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UNIFICATION OF PRICING, VALUATION, AND MANAGEMENT-BASIS FINANCIALS FOR PARTICIPATING AND NONGUARANTEED ELEMENT CONTRACTS

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ABSTRACT

The impetus for this paper is a concern that a unification of pricing, valuation, and management-basis financials may be essential to ensure the viability of life insurance companies in the variable economic environment of the foreseeable future, which will have serious effects on interest-sensitive products. The paper offers hypotheses on which any unification should be based. The proposed unification is structured on the equity-capital management approach and on a variation of the level-return-on-equity accounting method. Pricing is based on an ideal generalized dividend/credits formula, applicable to both participating and nonguaranteed element contracts, with actual payable dividends/credits related to the ideal generalized dividends/credits by an equivalence equation. The valuation actuary process for establishing reserves and risk (contingency) surplus needed for the C-1, C-2, and C-3 risks is fundamental. The management-basis financials are driven by pricing and not by a plan emphasizing emergence of net income. Reserves are a given in the management-basis financials and are determined independently to reflect reasonable deviations from expected values arising from the C-1, C-2, and C-3 risks. Thus, the dividends/credits are the sole release-from-risk mechanism. Because general reasoning is not entirely trustworthy, detailed mathematical analysis is included for thorough understanding of the complex dynamics and for assurance of throughput in practice.

1. OBJECTIVES

A unification of pricing, statutory valuation, and management-basis financials for control and planning may be essential for the ongoing viability of life insurance companies offering interest-sensitive products in the variable economic environment of the foreseeable future. The unification is based on the following hypotheses:

1. The valuation actuary offers analyses and recommendations on (a) adequacy of assets equal to statutory reserves for reasonable deviations from expected experience on C-1, C-2, and C-3 risks and (b) adequacy of total

assets for plausible deviations from expected experience on such risks to provide for obligations and expenses on in-force business and for financial plans for future business. But these are insufficient of themselves to ensure the future viability of the company. Such analyses must also be an ingredient of both advance pricing and ongoing management-basis financials.

2. Management acceptance of the recommendations of the pricing-product actuary is essential to viability. The recommendations not only should encompass the traditional factors, but also should provide explicitly for appropriate profit charges, including charges for risk (contingency) surplus utilized against the C-1, C-2, and C-3 risks for plausible deviations from expected experience, quantified by the processes used by the valuation actuary. See [5] and [8]. Thus, pricing should specifically provide for growth in surplus compatible with long-range planning.

3. Management-basis financials should be designed for management control of optimum use of capital and for measurement of performance. Such financials should be price-driven. They should not be driven by a plan for the emergence of profit accomplished by adjustments of reserves because of the danger of premature emergence of profit with high probability of later losses; such design is potentially misleading to management. Thus, reserves in management-basis financials should be determined independently and should include a fixed schedule for amortization of acquisition expenses.

4. The release-from-risk mechanism for participating contracts is the dividend, and that for nonguaranteed element contracts is the policyholder credit. The reserve is a given element in the structure and should not be chosen as another release-from-risk mechanism.

5. The unification should be applicable to participating and nonguaranteed element contracts.

6. Although the intimate relationship of pricing, valuation, and management-basis financials is generally recognized, the unification needs to be precise and complete, based on detailed formulation, and firmly controlled.

This paper describes in detail a structure of unification that conforms to these hypotheses. The characteristics are typical of any structure satisfying the hypotheses, and the details are not claimed to be unique.

2. OVERVIEW OF THE APPROACH

Before we examine the details of the unification, it is appropriate to review the relationship of pricing and management-basis financials.

Management-basis financials designed to provide, on the assumption of expected experience, for net income equal to a level percentage return, r, on equity (capital advanced from corporate entity surplus) are described in a report from the Financial Reporting Section of the Society of Actuaries [7] for participating life insurance in mutual companies. Rate r is referred to in the literature variously as internal rate of return (IRR), target return on equity, and risk interest rate. Some salient characteristics of these management-basis financials are as follows, where, for simplicity, the benefit reserve is taken as the statutory reserve held, V_n^S:

- They maintain invested assets equal to statutory reserves, V_n^S, plus contingency surplus needed, CSN_n^S, at duration $n \geq 0$.
- Maintenance of $V_n^S + CSN_n^S$ is a capital transaction between the contract account and corporate entity surplus. The initial capital advanced from the corporate entity surplus is the deferred acquisition cost, DAC_0, plus CSN_0^S.
- Pricing (actual dividends/credits, D_n') is designed to compensate corporate entity surplus for the above capital transactions at the IRR over the lifetime of the contract class, assuming expected experience. (Or, given the pricing, the IRR is determined.) This is an elaboration of the Anderson method.
- Assuming expected experience, the relationship of pricing, reserves, and net income taken to the capital transactions, based on the IRR, allows reasonable arbitrary choices of reserves and net income taken.
- Thus, the net liability, V_n^M, equal to V_n^S minus DAC_n, can be determined so that net income for year n in the contract account equals $r(DAC_{n-1} + CSN_{n-1}^S)$ based on expected experience.
- Because V_n^S is fixed, this determination of V_n^M constitutes a redetermination of DAC_n and in effect adjusts the schedule of amortization of DAC_0.
- When D_{n+t}' are changed at duration n ($t \geq 0$) to reflect a change in future expected experience, the IRR is redetermined to the extent indicated by the new D_{n+t}' dividend/credits scale.

As shown in [7], this process is straightforward in practice. However, the process is trustworthy only to the extent that the expected experience on which the dividends/credits are based is trustworthy, so that net incomes can be rearranged by duration based on the actuarial equivalence at interest rate r.

It is hypothesized that while the process probably is satisfactory for traditional participating contracts in mutual companies with dividends based on the contribution principle, it probably is not satisfactory for nonguaranteed element, interest-sensitive contracts, for which credits are affected significantly by factors outside the financial dynamics of the contract class. Chapter III of the 1987 *Valuation Actuary Handbook* [9] illustrates the difficulty of establishing expected experience on interest-sensitive contracts.

Nor is the process likely to be satisfactory for traditional participating contracts for which dividends do not track experience closely. Thus, taking income early, based on the expectation of later recovery, can be illusory, and management can be misled by such an accounting system on interest-sensitive contracts.

Therefore, this paper looks to a generalized dividend/credits design, D_n, that automatically would produce net income equal to a level percentage return, r, on equity. The paper then examines the effects and implications of actual dividends/credits, D_n', which differ from D_n.

A generalized dividend/credits formula reflects the actual financial dynamics and experience of the contract class to the extent practically feasible and includes explicit charges for amortization of acquisition expenses and for profit. The particular formula in this paper provides for the following: (1) assets needed equal to statutory reserves plus contingency surplus needed, (2) contract net liability equal to pricing reserve less unamortized acquisition expenses based on a schedule established at issue and retained thereafter, and (3) a level percentage return on equity (capital advanced from corporate entity surplus equaling the contingency surplus needed plus unamortized acquisition expenses).

The differences between actual dividends/credits and generalized dividends/credits are accumulated with benefit of interest at rate r and survivorship. This accumulation is the key to control of pricing. If the accumulation is zero over the lifetime of the contract class, the pricing and profit objectives are realized, particularly if it oscillates around zero by periods of years.

The generalized dividends/credits are closely related to actual experience and do not refer to any estimated expected experience. They contain independently determined reserve factors and a definite acquisition expense amortization schedule. The management-basis financials are mirror images of the generalized dividends/credits and produce net income equal to the level percentage return on equity if actual dividends/credits equal the generalized dividends/credits. Actual net income is derived directly. The detailed algebraic analysis is provided in Sections 4, 5, and 6.

The approach to control through generalized dividends/credits moves pricing up front as the driving force in the management-basis financials and does not share the release-from-risk mechanism with reserve adjustments designed to force a pattern of net income assuming expected experience.

The generalized dividend formula is used in some form by at least one mutual company for traditional participating contracts, but, to my knowledge, has not been used in connection with interest-sensitive contracts.

