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MODIFIED COST METHODS FOR SMALL PENSION PLANS

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ABSTRACT

The purpose of this paper is to discuss modified cost methods that have been proposed for small-plan valuations. The first portion of the paper deals with criteria for choosing a cost method. The next portions deal with small-plan characteristics and the potential shortcomings of conventional cost methods when applied to these plans. The final portion discusses two classes of modified cost methods that have been proposed for circumventing these shortcomings.

1. INTRODUCTION

Actuarial cost methods for defined benefit pension plans have been discussed in numerous articles.[1] Almost without exception, however, the discussion has been within a large-plan context, so that there is very little specific information dealing with actuarial cost methods designed for small plans. This also is true for recognized texts in the area. Furthermore, some actuarial cost methods that are appropriate for small plans are not mentioned in the standard valuation literature.

This is an unfortunate state of affairs. Small plans have characteristics quite distinct from those of large plans, aside from the obvious difference in size, and there may be serious consequences if large-plan valuation methods are applied in a small-plan environment.

The purpose of this paper is to discuss modified cost methods that have been developed for small-plan situations. The first section presents criteria for choosing a cost method. The second section discusses small-plan characteristics. Next, a case study is used to explore potential shortcom-

[1] See, for example, Stephen L. Cooper and James C. Hickman, "A Family of Accrued Benefit Actuarial Cost Methods," *TSA*, XIX (1967), 53–59; Richard Daskais, *Aggregate Entry-Age Cost Methods of Pension Funds*, Society of Actuaries Study Note 81-21-76; Hilary L. Seal, "Acceptable Funding Methods for Self-insured Pension Plans," *PCAPP*, II (1952), 17–44; John R. Taylor, "The Generalized Family of Aggregate Cost Methods for Pension Funding," *TSA*, XIX (1967), 1–12; and Charles L. Trowbridge, "Fundamentals of Pension Funding," *TSA*, IV (1952), 17–43, and "The Unfunded Present Value Family of Pension Funding Methods," *TSA*, XV (1963), 151–69.

ings of conventional cost methods when applied to small plans. This is followed by a discussion of the distinctive characteristics of two classes of modified cost methods that have been proposed for the small-plan environment.

II. CRITERIA FOR CHOOSING AN ACTUARIAL COST METHOD

The first step in choosing a particular actuarial cost method is to select criteria to be met. Possible criteria are given in Table 1.

Adequacy

The first consideration is adequacy. An actuarial cost method satisfies the condition of adequacy if plan assets keep pace with the present value of accrued benefits (*PVAB*). The present value of accrued benefits is important because, if a plan is to be terminated and assets can be distributed to each participant equal to *PVAB* for that participant, there will be no discrimination.[2] In simple terms, if there is not enough money, there is a problem.

If the adequacy requirement is that the chance of insufficient funds at time t be less than $e(t)$, it can be expressed in the form

$$\Pr\{A(t, j) \geq PVAB(t, j)\} \geq 1 - e(t), \quad \text{for all } t, j,$$

where $A(t, j)$ is an appropriate asset allocation at time t to the jth participant, and $PVAB(t, j)$ is the present value of accrued benefits at that time

TABLE 1

CRITERIA FOR CHOOSING AN ACTUARIAL COST METHOD

Adequacy: Assets keep pace with the present value of the accrued [vested] benefits.

Consistency: The method develops an annual cost that is a level dollar amount or is a level percentage of total compensation.

Conditional consistency: If all assumptions materialize, the method develops an annual cost that is a level dollar amount or is a level percentage of total compensation.

Flexibility: The plan sponsor has some flexibility so far as annual contributions are concerned.

Robustness: The method automatically adjusts for adverse experience.

Precision: The method minimizes the difference between actual assets and required assets.

Definition: If all assumptions materialize, actual assets equal expected assets; that is, no gains or losses occur.

Deductibility: The annual cost developed by the method is deductible by the corporation for federal income tax purposes.

Simplicity: The method obviates cumbersome techniques.

[2] See Rev. Rul. 80-229. The reader might prefer to use alternative conditions for this bound, such as the PBGC constraints of sec. 4044(a) of ERISA.

for that participant. *PVAB* may be computed under any of the methods allowed by the IRS,[3] but in the small-plan area it is generally computed by using the fractional rule.

Benefits are fully accrued at the normal retirement age. Thus, for example, in a professional corporation the adequacy constraint ensures that, with a stated probability, the funds on hand will be adequate to retire the plan sponsor(s).[4]

In a small plan, it is difficult to measure the probability of adequate funds at each duration. An alternative approach would be to trace the results of a number of sensitivity studies, and then to form a subjective judgment regarding probable adequacy. This approach is taken in the current study.

Consistency

A second consideration is consistency. A method has consistency if it develops an annual cost that is a level dollar amount or is a level percentage of compensation. There likely will be dissatisfaction from the accountant or plan sponsor, or both, if annual contributions are erratic because of the actuarial cost method.

Let $C(t)$ denote the annual contribution to a pension plan at time t, let $S(t)$ denote the annual covered compensation at time t, and let $k(t)$ denote an acceptable upper bound on the absolute variability in cost between time $t - 1$ and t. Then the consistency constraint can be written in the form

$$|\nabla C(t)| \leq k(t)$$

in the case of level dollar costs, and

$$|\nabla[C(t)/S(t)]| \leq k(t)$$

in the case of cost as a level percentage of compensation.

