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POLICY RESERVES IN GROUP INSURANCE

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ABSTRACT

The theory of policy reserves for future claims and dividends under retrospectively experience-rated group insurance contracts is developed. Termination cost reserves are defined and related by formula to the policy reserve for future claims and dividends. The formula provides an interpretation of the policy reserve for future claims and dividends in terms of the amount of the current deficit expected to be recovered in the future or the amount of the current contingency fund balance. Sample numerical values of the policy reserve for future claims and dividends are presented.

The policy reserve for future claims and dividends developed in this paper is a true policy-year benefit reserve and should not be confused with either the statutory rating refund liability or the claim reserves arising under group insurance contracts.

INTRODUCTION

This paper deals with the policy reserves for future benefits that arise under retrospectively experience-rated group insurance policies. Specifically, we develop the theory of policy reserves for the combined benefits of claims and policyholder dividends[1] (experience-rating refunds) payable under such policies. The reserves discussed are true policy-year reserves and should not be confused with the statutory rating refund liability, which represents the amount of the current policy year's dividend or experience-rating refund earned between the last policy anniversary and December 31.

The traditional definition of a policy-year benefit reserve is the present value of future benefits less the present value of future net premiums. In this paper, we use a generalized version of this definition, based on Gerber [2]. We define at time t, the policy reserve for future claims and dividends to be the present value of expected future claims and dividends less the

[1] Throughout this paper, the terms "dividends" and "policyholder dividends" are used interchangeably. Also, the term "policy" is restricted to "retrospectively experience-rated group insurance policy."

present value of expected future net premiums for claims and dividends, given the history of the policy. This generalized definition must be used because the future claims and dividends payable depend on the deficit level or contingency fund balance of the policy at the current time.

Before the development of the policy reserve for future claims and dividends can take place, it is necessary to determine the net premium for claims and dividends. We show that when the gross premium is calculated in the usual manner, the net premium is equal to the sum of the expected incurred claims, the dividend loading, and the risk charge. This fact will enable us to write down a general expression for the policy reserve for future claims and dividends.

The risk charge is a component of the retention charged that is designed to compensate the insurer for losses sustained upon cancellation of a policy in a deficit position. This has been the subject of lively debate among group actuaries in recent years. Bolnick [1] argued that it is not possible to determine accurately a proper risk charge, because of the uncertain nature of the cancellation process. Others have disagreed with Bolnick's point of view, maintaining that it is possible to make reasonable assumptions about the effect of the deficit level on the cancellation rate. We are taking the position that a risk charge can be determined so that the present value, at issue, of expected future termination costs (deficits lost upon cancellation) equals the present value of expected future risk charges.

The relation satisfied by the risk charge at issue leads naturally to the consideration of what we call a termination cost reserve. We define, at time t, the termination cost reserve to be the present value of expected future termination costs less the present value of expected future risk charges, given the history of the policy.

By a careful examination of the retrospective experience-rating mechanism, we relate the policy reserve for future claims and policyholder dividends at time t to the termination cost reserve at time t. This relation allows us to give an interesting interpretation of the policy reserve for future claims and dividends. We show that if the policy has an outstanding deficit at time t, the policy reserve for future claims and dividends at time t can be interpreted as an asset equal to the amount of the deficit expected to be recovered in the future, less a correction term. Similarly, if the policy has a contingency fund balance at time t, the policy reserve for future claims and dividends can be interpreted as a liability equal to the entire contingency fund balance less a correction term. These interpretations are satisfying because they allow us to think of the policy reserve for future claims and dividends under retrospectively experience-rated group insurance contracts in terms of the asset or liability that common sense tells us we should hold for the policy.

To illustrate the numerical value of policy reserves for future claims and dividends, we use an example that has appeared previously in the literature. We employ the methods developed by Panjer and Mereu [3] to calculate the policy reserves for future claims and dividends for the group of 1,050 lives considered by Bolnick [1]. The coverage on this group is life insurance only, and it is assumed that the group's aggregate claims distribution remains the same from year to year.

The example illustrates that the policy reserve for future claims and dividends can assume a significant size, and so the decision whether or not to hold such reserves for retrospectively experience-rated group insurance policies is an important one, which could have major effects on an insurer's financial statements.

We conclude the paper with a discussion of some considerations involved in the computation of the policy reserves for future claims and dividends and the appropriateness of their use in the various financial statements prepared by group insurers.

NOTATION AND ASSUMPTIONS

Notation

Each of the following is considered a nonnegative random variable pertaining to one retrospectively experience-rated policy issued at time $t = 0$:

E_t = Expected incurred claims in policy year t;
CC_t = Claims charged in dividend formula in policy year t;
C_t = Actual incurred claims in policy year t;
DL_t = Dividend loading in policy year t (also called margin);
RC_t = Risk charge in policy year t;
EC_t = Commission, expense, and tax charge in policy year t;
PC_t = Profit or contribution to surplus charge in policy year t;
IC_t = Investment income credit in policy year t;
GP_t = Gross premium in policy year t;
EA_t = Actual commission, expenses, and taxes in policy year t;
IA_t = Actual investment income earned on cash flow in policy year t;
PD_t = Actual profit disbursed or contribution to surplus made in policy year t;
GS_t = Surplus generated in policy year t;
GD_t = Deficit generated in policy year t;

DR_t = Deficit recovered in policy year t;
D_t = Dividends paid in policy year t;
D_t^1 = Dividends paid as a result of surplus generated in policy year t;
D_t^2 = Dividends paid as a result of return of outstanding contingency fund balance on case canceling at end of policy year t.

(Note that $D_t = D_t^1 + D_t^2$.)

CF_t = Contingency fund balance at end of policy year t;
ΔCF_t = Addition to contingency funds in policy year t from surplus generated in policy year t;
ΔCF_t = Withdrawal from contingency funds in policy year t;
ΔCF_t^1 = Withdrawal from contingency funds in policy year t to offset a deficit generated in policy year t;
ΔCF_t^2 = Withdrawal from contingency funds in policy year t to pay terminal dividend to a case canceling at end of policy year t.

(Note that $\Delta CF_t = \Delta CF_t^1 + \Delta CF_t^2$ and that $\Delta CF_t^2 = D_t^2$.)

R_t = Deficit level at end of policy year t;
TC_t = Termination costs in policy year t.

We assume that R_t and CF_t are measured after the financial accounting for policy year t has taken place and after the decision whether or not to cancel has been made.

Each of the following is assumed to be a fixed constant:

w = Time by which cancellation is assumed to be certain;
i = Interest rate at which contingency funds and outstanding deficits are carried forward and all present-value calculations are performed;
$v = 1/(1 + i)$.

