An Option-Based Operational Risk Management on Pandemics

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February, 2007

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Abstract:

In this paper, we employ the theory of real option pricing to address problems in the area of operational risk management. Particularly, we develop a two-stage model to help firms determine the optimal triggers in the event of an influenza pandemic. In the first stage, we propose a regime-dependent epidemic model to simulate the spread of the virus, depending on whether the firm is active or inactive. In the second stage, we view the reactivation decision as a call option and the suspension decision as a put option, and use dynamic programming method to determine the optimal switching thresholds. Our numerical experiments suggest that given the parameter values in our paper, it is optimal for the firm to suspend the business (or parts of its business) when the fraction of infected employees is higher than 18%, and to reactivate the operation anytime the fraction drops to 3%. When considering the uncertainty in the future, firms are more conservative about the decisions of suspension and reactivation. If the firm incurs switching costs, the suspension threshold increases with costs, while the reactivation threshold decreases with costs. By implementing policies to control the disease, firms can meet their social obligations and in the meantime, increase their values in both regimes.

Key Words: Real Option Valuation, Epidemic Risk, Operational Risk Management, Regime-Switching Model, Dynamic Programming

1. Introduction

There were totally 31 pandemics occurring in the past 500 years and 3 in the past century, of which the 1918-1919 “Spanish flu”, the most severe one, killed up to 50 million people worldwide and 500,000 in the United States (Rasmussen 2005, Robert Arnold et. al. 2006). Historic data have shown that influenza pandemics happen with frightening regularity and occur every 30 to 50 years. Given this pattern, the possibility of another pandemic attack is not considered remote. Ever since the isolated outbreaks of avian influenza in 2003, scientists have been particularly worrying about the influenza A (H5N1) virus. The Centers for Disease Control and Prevention (CDC) predicts 2 to 7 million deaths and medical treatment for tens of millions people even in a moderately severe scenario (Jia and Tsui 2005). The business impacts are startling as well. The World Bank estimates if the impacts of a moderately severe pandemic were to last for a year, the economic loss would be between $100 to $200 billion for the US, and around $800 billion globally (WBEAPR 2005).

Dynamics of human epidemics is an important topic in epidemiology and mathematical biology. An enormous literature has developed in this field, the history of which can be traced 100 years back to pioneers such as Kermack and McKendrick (1927). Ever since the publication of Bailey (1957), mathematical epidemiology has grown overwhelmingly. A wide variety of epidemic models have been mathematically formulated, analyzed and empirically fitted (see reviews in Dietz 1967, Wick wire 1977, Becker 1978, Dietz and Schenzle 1985, Hethcote 1994, etc). Basically, they can be classified into two main streams: deterministic models and stochastic models. In essence, the spread of a disease through a given population is a discrete stochastic process. Although continuous deterministic models are relatively easy to work with and often used to obtain acceptable approximations for relatively large populations, we prefer stochastic models since they can usually provide more information about the intrinsic variability of the system.

Recently, new interest arises in research to use epidemic modeling as decision aids for optimal control policies such as immunization, worker furloughs, and quarantines. Based on a deterministic epidemic model, Finkelstein et al (1981) construct a decision system under which alternative public immunization strategies can be compared. They find that vaccinating the population at large is sometimes favored over targeting at the highest-risk groups. Meltzer, Cox, and Fukuda (1999) employ Monte Carlo mathematical simulations and reach the same conclusion. Jia and Tsui (2005) use SARS as a case study to quantify the impact of various control measures. These models, however, only evaluate the effectiveness of control measures on pandemics based on national needs, and conduct the cost/benefit analysis from the macroeconomic perspective. They provide neither operating instructions for large businesses to prepare for pandemic risks, nor any insights as to the triggers for implementing the optimal control strategies.

In the event of influenza pandemics, businesses play an especially important role in protecting employees' health as well as minimizing the economic losses to the whole society. Business continuity planning has become a key component of operational risk management. It emphasizes the maintenance of critical operations and services during a crisis or a timely recovery of business after a disruption. Companies that provide infrastructure services, such as power and telecommunications, should devote significant resources to ensure continued operations during a crisis. Firms in financial sectors, like banks or insurance companies, also have a special responsibility to plan for business continuity and maintain the stability of the financial system. In order to assist businesses to plan for the outbreak of a pandemic, the HHS

(Department of Health and Human Services) and the CDC have developed a checklist, which identifies necessary activities for large businesses to prepare for the impact and establish policies for implementation during a pandemic.[1] In particular, it requires businesses to "set up authorities, triggers, and procedures for activating and terminating the company's response plan, altering business operations (e.g. shutting down operations in affected areas), and transferring business knowledge to key employees."

This paper is motivated by current concerns about the possible outbreak of avian influenza pandemics and the CDC's instructions for large businesses. Basically, the questions our paper addresses are:

1. In the event of an infectious disease such as an influenza epidemic, should a profit maximizing firm continue to operate with the loss of productivity of its employees, or suspend the business (or parts of its business) temporarily in order to avoid the contagion?
2. Does a firm's intention to maximize its value contradict with its social obligation to control the disease?
3. And most important of all, what are the optimal triggers to implement these strategies?

In order to answer these questions, we propose a two-stage model in this paper. The intuition is straightforward. In the first stage of the model, we adapt a stochastic model to describe the dynamics of an epidemic that spreads in a given company (or parts of its businesses). The productivity of an employee is reduced once he/she gets infected. The disease will spread and the fraction of the infective is increasing over time, which diminishes the revenue of the company due to the decrease in average productivity. When the fraction goes above a certain high threshold (we call it mothballing threshold in the following), the manager may want to

[1] See CDC. 2005. Business Pandemic Influenza Planning Checklist. Available at http://www.pandemicflu.gov/plan/business/businesschecklist.html

temporarily suspend its business (or parts of its business in the most affected areas) and send employees, whether infected or non-infected, back home. The separation of the employees may help to control the disease. As long as the fraction of the infective drops to a certain low threshold (reactivation threshold), the manager may want to call the employees back to work and continue the business. Therefore, a regime-switching model is employed in the second stage to determine these optimal switching thresholds. Regime switching model is based on the theory of real option valuation and can be solved by dynamic programming methods. I will discuss this in more details in the methodology part.