There are a number of factors that mitigate against level costs: the maximum dollar benefit cannot be anticipated, so there is a built-in step function where large benefits are involved; social security offsets may

[3] See IRC, secs. 411(b)(1)(A)–(C).

[4] While the investment of assets generally lies outside the purview of the actuary, the adequacy condition depends on there being sufficient recognition of the cash-flow needs of the plan. In this regard, things like liquidity must be considered. If assets have to be unloaded because of the retirement of a key person, for example, it could have a serious impact on adequacy.

decrease retirement benefits for active participants; investment results and, indeed, actual experience from all sources may affect the contribution by causing gains and losses reconciliation; if a plan appears to be fully funded, contributions must be curtailed or reduced; new participants, since they result in little additional cost while increasing the total compensation base, may reduce the level of contributions; and the retirement of key people may reduce the contribution considerably.

Conditional Consistency

A related consideration is conditional consistency. A method has conditional consistency if, given that all assumptions materialize, it has consistency.

If $a(t, j)$ is the value of the jth parameter at time t, then the conditional consistency constraint can be expressed in the form

$$\hat{a}(t, j) = E\{a(t, j)\} \Rightarrow |\nabla C(t)| \leq k(t)$$

in the case of level dollar costs, where a circumflex indicates an actual value, and

$$\hat{a}(t, j) = E\{a(t, j)\} \Rightarrow |\nabla[C(t)/S(t)]| \leq k(t)$$

in the case of cost as a level percentage of salary.

Note that an actuarial cost method can have conditional consistency without having consistency. The converse, however, is not true.

Flexibility

An actuarial cost method has flexibility if the plan sponsor has some control over the incidence of the contributions. The flexibility constraint may be expressed in the form

$$\tilde{C}(t) - L(t) \leq C(t) \leq \tilde{C}(t) + U(t) ,$$

where $\tilde{C}(t)$ is the base cost for the plan at time t, and $L(t)$ and $U(t)$ are the lower-bound factor and upper-bound factor, respectively.

One source of flexibility is the separate amortization of past-service liability. In this case, for example, $\tilde{C}(t)$ might be the normal cost, $L(t)$ the contribution required to amortize the past-service liability over thirty years, and $U(t)$ the contribution required to amortize the past-service

liability over ten years.[5] Some additional flexibility may be obtained if gains and losses are involved, the extent of which will depend on the way in which gains and losses from various years are combined and the duration over which they are amortized.

Quite often, there is limited opportunity for flexibility in the small-plan area. This is particularly true with professional corporations, where the sponsor is within ten or fifteen years of retirement at the inception of the plan. In this type of situation, commonly suggested vehicles for flexibility, such as the independent amortization of the past-service liability, often are not feasible. Also, the cost of professional services varies directly with the complexity of the flexibility program. Affordability can be an important consideration.

Robustness

An actuarial cost method has robustness if it can automatically adjust for unanticipated experience. In the small-plan environment it is not unusual for actual experience to vary considerably from projected experience, and it is comforting to know that the cost method will not break down under adverse conditions. Thus, among other things, if there is a significant asset loss in the year immediately preceding retirement, the cost method will automatically make up the deficiency. To a large extent, the concern for robustness is the impetus for modifying traditional cost methods.

Robustness guarantees that at the retirement of any participant there will be sufficient funds. That is, at the time of retirement of the jth participant,

$$A(t, j) \geq PVRB(t, j)$$

and

$$A(t, i) \geq PVAB(t, i) , \quad \text{for all } i \neq j ,$$

where $PVRB(t, j)$ is the present value of the retirement benefits at time t for that participant.

A necessary condition for a cost method to be robust is that it explicitly recognize the current assets. This is not, however, a sufficient condition. Additionally, the fact that a method is robust does not guarantee sufficient funds at each point in time. If, for example, an asset loss or a benefit liberalization took place in a given year, it is conceivable that assets would be less than *PVAB* for the next few years.

[5] See IRC, secs. 401(a) and 412(a).

Precision

The next consideration is precision. The higher the precision of a cost method, the smaller the difference between the actual assets and the required assets at any point in time. The precision constraint takes the form

$$|A(t, j) - PVAB(t, j)| \leq k(t) , \quad \text{for all } t, j ,$$

or

$$|A(t, j) - PVAB(t, j)| \leq k(t)PVAB(t, j) , \quad \text{for all } t, j ,$$

and may be regarded as the inverse of the variability of the cost method. Note that the required assets are defined as equal to *PVAB*, and again the reader may prefer to use some other definition.

The precision constraint is particularly important where the employer is concerned with fine-tuning the required pension contribution. Thus, the employer may want to have sufficient funds on hand but at the same time not want to be "overfunded."

Definition

The next consideration is that the cost method be well defined, in the sense that if all assumptions materialize, actual assets will equal expected assets and no gains or losses will occur. This constraint is important, since a consistent significant gain or loss, for other than nonrecurring causes, suggests a potential problem with IRS auditors.

If $a(t, j)$ is the actual value of the jth parameter at time t, then an actuarial cost method is well defined if

$$\hat{a}(t, j) = E\{a(t, j)\}, \quad \text{all } j \Rightarrow \hat{A}(t) = E\{A(t)\} .$$

Note that this condition refers to each individual parameter, and not to the parameters in the aggregate.

An acceptable or reasonable actuarial cost method is recognized by IRS regulations as one that has conditional consistency as far as normal cost is concerned, and is well defined.[6]

Deductibility

Another consideration, for other than nonprofit organizations, is deductibility as a business expense. A cost method whose cost does not fall within deductibility limits is ineffective.