Nevertheless, any comprehensive pricing model for interest-sensitive contracts that incorporates scenarios of future happenings and reactions inherently involves the concept. This paper offers the concept as a useful insight into disciplined pricing and management-basis financials.

3. THE STRUCTURE OF THE UNIFICATION

3.1 *Entity Capital Approach to Financial Capital Maintenance*

The financial capital maintenance structure of the insurance company, common to pricing, valuation, and management-basis financials, is on the entity capital model. This structure is described in [7] and [8]. The life insurance company, mutual or stock, is assumed to have entity capital equal to statutory surplus, including mandatory securities valuation reserve (MSVR) and similar contingency funds, plus capital invested in new business, in the growth of existing business, in new lines, in new products, and in new marketing and administrative systems, where such invested capital is expected to be repaid in the future by charges made in pricing. In addition, the pricing should include profit charges for the advance of such invested capital, for risk undertaken, and for growth in entity capital.

Statutory surplus (including MSVR and similar contingency funds) can be divided into two parts: (a) risk (contingency) surplus needed that is utilized by in-force business, and (b) the balance. In this paper, this balance is called "corporate entity surplus." Corporate entity surplus in statutory financials thus equals the above entity capital less the contingency surplus needed in contract accounts and less the invested capital items. Corporate entity surplus in management-basis financials, however, could include some of the invested capital items.

3.2 *The Level-Return-on-Equity Accounting Method*

This accounting method is described in varying detail in [7] and [8], along with other methods. It fits well with the entity capital approach to financial capital maintenance, because it provides for a level percentage of return r on capital advanced from corporate entity surplus, assuming expected experience. However, as explained in Section 2, the application of the method in this paper is different from that in [7] and [8]. Here, actual net income after dividends/credits and after federal income tax (FIT) is r times capital advanced from corporate entity surplus when actual dividends/credits equal generalized dividends/credits. If the actual dividends/credits and generalized dividends/credits are not equal but they conform to the equivalence equation, the actual net income will be a varying rate of return on capital advanced equivalent to r over periods of time. Loss recognition is indicated when the present value of future actual net incomes is likely to be negative. Thus, the accounting method of this paper relates directly to dividends/credits; it does not relate to expected experience on an ongoing basis.

3.3 *Pricing by the Generalized Dividend/Credits Formula*

The ideal basis of pricing of participating contracts in mutual companies is the generalized dividend formula, described in detail in my previous work [2], [3], and [4]. It is a precise and comprehensive expression of the contribution principle for bundled participating contracts in mutual companies, as set forth in the Academy's "Recommendations and Interpretations for Dividends and Other Non-Guaranteed Elements" [1]. It is also an ideal, precise, and comprehensive expression for policyholder credits on unbundled nonguaranteed contracts, consistent with the Interim Actuarial Standards Board's "Recommendations Concerning the Redetermination (or Determination) of Non-Guaranteed Charges and/or Benefits for Life Insurance and Annuity Contracts" [6].

In Section 4 of this paper, the generalized dividend formula developed earlier is revised to incorporate the level-return-on-equity design of profit charges and a risk surplus in keeping with the entity capital approach; it also is extended to the ideal credits for nonguaranteed element contracts.

Pricing on the basis of the ideal generalized dividends/credits formula directly reflects the dynamics of the underlying management-basis financials. Actual dividends/credits should relate to the ideal generalized dividends/credits by an "equivalence equation." Where actual dividends/credits are larger than those conforming to the equivalence equation, profits according to the financial plan are reduced and losses may be indicated, calling for loss recognition.

3.4 *Valuation (Reserves and Risk Surplus)*

The extensive research findings on theory and practice in the valuation actuary effort are set forth in the 1987 *Valuation Actuary Handbook* [9] and in papers and discussions in the *Record* and the *Transactions*.

Assets equal to statutory reserves on in-force business should be adequate to provide for obligations and expenses for reasonable deviations from expected experience on C-1, C-2, and C-3 risks. Assets equal to statutory reserves plus risk surplus on in-force business should be adequate to provide for obligations and expenses under plausible deviations from expected experience on such risks. Invested assets equal to reserves and risk surplus in management-basis financials should be equal to such assets in statutory financials. The risk surplus used in pricing should be based on the same valuation actuary processes. A risk surplus set at some arbitrary percentage of reserves in pricing and in management-basis financials is unacceptable.

Reserves used in pricing and in management-basis financials are not necessarily statutory reserves, although my earlier work [2], [3], [4] and Section 5.4 in this paper show that they should be statutory reserves in traditional participating contracts in mutual companies where dividend formulas relate to statutory reserves. Statutory reserves on nonguaranteed element contracts are not well-defined at this time, so that reserves in pricing and management-basis financials should be defined objectively to meet the valuation actuary objectives; however, because invested assets equal to statutory reserves plus risk surplus must be identical on statutory and management-basis financials, any differences in reserves are reflected oppositely in risk surpluses.

3.5 *Management-Basis Financials*

Statutory financials (S-basis financials) are prescribed by the National Association of Insurance Commissioners (NAIC), and S-basis accounting is permitted by the American Institute of Certified Public Accountants (AICPA). The purpose is to test solvency and to ensure solidity.

Stock GAAP financials are prescribed by the AICPA and the Financial Accounting Standards Board (FASB). They are general-purpose public financials for use by stock companies.

Management-basis financials (M-basis financials) are going-concern financials for internal use by management for financial planning and control and for surplus management generally. The purpose is to optimize corporate vitality by allocating capital to products and lines, marketing systems, administrative systems, ventures, subsidiaries, and so forth to enhance profits and corporate entity surplus. They also are a means of monitoring the progress of, and providing incentives to, product and line managers with regard to profitable planning and productivity, using suitable rewards and punishments.

M-basis financials are usually modifications of S-basis financials in mutual companies and modifications of GAAP financials in stock companies. They contain many, sometimes most, GAAP-type adjustments, for example, an unamortized acquisition expense asset or negative reserve adjustment and MSVR as a surplus item. However, M-basis financials, subject to auditability, include other adjustments that are not necessarily permitted by GAAP but are needed for internal financial planning, such as different real estate and other equity investment accounting; amortization of realized capital gains and losses; capitalization and amortization of investments in product, marketing, and administration systems; allocation of corporate surplus as risk surplus to lines and products against C-1, C-2, and C-3 risks; accounting by-products and lines; and a corporate entity surplus.

The analysis in this paper is restricted to the management-basis financials applicable to contract classes. The overall structure of management-basis financials, with its extensive design details and dynamics, is not treated. Factors such as interest earned are oversimplified, and the analysis of results on the corresponding S-basis financials ignores obvious differences between factors on the M-basis and the S-basis. Nevertheless, these underlying inconsistencies should not detract from the dependability of the analysis and the conclusions.

3.6 *Detailed Structure of the Unification and Certain Key Relationships*

The rest of the paper derives and explains the details of the structure of unification. Much of it seems obvious by general reasoning, but general reasoning is often less than satisfactory for understanding the complex interrelationships.

Several concepts, definitions, and details not highlighted above are central to the unification:

- The ideal generalized dividend/credits formula and the M-basis financials are mirror images of each other. If the ideal generalized dividends/credits were actually paid, the financial plan would be precisely realized in the M-basis financials.
- Actual dividends/credits must relate to the ideal generalized dividends/credits by the equivalence equation, if the financial plan is to be realized on the average in the M-basis financials. (See Section 6.2 for definition of this equation.)
- The benefit reserves, called V_n, are identical in the ideal generalized dividend/credits formula and in the M-basis financials. They should satisfy the valuation actuary process for reasonable deviations from expected experience on C-1, C-2, and C-3 risks. On traditional participating contracts, this desideratum is easily satisfied. On unbundled, nonguaranteed element, interest-sensitive contracts, the desideratum probably will be compromised, even though the looser relationship between the ideal generalized credits and actual credits and the larger effects of scenarios of the C-1, C-2, and C-3 risks on reasonable deviations would call for more conservative reserves than on traditional contracts. A practical choice of V_n on nonguaranteed element, interest-sensitive contracts is the "policy value." Because required invested assets, equal to the sum of benefit reserve and contingency surplus needed, are identical in S-basis financials and

M-basis financials, any shortcoming in level of V_n is offset by an increase in contingency surplus needed.