Assumptions

A retrospectively experience-rated group insurance policy is one for which a financial accounting takes place at the end of each policy year. The value of gross premium received less claims charged less retention is determined for the policy. The retention consists of a commission, expense, and tax charge, a risk charge, a profit or contribution to surplus charge, and an investment income credit. If the result of the calculation is positive, it is called a surplus, which is used first to offset any outstanding deficit attributable to the policy, then to build up the policy's contingency fund to a desired level, and finally to pay a dividend to the policyholder. If the result of the calculation is negative, its absolute value is called a deficit, which is first offset to the extent possible by any of the policy's outstanding contingency funds, with the remainder carried forward to future accounting periods. Both outstanding deficits and contingency funds are carried forward with interest. Upon cancellation of the contract, any contingency funds are returned to the policyholder as a terminal dividend, and any outstanding deficit is permanently lost. The deficits lost at cancellation are called termination costs. We assume cancellation is a decision process possibly influenced by the duration since issue and the outstanding deficit level or contingency fund balance.

The gross premium for a retrospectively experience-rated group insurance policy is assumed equal to the sum of the expected incurred claims, the dividend loading, and the retention. We write this symbolically as

$$GP_t = E_t + DL_t + EC_t + RC_t + PC_t - IC_t \,. \tag{1}$$

If we assume that the commission, expense, tax, risk, and profit or contribution to surplus charges and the investment income credits are the same in the premium as in the dividend calculation, then the result of the financial accounting may be expressed as the expected claims plus the dividend loading less the claims charged. Thus the surplus generated in policy year t can be expressed as

$$GS_t = E_t + DL_t - CC_t \quad \text{if positive} \,, \quad 0 \text{ otherwise} \,; \tag{2}$$

and the deficit generated in policy year t can be expressed as

$$GD_t = CC_t - E_t - DL_t \quad \text{if positive} \,, \quad 0 \text{ otherwise} \,. \tag{3}$$

Combining these two expressions, we obtain

$$GS_t - GD_t = E_t + DL_t - CC_t \,. \tag{4}$$

RISK CHARGE

The risk charge is a component of the retention designed to compensate the insurer for losses sustained upon cancellation of a policy with an outstanding deficit. This relationship may be expressed by saying that the present value, at issue, of expected future termination costs must equal the present value, at issue, of expected future risk charges.

We assume that the risk charge is payable at the end of the policy year, when the dividend calculation takes place, by all policies entering the policy year. We write

$$\sum_{n=1}^{w} v^n \mathscr{E}(TC_n) = \sum_{n=1}^{w} v^n \mathscr{E}(RC_n) . \tag{5}$$

We use expected value notation in equation (5) because this is a succinct way of writing the equation properly and because this notation will be essential in the later development of reserves. This type of notation was discussed by Gerber [2]. The familiar persistency and lapse symbols do not appear at all in the equations, but rather are embodied in the expected values. For example, if the value of the risk charge made to an active policy in policy year n is denoted rc_n, then the possible values of RC_n are rc_n and 0. The probability that $RC_n = rc_n$ is just the probability that the policy is active at the beginning of policy year n, which we may write as P_{n-1}. The probability that $RC_n = 0$ is just $1 - P_{n-1}$. Therefore, $\mathscr{E}(RC_n) = P_{n-1} rc_n$.

NET PREMIUM FOR CLAIMS AND DIVIDENDS

We show that the net premium for claims and dividends is equal to the component of the gross premium represented by the sum of the expected incurred claims, the dividend loading, and the risk charge. To do this, we show that

$$\sum_{t=1}^{w} v^t \mathscr{E}(C_t + D_t) = \sum_{t=1}^{w} v^t \mathscr{E}(E_t + DL_t + RC_t) .$$

We start by considering what can happen to the surplus generated in policy year t, GS_t. According to our assumptions, it is either used to offset past deficits, added to the contingency fund, or paid as a dividend. We write

$$GS_t = DR_t + \Delta CF_t + D_t' . \tag{6}$$

We next consider the deficit level at time t, R_t. To arrive at R_t, the past deficit R_{t-1} is carried forward with interest and is then modified by the financial results of the policy year. If a surplus is generated, the deficit level will be reduced by the deficit recovered. If a deficit is generated, part of the deficit may be reduced by drawing down the contingency fund. The remainder is added to the deficit level. However, if a case terminates with a deficit at time t, the deficit level is reduced to zero—that is, reduced by the termination cost. We summarize these observations as follows:

$$R_t = R_{t-1}(1 + i) - DR_t + GD_t - \Delta CF_t^1 - TC_t\,. \qquad (7)$$

Combining equations (6) and (7), we obtain

$$GS_t - GD_t = \Delta CF_t + D_t^1 - [R_t - (1 + i)R_{t-1}] - \Delta CF_t^1 - TC_t\,. \qquad (8)$$

If we add D_t^2 and subtract ΔCF_t^2 from the right-hand side of equation (8) and simplify, we obtain

$$GS_t - GD_t = (\Delta CF_t - \Delta CF_t) - [R_t - (1 + i)R_{t-1}] - TC_t + D_t\,. \qquad (9)$$

Combining equations (4) and (9), and using the fact that $\Delta CF_t - \Delta CF_t = CF_t - (1 + i)CF_{t-1}$, we obtain

$$\begin{aligned} E_t + DL_t - CC_t = D_t - TC_t &+ [CF_t - (1 + i)CF_{t-1}] \\ &- [R_t - (1 + i)R_{t-1}]\,. \end{aligned} \qquad (10)$$

Adding RC_t to each side of equation (10) and rearranging, we obtain

$$\begin{aligned} E_t + DL_t + RC_t = C_t + D_t &+ (CC_t - C_t) + (RC_t - TC_t) \\ &+ [CF_t - (1 + i)CF_{t-1}] - [R_t - (1 + i)R_{t-1}]\,. \end{aligned} \qquad (11)$$

By taking expected values of each side of equation (11), multiplying by v^t, and summing from $t = 1$ to w, we obtain

$$\begin{aligned} \sum_{t=1}^{w} v^t \mathcal{E}(E_t + DL_t + RC_t) = \sum_{t=1}^{w} v^t \mathcal{E}(C_t + D_t) & \\ + \sum_{t=1}^{w} v^t \mathcal{E}(CC_t - C_t) & \\ + \sum_{t=1}^{w} v^t \mathcal{E}(RC_t - TC_t) & \qquad (12) \\ + \sum_{t=1}^{w} v^t \mathcal{E}[CF_t - (1 + i)CF_{t-1}] & \\ - \sum_{t=1}^{w} v^t \mathcal{E}[R_t - (1 + i)R_{t-1}]\,. & \end{aligned}$$

We make the following observations:

A1. $\mathscr{E}(CC_t) = \mathscr{E}(C_t)$ for each t; otherwise, the formula for claims charged is inappropriate.