[6] See Reg., secs. 1.412(c)(3)-1(b) and (c).

In general, contributions are deductible if they are computed by an acceptable cost method. However, contributions must be curtailed if the plan becomes fully funded, the test of which is based on an upper bound called the full-funding limitation (*FFL*). If the potential contribution exceeds the full-funding limitation, the excess is not deductible during the current year.[7]

The full-funding limitation at the end of a plan year for actuarial cost methods that develop an accrued liability is equal to

$$FFL(t + 1^-) = AL(t) + NC(t) + P(t) + I(t + 1) - A(t) ,$$

where $t + 1^-$ denotes the end of the $(t + 1)$st plan year, $AL(t)$ is the accrued liability at time t, $NC(t)$ is the normal cost at time t, $P(t)$ is the insurance premium if any at time t, and $I(t + 1)$ is the accrued interest over the interval t to $t + 1$. If the cost method does not develop an accrued liability, the full-funding limitation is based on the entry-age normal cost method.[8]

Other things being equal, anything less than the full-funding limitation is deductible as a contribution; anything more is not. Thus the deductibility constraint at the end of the plan year is given by

$$C(t + 1^-) \leq FFL(t + 1^-) .$$

It is worth noting that the effect of the full-funding limitation is the immediate recognition of unanticipated increases in asset values. This can have an erratic effect on the maximum deductible contribution, depending on investment experience among other things.[9] It also means that exogenous factors can limit the contribution.

Simplicity

A final consideration is simplicity, in the sense that the method obviates cumbersome techniques. Depending on the circumstances, such techniques may include gain and loss analysis and amortization and the maintenance of extensive employee census data.[10]

[7] This is a particular problem in an inflationary environment.

[8] IRC, secs. 412(c)(6) and (7).

[9] This same problem can occur in a plan where there is considerable turnover among older nonvested employees. If the employer hires middle-aged or older people who never become vested, gains may act to accelerate full-funding limitation problems.

[10] There are many instances in small-plan valuations where simplicity of computation is

Comment

It should be obvious that many of the foregoing attributes are not compatible. A cost method generally will not have both robustness and consistency, for example. Therefore, given a funding policy, the problem becomes one of choosing those attributes that are most consistent with that policy.

The purpose of this paper is to discuss cost methods that are appropriate for small-plan valuations. Before doing so, it is appropriate to consider the salient features of small plans.

III. SMALL-PLAN CHARACTERISTICS

There is no generally accepted definition of a small pension plan. Possible definitions include plans with less than a given number of active or total participants, such as twenty-five;[11] plans with an annual contribution of less than a given amount ($20,000, for example);[12] plans in which more than some percentage of the contribution is made on behalf of the plan sponsor(s);[13] and some combination of these three. The last is probably the most appropriate.

Whatever definition of small plans is used, such plans usually have three unique characteristics. The first has to do with permanency. The second characteristic is concerned with the demographic makeup of the plan. The third characteristic relates to the future experience of the plan.

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an overriding consideration. The proliferation of affordable computers ultimately may eliminate this problem, however.

[11] In its annual reporting requirements, the federal government segregates plans into two major groups, those with less than 100 participants and those with 100 or more participants. Purportedly, since plans with less than 100 participants have less stringent reporting requirements, the federal government regards these as small plans. The practitioner, however, generally regards twenty-five or so to be an upper limit. According to the PBGC, about 56.6 percent of the single employer plans fall into this category. See *Analysis of Single Employer Defined Benefit Plan Terminations, 1977* (Washington, D.C.: PBGC, November 1978), p. 18.

[12] This definition of a small plan can be misleading. It would lead to the conclusion, for example, that a plan with fifty participants and a $20,000 annual contribution was a small plan, while a plan with a single participant and a $150,000 contribution was a large plan. Few small-plan practitioners would support such a dichotomy.

[13] Many small pension plans are established because the plan sponsor(s) understands that most of the contribution will be made on his behalf. Often, for example, the benefit formula is structured so that 70 percent or more of the contribution is implicitly allocated to the plan sponsor(s).

Quasi-Permanency

While most plan sponsors intend their plan to be permanent,[14] and thereby in compliance with IRS requirements, it is questionable whether small plans generally have this attribute. The termination rate of small plans may be many times greater than that of large plans.[15] Furthermore, about half the terminated plans in the post-ERISA period have been maintained for fewer than eight years, and a disproportionate percentage of these are small plans.[16] These observations would seem to argue against the permanency of many small plans.

Nonhomogeneity

In many small plans the bulk of the total annual contribution is attributable to the funding of anticipated benefits of a few owner-employees. The financial demands on plan assets generally are dominated by the specific experience of these few. Such nonhomogeneity can disturb cash flow.

Limited Experience

In small plans, actual experience is likely to deviate substantially from expected experience. There are two main reasons for this. First, the historical data generally are insufficient to develop such things as decrement factors. Where these factors are relevant, published tables, rather than the firm's own experience, must often be used. Second, even if these factors were known, the limited exposure would undoubtedly lead to considerable variation from anticipated experience.

[14] Under income tax regulations issued by the Treasury Department, sec. 1.401-4(c)(2)(1), a plan may be disqualified if it terminates prior to ten years and discriminates in favor of high-paid employees. Under Rev. Rul. 69-25, 1969-1, C.B. 113, if a plan is discontinued within a few years after its adoption, it may be assumed not to have been a permanent commitment of the employer, unless terminated because of business necessity that could not reasonably have been foreseen when the plan was adopted.