- The deferred acquisition expenses are amortized by an independent formula, with the original schedule of amortization retained with duration.
- Contingency surplus needed should be based on plausible deviations from expected, with special attention to poorly matched asset/liability cash flows (C-3 risk), heavy use of junk bonds (C-1 risk), and slim pricing margins (C-2 risk).
- The unification structure is price-driven with reserves predetermined. This necessitates careful observation of the relationship of actual dividends/credits to ideal generalized dividends/credits. Advancement of additional capital because actual dividends/credits fail adversely to satisfy the equivalence equation should provide early warning of a breakdown in the financial plan, with loss recognition indicated unless actual dividends/credits can be redetermined.
- The ideal generalized dividend/credits formula and the formula for the additional capital advanced, representing the accumulated excess of actual dividends/credits over ideal dividends/credits, are, of course, analytical surrogates for the dynamics of the M-basis financials. They provide detailed insight into what is developing in aggregate in the financials. Thus they can show exactly where corrective action is needed.
- There is no suggestion that the ideal generalized dividends/credits be calculated on every contract, although for traditional participating contracts, actual dividends may prove to be feasible, close to the ideal generalized dividends smoothed by use of the equivalence equation. However, calculation of such dividends/credits on a grid of contract classes and scenarios may be fruitful in the planning stage. Also, an ongoing determination of ideal generalized dividends/credits and additional capital advanced (or repaid) due to differences between actual and ideal generalized dividends/credits on a similar grid seems desirable so that detailed corrective actions can be taken.

4. PRICING: THE IDEAL GENERALIZED DIVIDENDS/CREDITS STRUCTURE

Although this structure has been fully developed for traditional participating contracts in mutual companies [2], [3], and [4], the treatment below is fairly self-contained. It also extends the structure to ideal generalized credits on nonguaranteed element, interest-sensitive contracts, introduces contingency (risk) surplus, and changes the profit charge design to the level-return-on-equity method. The capital advanced for unamortized acquisition expenses and for the contingency surplus needed for the C-1, C-2, and C-3 risks is charged for at rate r, the level return on equity. At the end of each year, charges in the dividend for amortization of acquisition expenses and for profit, together with the decrease in contingency surplus needed, are credited to corporate entity surplus, so that the only surplus left in the contract account is the contingency surplus needed. In other words, net income is credited to corporate entity surplus at the end of each year, as required by the equity capital management approach to capital maintenance.

4.1 *Ideal Generalized Dividend/Credits Formula*

The formula is applicable to life insurance and deferred annuity contracts on both the bundled traditional participating design and the unbundled non-guaranteed element, interest-sensitive design:

$$
\begin{aligned}
D_n = &\left[\left(\pi_n + V_{n-1}\right)\left(1 + i_n\right) - V_n\right] \\
&- \left[E'_n\left(1 + i_n\right) + E''_n\left(1 + \frac{1}{2}i_n\right)\right] - \mathrm{FIT}_n^{\mathrm{ins}} \\
&- q_{n-1}\left[DB_n\left(1 + \frac{1}{2}i_n\right) - V_n\right] - w_{n-1}\left[C_n - V_n\right] \\
&- A_n - B_n - X_n
\end{aligned}
$$

where

A_n = charge for advance of CSN_{n-1}, the contingency surplus needed (target surplus).
B_n = charge for amortization of deferrable acquisition expenses.
X_n = charge for profit to cover risk, growth, and change.

Other factors are defined and discussed below.

4.2 *Definitions and Derivations of Factors*

D_n = generalized dividends/credits payable to all entering year n.
D'_n = corresponding actual dividends/credits.
n = duration or contract year.
ω = terminal duration of contract.

M, *S* superscripts denote M-basis financials and corresponding S-basis financials, respectively.

Contract Factors

π_n = gross premium, if any, paid at beginning of year n, including D'_{n-1}, where D'_{n-1} is credited to the contract, i.e., not paid in cash.

DB_n = actual death benefit.

C_n = actual cash value, assumed paid at year-end.

Experience Factors

E'_n, E''_n = actual expenses, commisions, and non-FIT taxes payable or allocable at the beginning of the year and during the year ($n > 0$), respectively.

E_0 = acquisition expenses.

$\text{FIT}_n^{\text{ins}}$ = FIT, excluding marginal interest FIT rate.

e = $P_2 - P_1 + D_1 (1 + \bar{i}i_1)^{-1}$ = credit to acquisition expenses E_0 for CRVM design, if applicable, and nonpayment of D_1, if applicable.

$E_0 - e$ = amortizable net acquisition expenses.

NOTE: Where CRVM design of V_n applies, E'_1 is increased by $P_2 - P_1$. P_1 and P_2 are net premiums on the CRVM basis (similarly for other designs of e).

i_n = actual interest rate after marginal FIT interest factor.

NOTE: In this formula, i_n is defined as a weighted average of (a) the actual portfolio interest rate in the real or notional asset segment for the line and (b) the actual investment year method (IYM) interest rate for the contract class, with weights a and $1-a$, where $0 \leq a \leq 1$; there also is an adjustment for policy loans. In M-basis financials, the interest rate credited, also designated by i_n, probably would be the portfolio rate for the asset segment. In later sections this difference is ignored; the resultant error will, of course, average out over the whole line.

q_{n-1} = actual mortality rate.

w_{n-1} = actual termination rate.

$\bar{i}_n$, $\bar{q}_{n-1}$, and $\bar{w}_{n-1}$ are corresponding rates assumed at issue to establish the schedule for amortizing acquisition expenses, which is kept invariant as experience changes.

All experience factors are based on the very latest experience available.

Note especially the careful development of expense factors in Cody [2], based on the latest companywide cost accounting expense matrices, including the suggested capitalization and amortization of investments in new and changed systems of marketing, products, and administration.

Design Factors

Assigned factors are the elements that control the amount and timing of the emergence of net income.

a) Pricing Reserve, V_n, *and Contingency Surplus Needed,* CSN_n

V_n^S = statutory reserve.

V_n = pricing reserves = $V_n^S - \delta_n$, $\delta_n \geq 0$. V_n is designed to retain all or some of the margins for reasonable deviations from expected experience contained in V_n^S. (See Section 3.)

CSN_n^S = contingency surplus needed (risk surplus, target surplus, benchmark surplus) corresponding to V_n^S. CSN_n^S is determined by C-1, C-2, and C-3 risk analysis used by the valuation actuary.

CSN_n = contingency surplus needed corresponding to V_n.

IA_n = invested assets needed = $V_n^S + CSN_n^S = V_n + CSN_n$ because invested assets needed are identical on all financial bases. Thus, $CSN_n = CSN_n^S + \delta_n$.

Note that neither the net premium nor the expected experience appears explicitly in D_n. The definition of V_n chosen also is used in M-basis financials.

On participating contracts for which the dividend formula incorporates V_n^S, V_n equals V_n^S. The characteristics of the dividend financial structure on such contracts are fortunate, because the M-basis financials are closely tied to the S-basis financials and $V_n = V_n^S$ determines solvency and is the basis from which solidity is measured.

On other participating contracts for which by actuarial choice the dividend formula does not incorporate V_n^S, and on many nonguaranteed element contracts, where V_n^S may not be acceptable for pricing and M-basis financials, $V_n \neq V_n^S$. Here

$V_n = V_n^S - \delta_n$

$CSN_n = CSN_n^S + \delta_n$

where $\delta_n \geq 0$. V_n may be calculated on a net or gross premium basis, with provision against C-1, C-2, C-3 risks of reasonable future deviations from expected. As discussed in Section 3, a practical choice for V_n on nonguaranteed element, interest-sensitive contracts is the "policy value."