A2. $$\sum_{t=1}^{w} v^t[CF_t - (1+i)CF_{t-1}] = \sum_{t=1}^{w} v^t CF_t - \sum_{t=0}^{w-1} v^t CF_t$$
$$= v^w CF_w - CF_0 .$$

But, since w was the time by which the policy is certain to have canceled, $CF_w = 0$. Also, the policy starts out at time 0 with no contingency fund, so $CF_0 = 0$. Thus,

$$\sum_{t=1}^{w} v^t [CF_t - (1+i)CF_{t-1}] = 0 .$$

A3. By an analysis similar to that used in observation A2, we may show that

$$\sum_{t=1}^{w} v^t[R_t - (1+i)R_{t-1}] = 0 .$$

Using observations A1, A2, and A3 and equation (5), we see that the right-hand side of equation (12) reduces to $\Sigma_{t=1}^{w} v^t \mathscr{E}(C_t + D_t)$. Therefore,

$$\sum_{t=1}^{w} v^t \mathscr{E}(E_t + DL_t + RC_t) = \sum_{t=1}^{w} v^t \mathscr{E}(C_t + D_t) . \tag{13}$$

Equation (13) establishes that the net premium for claims and dividends is equal to the component of the gross premium represented by the sum of the expected incurred claims, the dividend loading, and the risk charge. The objection might be raised that discounting in equation (13) takes place at the end of each policy year, whereas premiums are received periodically throughout and possibly following the policy year, incurred claims are paid periodically throughout and following the policy year, and dividends are typically paid several months after the end of the policy year. This objection may be overcome by considering the investment income credit IC_t, which is part of the gross premium.

The investment income credit IC_t represents the overall investment income incurred in policy year t, taking into account the timing of each cash inflow and outflow incurred in policy year t. Therefore, the interest adjusted values, at the end of policy year t, of the disbursements incurred in policy year t less the premium incurred in policy year t, is given by

$$C_t + D_t + EA_t + PD_t - GP_t - IC_t \,. \tag{14}$$

Substituting the value of GP_t from equation (1) into expression (14) and rearranging, we obtain

$$(C_t + D_t - E_t - DL_t - RC_t) + (EA_t - EC_t) + (PD_t - PC_t) \,. \tag{15}$$

If we ignore differences between actual and charged commissions, expenses, and taxes and profit or contributions to surplus disbursed and charged, expression (15) indicates that the interest-adjusted value, at the end of policy year t, of the claims and dividends disbursed less the gross premium received other than the commission, expense, and tax charge and the profit or contribution to surplus charge, is given by

$$C_t + D_t - (E_t + DL_t + RC_t) \,. \tag{16}$$

Since a proper way to discount cash flows incurred in policy year t back to time 0 is to accumulate or discount them to the end of policy year t and then apply the discount factor v^t, expression (16) indicates that the present value, at time 0, of the claims and dividends incurred in policy year t less the gross premiums, other than the commission, expense, and tax charge and the profit or contribution to surplus charge, incurred in policy year t is given by

$$v^t(C_t + D_t) - v^t(E_t + DL_t + RC_t) \,. \tag{17}$$

Therefore, the discounting in equation (13) correctly recognizes the timing of the cash flows involved.

POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

The basic definition that we will use for the policy reserve for future claims and dividends at time t is the present value of expected future claims and dividends less the present value of expected future net premiums for claims and dividends, given the history of the policy. In the previous section, we showed that the net premium for claims and dividends in policy year t is given by $E_t + DL_t + RC_t$. We will assume that the relevant history of the policy can be summarized by the following pieces of information: whether the policy is active at time t, and, if so, the outstanding deficit level or contingency fund balance at that time.

A policy that is not still active at time t clearly has the policy reserve for future claims and dividends at time t equal to zero. If we let x be the deficit level and y the contingency fund balance of an active policy at time t, then the policy reserve for future claims and dividends at time t, denoted ${}_tV(x, y)$, is, according to the above remarks, given by the following equation:

$$\begin{aligned} {}_tV(x, y) = & \sum_{n=t+1}^{w} v^{n-t}\mathscr{E}[(C_n + D_n)|\text{policy is active at time } t, \\ & \qquad\qquad R_t = x, \quad CF_t = y] \\ & - \sum_{n=t+1}^{w} v^{n-t}\mathscr{E}[(E_n + DL_n + RC_n)|\text{policy is active at time } t, \\ & \qquad\qquad R_t = x, \quad CF_t = y]. \end{aligned} \tag{18}$$

The interest discounting in equation (18) takes place from the end of each policy year. This is justified by the same reasoning explained in the previous section for equation (13).

Since an active policy cannot have both a positive outstanding deficit level and a positive contingency fund balance, one of x and y must always be zero.

TERMINATION COST RESERVE

Equation (5) indicates that the risk charge is the net premium for the termination costs. We define the termination cost reserve at time t to be the present value of expected future termination costs less the present value of expected future risk charges, given the history of the policy. As with the policy reserve for future claims and dividends, it is assumed that the history of the policy can be summarized by the following pieces of information: whether the policy is active at time t, and, if so, the outstanding deficit level or contingency fund balance at that time.

A policy that is not still active at time t clearly has the termination cost reserve at time t equal to zero. If we let x be the deficit level and y the contingency fund balance of an active policy at time t, then the termination cost reserve at time t, denoted ${}_tV^{TC}(x, y)$, is given by the following equation:

$$\begin{aligned} {}_tV^{TC}(x, y) = & \sum_{n=t+1}^{w} v^{n-t}\mathscr{E}(TC_n|\text{policy is active at time } t, \\ & \qquad\qquad R_t = x, \quad CF_t = y) \\ & - \sum_{n=t+1}^{w} v^{n-t}\mathscr{E}(RC_n|\text{policy is active at time } t, \\ & \qquad\qquad R_t = x, \quad CF_t = y). \end{aligned} \tag{19}$$

The remarks made after equation (18) apply to equation (19) as well.

RELATIONSHIP BETWEEN ${}_tV(x, y)$ AND ${}_tV^{TC}(x, y)$

We will derive a relationship between ${}_tV(x, y)$ and ${}_tV^{TC}(x, y)$ by considering equation (11). First, we rearrange equation (11) and substitute the index n for the index t:

$$\begin{aligned} D_n + C_n - (E_n + DL_n + RC_n) = {} & TC_n - RC_n \\ & + (C_n - CC_n) \\ & - [CF_n - (1 + i)CF_{n-1}] \\ & + [R_n - (1 + i)R_{n-1}] \, . \end{aligned} \tag{20}$$

By taking expected values of each side of equation (20), given that the policy is active at time t with outstanding deficit level x and contingency fund balance y, then multiplying by v^{n-t} and summing from $n = t + 1$ to w, we obtain the following:

$$\begin{aligned} & \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}(D_n + C_n)|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y] \\ & \quad - \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}[(E_n + DL_n + RC_n)|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y] \\ & = \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}(TC_n|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y) \\ & \quad - \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}(RC_n|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y) \\ & \quad + \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}(C_n - CC_n)|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y) \\ & \quad - \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}\{[CF_n - (1 + i)CF_{n-1}]|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y\} \\ & \quad + \sum_{n=t+1}^{w} v^{n-t} \mathscr{E}\{[R_n - (1 + i)R_{n-1}]|\text{policy is active at time } t \, , \\ & \qquad\qquad R_t = x \, , \quad CF_t = y\} \, . \end{aligned} \tag{21}$$

We make the following observations:

B1. $\mathscr{E}(C_n|\text{policy is active at time } t,\ R_t = x,\ CF_t = y) = \mathscr{E}(CC_n|\text{policy is active at time } t,\ R_t = x,\ CF_t = y)$, for each n; otherwise, the formula for claims charged is inappropriate.

