respectively. Expected values yield net single premiums for various standard life

insurances, and one can proceed to variances, or higher moments. In a final section on recursive formulas, the interesting relation

$$A_{x} = \sum_{k=0}^{x} v^{k} v(1-A_{x+k+1})q_{x+k}, k = 0,1, \dots$$

is obtained.

Chapter 4 on life annuities begins with the random variable

$$Y = \ddot{a}_{\overline{K+1}}$$

and

$$\ddot{a}_x = E(Y) = \sum_{k=0}^{\infty} \ddot{a}_{\overline{K+1}} \,_k p_x \, q_{x+k},$$

and proceeds to annuities with payments $z_{0}, z_{1/m}, z_{2/m}, \ldots$ at times $0, 1/m, 2/m, \ldots, K+S^{(m)} - 1/m$, where $S^{(m)}$ is obtained by rounding S to the next *m*th. Recursive formulas and inequalities are further topics. The exact formula, under uniform distribution of deaths in the years of age, for $\ddot{a}_{x+u}, 0 < u < 1$, is given as a weighted mean of \ddot{a}_x and \ddot{a}_{x+1} .

In Chapter 5, the discussion of net annual premiums, Π_k , leads to the general loss random variable

$$L = c_{K+1} v^{K+1} - \sum_{k=0}^{K} \prod_{k} v^{k}$$

and the condition E(L) = 0. From this, the endowment net annual premium $P_{x:\overline{n}}$ may be formulated by setting $c_1 = c_2 = \ldots = c_n = 1$, $c_{n+1} = c_{n+2} = \ldots = 0$; $\Pi_0 = \Pi_1 = \ldots = \Pi_{n-1} = P_{x:\overline{n}}$, $\Pi_n = -1$, $\Pi_{n+1} = \Pi_{n+2} = \ldots = 0$. The chapter ends with a discussion of stochastic interest, and points out the lack of validated models for the long term, and the dependence that would follow for the individual insurance risks.

Chapter 6 on net premium reserves is a smorgasbord of formulas and their interpretations, including Hattendorf's Theorem and a brief reference to universal life insurances. The net annual premium Π_k is split into savings and risk components

$$\Pi_{r}^{s} = {}_{k+1}V v - {}_{k}V,$$
$$\Pi_{k}^{r} = (c_{k+1} - {}_{k+1}V)v q_{k+k}.$$

Then the technical gain, G_{k+1} , in policy year k+1 is split into corresponding components, G_{k+1}^{s} and G_{k+1}^{r} . One possible allocation of the gain can be accomplished by increasing both the future amounts of insurance and the future net annual premiums by the factor v(1+i') where i' is the effective annual rate of interest earned in year k+1. The chapter ends with a continuous model of a tontine-type fund for a closed group of annuitants for which mortality gains or losses are used to adjust the rate of annuity income.

One variation in Chapter 7 on multiple decrement models is the denoting of components related to the *j*th cause of decrement by an additional lower right subscript rather than by a superscript, e.g., $q_{j,x}$ in place of $q_x^{(j)}$. Notable also is the omission of discussion of the abstraction, $q_x^{(ij)}$, the absolute annual rate of decrement if only cause (*j*) is operating. The author's treatment of premiums and reserves for a general insurance covering multiple

causes of decrement is of special interest to this reviewer because of earlier work by W. S. Bicknell (*TSA* VIII) and by J. C. Hickman (*TSA* XVI). These authors discussed component level premiums and corresponding reserves for the various causes of decrement. Professor Gerber begins by defining a savings premium to accumulate the total reserve, $_{k}V$, and component risk premiums

$$\Pi_{j,k}^{r} = (c_{j,k+1} - {}_{k+1}V)vq_{j,x+k}$$

This is close to the concept of dependent premium and reserve components discussed by the earlier authors but seems more flexible. It does not, however, define component reserves explicitly.

In Chapter 8, after an introduction to probability theory for joint life and last survivor statuses, the author discusses general symmetric statuses. Let B_1, B_2, \ldots, B_m denote m events of interest, and

$$S_k = \sum Pr(B_{j_1} \cap B_{j_2} \cap \ldots \cap B_{j_k})$$

where the sum extends over all $\binom{m}{k}$ combinations of k events from the m given ones. Let N denote the number of events that occur. A key relation, developed by D. R. Schuette, states that for any coefficients c_0, c_1, \ldots, c_m

$$\sum_{n=0}^{m} c_n Pr(N=n) = \sum_{k=0}^{m} \Delta^k c_0 S_k.$$

Professor Gerber gives a neat proof of this theorem using indicator random variables and not requiring independence of the events B_1, B_2, \ldots, B_m . The chapter gives applications of this theorem, and then discusses values of asymmetric annuities e.g. $a_{\overline{wx},\overline{yz}}$, and of asymmetric insurances, e.g. \overline{A}_{wxy^2} .