Our results suggest that given the parameter values in our paper, it is optimal for the firm to lay up the business (or parts of its business) when the fraction of infected employees is higher than 18%, and to reactivate the operation anytime the fraction drops to 3%. When considering the uncertainty in the future, firms are more conservative about the decisions of lay-up and reactivation. Upon the condition that firms incur lump sum costs when switching between regimes, the sensitivity analysis shows that the mothballing threshold increases with the mothballing cost and reactivation cost. On the contrary, the reactivation threshold decreases with the costs. By implementing measures to control the disease, firms can increase their values in both regimes, and thus meet their social obligations at the same time of maximizing their profits.

The rest of this paper is organized as follows. In Section 2, we describe the two-stage model we are going to use. In Section 3, we discuss the theoretical framework and the methodologies to determine the optimal triggers of control policies. In Section 4, we report the numerical results and conduct sensitivity analysis. Section 5 is concluding remarks.

2. The Two-Stage Regime Switching Model

In this part, we develop a two-stage model in this paper to determine the triggers for firms to activate or terminate the optimal response policy. In the first stage, we use a stochastic model to simulate the spread of the virus, which may depend on the regime that the firm is currently in. In the second stage, we employ a regime-switching model under uncertainty to determine the optimal switching thresholds. It is noteworthy to mention that although we consider the case of a manufacturing firm in this paper, the same model can be applied to firms providing critical services, such as infrastructure services and financial services. Actually, it is more reasonable to regard the firm as a division or parts of a large business that is in the affected areas.

2.1. Stage I: The Regime-Dependent Epidemic Model

Almost all the epidemic models share the common feature, that is, dividing the modeled population into different groups (passively immune, susceptible, exposed, infective and removed), and studying the disease transmission between different groups. In this paper, we choose to adapt a simple stochastic epidemic model originally proposed by Bailey (1957) and expanded later by Bartholomew (1973) in social science. Noting that the process of the epidemic may tie up with the choice the manager makes, we extend the model to two regimes, depending on whether the firm is active or mothballing.

To make the analysis simple, we assume the total number of employees in a given firm remains constant during an epidemic. There is no entry into or departure from the working force. Moreover, there are no deaths during the epidemic. This assumption may seem strong, but is still reasonable. On one hand, the time scale of an epidemic is generally shorter than the demographic time scale, the natural deaths are thereby negligible. On the other hand, provided our health care system is advanced enough, any infected employee can get appropriate medical treatment and recover from the disease. The possibility of death from the disease is also ruled out.

We classify the total working force into two categories, susceptible and infective. Although the most popular epidemic model is SIR model, we adapt this model by eliminating the removed class (R). The class R refers to those who have either died or recovered from the disease and thereby acquire immunity from infection. It is not only because the death from the disease is negligible based on the arguments above, but also because the recovered people may have access to the virus and are likely to get infected again. This recurrence of a disease is particularly common under the attack of an influenza pandemic.

Let $S(t)$ be the fraction of employees that has an infectious disease at time t. It is obvious that $S(t) \in [0,1]$, where $S = 0$ denotes nobody is infected in the firm and $S = 1$ shows another extreme case that all the employees are infected. It is reasonable to assume that the random variation is greater in the center region than in the extreme cases, therefore the variance term is proportional to $S(1-S)$. When the firm is active, the change in $S(t)$ usually follows the rule such as:

$$dS = [aS(1-S) - bS + c(1-S)]dt + \sqrt{\varepsilon S(1-S)}dw_t, \ a,b,c,\varepsilon > 0$$

where a is the rate of person-to-person transmission inside the company, b the rate of recovery, c the rate of transmission from an external source, ε a small positive constant, and w_t a standard Brownian motion.

Firms can use some control policies to alter the value of parameters, such as a, b and c, and reduce the spread of the contagion. For instance, when the epidemic breaks out, the firm can adopt some immunization programs to lower the rate of transmission between the infective and the susceptible. When some employees in the company get infected, the manager can screen the suspected infective and mandate the immediate full-paid leave. Furthermore, if the number of the infective increases and hits a predetermined point, the firm can temporarily close the business and send all the employees back home. In this way, the access of person-to-person transmission is cut off. The contagion will be controlled and the infective will recover from the disease with a higher recovery rate d . Assuming the external transmission rate and the random variations remain the same, the dynamics of the disease turns out to be:

$$dS = [-dS + c(1-S)]dt + \sqrt{\varepsilon S(1-S)}dw_t \,,\ d, c, \varepsilon > 0$$

where $d > b > 0$ is the recovery rate when the workers stay at home.

To conclude, the epidemic model can be described as:

$$dS = \mu(S,r)dt + \sigma(S,r)dw_t \,,$$

where we denote by r the regime of the firm ($r = 1$ if the firm is in suspension and $r = 2$ if the firm is active). Obviously, we have

$$\mu(S,r) = \begin{cases} aS(1-S) - bS + c(1-S), r = 2 \\ -dS + c(1-S), r = 1 \end{cases} \text{ and } \sigma(S,r) = \sqrt{\varepsilon S(1-S)}\,.$$

2.2. Stage II: Regime-Switching Model

Suppose that the manager cannot tell if an employee is infected individually (it may be due to lack of expertise, or too costly to do so), but he has some technique that can help him know the fraction of the infective $S(t)$.[2] As mentioned before, the manager wants to determine two optimal switching thresholds, S_H and S_L, such that if $S(t)$ is above S_H, the manager suspends the

[2] For example, he may use daily released data of disease cases, outpatient visits or hospitalization in this affected region to obtain a proxy of the fraction, which is acceptable especially when the disease is spreading rapidly in the region.

production temporarily and offers full-paid leave for all employees[3], and if $S(t)$ is lower than S_L, the manger calls back all the employees and reactivates the production. We make the above assumptions in order to avoid adverse selection and moral hazard problem. Otherwise, some of the infective will pretend to behave normally if they cannot get paid during the suspended period, and the non-infected employees will pretend to be infected in order to enjoy more leisure without being detected.