B2. $$\sum_{n=t+1}^{w} v^{n-t}[CF_n - (1+i)CF_{n-1}] = \sum_{n=t+1}^{w} v^{n-t}CF_n - \sum_{n=t}^{w-1} v^{n-t}CF_n$$

$= v^{w-t}CF_w - CF_t = -CF_t$, since we assume the policy has canceled by time w, so $CF_w = 0$. Therefore,

$$\sum_{n=t+1}^{w} v^{n-t}\mathscr{E}\{[CF_n - (1+i)CF_{n-1}]|\text{policy is active at time } t,\ R_t = x,\quad CF_t = y\}$$

$$= -y\ .$$

B3. By an analysis similar to that used in B2, we may show that

$$\sum_{n=t+1}^{2} v^{n-t}\mathscr{E}\{[R_n - (1+i)R_{n-1}]|\text{policy is active at time } t,\ R_t = x,\quad CF_t = y\}$$

$$= -x.$$

By using observations B1–B3 and equations (18) and (19), we may restate equation (21) as follows:

$$_tV(x, y) = {_tV^{TC}}(x, y) + y - x\ . \tag{22}$$

Equation (22) states that, at time t, the policy reserve for future claims and dividends is equal to the termination cost reserve plus the contingency fund balance less the outstanding deficit level.

INTERPRETATION OF POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

When an active retrospectively experience-rated group insurance policy has an outstanding deficit x at time t, equation (22) states that the policy reserve for future claims and dividends at time t equals $-x + {_tV^{TC}}(x, 0)$. We consider the negative of the reserve, $x - {_tV^{TC}}(x, 0)$, as the asset we should hold at time t on account of this policy. We write

$$_t\text{Asset}_x = x - {_tV^{TC}}(x, 0)\ . \tag{23}$$

The current deficit x may be expressed as the sum of that portion of x which we expect will be recovered with interest in the future, x_R, and that portion of x we expect will be lost with interest in the future upon cancellation, x_L. We write

$$x = x_R + x_L . \tag{24}$$

The termination cost reserve ${}_tV^{TC}(x, 0)$ at time t is equal to the present value of expected future termination costs less the present value of expected future risk charges, given that the policy is active with deficit level x. We write

$${}_tV^{TC}(x, 0) = PVFTC_x - PVFRC_x . \tag{25}$$

The present value of future termination costs at time t, given that the policy is active with deficit level x, can be expressed as the sum of that portion of the current deficit x we expect will be lost with interest at cancellation, x_L, and the present value, at time t of expected subsequent termination costs arising from deficits generated, given that the policy is active with deficit level x. We denote the latter amount f_x and write

$$PVFTC_x = x_L + f_x . \tag{26}$$

Combining equations (23), (24), (25), and (26), we obtain

$${}_t\text{Asset}_x = x_R - (f_x - PVFRC_x) . \tag{27}$$

Equation (27) may be verbalized by saying that the asset we should hold at time t on account of an active policy with an outstanding deficit is that portion of the deficit we expect to recover in the future less a correction term. The correction term represents the amount by which the present value, at time t, of expected future termination costs arising from future deficits exceeds the present value, at time t, of expected future risk charges collected for this particular policy.

In other words, we may anticipate recovering a portion of the current outstanding deficit, but we must at the same time provide for any inadequacies of the future risk charges to finance termination costs arising from deficits generated after time t. One reason why future risk charges may be inadequate to finance termination costs arising from deficits generated after time t is that the current deficit will have to be completely recovered before any deficit generated after time t can be recovered.

When an active policy has a contingency fund balance y at time t, equation (22) states that the policy reserve for future claims and dividends at time t is equal to $y + {}_tV^{TC}(0, y)$. We consider the reserve as the liability we should hold at time t on account of this policy. We write

$$ {}_t\text{Liability}_y - y + {}_tV^{TC}(0, y) \ . \tag{28}$$

The termination cost reserve ${}_tV^{TC}(0, y)$ at time t is equal to the present value of expected future termination costs less the present value of expected future risk charges, given that the policy is active with contingency fund balance y. We write

$$ {}_tV^{TC}(0, y) = PVFTC_y - PVFRC_y \ . \tag{29}$$

Since the policy has no current deficit level, the present value, at time t, of expected future termination costs, given that the policy is active with contingency fund balance y, is equal to the present value of expected future termination costs arising from future deficits, given that the policy is active with contingency fund balance y. We denote this amount f_y and write

$$ PVFTC_y = f_y \ . \tag{30}$$

Combining equations (28), (29), and (30) we obtain

$$ {}_t\text{Liability}_y = y - (PVFRC_y - f_y) \ . \tag{31}$$

Equation (31) may be verbalized by saying that the liability we should hold at time t on account of an active policy with a contingency fund balance is the entire contingency fund balance less a correction term. The correction term represents the amount by which the present value, at time t, of expected future risk charges collected exceeds the present value, at time t, of expected future termination costs arising from deficits generated after time t for this particular policy.

In other words, we must provide for the complete liquidation of the contingency fund, because any portion of it not used to offset future deficits generated will be returned as a terminal dividend. However, we may offset this liability by any expected excess of future risk charges over future termination costs arising from future deficits generated. One reason why future risk charges may be in excess of future termination costs arising from deficits generated after time t is that future deficits generated will be immediately recovered up to the balance in the contingency fund.

NUMERICAL EXAMPLE

We will evaluate sample policy reserves for future claims and dividends for the group of 1,050 lives considered by Bolnick [1]. The approach used will be to calculate termination cost reserves for the policy and then use equation (22) to calculate the policy reserves for future claims and dividends. The termination cost reserves will be calculated using the methods developed by Panjer and Mereu [4]. The reader is urged to refer to this paper for a complete understanding of the methods used; however, for the sake of completeness a brief description of their methods will be given below.

The type of group policy considered by Panjer and Mereu in [4] was retrospectively experience rated but not prospectively experience rated. The gross premium, expected incurred claims, aggregate claims distribution, and dividend loading were assumed to be the same in each policy year. Cancellation was assumed to depend only on the deficit level, and a deficit level R was assumed at which cancellation was certain. Contingency funds were assumed to be built up to a maximum level M before dividends were paid. By considering a contingency fund as a negative deficit, by dividing up the possible range of deficits $[-M, R]$ into n subintervals, and by representing each subinterval by one particular deficit level included in that subinterval, Panjer and Mereu applied linear algebraic techniques to calculate a level risk charge.