In this paper, the design for V_n involving δ_n is used; if δ_n is set equal to zero in the formulas, the results for $V_n = V_n^S$ emerge.

The ideal generalized dividend/credits formula provides within the contract financial structure for funding V_n and for recovery of acquisition expenses. However, CSN_n is not funded within the contract financial structure; CSN_n is provided by capital advanced from corporate entity surplus. The generalized dividend/credits formula makes a charge A_n, credited annually to corporate entity surplus, for this use of capital advanced. The formula also charges for the capital advanced for unamortized acquisition expenses. At the end of each year, $\Delta\, CSN_n$, the change in CSN_n, is charged to corporate entity surplus, if positive, and is returned to corporate entity surplus, if negative. Here, $\Delta CSN_{n-l} = (1 - q_{n-1} - w_{n-1})\, CSN_n - CSN_{n-1}$. Note that in this paper, Δ is not the conventional difference operator (see Section 5.2).

b) Rate of Return on Equity, r

The rate of return on equity is the rate charged, after marginal FIT, for capital advanced, CA_{n-1}. The charge rCA_{n-1} serves two purposes: profit for corporate growth and change and a charge for risk-taking. The rate r is typically 10 percent to 15 percent after FIT.

c) The Level-Percentage-Return-on-Equity Design of $A_n + B_n + X_n$

A_n $= (r - i_n)\, CSN_{n-1}$ = charge for advance of CSN_{n-1} from corporate surplus.

B'_n $= B_n + X_n$ = the combined charges for profit and amortization of acquisition expenses,

$$= (1 - q_{n-1} - w_{n-1})\, S_n - (1 + r)\, S_{n-1}$$

$$= \Delta S_{n-1} - r\, S_{n-1}.$$

$$S_n = -\frac{E_0 - e}{a_{\overline{\omega}|}}\, a_{\overline{\omega - n}|} \qquad S_0 = -(E_0 - e).$$

$a_{\overline{w}|}$ and $a_{\overline{w-n}|}$ are defined in the Appendix based on interest rate r and survivorship assumed at issue.

$\Delta S_{n-1} = (1 - q_{n-1} - w_{n-1})\, S_n - S_{n-1}$.

This design thus amortizes the acquisition expenses on the schedule established at issue at an interest rate equal to the return-on-equity rate, r. In addition, it charges at rate r for the advance of CSN_{n-1}.

5. M-BASIS FINANCIALS WHERE $D_n' = D_n$

5.1 *General Description*

When $D_n' = D_n$, the M-basis financials will show a level percentage return on equity (capital advanced) annually in the contract account. This is to be expected, because the M-basis financials are a mirror image of the ideal generalized dividends (credits), D_n.

The accounting dynamics of the contract account in the M-basis financials are as follows:

- At the beginning of the contract year, the required invested assets, IA_{n-1}, equal $V_{n-1} + CSN_{n-1}$.
- During the contract year, the net income is the excess of the sum of gross premium, if any, and investment income over the sum of claims, cash value paid, expenses, taxes, dividends/credits, D_n, and the increase in the reserve for benefits and acquisition expenses, all after FIT.
- At the end of the year, net income, if positive, is transferred to corporate entity surplus or, if negative, is advanced from corporate entity surplus.
- Then, ΔCSN_{n-1}, if positive, is advanced from corporate entity-surplus or, if negative, is returned to corporate entity surplus.
- The contract account then enters the next contract year with required invested assets $IA_n = V_n + CSN_n$.

Although the above is perhaps evident, it is desirable to explore the details for purposes of a clear understanding and to derive explicit relationships needed in Section 6.

The following listing is complete, with repetition of some definitions from Section 4 for convenient reference.

5.2 *Definitions and Relationships*

V_n^S = reserve on S-basis.

CSN_n^S = contingency surplus needed on S-basis.

V_n = (benefit) reserve on pricing basis, used in the ideal generalized dividend formula.

CSN_n = corresponding contingency surplus needed.

V_n = $V_n^S - \delta_n \quad (\delta_n \geq 0)$.

IA_n = required invested assets = $V_n^S + CSN_n^S = V_n + CSN_n$.

CSN_n = $CSN_n^S + \delta_n$.

V_n^M = reserve on M-basis, including deferred acquisition expenses reserve, S_n.
= $V_n + S_n = V_n^S - \delta_n + S_n$.

S_n = $-\dfrac{E_0 - e}{a_{\overline{w}|}} a_{\overline{w-n}|}$ where the deferred acquisition expenses are $E_0 - e$.

e = $(P_2 - P_1) + D_1 (1 + \bar{i}_1)^{-1}$, typically.
NOTE: It is possible to define functions that will defer all acquisition expenses, but I think that in M-basis financials any credits arising from CRVM, front-end loads, or non-payment of first-year dividend should be applied to reduce deferred acquisition expenses.

CA_n = capital advanced = $CSN_n - S_n = CSN_n^S - S_n + \delta_n$.

ΔF_{n-1} = $(1 - q_{n-1} - w_{n-1}) F_n - F_{n-1}$ where F_{n-1} is any of the following functions: IA_{n-1}, V_{n-1}, V_{n-1}^S, V_{n-1}^m, CSN_{n-1}, CSN_{n-1}^S, S_{n-1}, DS_{n-1}, or δ_{n-1}. Note that Δ is not the conventional difference operator; here Δ contains a persistency factor, because F_n applies to units of contract persisting to duration n. Letting l_n equal number of units of contract persisting to duration n:

$$l_n = (1 - q_{n-1} - w_{n-1})\, l_{n-1} \quad \text{so that}$$

$$l_{n-1}\, \Delta F_{n-1} = l_{n-1} (1 - q_{n-1} - w_{n-1}) F_n - l_{n-1} F_{n-1}$$
$$= l_n F_n - l_{n-1} F_{n-1}.$$

B'_n = $B_n + X_n = \Delta S_{n-1} - r S_{n-1}$.

B_n = $\Delta S_{n-1} - i_n S_{n-1}$.

X_n = $(r - i_n)(-S_{n-1})$.

NI_n^M = net income on M-basis in year n
= $\Delta' IA_{n-1} - \Delta V_{n-1}^M \; (n>0)$.

NI_n^S = net income on S-basis in year n
= $\Delta' IA_{n-1} - \Delta V_{n-1}^S \; (n>1)$.

$\Delta' IA_{n-1}$ = increase in required invested assets from cash flow in year n, as defined in Subsection 5.3.

5.3 *M-Basis Net Income and Return on Equity*

$$NI_n^M = \Delta' IA_{n-1} - \Delta V_{n-1}^M.$$

$$\begin{aligned}\Delta' IA_{n-1} &= i_n\, IA_{n-1} + \pi_n \left(1 + i_n\right) \\ &\quad - E'_n \left(1 + i_n\right) - E''_n \left(1 + \frac{1}{2} i_n\right) \\ &\quad - q_{n-1}\, DB_n \left(1 + \frac{1}{2} i_n\right) \\ &\quad - w_{n-1}\, C_n - \mathrm{FIT}_n^{\mathrm{ins}} - D_n.\end{aligned}$$

$$\Delta V_{n-1}^M = \Delta V_{n-1} + \Delta S_{n-1}.$$

Now, introduce the expression for D_n from Section 4.1 and collect terms:

$$\begin{aligned}NI_n^M &= i_n\,(IA_{n-1} - V_{n-1}) + \Delta V_{n-1} - \Delta V_{n-1}^M + A_n + B_n + X_n \\ &= i_n\, CSN_{n-1} - \Delta S_{n-1} + A_n + B'_n \\ &= i_n\, CSN_{n-1} - r\, S_{n-1} + (r - i_n)\, CSN_{n-1} \\ &= r\, CA_{n-1}.\end{aligned}$$

$$ROI_n^M = \text{return on investment on the M-basis} = \frac{NI_n^M}{CA_{n-1}} = r.$$

This result proves that the ideal generalized dividend (credits) formula in Section 4 provides for a level percentage rate of return, r, on equity on the M-basis financials designed on the entity-capital management approach, as expected.