[15] In plans with 1–5, 6–9, and 10–25 participants, for example, the termination rates are approximately 0.14, 0.13, and 0.09, respectively, as compared with a termination rate of 0.007 for plans with 500 or more participants. See PBGC, *Analysis of Single Employer Defined Benefit Plan Terminations, 1977*, cited above.

[16] In the PBGC study cited above, 87.1 percent of the terminations were plans with twenty-five or fewer participants, while these comprised only 56.6 percent of all plans. Among the major non-ERISA reasons for termination were change in ownership, closing, and adverse business conditions, which accounted for 10, 10, and 22 percent of the terminations, respectively.

It seems clear that the basic assumptions underlying large-plan valuations (permanency, homogeneity, and large exposure) can be inappropriate for small plans, and that special valuation techniques may be required.

IV. SHORTCOMINGS OF CONVENTIONAL COST METHODS

For the purpose of exploring the shortcomings of conventional cost methods when applied in a small-plan environment, consider a plan that provides only retirement benefits. The salient features of the plan are that the monthly retirement pension is 20 percent of final five-year average monthly salary plus 30 percent of such salary above $600, and vesting is on the basis of the 4-40 rule.[17]

The firm starts with seven employees at the inception of the plan. In subsequent years employees terminate and new employees are hired. It is anticipated that each employee who terminates and is not an owner or key person will be replaced by another employee. The new employee will have the same characteristics as the terminating employee, except that the attained age and salary of the new employee will be the hiring age and salary of the terminated employee. The characteristics of the employees at the inception of the plan are shown in Table 2.

TABLE 2

EMPLOYEE CHARACTERISTICS AT INCEPTION OF PLAN

Name	Sex	HA	EA	PA	AA	IVA	ERA	NRA	Salary
1. Owner 1	M	41	41	49	49	45	59	65	$42,000
2. Owner 2	M	26	27	57	57	30	65	65	48,000
3. Key person 1	M	29	29	33	33	33	55	65	24,000
4. Key person 2	F	23	24	54	54	27	64	65	12,000
5. Clerical 1	F	22	24	24	24	26	55	65	8,000
6. Clerical 2	F	27	28	29	29	31	55	65	8,000
7. Clerical 3	F	20	24	24	21	24	55	65	8,000

NOTE.—HA = age at hire; EA = age at participation if plan had always been in effect; PA = age at participation (age 24 and six months' service); AA = attained age at valuation date; IVA = initial vesting age (4-40 rule); ERA = early retirement age (age 55 and ten years' participation); and NRA = normal retirement age (age 65).

[17] Under the 4-40 vesting rule, an employee who has completed at least four years of service has a nonforfeitable right in his accrued benefits as shown below:

Years of Service	Nonforfeitable Percentage	Years of Service	Nonforfeitable Percentage
4	40%	8	70%
5	45	9	80
6	50	10	90
7	60	11 or more	100

It is important to note that the plan under investigation presents a distinct problem from a funding point of view. The plan sponsor is only eight years away from retirement. This being the case, any funding method which anticipates that future contributions will be higher than current contributions is likely to promote underfunding. Similarly, any funding method that involves funding over an extended period of time is unlikely to be satisfactory.

It is anticipated that the firm will grow, and that by the end of the eighth year (just prior to the retirement of the plan sponsor) there will be twelve employees. All future employees will be clerical, however, and their expected active lifetime is anticipated to be two years if their hiring age is 24 or under, and five years otherwise. As with the employees at the inception of the plan, it is anticipated that when these future employees terminate they will be replaced with employees having similar hiring-age characteristics.

Actuarial Assumptions

It is assumed that the effective annual interest rate earned on invested funds is 6 percent. Depending on the nature of the actuarial cost method, the annual rate of salary increment is taken to be 0 or 5 percent. Preretirement mortality, if any, is assumed to follow the 1971 GAM Table. Termination rates, if any, are assumed to be approximately 25 percent at age 25, dropping off to 0 percent at about age 55.

Scenarios

The sensitivity analysis of the cost methods is done by imposing various scenarios on both the investment income and projected salary. The combinations used include an interest rate and salary scale of 6 percent, an interest rate of 5 percent and a salary scale of 6 percent, and then finally a 3 percent interest rate and 10 percent salary scale. The last is intended to test the robustness of the cost methods. The impact of various amortization periods is also considered.

Analysis

The analysis involves two distinct considerations. The first is the annual contribution as a percentage of salary for the first ten plan anniversaries. This provides insight into the consistency of the contributions under various conditions. The second is plan assets as a percentage of the present value of vested accrued benefits (PVVAB). If assets drop below 100 percent of the PVVAB, there are insufficient assets to terminate the plan properly. A plan is in a ruin condition if its assets are depleted and there

are unfunded vested benefits outstanding. Table 3 summarizes the findings for the criteria of adequacy, consistency, flexibility, and robustness based on a subjective classification scheme[18] for the primary ERISA actuarial cost methods.[19] Consideration was limited to these criteria, since they are the ones most often discussed.[20]

The accrued benefit cost method (ABCM) without supplemental liability does not appear to be suitable for the type of plan under consideration. Its attributes are that it provides some flexibility owing to gain and loss amortization, and it is somewhat responsive to the precision criterion.