We normalize the productivity of a non-infected employee to unity, and assume that the productivity will drop to a given level $\alpha < 1$ once the employee gets infected. N is the total number of employees in the company. The price of the product, P, can be viewed as given, because we only consider one firm in a competitive market. The variable cost and fixed cost are denoted by C and K, respectively. (Note wages for the workers are contained in the fixed cost K because we consider full-paid leaves.) In addition, there is a penalty cost E to the firm for every infected employee when the firm is in active regime. The penalty cost may come from the employees' complain and reluctance to work, or the firm's loss of reputation in the future. Therefore the cash flow function can be defined as:

$$f(S,r) = \begin{cases} [\alpha SN + (1-S)N](P-C) - K - ESN, r = 2(active) \\ -K, r = 1(inactive) \end{cases}$$

Suppose there is a lump-sum cost of M when the firm is switching from the active to mothballing regime, and the reactivation cost is R when the firm resumes operation. The cost matrix is hence defined as $C = \begin{pmatrix} 0 & M \\ R & 0 \end{pmatrix}$. [4]

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[3] The full-paid leave is also suggested by the HHS and CDC in the Business Pandemic Influenza Planning Checklist.
[4] To avoid the possibility of "an infinite money machine", we assume $R + M \geq 0$.

The manager desires to maximize the expected discounted payment less any switching costs incurred over an infinite time horizon, by choosing the optimal regime at each moment. That is,

$$\max V(S,r) = E\left[\int_0^{\infty} e^{-\rho t} f(S_t, r_t)dt - \sum_{k=1}^{\infty}\sum_{i=1}^{m}\sum_{j=1}^{m} e^{-\rho t_k^{ij}} C_{ij}\right]$$

where t_k^{ij} are the times at which the agent switches from regime i to j.

3. The Methodology: Real Option Valuation and Dynamic Programming Method

The theoretical framework of the regime-switching model is based on real option valuation. Mossin (1968) launched the first discussion on this issue, followed by Brennan and Schwarz (1985) and Dixit (1989) who present the formal and complete regime-switching model. Based on the contingent claim theory, as McDonald and Siegel (1986) have done, Brennan and Schwarz apply the Black-Scholes-Merton formula to evaluate the active and inactive firms. They argue that the inactive firm has the option to invest and its value is equivalent to the value of this call option, with strike price equal to the entry cost. Likewise, the active firm has the option to exit the market and its value is determined by the current profit and the option to abandon. Considering the value-matching conditions and the smooth-pasting conditions, Dixit (1989) further obtains a pair of price thresholds for the entry and exit decision. During the 1990's, the baseline Dixit's model was extended in many directions. Allowing the possibility of laying-up or scrapping the project, Dixit and Pindyck (1994) consider four prices thresholds for investment, laying-up, reactivation and scrapping. Ekern (1993) relaxes the extreme assumption of complete irreversibility, and assumes restricted number of switches between states. Brekke and Oksendal (1994) introduce diminishing production capacity over time into the model and solve the model as a special case of sequential optimal stopping problem. Bar-Ilan and Strange (1996) consider the time delay between the decision to invest and the start of the production, and include one more state of nature (under construction) into the model.

Practically, the regime-switching model can be solved by dynamic programming methods. Miranda and Fackler (2002) and Fackler (2004) have a detailed discussion on this issue. Suppose that there are m regimes (i.e., $r = \{1,...,m\}$). The agent obtains a reward of payments $f(S,r)$ per unit time, which depends on both the discrete regime variable r and on a continuous state variable S. The dynamics of the state variable S can be described by

$$dS = \mu(S,r)dt + \sigma(S,r)dW_t .$$

The agent can move from regime i to j at a cost of C_{ij}, but there is no cost to remain in the current regime, i.e., $C_{ii} = 0$. The discount rate is $\rho(S)$.

In the interior of the no-switch regions, we have the so-called Feynman-Kac equation:

$$\rho(S)V(S,r) = f(S,r) + \mu(S,r)V_S(S,r) + \frac{1}{2}\sigma^2(S,r)V_{SS}(S,r)$$

The economic intuition underlying this condition is easier to understand if we rewrite the Feynman-Kac equation into the following form:

$$\rho(S)V(S,r) = f(S,r) + \frac{dE[V(S,r)]}{dt} ,$$ [5]

[5]. By Ito's lemma, the expected rate of appreciation of the asset $V(S,r)$ is

$$\begin{aligned} dV(S,r) &= V_S(S,r)dS + \frac{1}{2}V_{SS}(S,r)dSdS \\ &= V_S(S,r)(\mu(S,r)dt + \sigma(S,r)dw_t) + \frac{1}{2}\sigma^2(S,r)V_{SS}(S,r)dt \\ &= (\mu(S,r)V_S(S,r) + \frac{1}{2}\sigma^2(S,r)V_{SS}(S,r))dt + \sigma(S,r)V_S(S,r)dw_t \end{aligned}$$

Taking expectation and dividing both sides by dt, we get

$$\frac{dE[V(S,r)]}{dt} = \mu(S,r)V_S(S,r) + \frac{1}{2}\sigma^2(S,r)V_{SS}(S,r)$$

Plugging above equation into the Feynman-Kac condition, we have

$$\rho(S)V(S,r) = f(S,r) + \frac{dE[V(S,r)]}{dt}$$

which means the total rate of return in regime r equals the current reward $f(S,r)$ plus the expected rate of capital appreciation $\frac{dE[V(S,r)]}{dt}$.