5.4 *Benefit Reserves and Expense Reserves in the M-Basis Financials*

The following manipulation of the formula for D_n in Section 4 shows that the benefit reserves and expense reserves in the M-basis financials equal V_n and S_n, respectively, with an explicit adjustment involving e. If $e = 0$, the adjustment is zero. (This relationship is discussed in Cody [4].)

Solve the equation for D_n in 4.1 for V_{n-1} as follows:

$$V_{n-1} = v_n \left\{ D_n + E'_n \left(1 + i_n\right) + E''_n \left(1 + \frac{1}{2} i_n\right) + \mathrm{FIT}_n^{\mathrm{ins}} \right.$$
$$+ q_{n-1}\, DB_n \left(1 + \frac{1}{2} i_n\right) + w_{n-1}\, C_n$$
$$\left. - \left[\pi_n \left(1 + i_n\right) - B_n \right] + A_n + X_n \right\} + {}_1p_{n-1}\, V_n$$

where v_n and ${}_1p_{n-1}$ are defined in the Appendix.

Now, substitute the expression for V_n, obtained by solving the D_{n+1} equation for V_n, into the above equation and reiterate for $V_{n+1}, V_{n+2}, \ldots, V_{\omega-1}$. Then, substituting V_n for V_{n-1}, the resulting equation is as follows:

$$V_n = \sum_{t=1}^{\omega-n} v_n^t\, {}_{t-1}p_n \left\{ q_{n+t-1}\, DB_{n+t} \left(1 + \frac{1}{2} i_{n+t}\right) + w_{n+t-1}\, C_{n+t} \right.$$
$$+ E'_{n+t} \left(1 + i_{n+t}\right)$$
$$+ E''_{n+t} \left(1 + \frac{1}{2} i_{n+t}\right) + \mathrm{FIT}_{n+t}^{\mathrm{ins}} + D_{n+t} + A_{n+t}$$
$$\left. + X_{n+t} - \left[(1 + i_{n+t})\pi_{n+t} - B_{n+t} \right] \right\}$$

Thus, V_n is seen to be the benefit reserve including profit $A_{n+t} + X_{n+t}$ with a benefit premium of $\pi_{n+t} - B_{n+t}(1 + i_{n+t})^{-1}$.

The corresponding unamortized acquisition expense asset is:

$$\frac{E_0 - e}{a_{\overline{\omega}|}}\, a_{\overline{\omega-n}|} \text{ with premium } B_{n+t}(1 + i_{n+t})^{-1}.$$

If the equation for D_n is solved for V_n and the result is iterated retroactively, V_n is seen to be the retroactive accumulation of the excess of premiums over benefits, expenses, dividends, profit charges, and charges for amortization of acquisition expenses.

5.5 *Corresponding S-Basis Financials*

$$\begin{aligned} NI_n^S &= \Delta' IA_{n-1} - \Delta V_{n-1}^S \; (n>1) \\ &= NI_n^M + (\Delta V_{n-1}^M - \Delta V_{n-1}^S) \\ &= NI_n^M + (\Delta V_{n-1} + \Delta S_{n-1}) - (\Delta V_{n-1} + \Delta\delta_{n-1}) \\ &= NI_n^M + \Delta S_{n-1} - \Delta\delta_{n-1}. \end{aligned}$$

Because ΔS_{n-1} is positive and $\Delta\delta_{n-1}$ is usually negative, NI_n^S is larger than NI_n^M for $n>1$. NI_n^S also can be expressed as follows:

$$NI_n^S = r\,CA_{n-1} + B_n' + r\,S_{n-1} - \Delta\delta_{n-1} = \mathrm{B}_n' + r\,CSN_{n-1} - \Delta\delta_{n-1} \quad (n>1).$$

For $n = 1$, there are additional cash flow items, so that

$$NI_1^S = B_1' + r\,CSN_0 - \Delta\delta_0 - (E_0 - e)\,(1 + i_1).$$

For $n = 1$, it is reasonable to set $i_1 = \bar{i}_1$ and $B_1' = \dfrac{E_0 - e}{a_{\overline{\omega}|}}$, so that

$$NI_1^S = -\left(E_0 - e\right)\left[\left(1 + \bar{i}_1\right) - \frac{1}{a_{\overline{\omega}|}}\right] + r\,CSN_0 - \Delta\delta_0 \quad (n = 1).$$

6. M-BASIS FINANCIALS WHERE $D_n' \neq D_n$

6.1 *Actual Dividends (Credits)* D_n'

On traditional participating contracts, D_n' is in essence based on D_n, usually on a simpilifed three-factor formula, with the effects of cash values, termination rates, and profit charges absorbed in the three factors. Like D_n, they are in accordance with the contribution principle and in keeping with the Academy's "Recommendations and Interpretations for Dividends and Other Non-Guaranteed Elements" [1]. Close relationship of D_n' to D_n is mandated [2], [3]. The bundled nature of D_n' subjects them to less competitive scrutiny, enabling this close relationship, based on equity principles.

On unbundled interest-sensitive contracts with nonguaranteed elements, the relationship of D_n' to D_n is more tenuous. The determination and redetermination policy for pricing, as recommended by the Interim Actuarial Standards Board [6], should be based on the financial plan for the contracts.

Although equity may be desirable, disclosure, regularity, and ethical performance of pricing changes (changes in D_n') are the primary considerations, together with the underlying solvency, profit, and marketing objectives. Subject to these considerations and objectives, D_n' relates not only to contract experience but also to other inputs such as outside indexes, new money rates, term structure of interest rates, rates used by competitors, previous level of rates, and so forth. The unbundled nature of D_n' allows contract-holders, agents, and brokers to form judgments easily, and there is a tendency to make credits on new offerings more attractive than the present and future environments are likely to support. Also, the unbundling restricts the design of D_n' on interest-sensitive products to three stand-alone factors—expense charges, mortality charges, and interest rate spread—applied in a structure involving a formulated contract account and surrender charges; the flexibility of premium payments is an additional complication.

Nevertheless, on interest-sensitive, nonguaranteed element contracts, a well-conceived financial plan for the contract must explicitly or implicitly call for D_n' to interweave around D_n and to relate to D_n by the equivalence equations set out in Section 6.2.

6.2 *Equivalence Equations Relating* D_n' *to* D_n

The basic equivalence equation is this:

$$\sum_{t=1}^{\omega} v'^{\,t-1}\,{}_{t}p_0\,(D'_t - D_t) = 0 \qquad \text{(A)}$$

where ${}_tp_0$ (defined in the Appendix) involves actual survivorship experience and $v = (1 + r)^{-1}$.

An important practical second equivalence equation is this:

$$\sum_{t=1}^{n'-n} v'^{\,t-1}\,{}_{t}p_n\,(D'_{n+t} - D_{n+t}) = 0 \qquad \text{(B)}$$

where ${}_tp_n$ (also defined in the Appendix) is the probability of actual survivorship from n to $n+t$. Equation (B) is important because it is unlikely that equation (A) will be realized unless $(D_n' - D_n)$ oscillates around zero over short periods of $(n' - n)$ years.

The accumulation of $(D'_n - D_n)$ from issue to n is defined as DS_n:

$$DS_n = \frac{1}{v^n \, {}_nP_0} \sum_{t=1}^{n} v^t \, {}_{t-1}p_0 \, (D'_t - D_t).$$

$$DS_\omega = 0 \quad \text{(a form of equation A).}$$

$$\Delta DS_{n-1} = (1 - q_{n-1} - w_{n-1}) \, DS_n - DS_{n-1} = (D'_n - D_n) + r \, DS_{n-1}. \qquad \text{(D)}$$

ΔDS_{n-1} must be advanced by corporate entity surplus (or credited to corporate entity surplus, if negative). Hence capital advanced becomes as follows:

$$CA_n = CSN_n - S_n + DS_n.$$

Equation (B) indicates that DS_n will oscillate around zero, and equation (A) means that $DS_\omega = 0$. It follows that CA_n will oscillate around $CSN_n - S_n$, its value for $D'_n = D_n$.