The possible deficit levels after the approximation were denoted r_i for $1 \leq i \leq n$, where $r_1 = -M$ and $r_n = R$. Formulas for a transition matrix $T = (t_{ij})$ were given, where t_{ij} represents the probability that a policy, active at the beginning of the policy year with deficit level r_i, will have deficit level r_j at the end of the policy year before the decision whether or not to cancel has been made. A diagonal persistency matrix $P = (p_{ij})$ was defined, where p_{ii} represents the probability that a policy with deficit level r_i will not cancel. A diagonal termination matrix $Q = (q_{ij})$ was defined by $Q = I - P$, where I is the identity matrix.

With these tools, Panjer and Mereu were able to calculate the present value, at issue, of expected future termination costs and the present value, at issue, of expected future risk charges, based on a level risk charge made to an active policy in each policy year. By equating these two expressions, they solved for the level risk charge. The resulting formula, from [4], is the following:

$$DRC = \frac{\mathbf{a}_0[(1+i)I - TP]^{-1}TQ\mathbf{r}^*}{\mathbf{a}_0[(1+i)I - TP]^{-1}\mathbf{1}} \, .$$

In this formula, DRC represents the risk charge, and $\boldsymbol{a}_0$ represents the row vector (0, 0, . . . , 0, 1, 0, . . . , 0), where the 1 corresponds to deficit level zero. All other symbols in the equation are given in [4].

In Appendix A we show how the techniques developed by Panjer and Mereu may be used to calculate the termination cost reserves ${}_tV^{TC}(x, y)$. Using the above assumption and techniques, the termination cost reserve depends on the deficit level or contingency fund balance but is independent of the duration since issue.

The main result of Appendix A is that, under the stated assumption, the termination cost reserve is given by

$$\begin{aligned} {}_tV^{TC}(x, y) &= \boldsymbol{z}[(1 + i)I - TP]^{-1}TQ\boldsymbol{r}^* \\ &\quad - \boldsymbol{z}[(1 + i)I - TP]^{-1}\mathbf{1}DRC\,. \end{aligned} \tag{32}$$

In equation (32), the row vector $\boldsymbol{z}$ is equal to (0, 0, . . . , 0, 1, 0, . . . , 0), where the 1 corresponds to deficit level x if $x \neq 0$ or $-y$ if $y \neq 0$. The other symbols in equation (32) are given in [4]. The reader familiar with these techniques will realize that equation (32) will produce equal values of the termination cost reserve for all deficit levels contained in the subinterval represented by r_i $(1 \leq i \leq n)$. In practice, it would be desirable to interpolate between the values of the termination cost reserves produced by equation (32) for deficit levels between r_i and r_{i+1} $(1 \leq i \leq n - 1)$. To avoid obfuscating our examples, we have chosen to use equation (32) directly in our calculations without any interpolation.

Once the termination cost reserve ${}_tV^{TC}(x, y)$ has been calculated for a particular deficit level or contingency fund balance, equation (22) is used to calculate the policy reserve for future claims and dividends. Tables 2–5 show the values of the policy reserve for future claims and dividends for selected deficit levels and contingency fund balances, calculated using the above techniques, for the group of 1,050 lives considered by Bolnick [1]. The reserves are expressed as a percentage of the corresponding deficit level or contingency fund balance, with the algebraic sign indicating a liability (+) or an asset (−). The average value of the reserve for an active policy at anniversaries 1, 2, and 4 are also shown, based on an expected distribution of deficit levels and contingency fund balances for an active policy at these anniversaries. The formula used to calculate these average values is given in Appendix A. In Tables 1–5, these average reserves are expressed as a percentage of expected incurred claims.

To use Panjer and Mereu's techniques, it is necessary to specify a number of parameters. In each example, R, the deficit level at which

TABLE 1

RISK CHARGE AND POLICY RESERVES FOR FUTURE CLAIMS AND DIVIDENDS

Parameters: k Varying; $DL = 5$; $M = 25$; $S = 200$; $R = 300$; Termination Rates of 5 Percent up to Deficit Level R)

A. RISK CHARGE

(Percentage of Expected Incurred Claims)

$k=25$	$k=50$	$k=100$
.2	1.7	6.2

B. POLICY RESERVES* FOR FUTURE CLAIMS AND DIVIDENDS

Deficit Level	$k=25$	$k=50$	$k=100$
−25	+94.3	+86.3	+78.7
−10	+89.3	+78.2	+71.5
10	−90.9	−86.2	−82.9
30	−79.4	−74.8	−73.5
50	−68.4	−66.8	−67.8
70	−58.7	−59.0	−62.5
90	−50.2	−52.3	−57.6
110	−43.1	−46.1	−52.2
130	−36.4	−40.5	−47.7
150	−30.5	−35.3	−43.5
170	−25.2	−30.6	−39.4
190	−21.2	−26.3	−35.9
210	−17.8	−22.5	−30.5
230	−14.7	−19.1	−27.7
250	−11.8	−15.5	−24.8
270	− 8.9	−13.2	−22.4
290	− 7.0	−11.5	−21.8

* Absolute value of reserves expressed as a percentage of absolute value of corresponding deficit level. Algebraic sign on reserve represents liability (+) or asset (−).

C. AVERAGE POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

(Percentage of Expected Incurred Claims)

Anniversary	$k=25$	$k=50$	$k=100$
1	+3.3	+1.8	− 3.3
2	+6.7	+1.5	− 7.9
4	+9.7	− .1	−14.6

TABLE 2

RISK CHARGE AND POLICY RESERVES FOR FUTURE CLAIMS AND DIVIDENDS

Parameters: $k = 100$; DL Varying; $M = 25$; $S = 200$; $R = 300$; Termination Rates of 5 Percent up to Deficit Level R)

A. RISK CHARGE

(Percentage of Expected Incurred Claims)

$DL=0$	$DL=1$	$DL=2$	$DL=5$	$DL=10$
8.5	8.0	7.5	6.2	4.4

B. POLICY RESERVES* FOR FUTURE CLAIMS AND DIVIDENDS

Deficit Level	$DL=0$	$DL=1$	$DL=2$	$DL=5$	$DL=10$
−25	+74.0	+75.0	+76.4	+78.7	+84.7
−10	+63.6	+65.2	+66.9	+71.5	+78.7
10	−78.7	−79.5	−80.4	−82.9	−86.7
30	−66.1	−67.4	−69.7	−73.5	−79.4
50	−60.1	−61.5	−62.8	−67.8	−74.6
70	−54.6	−56.1	−57.6	−62.5	−69.5
90	−49.4	−51.1	−52.5	−57.6	−65.0
110	−44.7	−46.3	−47.6	−52.2	−60.2
130	−40.1	−41.9	−43.1	−47.7	−55.6
150	−36.2	−37.7	−39.1	−43.5	−51.4
170	−32.9	−34.2	−35.2	−39.4	−47.0
190	−28.0	−29.4	−30.3	−35.9	−42.6
210	−25.5	−26.5	−27.2	−30.5	−36.7
230	−23.0	−23.8	−24.5	−27.7	−32.6
250	−21.0	−22.3	−22.8	−24.8	−29.6
270	−20.2	−20.6	−21.0	−22.4	−26.3
290	−19.3	−19.6	−19.8	−21.8	−23.7

* Absolute value of reserves expressed as a percentage of absolute value of corresponding deficit level. Algebraic sign on reserve represents liability (+) or asset (−).