At the boundary point S^*, supposing it is optimal to switch from regime r to regime q, the value function must satisfy two conditions at such a point. The first is value-matching condition, which will hold no matter whether the switching points are optimal or not. Namely, the value before switching must be equal to the value after switching less the switching cost.

$$V(S^*,r) = V(S^*,q) - C_{rq}(S^*)$$

The second is smooth-pasting condition that is satisfied at the optimal switching points, that is, the marginal values before switching must equal the marginal value after switching minus the marginal cost of switching.

$$V_S(S^*,r) = V_S(S^*,q) - C'_{rq}(S^*)$$

Optimal switching models generally require numerical approximations. We can approximate the value function by a family of approximation functions, i.e., $V(S,r) \approx \phi_r(S)\theta_r$, where $\phi_r(S)$ defines a set of n basis functions and θ_r is a vector of coefficients in regime r. If we take the switching points as given, the values of θ_r can be obtained by solving the Feynman-Kac equation

$$[\rho\phi_r(s_i) - \mu(s_i,r)\phi'(s_i,r) + \frac{1}{2}\sigma^2(s_i,r)\phi''(s_i,r)]\theta_r = f(s_i,r)$$

at a set of nodal values s_i, along with the non-optimality side conditions. And the optimality conditions can be used to solve the switching points by a root-finding algorithm.

4. Numerical Analysis

The calibration of the parameter values in this model proves to be difficult. We choose the disease parameter values that are both compatible with the empirical influenza pandemic data and based on the suggestions in other empirical works. Finkelstein et al. (1981) set the person-to-person transmission rate to be 0.75 in their paper. In order to reflect scientists' concern that the H5N1 virus will evolve in a way that allows for efficient human-to-human transmission, we set the transmission rate a at 1 in this paper. As to the recovery rate, it is defined as the reciprocal of the average number of days of the infective period. The commonly reported duration of influenza ranges from 1 to 5 days, therefore the recovery rate should change from 0.2 to 1. We select the recovery rate in the active regime (b) to be 0.4 and that in the mothballing regime (d) equal to 0.6 to indicate that the recovery rate should be higher when workers are separated from each other. The values of external transmission rate c and the volatility coefficient ε are selected based on Cobb (1998), where he suggests $c = 0.02$ and $\varepsilon = 0.1$.

The values of the other parameters used in this paper are set as follows. We choose these values both for making economic sense and for making the sensitivity analysis in the later session more evident: [6]

Discount rate: $\rho = 0.05$

Productivity of an infective: $\alpha = 0.5$

Total number of employees: $N = 1000$

Price of product: $P = 3$

Variable cost: $C = 1$

[6] For example, we could choose a relatively bigger value for the switching costs. At this time, our economic rational on the sensitivity analysis is still correct. However, we may get a zero reactivation threshold and it remains unchanged when we try to increase the switching cost. We, therefore, may not see the expected effect of increasing the switching costs.

Penalty cost: $E = 5$

Fixed cost: $K = 150$

Mothballing cost: $M = 300$

Reactivation cost: $R = 300$

4.1. The Stationary Probability Distribution of the Epidemic Process

People are concerned what will happen to the disease if no action is taken, i.e. no regime switching and no other controls. Will it spread over the population or die out gradually? What's the possible fraction of people who are infected by the disease? In order to answer these questions, we need to examine the distribution of $S(t)$ and get some statistical information.

Recall the variable we are interested in, $S(t)$, behaves according to a stochastic differential equation:

$$dS = \mu(S)dt + \sigma(S)dW_t ,$$

where $\mu(S) = aS(1-S) - bS + c(1-S)$ and $\sigma^2(S) = \varepsilon S(1-S)$

The probability density function of such a random variable depends not only on the random variable itself but also on time t, i.e. $f(S,t)$. The evolution of the probability density function is presented in the form of a partial differential equation:

$$\frac{\partial f(S,t)}{\partial t} = \frac{\partial}{\partial S}(\mu(S) f(S,t)) + \frac{\partial^2}{\partial S^2}(\sigma^2(S) f(S,t))$$

Generally speaking, an explicit solution to this equation is not available. We, therefore, turn to the stationary probability density function when the process reaches the equilibrium (i.e. when $\frac{\partial f(S,t)}{\partial t} = 0$). Wright (1938) has developed a formula to calculate the stationary probability density function:

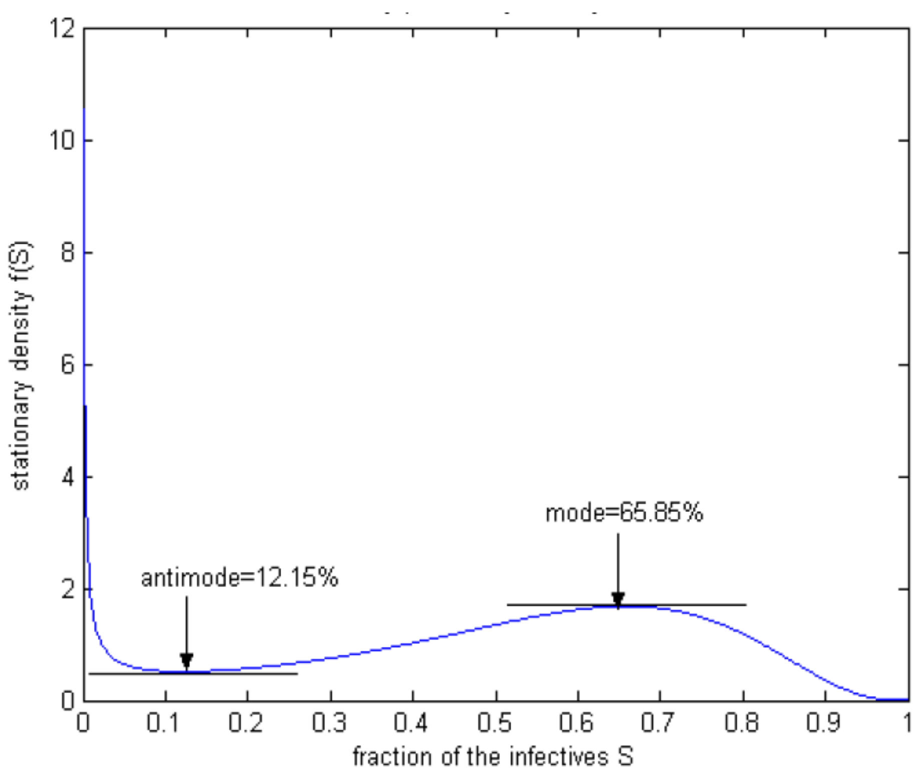
$$f(S) = \frac{\psi}{\sigma^2(S)} \exp\left\{ \int_{-\infty}^{S} \frac{\mu(x)}{\sigma^2(x)} dx \right\},$$

where ψ is a constant such that $\int_{-\infty}^{\infty} f(s)ds = 1$

As in our example, $f(S) = \frac{\psi}{S(1-S)} \exp\left\{ \int_{0}^{S} \frac{ax(1-x) - bx + c(1-x)}{\varepsilon x(1-x)} dx \right\}$

$$= \psi S^{-1+c/\varepsilon} (1-S)^{-1+b/\varepsilon} e^{ax/\varepsilon}$$

Figure 1: stationary probability density function



In Figure 1, the points s_1 and s_2 are the antimode and mode of the stationary probability density function respectively[7], and have important economic meanings. The antimode indicates a