6.3 *M-Basis Financials Assuming* $D'_n - D_n$ *Conforms to Equivalence Equations*

Similarly to the development in Section 5.3, the net income becomes as follows:

$$NI_n^M = r \, (CSN_{n-1} - S_{n-1}) - (D'_n - D_n)$$
$$= r \, CA_{n-1} - r \, DS_{n-1} - (D'_n - D_n),$$

and

$$DS_n + \sum_{t=1}^{\omega-n} v^t \, {}_{t-1}p_n \, (D'_{n+t} - D_{n+t}) = v^{\omega-n} \, {}_{\omega-n}p_n \, DS_\omega.$$

Hence, NI_n^M oscillates around NI_n^M *for* $D'_n = D_n$, and because $DS_\omega = 0$, the financial plan is realized over short periods of time and over the lifetime of the contract.

6.4 *M-Basis Financials Where* $D'_n - D_n$ *Does Not Conform to Equivalence Equations*

If $DS_\omega < 0$, excess profits beyond the financial plan have been realized; these excess profits equal DS_ω. If $DS_\omega > 0$, profits may have been realized that are lower than anticipated in the financial plan. Or, worse, losses have been taken. Failure of the financial plan is characterized by the following expression for the present value of future losses and/or reduction in future profit anticipated under the financial plan, looking forward from duration n:

$$\sum_{t=1}^{\omega-n} v^t \,_{t-1}p_n \,(D'_{n+t} - D_{n+t}) = v^{\omega-n} \,_{\omega-n}p_n \, DS_\omega - DS_n > 0. \qquad \text{(E)}$$

Expression (E) applies in aggregate over the contract class in the domain of reasonable deviations from expected in the future from C-1, C-2, and C-3 risks, with DS_n equaling actual accumulated past excesses of D'_n over D_n. Future D'_{n+t} would be assumed to be reduced as far as practicable, and future policy as to matching of asset/liability cash flows would be expected to be optimized.

For purposes of loss recognition, expression (E) is of theoretical interest only. Unreasonable and unrecoverable increases in DS_n appear to be the danger signal for considering loss recognition in M-basis financials:

- Loss recognition is indicated when the excess of (a) the present value of the sum of future benefits, future expenses, and future D'_{n+t} over (b) the present value of future gross premiums exceeds $V_n = V_n^M + S_n$ in aggregate over the domain of reasonable deviations from expected experience from C-1, C-2, and C-3 risks, assuming all possible steps are taken to reduce future D'_{n+t} as far as practicable and to make optimum change in investment policy, etc.
- The first step in loss recognition is to write down partially or completely the unamortized expense asset, $-S_n$. If this is inadequate, $V_n^M = V_n$ should be appropriately increased; and V_n^S may have to be increased as well.
- After loss recognition, CSN_n continues to be needed against plausible deviations from expected experience on C-1, C-2, and C-3 risks.

Some such loss recognition rules should be carefully formulated and rigorously followed as an essential part of the M-basis financial structure.

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APPENDIX

DEFINITIONS OF PROBABILITY OF SURVIVING AND DISCOUNT FUNCTIONS

i_n, q_{n-1}, and w_{n-1} are actual interest, mortality, and termination rates.

$\bar{i}_n$, $\bar{q}_{n-1}$, and $\bar{w}_{n-1}$ are the corresponding rates assumed at issue.

The probability of surviving from duration n to duration $n + t$ is ${}_tp_n$, where

$${}_tp_n = {}_{t-1}p_n\,(1 - q_{n+t-1} - w_{n+t-1}) \text{ where } {}_0p_n = 1.$$

$${}_t\bar{p}_n = {}_{t-1}\bar{p}_n\,(1 - \bar{q}_{n+t-1} - \bar{w}_{n+t-1}) \text{ where } {}_0\bar{p}_n = 1.$$

Discount factors from $n + t$ back to n are as follows:

$$v_n^t \quad = v_n^{t-1} (1 + i_{n+t})^{-1} \text{ where } v_n^0 = 1.$$

The discount factor used in corporate entity surplus advanced is:

$$v^n \quad = (1 + r)^{-n}.$$

Annuity values used to amortize $(E_0 - e)$ are:

$$a_{\overline{\omega-n|}} \quad = \sum_{t=1}^{\omega-n} v^t \, {}_{t-1}\bar{p}_n.$$

DISCUSSION OF PRECEDING PAPER

DONALD R. SONDERGELD:

Many of the concepts contained in this fine paper have been used by some stock life insurance companies for years. However, I found the treatment of dividends of special interest, as my 1982 *TSA* paper [3] did not discuss policyholder dividends.

Management financials are generally based on GAAP for stock life insurance companies. GAAP accounting rules are not overly flexible, but in areas in which there is flexibility, heavy weight is usually given to the pattern of the emergence of net income, as opposed to the author's emphasis on pricing for mutual companies. It might be helpful if the author would expand somewhat on what he means by the term "pricing."

Although stock life companies must follow GAAP, many attempt to produce level return results under GAAP. The reader may wish to refer to my 1974 paper [1], which described the internal rate of return method of accounting (IRRMA). It is quite similar to the level return on equity accounting method.

In a discussion of Robin Leckie's 1979 *TSA* paper [2], I commented that the nonrefundable charge made to participating policyholders should be for the use of capital and the rate of return on the capital should be commensurate with the risk. If I correctly interpret Mr. Cody's paper, he is also suggesting there be a nonrefundable charge for the use of capital.

Comments on Formulas

In Section 4.1 the author uses three terms in his dividend formula:.

$$-A_n - B_n - X_n,$$

where
$$A_n = (r - i_n)(CSN_{n-1})$$
$$= (r - i_n)(CSN^S_{n-1} + V^S_{n-1} - V^M_{n-1} + S_{n-1})$$
$$B_n = \Delta S_{n-1} - i_n S_{n-1}$$
$$X_n = (r - i_n)(-S_{n-1}).$$

Let's define

$$-A_n - B_n - X_n = +AA_n - BB_n - XX_n,$$

where $AA_n = (i_n)\,(CSN)_{n-1}$

$BB_n = \Delta\, S_{n-1}$

$XX_n = (r)\,(CA_{n-1}) = (r)\,(CSN^S_{n-1} + V^S_{n-1} - V^M_{n-1})$

$= NI^M_n.$

By examining these three factors, we see that:

- AA_n is the credit the participating profit center gets for interest earned on contingency surplus in year n.
- BB_n is the charge for the amortization of the deferred acquisition expense in year n. Note that $\Sigma BB_t = S_o$.
- XX_n is the nonrefundable charge the corporate line should receive from the participating profit center for the use of capital in year n.

In Section 5.3, we find

$$NI^M_n = i_n\, CSN_{n-1} - \Delta\, S_{n-1} + A_n + B_n + X_n$$

$$NI^M_n = i_n\, CSN_{n-1} - \Delta\, S_{n-1} - AA_n + BB_n + XX_n$$

$$NI^M_n = i_n\, CSN_{n-1} - \Delta\, S_{n-1} - (i_n)\,(CSN_{n-1}) + \Delta\, S_{s-1} + NI^M_n$$

$$NI^M_n = NI^M_n$$

Comments on DS_n

In Section 6.2, we can write CA_n as follows:

$$CA_n = CSN^S_n + V^S_n - V^M_n + DS_n.$$

This is because

$$CSN_n - S_n = CSN^S_n + V^S_n - V^M_n.$$

This says that "management capital" equals statutory surplus, plus the management (or GAAP) adjustments.

This means that capital advanced on the management basis equals contingency surplus needed on a statutory basis plus the difference between statutory and management reserves (where the management reserve includes the deferred acquisition expense reserve S_n) plus a smoothing adjustment.