TABLE 3

SUMMARY OF RESULTS

Cost Method	Adequacy	Consistency	Flexibility	Self-correction
Accrued benefit:				
Without supplemental liability	Poor	Poor	Fair	Poor
With supplemental liability	Ruin	Fair	Good	Poor
Entry-age normal	Poor	Poor	Good	Poor
Individual level premium	Good	Poor	Fair	Fair
Aggregate	Poor	Good	Poor	Poor
Frozen initial liability	Fair	Poor/Fair	Good	Poor/Fair

[18] The classification schemes for these criteria are as follows. For adequacy: *ruin*, if there is a depletion of assets in at least one case; *poor*, if there are insufficient assets in all cases; *fair*, or *marginal*, if there is at least one case when assets are sufficient; *good*, if assets are insufficient only in the adverse case; and *very good*, if assets are always sufficient. For consistency: *poor*, if the contribution as a percentage of salary varies significantly under all scenarios; *fair*, if the ratio is relatively constant in all but the adverse case; and *good*, if the ratio is relatively constant in all cases. For flexibility: *poor*, if there is not amortization of gains or losses or supplemental liability; *fair*, if there is amortization of gains or losses only; and *good*, if there is amortization of gains or losses and supplemental liability. For robustness: *poor*, if there are insufficient assets for the retirement of all participants at normal retirement age in all cases; *fair*, if there are insufficient assets only in the adverse case; and *good*, if there are always sufficient assets. Finally, for precision: *poor*, if the difference between actual assets and required assets exceeds 20 percent of the required assets in all cases; *fair*, if this happens only in the adverse case; and *good*, if this never happens.

The deductibility question is not considered here, since the scenarios are not conducive to deductibility problems.

[19] ERISA, sec. 3(31). The attained-age cost method, which is the other cost method mentioned in ERISA, is not considered in this study, since it is seldom, if ever, advocated for the small-plan area.

[20] The investigation of a model based on decisions with multiple objectives, which involves all criteria of Table 1, is currently being undertaken by the author and will form the basis of a sequel to the present study.

This latter is not surprising, since, if all assumptions materialized, this method would have perfect precision. All other cost methods were poor with respect to precision. The ABCM without supplemental liability is deficient in that assets may be insufficient to support completely the retirement of the plan sponsor.

Similarly, the ABCM with supplemental liability seems unsuitable. The use of the method is particularly questionable as the amortization period is increased. In the limiting case, funding the retirement benefits of the plan sponsor would deplete plan assets completely, resulting in a ruin situation.

The entry-age normal cost method, where increases in benefits are funded through an incremental normal cost based on attained age, fares poorly on all accounts except flexibility.

The individual level premium cost method, where increases in benefits are funded through an incremental normal cost based on attained age, has good adequacy and is fairly robust. However, it has poor consistency and only marginal flexibility.

The traditional aggregate cost method is satisfactory only with respect to consistency, and is unacceptable on all other counts.

The frozen initial liability cost method is marginal with respect to adequacy and is unsatisfactory with respect to robustness. Its primary attribute is flexibility.

Comment

The criteria of adequacy and robustness are often the most important considerations for small plans. For the plan in question with respect to these criteria, the foregoing cost methods seem deficient except for the individual level premium method.

The next section describes two actuarial cost methods that appear to overcome these deficiencies.

V. MODIFIED COST METHODS

Two cost methods that have been suggested for the valuation of small plans are the modified aggregate cost method (MACM) and the individual aggregate cost method (IACM). These cost methods are defined in this section.

Modified Aggregate Cost Method

In general, contributions under aggregate cost methods are equal to the difference between the total present value of future benefits and the net

assets divided by the sum of weighted annuities. The net assets are the current assets minus accumulated credit balances plus accumulated funding deficiencies.[21] Thus, they have the form

$$\frac{\Sigma\ PVFB\ -\ \text{Net assets}}{\Sigma\ PV\ \text{(weighted annuity)}}\ .$$

The weighted annuities have the general form

$$w(j)\ \Sigma_t\ v^t\ {}_tp_x^{aa}\ (1\ +\ s)^t\ ,$$

where $w(j)$ is the weight associated with the jth participant, s is an effective salary increase rate per annum,[22] and v and p are the discount factor and the persistency factor, respectively.

The distinguishing feature of the MACM is the definition of the weighted annuities. When the traditional aggregate cost method (ACM) is used in small plans, the weight is usually the compensation at the current attained age:

$$w(j)\ =\ AS(t,\ j)/\Sigma_j\ AS(t,\ j)\ ,$$

where $AS(t,\ j)$ is the actual salary at time t for the jth participant. In contrast, the annuities of the MACM are weighted by tabular normal cost (TNC), where the tabular normal cost is defined as the present value of future benefits divided by the present value of an active-life annuity, weighted by a salary-scale function, as of some pivotal age:

$$w(j)\ =\ TNC(j)/\Sigma_j\ TNC(j)\ ,$$

where

$$TNC(j)\ =\ PVFB(j,\ y)/{}^s\ddot{a}_y^{aa}$$

and ${}^s a_y^{aa}$ is an active-life annuity for the jth participant, from the pivotal age to the normal retirement age.[23] Suggested pivotal ages are the entry

[21] Rev. Proc. 80-50, sec. 3.02(2). Briefly, credit balances are past contributions that were in excess of minimum funding requirements, and funding deficiencies result from contributions that were less than minimum required contributions.

[22] Salary scales, themselves, may be regarded as weights that anticipate compensation increases in the future. In this regard, salary-scale factors are occasionally used.