[7] By solving the equation $f'(S) = 0$, we can get the antimode $s_1 = d - \sqrt{d^2 - (\varepsilon - c)/a}$, mode $s_2 = d + \sqrt{d^2 - (\varepsilon - c)/a}$, where $d = (a - b - c + 2\varepsilon)/2a$

threshold beyond which the epidemic is likely to spread, while the mode is the most likely fraction of the infective in the whole population. For the parameter values set in this paper, we obtain $s_1 = 12.51\%$ and $s_2 = 65.85\%$. The epidemic is unlikely to spread unless more than 12.51% of the workers are infected. And it is most likely that 65.85% of the population will get infected once the epidemic spreads.

4.2. Effect of Uncertainty: Stochastic versus Deterministic

Based on the MATLAB implementation proposed by Fackler (2004), we solve the above regime-switching problem by dynamic programming. The value functions and marginal value functions are displayed in Figure 2 and 3.

Given the parameters above, it is optimal for the firm in the active regime to close the business temporarily when the fraction of infected employees is higher than 18%. In the inactive regime, however, it is optimal to reactivate anytime the fraction drops to 3%. If there is no uncertainty in regard to the dynamic of the epidemic, i.e. $\varepsilon = 0$, the epidemic model in stage I becomes deterministic. The corresponding thresholds are 23% and 6% respectively, by our calculation.

We can see firms are more conservative about the decisions on suspension and reactivation when they take into account the uncertainty in the future: in the stochastic case, firms tend to suspend the business even when the fraction of infected employees is 5 points lower than that in the deterministic case, and won't come back to operation until the fraction drops by 3 points below 6% (the reactivation threshold in the deterministic case).

Figure 2: Value Function of the Stochastic Model

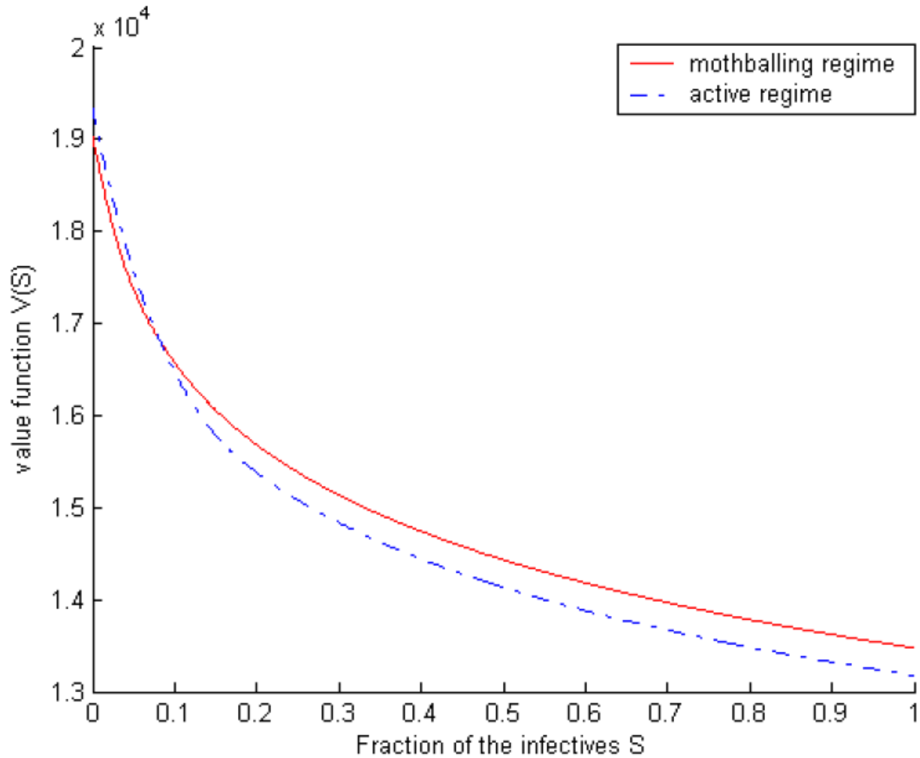
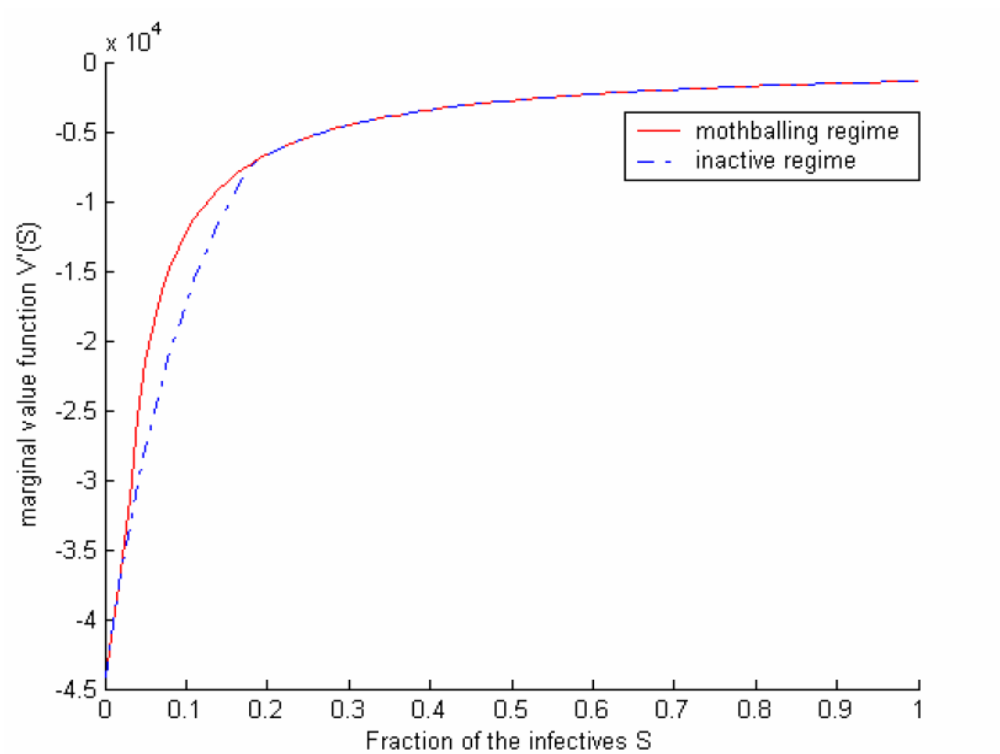