Therefore,

$$NI^M_n = r(CSN^S_n + V^S_n - V^M_n + DS_n).$$

I was intrigued by the use of DS_n, which is a cumulative adjustment made to the dividend scale for various reasons, which include smoothing out chance fluctuations and competitive pressures. However, I doubt that DS_ω will equal zero in all cases. Presumably DS_ω is determined separately for various classes of business. Some business in a class may experience expenses much larger, for example, than implicit in the dividend scale illustrated. It would seem logical that a company might elect to reduce its return in lieu of reducing dividends. In that case, DS_ω is greater than zero.

My interpretation of Section 6 of the paper is that if it is expected that DS_ω will equal zero, then DS_n is treated as an asset or liability depending upon its algebraic sign. This results in the price for level return on equity being reported to management each year. However, when it is first recognized that DS_ω is not expected to be zero, the author suggests that an adjustment to income and surplus be made. The adjustment is the present value of DS_ω after tax. After the adjustment is made, the expectation is that DS_ω will equal zero.

However, if there are dividends expected in the future, it would seem there are at least two possibilities for making the adjustment: (a) at one time as the author suggests or (b) in installments. If $r = 12$ percent, it might mean that if (a) is used, the ROE reported to management might equal a small number, for example, 1 percent, the year the adjustment is made, with a 12 percent ROE expected in the future. On the other hand, if (b) is used, ROE might become smaller than 12 percent, say 9 percent, and be reported in the current year and expected to be reported in future years.

I strongly support the author's view that

> "A unification of pricing, statutory valuation, and management-basis financials for control and planning may be essential for the ongoing viability of life insurance companies offering interest-sensitive products in the variable economic environment of the foreseeable future."

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JOSEPH H. TAN:

I would like to congratulate the author for writing a fine paper on the relationship of pricing, valuation, and management-basis financials. My discussion focuses on the level-return-on-equity accounting method.

In his 1974 *TSA* paper [3], Sondergeld discusses an accounting method that yields a level return on equity. Mr. Sondergeld calls it the internal rate of return method of accounting (IRRMA). By computing the insurance surplus account as the present value at the internal rate of return of future statutory book profits, Mr. Sondergeld shows that the earnings under IRRMA emerge as a fixed percentage of the beginning of year (*BOY*) insurance surplus. Because the contingency surplus needed (*CSN*) for the insurance account is set to zero, the capital advanced (*CA*) at *BOY* is equal to the insurance surplus.

This same idea, with some modification, is also discussed in two other papers [1] and [2]. Both of these discuss a level return on equity (ROE) method, which is based on a net reserve that is equal to (*a*) minus (*b*) where

(*a*) is the outstanding required asset. We can view this as the sum of statutory reserve (V^s) and *CSN*. For instance, Appendix C of the Committee Report [1] assumes (*a*) is equal to 105 percent of the statutory reserve.

(*b*) is the present value of future capital flows. The discount rate used is the level return on equity, or the internal rate of return. We can view capital flow as statutory book profit less the increase in *CSN*.

We note that the level ROE method as discussed by Ramsey [2] and in the Committee Report [1] is the same as the IRRMA except for the presence of *CSN*. The equity of the adjusted statement (management-basis statement) is computed as the present value at the internal rate of return of future statutory book profits (with or without the increase in *CSN*). Is this the only way to arrive at an adjusted statement profit that is a level percentage of equity every year?

Mr. Cody's paper shows that the answer is no. At first, it would seem that unless the equity of the management-basis (M-basis) statement is defined as the present value at the internal rate of return of future statutory book profits (net of the increase in *CSN*), the resulting M-basis net income (NI^M) would not be a level percentage of *BOY* equity. Then why does the author's model result in level return of equity (as shown in Section 5.3 of the paper)?

The answer lies in the fact that the author's model deals with participating and nonguaranteed element contracts only. For guaranteed nonparticipating contracts, it is difficult for an insurance company to alter the incidence of the cash-flow items (premium, death benefit, and so on). Adjusting the incidence of reported profit to yield a level ROE (that is, to produce an adjusted statement) can only be accomplished through the adjusted reserve mechanism. Hence, level ROE is accomplished in the three papers (that is, Sondergeld's paper [3], Ramsey's discussion [2], and the Committee Report [1]) by computing the net adjusted reserve (and hence adjusted equity) in a certain manner.

Participating and nonguaranteed element contracts have an additional feature that can be used to alter the incidence of reported profit, namely, dividend and nonguaranteed element credit. This feature, together with the author's method for computing capital advanced (*CA*), is the mechanism used to produce a level ROE. As we see, even though the adjusted equity (equal to *CA* in the author's paper) is defined differently from the above three papers, level ROE also emerges.

Let us examine why the author's method results in a profit emergence that is a level percentage of equity (*CA*). The following notations, in addition to those of the author's, will be used:

CF_n = Insurance cash flow before dividend
= Gross Premium − Surrender Benefit − Death Benefit − Expenses and Taxes

NII_n = Net investment income
$= i(IA_{n-1}) + i(CF_n)$
$= i(V_{n-1} + CSN_{n-1}) + i(CF_n)$

Consistent with the author's development, dividend is assumed paid at the end of the year and hence does not generate investment income.

We can write dividend D_n and M-basis net income NI_n^M as:

$$(1): D_n = CF_n + iV_{n-1} + iCF_n - \Delta V_{n-1} - A_n - B'_n$$

$$(2): NI_n^M = CF_n + i(V_{n-1} + CSN_{n-1}) + iCF_n - \Delta V_{n-1}^M - D_n$$

Equation (1) is similar to the equation under Section 4.1 of the author's paper, and Equation (2) is similar to the first two equations under Section

5.3 of the author's paper. Substituting Equation (1) into Equation (2) and simplifying, we have

$$(3): NI_n^M = i(CSN_{n-1}) - \Delta S_{n-1} + A_n + B_n'$$

To yield an adjusted profit equal to r percent of beginning of year CA

$$(4): r(CA_{n-1}) = r(CSN_{n-1} - S_{n-1}),$$

it is easy to see that $A_n + B'_n$ should be equal to the sum of the following two terms:

(i) $(r - i)(CSN_{n-1})$,

and

(ii) $\Delta S_{n-1} - rS_{n-1}$.

The author has defined (i) as A_n and (ii) as B_n'.

Let us examine the above algebraic manipulation. A_n and B'_n are charges in the dividend formula. Regarding A_n, because Equation (3) (that is, NI^M) already contains interest i on CSN, we only need the excess of r over i times CSN in the dividend formula to obtain $r(CSN)$ as required in Equation (4). Regarding B_n', because Equation (3) subtracts out ΔS, we need to add it back and subtract out rS in the dividend formula to obtain $-rS$ as required in Equation (4). Note that $-\Delta S$ exists in Equation (3) because NI^M is the M-basis net income (that is, adjusted profit).

Note the following observations:

1. Because the mechanism works regardless of what r is, r need not be the internal rate of return (IRR) for level ROE to emerge. For instance, the IRR could be 15 percent and the level ROE be 12 percent. This is quite different from the level ROE method discussed in the three other papers. In those three papers, the level ROE rate equals IRR.
2. Because the $-\Delta S_{n-1}$ included in NI^M of Equation (3) is offset by the ΔS_{n-1} included in B_n' of the dividend formula, level ROE will result, regardless of how ΔS_{n-1} is expressed. The author chose to set ΔS_{n-1} equal to the nth year amortized amount of the unamortized deferred acquisition expenses $(E_o - e)$. That is, the author's method:

   (a) Computes ΔS_{n-1} based on the initial deferred acquisition expenses $(E_o - e)$, and

(b) Computes the amortization at a constant rate based on the present value at rate r of future in force units. Hence,

$$S_n = -\frac{E_0 - e}{a_{\overline{\omega}|}} a_{\overline{\omega - n}|}$$

where $a_{\overline{\omega-n}|} = \sum_{t=1}^{\omega-n} \left(\frac{1}{1+r}\right)^t {}_{t-1}\bar{p}_n$

Following our discussion earlier, we see that one can compute (*a*) and (*b*), and hence $\Delta\ S_{n-1}$, differently and still obtain a level ROE.