[23] It is worth noting that the traditional frozen initial liability cost method may be modified in much the same way as the MACM, and for the same reason. The advantage of doing this may be dampened considerably, however, depending on the extent of the frozen initial liability.

age, the participation age, and the attained age. For the purpose of this paper the participation age is used.

A variation of this cost method that has been advocated by the IRS requires that the tabular normal cost be modified by the incremental tabular normal cost after the first year.[24] Then, in any year after the first, if x is the attained age of the jth participant,

$$TNC(t, j) = TNC(t - 1, j) + \Delta TNC(t - 1, j) ,$$

where

$$\Delta TNC(t - 1, j) = \Delta PVFB(t - 1, j)/{}^{s}a_{x}^{\text{an}} .$$

Individual Aggregate Cost Method

Under the IACM, the contribution on behalf of each participant is computed as the present value of future benefits, net of a hypothetical asset allocation (AA),[25] divided by an active-life annuity that is weighted by a salary-scale function. Specifically, the contribution in the tth year for the jth participant is

$$\frac{PVFB\ (t, j) - AA(t, j)}{PV(\text{weighted annuity})} .$$

The asset allocation, $AA(t, j)$, is developed from an asset allocation factor $AAF(t, j)$, and is given by

$$AA(t, j) = AAF(t, j) \times \text{Net assets} .$$

ALLOCATION ON THE BASIS OF EXPECTED ASSETS

A number of methods have been suggested for developing the asset allocation factor. One is based on the pro rata share of expected assets (EA) and is given by

$$AAF(t, j) = EA(t, j)/\Sigma_j\, EA(t, j) ,$$

[24] See Rev. Proc. 80-50, sec. 3.01(3). However, it does not have to be complied with unless automatic approval is desired for a change in cost method.

[25] It is important that participants are not confused by this terminology, and that they are not led to believe that the hypothetical asset allocation is, in fact, an account balance allocation. In a defined benefit plan, there is no such thing as an account balance, and there should be no confusion in this regard. One way around this problem is to use some other term to refer to the asset allocation. One possibility is the term "individual offset."

It should be mentioned that for IRC sec. 414(k) purposes (relating to defined benefit plans

where

$$EA(t, j) = [AA(t - 1, j) + NC(t - 1, j)](1 + i)/p_x .$$

Assuming that no active-life decrements are used, and recognizing that the interest factor cancels out, we have

$$EA(t, j) = [AA(t - 1, j) + NC(t - 1, j)] .$$

This latter form has been advocated by the IRS.[26]

Regardless of what basis is used to allocate assets in the first year of a takeover, in subsequent years it is common to allocate on the basis of expected assets.[27]

ALLOCATION ON THE BASIS OF PRESENT VALUE OF ACCRUED BENEFITS

A second method is to allocate on the basis of a pro rata share of the present value of accrued benefits. As mentioned previously, an allocation on this basis is commonly regarded as nondiscriminatory and equitable. In this case, the asset allocation factor takes the form

$$AAF(t, j) = PVAB(t, j)/\Sigma_j PVAB(t, j) .$$

ALLOCATION ON THE BASIS OF ACCRUED LIABILITY

Another alternative is to allocate on the basis of a pro rata share of the accrued liability under a relevant cost method. If this were done, the asset allocation factor would take the form

$$AAF(t, j) = AL(k, t, j)/\Sigma_j AL(k, t, j) ,$$

where the accrued liability under cost method k is $AL(k, t, j)$. If, for example, ERISA was the authority, and the cost method was the entry-age normal cost method, k would be equal to 2.[28]

The rationale for using this approach is that, other things being equal, the accrued liability represents a plan's potential reserve for a given par-

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that provide a benefit derived from employer contributions that is based partly on the balance of the separate account of a participant), there is an asset allocation to a participant's account.

[26] Rev. Proc. 80-50, sec. 3.02(4).

[27] This is mandated for plans that take advantage of the automatic approval provisions of Rev. Proc. 80-50. Beyond this, however, it has become a common practice even where automatic approval is not sought.

[28] See ERISA, sec. 3(31).

ticipant. In this regard, a common claim is that the accrued liability of the entry-age normal cost method is the most appropriate one to use. A strong argument in its favor is that it is the easiest to use, since it has to be calculated in determining the full-funding limit.[29] This argument has been so substantive for some sponsors that they have applied for and received a class ruling letter[30] that allowed an automatic change to this version of the IACM.

ALLOCATION ON THE BASIS OF ACCRUED PRESENT VALUE OF FUTURE BENEFITS

Another method for developing an asset allocation factor is to allocate on the basis of the pro rata share of the accrued present value of future benefits (*APVFB*). The asset allocation factor becomes

$$AAF(t, j) = APVFB(t, j)/\Sigma_j APVFB(t, j) ,$$

where

$$APVFB(t, j) = (AA - PA)PVFB(t, j)/(NRA - PA) .$$

REALLOCATION

In some instances, the asset allocation under the IACM will result in excessive assets being allocated to some individuals in the sense that their asset allocation will exceed *PVFB*. This is not uncommon, for example, in a takeover situation. Ultimately this will be corrected by the method being used. That is why, as is shown below, the method scores high on robustness. Given the opportunity, the method will obviate having to deal with this problem. In the year of occurrence of the problem, there is a choice between awaiting robustness and reallocating.

Two possibilities present themselves. First, the allocation approach can be changed. Instead of allocating on the basis of the present value of accrued benefits, for example, the allocation may be on the basis of accrued liability. This may be unacceptable, however, because it is incompatible with the choice of the original allocation method.