Figure 3: Marginal Value Function of the Stochastic Model



4.3. Effect of Switching Costs on Switching Thresholds

Our next interest is to examine effects of changing the mothballing cost and reactivation cost on the change of the two thresholds.

Table 1: Effect of changing switching costs

Initial parameters M=300, R=300	Mothballing threshold (S_H =18%)	Reactivation threshold (S_L =3%)	Conclusion
M=400, R=300	19%	2%	S_H increases with M, S_L decreases with M
M=300, R=400	19%	2%	S_H increases with R S_L decreases with R

First, let us consider the impact of an increase in the mothballing cost. As was shown in Table 1, the mothballing threshold S_H increases to 19% and the reactivation threshold S_L drops to 2%, when M rises to 400 and R remains at the original level. At the first glance, the change of the mothballing threshold is more understandable: when the mothballing cost increases, the firm needs to pay more when it switches to the inactive regime. Thus, the manager is more reluctant to suspend the business, and the mothballing threshold S_H increases as a result. The influence of the mothballing cost on the reactivation threshold S_L might need further consideration. Intuitively, the increase in the mothballing cost M decreases the reactivation threshold S_L according to the mirror image effect: the firm might reactivate the production with less willingness if it has to pay a large amount of lump sum cost once again when the fraction of the infective increases in the future, otherwise it would rather stay in the mothballing regime.

Similarly, if we increase the reactivation cost R to 400 while keeping M unchanged, the reactivation threshold S_L drops to a lower level, 2%. This is consistent with our prediction that the reactivation threshold S_L decreases with the reactivation cost R, because the firm is more reluctant to reactivate the production as the reactivation cost increases. Notice that the increase of reactivation cost also has an effect on the mothballing threshold: the mothballing threshold S_H increases to 19%. That's because the firm lays up the production with some reluctance to lose its option value. Considering the possibility that the fraction of the infective might drop in the near future, the firm could avoid paying the reactivation cost again by remaining in the active regime. Therefore, the larger is the reactivation cost, the larger is the option value and the greater is the reluctance to suspend.

The following graphs might help us observe the comparative results more clearly. In Figure 4, I keep the reactivation cost R unchanged and increase the mothballing cost M from 300 to 700. The switching costs changes like a step function. The mothballing threshold S_H increases from 0.18 to 0.19 when M rises to 370, to 0.20 when M is 530, and to 0.21 when M reaches 690. The reactivation threshold S_L decreases from 0.03 to 0.02 when M arrives at 340, and keeps constant afterwards.

Figure 5 shows the effect of the reactivation cost on the switching thresholds. We can see that the increase of the reactivation cost R almost leads to the same results as in Figure 4, except that the mothballing threshold S_H increases to 0.21 after R is 700.