Chapter 9 deals with total claims of a portfolio of n insurances. The claim under insurance h is denoted by the random variable S_h with possible values $0, s_{1h}, s_{2h}, \ldots, s_{mh}$, and probabilities

$$Pr(S_{h}=0)=p_{h}, Pr(S_{h}=s_{ih})=q_{ih},$$

 $j = 1, 2, \ldots, m, \quad h = 1, 2, \ldots, n$. The probability distribution of

$$S = S_1 + S_2 + \ldots + S_n$$

is sought. One approach is to use the normal distribution, another is the exact computation successively of the distributions of $S_1 + S_2$, $S_1 + S_2 + S_3$, etc. A third approach is by the compound Poisson model of collective risk theory which can be facilitated by use of the recursive formula

$$f(x) = \frac{1}{x} \sum_{j=1}^{m} j q_j f(x-j), \quad x = 1, 2, 3, \ldots,$$

where f(x) = Pr(S=x) and $q_j = \sum_{h=1}^{n} q_{jh}$. The chapter concludes with applications to excess-of-loss and stop-loss reinsurances.

Chapter 10 discusses premiums and reserves with allowance for expenses. These latter are classified as closing, collection and administrative. A sufficient premium is defined as

$$P^{a} = P + P^{\alpha} + P^{\beta} + P^{\gamma}$$

where P is the net premium, and $P^{\alpha}, P^{\beta}, P^{\gamma}$ are premium equivalents of the closing, collection and administrative expenses, respectively. Corresponding reserves are defined prospectively. In an illustration for an endowment insurance, by setting ${}_{1}V_{x;\overline{n}}^{\alpha}=0$, the sufficient reserves equal full preliminary term reserves.

A summary of some of the ideas of Chapter 11 regarding the estimation of probabilities of death is as follows. The author uses a Lexis diagram to illustrate observed life-years over a given observation period. If n lives are observed within the year of age from x to x+1, and if for life i, observation begins at age $x+t_i$ and ceases at $x+s_i$, $(0 \le t_i < s_i \le 1)$, then

$$E_x = \sum_{i=1}^n (s_i - t_i)$$

is the (exact) exposure of these lives within the year of age and the observation period. Let D_x represent the number of observed deaths within the same age interval-observation period region of the Lexis diagram. As a first estimator, expected deaths are calculated by using the Balducci assumption and equated to observed deaths to obtain

$$\hat{q}_x = \frac{D_x}{E_x + \sum_{i \in I} (1 - s_i)}$$

where I is the set of indices for the deaths. This estimator may be adequate for cases with numerous data but may fail for cases with small data. Also it does not facilitate statistical inference. The author prefers the maximum likelihood estimator

$$\hat{\mu}_{x+\frac{1}{2}} = \frac{D_x}{E_x}$$

where the force of mortality is assumed to be constant over the age interval. Then

$$\hat{q}_x = 1 - \exp\left[-\hat{\mu}_{x+\frac{1}{2}}\right].$$

By considering E_x as known, and D_x as an observation from a Poisson distribution with parameter $\lambda = \mu_{x+\frac{1}{2}} E_x$, the author develops confidence limits in turn for λ , $\mu_{x+\frac{1}{2}}$ and q_x .

The maximum likelihood estimation can be extended to the multiple decrement case to obtain

$$\hat{\mu}_{j,x+\frac{1}{2}} = \frac{D_{j,x}}{E_x}, j = 1,2,\ldots,m$$

$$\hat{q}_{j,x} = \frac{D_{j,x}}{D_x} \hat{q}_x.$$

where \hat{q}_x is the maximum likelihood estimator of the total probability of decrement.

The author also discusses another estimation approach by Bayesian inference for both single and multiple decrement cases.

Appendix A introduces commutation functions and their application. Appendix B discusses simple interest.

The text appears remarkably error-free. The reviewer has noted only two, a transposition of signs in formula (7.15) of Chapter 1 and a misplaced index in the formula $P_{x,\overline{n}}^1 \ddot{a}_{x+n}$ in Section 5.3.4.

There is no discussion of actuarial accumulated values or of retrospective formulas for reserves. Under present circumstances, these may be of less importance.

There are a number of enlightening interpretations of formulas and, from time-to-time, numerical examples to illustrate the mathematical theory. There are no exercises so this is not a textbook in the more usual format. But teachers and students will find the book to be an excellent stimulus for their own understanding of life insurance mathematics. Both pleasure and information await the interested reader.

CECIL J. NESBITT

B. Michael Pritchett, Financing Growth: A Financial History of American Life Insurance Through 1900, pp. 90, published by the S.S. Huebner Foundation for Insurance Education, Wharton School, University of Pennsylvania, Philadelphia, Pa. 19104, 1985.