For instance,

(*a*) Deferred amount could be defined as those acquisition expenses that vary with and can be attributed to new sales. It can even be defined arbitrarily as a certain percentage of first-year gross premium or some arbitrary number.

(*b*) Amortization could be computed based on the present value at rate r of future book profits, cash flows, or gross premiums. The amortization could even be computed based on the present value at rate r of \$1 per year. In algebraic form, $a_{\overline{\omega-n}|}$ can be an annuity certain

$$a_{\overline{\omega-n}|} = \sum_{t-1}^{\omega-n} \left(\frac{1}{1+r}\right)^t.$$

In the other three papers

(*a*) "Deferred amount" is defined as the initial statutory strain (including *CSN*). It can be shown that this is equal to the present value at the IRR rate of future statutory book profit (including ΔCSN) at issue.

(*b*) Amortization is computed based on the present value at the IRR rate of future statutory book profit (including ΔCSN).

Depending on the basis for amortizing S_n (for example, in-force unit, gross premium, book profit, or \$1), the incidence of declared dividends will be different. For instance, if the amortization is based on the present value of \$1 versus in-force units, the amortized amounts in the earlier years will be smaller. This will give rise to higher dividends in the earlier years and slower payback of the capital advanced (CA) from the corporate. Note that I am not suggesting that this result is better than the one based on in-force units (author's method). What I am suggesting is that the actuary should keep this in mind and determine the amortization pattern (and hence dividend incidence) that is best for the company. For instance, some actuaries may believe that amortization based on the present value of future book profit before dividend and acquisition expenses yields a more meaningful result. The latter is patterned after the amortization of deferred acquisition cost prescribed in the *Statement of Financial Accounting Standard No. 97 (FAS 97)* for universal life accounting, in proportion to estimated gross profit excluding acquisition cost.

Conclusion

As we have seen, level ROE can be accomplished in various ways. The actuary and management need to understand the various mechanisms for accomplishing level ROE and choose the methodology that best fits the company's goal.

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(AUTHOR'S REVIEW OF DISCUSSION)

DONALD D. CODY:

I am grateful to Mr. Sondergeld and Mr. Tan for their analyses. Mr. Sondergeld is a recognized authority in these areas, and Mr. Tan shows deep insights into various accounting systems.

Mr. Tan notes that the actuarial-accounting structure in my paper is different from others in the literature in at least two important respects:

(1) It does not manipulate DAC_n so as to control ROE.
(2) My r is not equal to IRR.

He also observes that my method allows for a range of definitions for the amortization schedule for S_0.

I was aware of (1); it is one of my basic objectives. But, as to (2), I had carelessly assumed that r is equal to IRR. I think that some of the seeming disagreements with Mr. Sondergold arise from the use of those other structures in the literature.

Actually IRR is quite irrelevant in my structure. However, IRR can be determined even though it is a curiosity. Let r' equal IRR in a financial structure containing CSN_n. Also, let v' equal $(1 + r')^{-1}$ and v equal $(1 + r)^{-1}$.

Then, in an ideal situation where $D'_n = D_n$, r' satisfies the following equation:

$$S_0 + \sum_{n=1}^{\omega} {}_{n-1}p_0\, v'^n \left\{ (1 + i_n)\, CF_n + \Delta CSN_{n-1}^S + i_n\, (V_{n-1}^S + CSN_{n-1}^S) - \Delta(V_{n-1}^S + CSN_{n-1}^S) - D'_n \right\} = 0.$$

Introducing the following relationships:

$$V_{n-1}^S + CSN_{n-1}^S = V_{n-1} + CSN_{n-1},$$

$$V_{n-1}^S = V_{n-1} + \delta_{n-1},$$

$$S_0 = -\sum_{n=1}^{\omega} {}_{n-1}p_0\, v^n B'_n,$$

and

$$D'_n = D_n = (1 + i_n)\, CF_n + i_n V_{n-1} - \Delta V_{n-1} - A_n - B'_n,$$

the equation reduces to this:

$$\sum_{n=1}^{\omega} {}_{n-1}p_0\, v'^n \left\{ r\, CSN_{n-1} - \Delta\delta_{n-1} + B'_n \left(1 - \frac{v^n}{v'^n}\right)\right\} = 0.$$

This equation can usually be satisfied only if $v > v'$, that is, $r' > r$.

In addition, the following characteristics also apply to r':

- $r' > r$ usually, if $DS_\omega = 0$
- r' will decrease, if $DS_\omega > 0$
- r' will increase, if $DS_\omega < 0$
- $r' = r$ if $CSN_{n-1} = 0$ and $\delta_{n-1} = 0$ and $D'_n = D_n$.

I would now like to return to Mr. Sondergeld's comments, keeping the above background in mind. First, as to what I mean by "pricing": narrowly it means determination of D'_n, C_n, and DB_n; more broadly, pricing comprehends all the formulas, factors, and procedures of Section 4 and Subsections 6.1 and 6.2.

As he infers, I draw little distinction between participating and nonguaranteed element contracts in mutuals and nonguaranteed element contracts in stocks. In particular, both mutuals and stocks must make nonrefundable charges for use of capital, commensurate with risk.

I found his AA_n, BB_n, and XX_n functions very interesting because they convey intrinsic meaning. I would note that because my

$$\Delta S_{n-1} = p_{n-1} S_n - S_{n-1},$$

it follows that

$$\sum_{n=1}^{\omega} {}_{n-1}p_0 \, BB_n = - S_0.$$

Mr. Sondergeld appears to believe that DS_n affects liabilities and/or assets. It does not in my system. More particularly, it does not affect IA_{n-1}, V_{n-1}, S_{n-1}, or D_n. It does appear in NI_n^M in the factor $(-r\, DS_{n-1})$, and in CA_n as the factor DS_n. DS_n is the fundamental early warning device that loss recognition may be needed. ROE_n equals r ideally; it varies around r, if D'_n and D_n satisfy the equivalence equation; and it becomes less than r, if $DS_\omega > 0$. Its general format is this:

$$ROE_n^M = \frac{NI_n^M}{CA_{n-1}} = r - \frac{r\, DS_{n-1} + (D'_n - D_n)}{CSN_{n-1} - S_{n-1} + DS_{n-1}}$$

Mr. Sondergeld also comments about loss recognition, which I described in general terms in Subsection 6.4. In preparing a response, I developed the theoretical algebraic structure, which I shall not record here because the conclusions are more or less self-evident and the actual theory would not be used in practice directly.

I agree that loss recognition would normally be taken in steps, considering the uncertain future relationship between D'_n and D_n. And because of the narrow margins in the pricing, the earlier the first step is taken, the better. At loss recognition at duration n, V_n^M (and later V_{n+t}^M) is increased to $V_n''^M$ to assure provision for future benefits, expenses, and actual dividend/credits. A characteristic of loss recognition is that r is reduced to a value, r'', no less than i_{n+t}. Indeed, if S_n turns out to be irrecoverable and V_{n+t} must also be increased, $r'' = i_{n+t}$. Another characteristic of loss recognition is that

$$DS_\omega \not> DS_n (1 + r'')^{\omega - n}.$$

I also believe that after loss recognition

$$CA_{n+t} = CSN_{n+t} - S''_{n+t} + {}_nDS''_{n+t} + (V''^M_n - V^M_n + DS_n)(1 + r'')^t$$

where the double primed functions are those applying after loss recognition, ${}_nDS_{n+t}$ being the accumulation of D'_{n+t} over D'_{n+t} and D''_{n+t} being a generalized dividend/credit applying after loss recognition. Thus, ROE_{n+t} will involve not only a reduced r'' but also an increased CA_{n-1}.

This unforgiving attitude as to emerging ROE on contract classes in which there has been loss recognition is, I believe, important so as to keep management aware of poor pricing in the past.

And last, but not least, I appreciate Mr. Sondergeld's strong support of my view that integration of pricing, statutory valuation, and management-basis financials for control and planning is necessary for viability of life insurance companies offering interest-sensitive products. That is the essential import of this paper, not the detail.