C. AVERAGE POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

(Percentage of Expected Incurred Claims)

Anniversary	$DL=0$	$DL=1$	$DL=2$	$DL=5$	$DL=10$
1	− 3.7	− 3.8	− 3.6	− 3.3	− 2.0
2	− 8.6	− 8.6	− 8.4	− 7.9	− 6.1
4	−15.6	−15.5	−15.3	−14.6	−12.2

TABLE 3

RISK CHARGE AND POLICY RESERVES FOR FUTURE CLAIMS AND DIVIDENDS

Parameters: k = 100; DL = 5; M = Varying; S = 200; R = 300; Termination Rates of 5 Percent up to Deficit Level R)

A. RISK CHARGE

(Percentage of Expected Incurred Claims)

$M=0$	$M=10$	$M=25$	$M=50$
7.4	6.9	6.2	5.4

B. POLICY RESERVES* FOR FUTURE CLAIMS AND DIVIDENDS

Deficit Level	$M=0$	$M=10$	$M=25$	$M=50$
−50	....	....	....	+80.3
−25	....	....	+78.7	+72.7
−10	....	+75.9	+71.5	+80.4
10	−73.9	−86.2	−82.9	−79.5
30	−75.1	−76.7	−73.5	−67.4
50	−71.4	−71.6	−67.8	−60.8
70	−67.3	−62.2	−62.5	−54.9
90	−58.7	−57.7	−57.6	−49.2
110	−54.9	−53.7	−52.2	−48.7
130	−51.2	−49.2	−47.7	−43.5
150	−47.5	−45.3	−43.5	−39.2
170	−40.2	−41.6	−39.4	−34.9
190	−35.5	−32.9	−35.9	−28.9
210	−32.0	−30.1	−30.5	−28.7
230	−26.8	−27.0	−27.7	−25.1
250	−24.1	−24.9	−24.8	−22.7
270	−23.0	−22.8	−22.4	−20.4
290	−22.5	−22.3	−21.8	−19.0

* Absolute value of reserves expressed as a percentage of absolute value of corresponding deficit level. Algebraic sign on reserve represents liability (+) or asset (−).

C. AVERAGE POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

(Percentage of Expected Incurred Claims)

Anniversary	$M=0$	$M=10$	$M=25$	$M=50$
1	−12.9	− 9.6	− 3.3	+4.3
2	−20.1	−15.7	− 7.9	+2.7
4	−28.3	−23.3	−14.6	−1.8

TABLE 4

Risk Charge and Policy Reserves for Future Claims and Dividends

Parameters: $k = 100$; $DL = 5$; $M = 25$; S Varying; $R = 300$; Termination Rates of 5 Percent up to Deficit Level R)

A. Risk Charge

(Percentage of Expected Incurred Claims)

$S = 100$	$S = 125$	$S = 200$	$S = \infty$
1.8	3.1	6.2	7.2

B. Policy Reserves* for Future Claims and Dividends

Deficit Level	$S = 100$	$S = 125$	$S = 200$	$S = \infty$
−25	+82.7	+80.7	+78.7	+80.2
−10	+74.3	+72.4	+71.5	+72.3
10	−83.8	−79.5	−82.9	−84.6
30	−73.3	−71.3	−73.5	−75.0
50	−65.7	−62.5	−67.8	−69.5
70	−57.6	−57.4	−62.5	−64.4
90	−49.0	−52.0	−57.6	−59.6
110	−43.9	−47.3	−52.2	−54.9
130	−38.9	−42.1	−47.7	−50.5
150	−34.5	−37.8	−43.5	−46.3
170	−30.0	−32.0	−39.4	−42.2
190	−25.8	−28.5	−35.9	−38.2
210	−20.3	−24.0	−30.5	−34.8
230	−18.2	−21.3	−27.7	−31.3
250	−14.6	−19.6	−24.8	−28.0
270	− 9.2	−14.1	−22.4	−26.0
290	−10.6	−14.0	−21.8	−23.7

* Absolute value of reserves expressed as a percentage of absolute value of corresponding deficit level. Algebraic sign on reserve represents liability (+) or asset (−).

C. Average Policy Reserve for Future Claims and Dividends

(Percentage of Expected Incurred Claims)

Anniversary	$S = 100$	$S = 125$	$S = 200$	$S = \infty$
1	−1.5	−1.9	− 3.3	− 3.1
2	−2.7	−3.7	− 7.9	− 8.0
4	−5.1	−7.9	−14.6	−15.1

TABLE 5

RISK CHARGE AND POLICY RESERVES FOR FUTURE CLAIMS AND DIVIDENDS

Parameters: $k = 100$; $DL = 5$; $M = 25$; $S = 200$; $R = 300$; Termination Rates of 5 Percent up to Deficit Level R Varying)

A. RISK CHARGE

(Percentage of Expected Incurred Claims)

TERMINATION RATES UP TO DEFICIT LEVEL R		
1%	2%	5%
5.1	5.4	6.2

B. POLICY RESERVES* FOR FUTURE CLAIMS AND DIVIDENDS

DEFICIT LEVEL	TERMINATION RATES UP TO DEFICIT LEVEL R		
	1%	2%	5%
−25	+81.4	+80.7	+78.7
−10	+75.0	+74.0	+71.5
10	−85.1	−84.5	−82.9
30	−76.7	−75.8	−73.5
50	−71.4	−70.4	−67.8
70	−66.5	−65.4	−62.5
90	−61.7	−60.6	−57.6
110	−56.3	−55.2	−52.2
130	−51.7	−50.6	−47.7
150	−47.1	−46.1	−43.5
170	−42.7	−41.8	−39.4
190	−38.8	−38.0	−35.9
210	−32.8	−32.2	−30.5
230	−29.5	−29.1	−27.7
250	−26.2	−25.8	−24.8
270	−23.3	−23.1	−22.4
290	−22.3	−22.2	−21.8

* Absolute value of reserves expressed as a percentage of absolute value of corresponding deficit level. Algebraic sign on reserve represents liability (+) or asset (−).