Figure 4: Effect of Increasing the Mothballing Cost

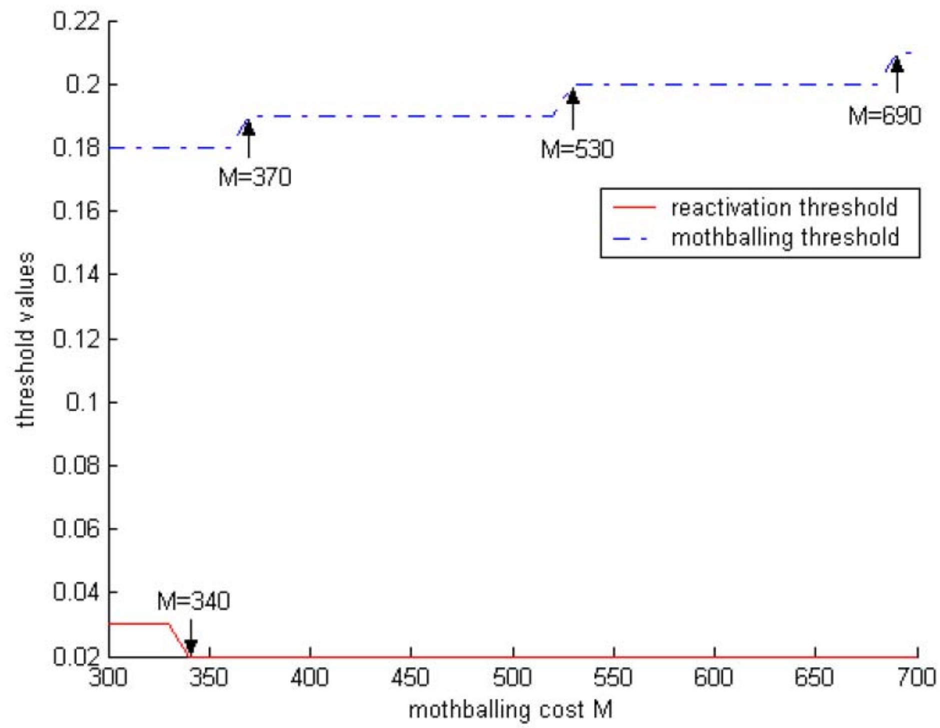
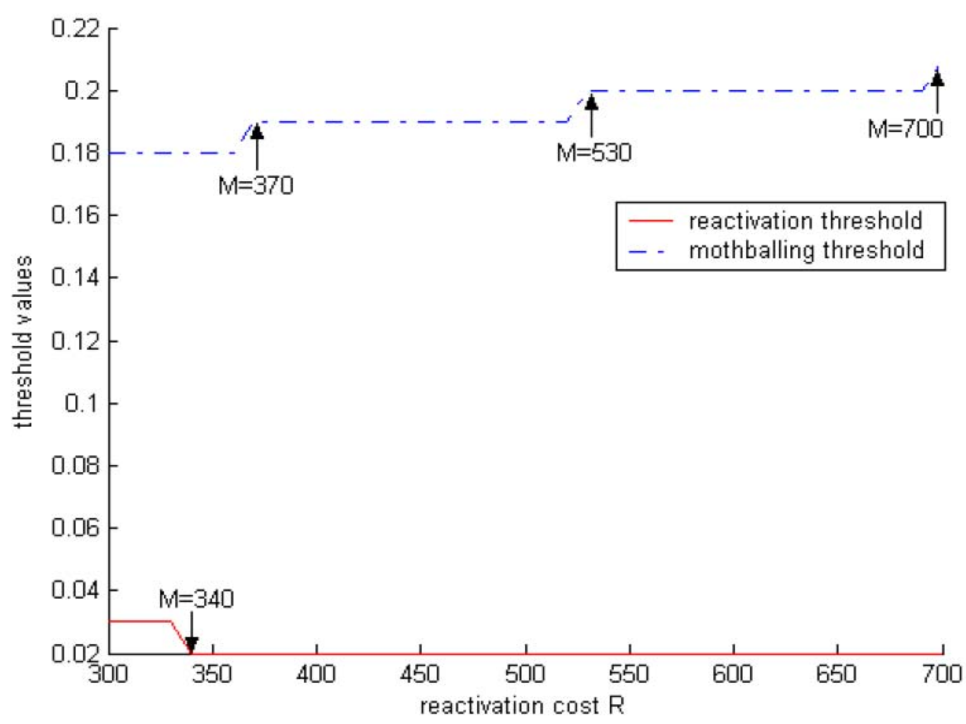


Figure 5: Effect of Increasing the Reactivation Cost



4.4. Effect of Disease Control Strategies

One of the purposes of modeling epidemics is to provide a rational basis for policies designed to control the spread of a disease. A firm could adopt different strategies which aim to alter the parameters in the epidemic model, so that the disease could be controlled. For instance, the firm could reduce the infectious contagion among its employees by adopting internet conference and phone meetings. It can screen the suspected infective and mandate an immediate leave for those who are thought to pose a risk. It could immunize some or all of the employees by vaccination. It can also initialize an information session to raise public awareness of higher disease prevalence and inform its employees of some preventive measures. All these control strategies are aiming at:

1. decreasing a, the rate of person-to-person transmission.
2. increasing b, the rate of recovery.
3. decreasing c, the rate of external transmission.

Interestingly, firms can implement these control policies at little cost, but can benefit a lot from these strategies. As shown in Figure 6, when the firm adopts some strategies to decrease the rate of person-to-person transmission a from 1 to 0.8, the value of the firm increases significantly in both regimes. Similar results can be obtained if we increase the rate of recovery b or decrease the rate of external transmission c (see Figure 7 and 8, respectively). Therefore, firms could maximize their values at the same time of meeting their social obligations of disease control.