Another possibility would be to reallocate the excess assets. Thus, the first allocation method would not be changed—it would merely be fine-tuned. Under this approach, the assets net of *PVFB* for the individual are reallocated among the remaining individuals according to the asset allo-

[29] It has been claimed that this approach is most effective in reducing full-funding limitation violations. However, the author is not aware of any proof of this assertion.

[30] See Rev. Proc. 78-37, sec. 5.

cation approach being used. To the extent that the excessive assets problem develops for some other participant, the reallocation process would be continued.

It is important to note that if a reallocation of the excess assets is not done, it may have adverse consequences. If, for example, the assets are simply disregarded, the total cost for the year will be increased because of the remaining participants. On the other hand, if a negative contribution is developed for that individual, the costs may decrease significantly because of a dollar-for-dollar trade-off with assets. This could easily happen if the overallocation was with respect to a participant who was within a year or two of retirement.

Finally, it is hard to imagine that the IRS would have trouble with a reallocation of excess assets. Among other things, it reduces costs.

Some Characteristics of the Modified Cost Methods

Consider now the extent to which the foregoing modified cost methods overcome the shortcomings of the traditional methods. Table 4 provides a summary of analysis in this regard.

The IACM, with an asset allocation based on the present value of accrued benefits, is adequate and robust except under adverse conditions. The contribution under the method varies considerably and provides no flexibility.

The IACM, with an asset allocation based on either accrued present value of retirement benefits or expected assets, is satisfactory from both an adequacy and a robustness point of view. Once again, consistency and flexibility are poor.

The MACM, with the tabular normal cost based on the participation age, is acceptable on the basis of both adequacy and robustness. While it is relatively consistent in all but the adverse case, it provides no flexibility.

TABLE 4

SUMMARY OF RESULTS

Cost Method	Adequacy	Consistency	Flexibility	Self-correction
Individual aggregate:				
PVAB	Good	Poor	Poor	Fair
APVFB	Very good	Poor	Poor	Good
EA	Very good	Poor	Poor	Good
Modified aggregate:				
TNC	Very good	Fair	Poor	Good

VI. CONCLUSION

The purpose of this paper has been to investigate some of the shortcomings of traditional actuarial cost methods as they relate to the valuation of small pension plans, and to discuss modified cost methods that seem more appropriate for this area. The results, of course, are not definitive, and there are many aspects of this question that have not been resolved or even raised. Nonetheless, the evidence seems to indicate that for two of the important small-plan criteria, adequacy and robustness, the modified cost methods are superior.

DISCUSSION OF PRECEDING PAPER

PETER A. CHRISTENSEN:

Mr. Shapiro has written a fine and useful paper. The methods promoted in his paper are important tools in the small plan actuary's repertoire.

I would like to augment the paper with a closer look at the operation of the individual level premium method (ILPM) with respect to small pension plans of a particular class. The cost methods suggested by Mr. Shapiro have the attribute of modifying the ILPM by spreading the effect of all actuarial experience as part of the plan normal costs. Although I suspect this is often a desirable modification, it may not always be so.

Many small plans are set up primarily as capital accumulation and tax deferral vehicles for the principals of the sponsoring company. As the drafters of the TEFRA top-heavy provisions saw, these plans are designed to maximize the allocation of benefits and employer contributions to the interest of the owner-employees. Many plans of this kind lack permanency. They tend to be highly volatile, and to reflect the business fortunes of their sponsors. As the company enjoys success, the salaries of its employees increase, but the rate of increase for the owner-employees far exceeds that of the rank and file. Since the costs of the plan are primarily devoted to the benefits of the principals, they will increase rapidly in these circumstances, regardless of the funding method employed. In such times the plan sponsor, rather than being dismayed by the increasing plan costs, often recognizes the tax-sheltering advantages of the plan and favors increased deductible contributions. In contrast, when financial hardship is encountered the plan may not long be tolerated. Future benefit accruals may quickly be curtailed and other cost-cutting measures employed. Often, if the slump is not soon overcome, the plan is terminated.

Plans of this type are commonly funded in accordance with the individual level premium method, using actuarial assumptions that individually are not the actuary's explicit best estimates, but which in aggregate meet the statutory criterion of reasonableness. The use of such implicit assumptions is often due to the economic circumstances of the small, principal-predominant plan: explicit assumptions will not be credible. Since the ILPM spreads the recognition of salary losses into the future, the result of its use with implicit assumptions is that the plan is routinely fully funded. This is because the interest assumption, salary scale, and turnover scale are usually understated (e.g., 5 percent interest, 2 percent salary scale, and no turnover). Emerging experience will usually produce a salary loss, which is spread, and interest and termination gains, which are im-

mediately recognized through the full-funding provision. Assets will usually exceed accrued liabilities.

In this situation it is not clear that the ILPM is deficient with respect to the adequacy criterion of Mr. Shapiro, when compared with the modified methods he suggests. First, the need for a fully robust method is somewhat more remote. Further, consistent plan costs in the small plan environment are not likely under any method. Regardless, it has not been definitively established that consistency is a characteristic of paramount concern to sponsors of small plans. The fact that the ILPM in combination with implicit assumptions produces a normal cost pattern that is expected to steadily increase is not necessarily inconsistent with the needs and concerns of the plan sponsor. If business remains good, the company principals' disposition to put cash into the plan often improves with advancing years. If business turns poor, normal cost pattern may be rendered meaningless by termination of the plan.