C. AVERAGE POLICY RESERVE FOR FUTURE CLAIMS AND DIVIDENDS

(Percentage of Expected Incurred Claims)

Anniversary	TERMINATION RATES UP TO DEFICIT LEVEL R		
	1%	2%	5%
1	− 3.7	− 3.6	− 3.3
2	− 8.7	− 8.5	− 7.9
4	−15.9	−15.5	−14.6

termination is guaranteed, is equal to 300 percent of expected incurred claims, i, the interest rate, is 6 percent, and n, the number of subdivisions, is 50. The other parameters are the credibility factor k, the dividend loading DL, the maximum contingency fund M, the stop-loss level S, and the termination rates. Tables 1–5 have been designed to show the effect of varying each of these parameters in turn. In Tables 1–5, k is given as a percentage, DL, M, S, and R are shown as percentages of expected incurred claims, and the termination rates are shown as percentages.

In Tables 1–3 of [4], risk charges were illustrated for sets of parameters, including all the ones exhibited in Tables 1–5 of this paper. However, the value of n used by Panjer and Mereu was 71, whereas we used $n = 50$. Consequently, for the same set of parameters, the risk charge shown in [4] will not, in general, be equal to the risk charge underlying our calculation of the policy reserve for future claims and dividends. For this reason, we have included in Tables 1–5 the risk charge calculated using $n = 50$ for each set of parameters exhibited. The risk charges shown are expressed as a percentage of expected incurred claims.

We make the following observations regarding the policy reserves for future claims and dividends illustrated in Tables 1–5:

1. The reserve is always positive, that is, a liability, for deficit levels < 0, that is, where there is a contingency fund. The reserve is always negative, that is, an asset, for deficit levels > 0. The reserve for deficit level zero, although not shown, is equal to zero.
2. When there is a contingency fund, the reserve expressed as a percentage of the contingency fund generally decreases as the size of the contingency fund decreases. When there is a deficit > 0, the absolute value of the reserve expressed as a percentage of the deficit level generally decreases as the size of the deficit level increases.
3. When the credibility factor k varies, the reserve, expressed as a percentage of the contingency fund, decreases as k increases, for a fixed contingency fund balance. When there is a deficit > 0, the absolute value of the reserve, expressed as a percentage of the deficit, follows no monotonic pattern for a fixed deficit.
4. When the dividend loading DL varies, the reserve, expressed as a percentage of the contingency fund, increases as DL increases, for a fixed contingency fund balance. When there is a deficit > 0, the absolute value of the reserve, expressed as a percentage of the deficit, increases as DL increases, for a fixed deficit.
5. When varying the maximum contingency fund M or the stop-loss level S, the reserve, expressed as a percentage of the deficit level or contingency fund balance, follows no monotonic pattern for a fixed contingency fund balance or deficit level.
6. When varying the termination rates, the reserve, expressed as a percentage of

the contingency fund, decreases as the termination rates increase, for a fixed contingency fund balance. When there is a deficit level > 0, the absolute value of the reserve, expressed as a percentage of the deficit, decreases as the termination rates increase, for a fixed deficit level.

7. The average value of the reserve can be either a liability or an asset, depending on the parameters and the anniversary. In general, the average reserve decreases, that is, becomes a smaller liability or a larger asset, as the time since issue increases. The only exception to this observation is in Table 1 for k = 25 percent.
8. For a particular deficit level or contingency fund balance, the reserves, whether assets or liabilities, are of a size that could have a material effect on a financial statement.

The reader should remember that the results presented in Tables 1–5 apply only to the particular group described in [1], and will not in general apply to any other group with a different aggregate claims distribution. Observations 1–8 above apply only to that group, also. Tables 1–5 are not meant to apply to all groups.

For a discussion of an apparent inconsistency between the assumptions of Panjer and Mereu and our assumptions, see Appendix B.

CONCLUSION

The theory of policy reserves for future claims and dividends developed in this paper has the potential for widespread use in the financial statements of group insurers. The theory as developed applies to all conventionally insured retrospectively experience-rated group insurance policies, whether prospectively experience-rated or not. The theory can be adapted to apply to some of the newer forms of funding retrospectively experience-rated group insurance policies that have appeared in the marketplace, such as the minimum premium funding method.

It should be noted, however, that the policy reserve for future claims and dividends has no place in an insurer's statutory statement. When a policy has an outstanding deficit, the reserve is generally negative and therefore not suitable as a statutory reserve. When a policy has a contingency fund balance, the reserve is generally positive but less than the full value of the contingency fund (see eq. [31]). Since the full amount of the contingency fund is already held in statutory statements as a liability, no further statutory adjustment need be made.

Although not useful in statutory statements, the policy reserve for future claims and dividends should be considered for inclusion in GAAP statements and in financial statements prepared for internal use by management. If included in a company's GAAP statement, provisions for adverse deviation would have to be made part of the calculation. This would in general be accomplished by using standard GAAP techniques with respect to the interest and persistency assumptions, and by using a more conservative aggregate claims distribution when calculating risk charges and termination costs. Quantifying the appropriate provisions for adverse deviation for these reserves would be a fruitful area for further research.

Proper use of the policy reserve for future claims and dividends in the GAAP or internal financial statements of group insurers, as with the use of any policy-year benefit reserve, will have the effect of a smoother distribution of earnings to various accounting periods. When a policy has an outstanding deficit, the reserve will generally be negative and can be thought of loosely as a deferred deficit recovery asset. In this situation, the reserve is similar to a negative expense reserve representing a deferred acquisition cost asset.

The idea of holding an asset in the GAAP statement for deficits expected to be recovered in the future is not a new one. The theory developed in this paper puts this idea into a traditional actuarial framework and provides the formula for the liability that must be held along with the asset (see eq. [27]).

The policy reserve for future claims and dividends will not fit directly into a calendar-year financial statement. The necessary adjustments to go from a policy-year reserve to a calendar-year reserve, as well as the integration of the policy reserve for future claims and dividends with the statutory dividend or rating refund liability (representing the dividend earned from the policy anniversary to December 31), are areas where future research would be useful.

When calculating the policy reserve for future claims and dividends, the group actuary should strive to maintain consistency between reserve assumptions and pricing methods. The development in this paper has assumed that a proper risk charge has been calculated for the policy. The risk charge would depend on whether the policy is prospectively experience rated, and, if so, the degree of credibility given to the past experience. The expected future termination costs would also depend on the prospective experience-rating mechanism employed. Therefore, for a proper calculation of the policy reserve for future claims and dividends, the pricing methods would have to be reflected in the reserve calculation.

This paper has dealt mainly with the theory underlying the termination cost reserves and policy reserves for future claims and dividends. The practical implementation of the theory has been discussed only in connection with the numerical example given. We realize that at many companies, risk charges are calculated in a rather approximate fashion, reflecting the recent deficit activity on the company's particular set of cases. Under this type of pricing mechanism, perhaps the best starting point for the approximation of the reserves would be equations (27) and (31). We hope that the recent research done by Panjer and Mereu [4] and Panjer [3] will help those actuaries trying to quantify risk charges and termination costs to do so in a more accurate way. As our techniques for calculating risk charges and termination costs become sharper, the practical implementation of the theory of termination cost reserves and policy reserves for future claims and dividends can proceed in a more scientific fashion.