Life insurance companies, some with hundreds of officers, have come a long way since the early days of the industry when a typical new company would have only three: President, Cashier and Actuary. Those early days are examined in some detail in *Financing Growth* by B. Michael Pritchett of Brigham Young University, an examination of the investment role played by life insurance companies prior to 1900.

By necessity, the book is in some respects a history of the early days of the industry itself, and a colorful history it was. The first company to issue policies was a branch of an English company with the incredible name of The Corporation for the Relief of Poor and Distressed Presbyterian Ministers and of the Poor and Distressed Widows and Children of Presbyterian Ministers. It issued its first policy on May 22, 1761, and, under the name of the Presbyterian Ministers Fund, is still operating from its office in Rittenhouse Square in Philadelphia.

Although several other companies entered the business in the following decades, the industry did not begin to flourish until the "mutual era," which began with New England Mutual (1835) and which soon included The Mutual Life Insurance Company of New York, New York Life, Connecticut Mutual, Penn Mutual and Mutual Benefit.

Active person-to-person solicitation began with the mutual companies. Earlier agents were involved mainly in arranging and buying mortgages on behalf of their companies and transacted sales by the more traditional (and presumably more respectable) method of waiting at their agencies for customers. The missionary zeal with which the newer companies pursued their goal was exemplified by this statement from the president of Berkshire Life to its policyholders:

Every policyholder should be a missionary of Life Insurance to aid in extending its benefits to others whose families need the protection. It is earnestly desired that each one will use his exertions to promote the interests and extend the business of the Company. If every member will secure but one other, the business of the Company would be rapidly increased, and the benefits of Life Insurance become widely spread.

In these years of rapid growth, there was a tremendous demand for agents, who were often the objects of bidding wars between the companies. The author quotes one incident in the 1860s in which "New York Life received notice one morning that their entire Manhattan office consisting of a general agent and over two hundred agents had gone over to Equitable."

There is a brief section on Henry Hyde of the Equitable and the tontine system which he introduced in 1867, under which the cash values of lapsed policies stayed in the tontine and were paid to the survivors of the tontine period (10, 15 or 20 years). The system did not last long since, as the author notes, it was "not in the spirit of the emerging forfeiture laws." Before these laws, if the insured did not have the premium in the agent's hands by the due date, the contract could be (and often was) terminated. Cash values reverted to the company, regardless of the number of premiums paid. The company was not required to notify the insured of the approaching due date. (The promise of the share of cash values on lapsed policies was one of the allures of the tontine.)

In spite of the obvious failings of the tontine system, the author notes that it made "faithful, regular savers of those who were not predisposed to such behavior," and thus contributed to making the industry a supplier of capital.

The author forcefully shows, both with text and with tables, the devastating effect of the protracted depression of the 1870s on the life insurance business. In 1870, the number of legal reserve life insurance companies was 135; by 1880, it was down to 69 (many of the companies that failed during this period were absorbed by others). Company failures were rampant, and state regulators (yes, even then they were a factor) and the press were not reluctant to criticize the management of the fallen companies. As the author states: "Men and firms lauded as statesmen and giants in one year were castigated as crooks and frauds in the next, with no factual change except the press of events and bankruptcy laying open their books to public scrutiny and hindsight."

The author notes how state regulation sometimes worked to the detriment of the companies whose financial solvency it was their duty to protect. State regulation emphatically encouraged investment in real estate; one large company's charter provided that it invest only in real estate or related assets except as otherwise specifically permitted in the charter. But real estate performed more poorly than many other types of investment, and in the 1870s, the companies found themselves saddled with real properties with severely depressed values, many of which were acquired by foreclosing on mortgages.

In this book, which contains 29 tables and 13 graphs, the author has sought to examine three hypotheses:

(1) life insurers mobilized a substantial pool of capital from a broad policyholder base,

(2) insurers supplied critical infusions of funds to new industries at a formative stage

of their development, and (3) the industry channeled capital to developing regions of the country roughly in proportion to regional premium volume.

The first hypothesis is shown to be valid, while the last two are not. It is hardly surprising that life insurance companies were not a source of venture capital, either then or now. The primary concern of life insurance investment departments has always been preservation of capital, with high rates of return (until recently at least) a secondary concern. (However, as the author notes, this did not stop some companies from trying to earn yields higher than those on investments permissible under law. Even in those days, companies used subsidiaries, which for this purpose invested in higher-risk, higheryield securities.) Further, state regulation often had the effect of requiring most investments to be made "close to home," with the effect that very little life insurance funds found their way into the less developed areas of the country, namely the South and the West.

Mr. Pritchett has produced an interesting and readable book on the early days of the business and a worthwhile addition to the archives of insurance industry history.

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