Figure 6: Effect of Decreasing the Person-to-person Transmission Rate

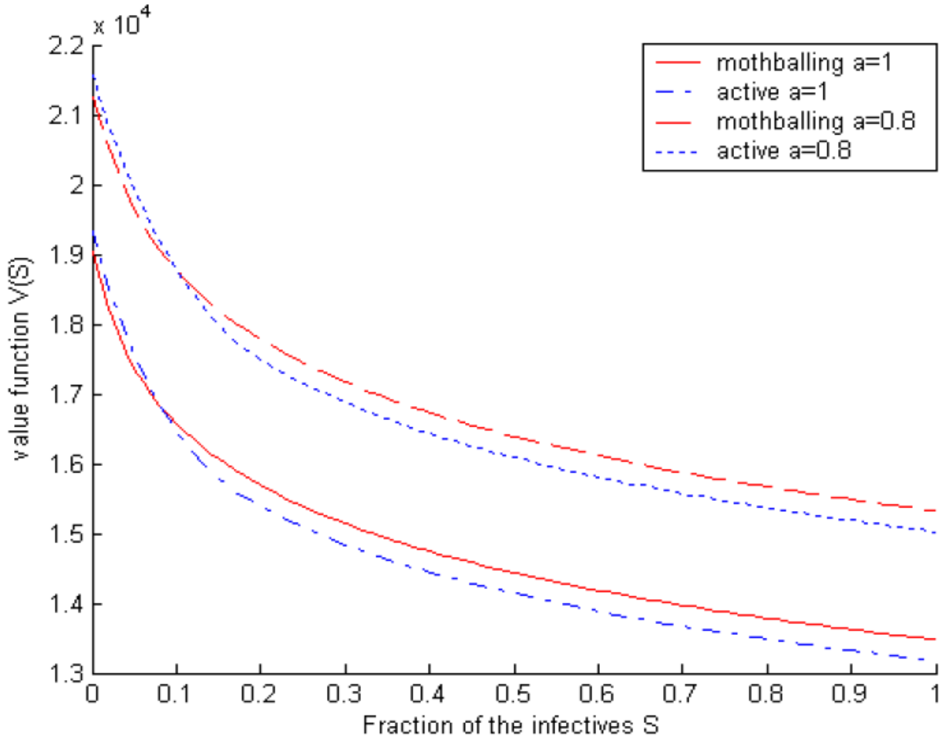


Figure 7: Effect of Increasing the Recovery Rate

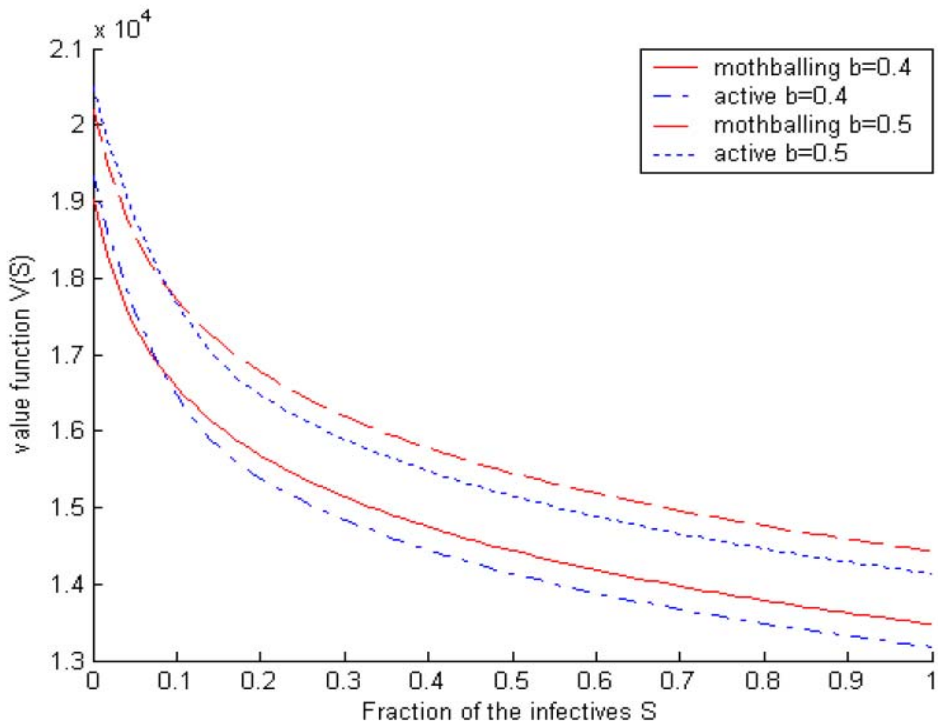
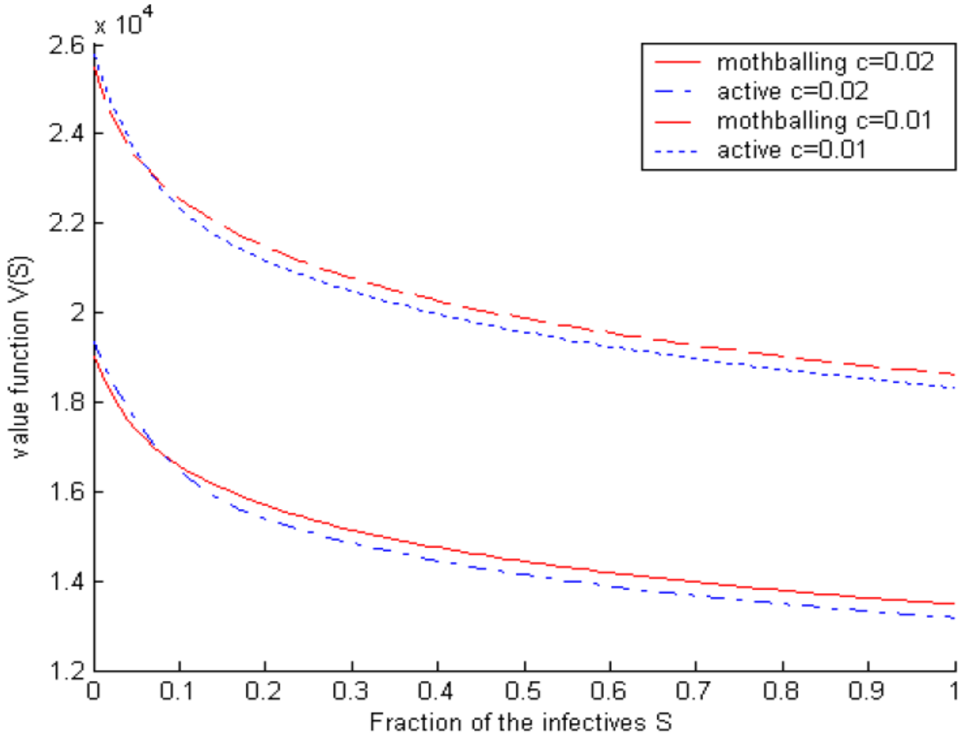


Figure 8: Effect of Decreasing the External Transmission Rate



5. Conclusion and Discussion

Dynamics of human epidemics is an important topic in epidemiology and mathematical biology. An enormous literature has developed in these fields, but it seems that little of it addresses epidemic risks facing private enterprises with large numbers of employees. Epidemic models may provide some insights as to the effectiveness of control measures such as immunization, worker furloughs, and quarantines. Modeling may lead to optimal rules for implementing these strategies. In this paper, we build a stochastic model to simulate the spread of an infectious disease, which is regime-dependent, and use the fraction of the infective in a given firm as a decision aid to construct an optimal control strategy. Dynamic programming is introduced and the switching points are found by numerical approximation.

Given the parameters set in our paper, the firm should suspend the business (or part of its business) when the infected workers account for 18% of the total work force, and reactive the operation as long as the fraction of the infective drops to 3%. Certainly, these triggers of suspension or reactivation are firm-specific: they rely on many factors involved in this model and are different for different firms. However, the economic rationale is universal. When faced with uncertainty in the future, firms are more conservative about the decisions of suspension and reactivation. If firms have to incur lump sum costs when switching regimes, the mothballing threshold increases with the switching costs, no matter it is mothballing cost or reactivation cost, and the reactivation threshold decreases with the costs.

In order to avoid the spread of the disease, we need to implement different strategies to decrease the rate of person-to-person transmission, increase the rate of recovery, or decrease the rate of external transmission. While these strategies aim at the control of the epidemic, the firms' value is also increasing in both regimes. Thus the social interests are in line with firms' own benefits. Disease control and firms' value maximization could be obtained simultaneously in our model.

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