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APPENDIX A

The development in this Appendix is based on the notation of Panjer and Mereu in [4], which will not be repeated here.

We first determine a formula for the termination cost reserve ${}_tV^{TC}(x, y)$ for an active policy with deficit level x or contingency fund balance y at time t, using the techniques of Panjer and Mereu.

Let $\boldsymbol{C}_s = (C_{s1}, C_{s2}, \ldots, C_{sn})$ denote the $1 \times n$ vector where C_{sj} represents the probability that a group, active at time t with deficit level x or contingency fund balance y, persists to time s and is at that time at deficit level r_j. Let $\boldsymbol{Z}$ denote the $1 \times n$ vector consisting of all zeros except a 1 in the position corresponding to deficit level x if $x \neq 0$ or deficit level $-y$ if $y \neq 0$.

Clearly, then,

$$\boldsymbol{C}_t = \boldsymbol{Z} ;$$

$$\boldsymbol{C}_{t+1} = \boldsymbol{C}_t\boldsymbol{TP} = \boldsymbol{ZTP} ;$$

$$\boldsymbol{C}_{t+2} = \boldsymbol{C}_{t+1}\boldsymbol{TP} = \boldsymbol{Z}(\boldsymbol{TP})^2 ;$$

$$\vdots$$

$$\boldsymbol{C}_{t+k} = \boldsymbol{Z}(\boldsymbol{TP})^k .$$

Let $\boldsymbol{d}_s = (d_{s1}, d_{s2}, \ldots, d_{sn})$ denote the $1 \times n$ vector where d_s represents the probability that a group, active at time t with deficit level x or contingency fund balance y, terminates at time s at deficit level r_j.

Clearly, then,

$$\boldsymbol{d}_{t+1} = \boldsymbol{C}_t TQ = \boldsymbol{Z}(TQ);$$

$$\boldsymbol{d}_{t+2} = \boldsymbol{C}_{t+1} TQ = \boldsymbol{Z}(TP)(TQ)\ ;$$

$$\boldsymbol{d}_{t+3} = \boldsymbol{C}_{t+2} TQ = \boldsymbol{Z}(TP)^2(TQ)\ ;$$

$$\vdots$$

$$\boldsymbol{d}_{t+k} = \boldsymbol{Z}(TP)^{k-1}(TQ^*)\ .$$

The termination cost reserve ${}_tV^{TC}(x, y)$ is defined as the present value, at time t, of expected future termination costs less the present value, at time t, of expected future risk charges, given that the policy is active at time t with deficit level x or contingency fund balance y. The former term is equal to

$$\begin{aligned} v\boldsymbol{d}_{t+1}\boldsymbol{r}^* + v^2\boldsymbol{d}_{t+2}\boldsymbol{r}^* + v^3\boldsymbol{d}_{t+3}\boldsymbol{r}^* + \ldots &= v\boldsymbol{Z}[I + vTP + v^2(TP)^2 + \ldots]TQ\boldsymbol{r}^* \\ &= v\boldsymbol{Z}(I - vTP)^{-1}TQ\boldsymbol{r}^* \\ &= \boldsymbol{Z}[(1+i)I - TP]^{-1}TQ\boldsymbol{r}^*\ , \end{aligned}$$

and the latter term is equal to

$$\begin{aligned} (v\boldsymbol{C}_t\mathbf{1} + v^2\boldsymbol{C}_{t+1}\mathbf{1} + v^3\boldsymbol{C}_{t+2}\mathbf{1} + \ldots)DRC &= v\boldsymbol{Z}[I + vTP + v^2(TP)^2 + \ldots] \\ &\quad \times \mathbf{1}DRC \\ &= v\boldsymbol{Z}(I - vTP)^{-1}\mathbf{1}DRC \\ &= \boldsymbol{Z}[(1+i)I - TP]^{-1}\mathbf{1}DRC\ . \end{aligned}$$

Therefore,

$$\begin{aligned} {}_tV^{TC}(x, y) = &\ \boldsymbol{Z}[(1 + i)I - TP]^{-1}TQ\boldsymbol{r}^* \\ &- \boldsymbol{Z}[(1 + i)I - TP]^{-1}\mathbf{1}DRC\ . \end{aligned}$$

This is the desired relationship. Of course, the outstanding deficit level x must be less than R for the equation to be meaningful.

Since the termination cost reserve is independent of the duration since issue according to the above formula, let $\boldsymbol{TCR} = (TCR_1, \ldots, TCR_n)$ denote the $1 \times n$ vector where TCR_j represents the termination cost reserve corresponding to deficit level r_j given by the above formula. From equation (22) it is clear that the policy reserve for future claims and dividends is also independent of duration since issue. It we let $\boldsymbol{V} = (V_1, V_2, \ldots, V_n)$ denote the $1 \times n$ vector where V_j represents the policy reserve for future claims and dividends corresponding to deficit level r_j, then equation (22) implies that

$$\boldsymbol{V} = -\boldsymbol{r} + \boldsymbol{TCR} \, .$$

Since it is assumed that there are no active policies with deficit levels R or greater, the nth component of $\boldsymbol{V}$ should be ignored.

The probability that a group persists to the end of t years and is at that time at deficit level r_j is defined in [4] as a_{tj}, and it is shown in [4] that

$$(a_{t1}, a_{t2}, \ldots, a_{tn}) = \boldsymbol{a}_t = \boldsymbol{a}_0(TP)^t \, .$$

Therefore, the average policy reserve for future claims and dividends among those policies active at time t is given by

$$\frac{\boldsymbol{a}_0(TP)^t\boldsymbol{V}}{\boldsymbol{a}_0(TP)^t\boldsymbol{1}} \, .$$

This is the formula used to calculate the average policy reserve for future claims and dividends at times 1, 2, and 4 shown in Tables 1–5.

APPENDIX B

There is an apparent inconsistency between the assumptions of Panjer and Mereu given in [4] and the assumptions made in this paper. We assumed a time w by which cancellation was certain, whereas Panjer and Mereu's model provided that a policy could persist an unlimited number of durations.

The only place in our development where we used the existence of w was in the derivations of observations A2, A3, B2, and B3, where it was essential to know that $\mathscr{E}(CF_n)$ and $\mathscr{E}(R_n)$ approach zero as n gets large. If we had substituted these conditions for the condition that such a w exists, we could have derived the same results.

Under Panjer and Mereu's assumption, the above conditions on $\mathscr{E}(CF_n)$

and $\mathscr{E}(R_n)$ are satisfied as long as each persistency factor p_{jj}, for $1 \leq j \leq n$, is strictly less than 1. For this reason, Panjer and Mereu's methods are not inconsistent with those of this paper.

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