PAULETTE TINO AND JAMES E. HOLLAND:

Mr. Shapiro discusses various funding methods commonly used for small pension plans. Some of the methods discussed could fail to satisfy the Income Tax Regulations, in which case they could not be used to determine the minimum funding standard and deductible limit for a plan. This discussion will identify one such unacceptable method, and will further analyze allocation and reallocation of assets. Certain asset allocation methods also fail to satisfy the Income Tax Regulations.

Under a reasonable funding method of the spread-gain type, the normal cost must be expressed either as a level dollar amount or as a level percentage of pay (see section 1.412(c)(3)-1(c)(2) of the Income Tax Regulations). This requirement is similar to the conditional consistency standard enumerated by Mr. Shapiro. (In Table 4, in which Mr. Shapiro rates his suggested methods, he does not rate them for his conditional consistency requirement.)

In addressing the modified aggregate cost method, the author suggests weighting the average temporary annuities on the basis of entry-age normal costs using the attained age as entry age. This would not satisfy the requirement noted above. The normal costs would remain level (assuming realization of the actuarial assumptions) only if the weights are kept the same each year. This is demonstrated below for a plan covering active employees only, none of whom are expected to retire or terminate employment. The notation is that of Mr. Shapiro.

At $t = 0$:

$$NC_0 = \frac{\Sigma PVFB_x(j) - A_0}{\Sigma w(j)\ddot{a}_x^{aa}(j)}, \quad \Sigma w(j) = 1.$$

At $t = 1$:

$$ENC_1 = \frac{\Sigma PVFB_x(j)(1 + i) - (A_0 + NC_0)(1 + i)}{\Sigma w(j)[\ddot{a}_x^{aa}(j) - 1](1 + i)}$$

$$= \frac{NC_0 \Sigma w(j)\ddot{a}_x^{aa}(j) - NC_0}{\Sigma w(j)[\ddot{a}_x^{aa}(j) - 1]} = NC_0.$$

And

$$NC_1 = (\Sigma PVFB_{x+1} - A_1)/\Sigma w(j)\ddot{a}_{x+1}^{aa}(j).$$

The expected normal cost ENC_1 is equal to the plan normal cost NC_1 only if the weights $w(j)$ are the same at $t = 0$ and at $t = 1$.

When entry-age normal costs with entry age taken as attained age are used to calculate the weights, we have for employee j_0 at $t = 0$

$$w(j_0)^0 = \frac{PVFB_x(j_0)/\ddot{a}_x^{aa}(j_0)}{\Sigma PVFB_x(j)/\ddot{a}_x^{aa}(j)} = \left[\sum \frac{PVFB_x(j)}{PVFB_x(j_0)} \frac{\ddot{a}_x^{aa}(j_0)}{\ddot{a}_x^{aa}(j)}\right]^{-1}.$$

At $t = 1$ we have

$$w(j_0)^1 = \left[\sum \frac{PVFB_x(j)}{PVFB_x(j_0)} \frac{\ddot{a}_x^{aa}(j_0) - 1}{\ddot{a}_x^{aa}(j) - 1}\right]^{-1}.$$

This shows that $w(j_0)^0$ and $w(j_0)^1$ are not equal.

In addressing the allocation of assets under the individual aggregate cost method, the author notes four approaches, including one based on the present value of accrued benefits, and one based on the "accrued present value of future benefits." A condition for acceptability under section 1.412(c)(3)-1 of the Income Tax Regulations is that an allocation method produce level normal costs when applied to a new plan (assuming realization of actuarial assumptions). These two approaches would not satisfy this condition. This is demonstrated below in the case where liabilities are discounted for interest only in the preretirement period. The normal cost, at $t = 0$, for a participant j in a new plan is

$$NC_0(j) = \frac{PVFB_x(j)}{\ddot{a}_x^{aa}(j)}.$$

The plan assets at the end of the first year will be equal to the accumulation of the normal costs at the valuation rate. If the asset allocated to employee j equals $NC_0(j)\ (1 + i)$, then the normal cost for j at $t = 1$, $NC_1(j)$, will be equal to the normal cost at $t = 0$. This is because

$$NC_1(j) = \frac{PVFB_1(j)(1 + i) - NC_0(j)(1 + i)}{(\ddot{a}_1^{aa}(j) - 1)(1 + i)} = NC_0(j) .$$

Any allocation resulting in different amounts being assigned to the participants would not produce level normal costs. An allocation on the basis of expected assets or on the basis of accrued liabilities calculated under the entry-age normal cost method with entry age equal to participation age would produce the desired amounts.

Under any asset allocation method, the assets should first be allocated to provide the present value of future benefits (*PVFB*) to any nonactive participant for whom an annuity has not yet been purchased. The remaining assets are then allocated to the active participants.

Mr. Shapiro notes a possible problem: The allocated assets (AA) for a participant may exceed the *PVFB* for that participant. In Mr. Shapiro's notation,

$$PVFB_j < AAF(t, j) \times \text{Net assets} .$$

If we let $AAF(t, j) = AAB_j/\Sigma AAB_j$, where *AAB* denotes an asset allocation basis suggested by Mr. Shapiro (*PVFB*, *AL*, *EA*, or *APVFB*), we have:

$$PVFB_j < \frac{AAB_j}{\Sigma AAB_j} \text{ Net assets } = AAB_j \frac{\text{Net assets}}{\Sigma AAB_j} .$$

Observing that each of the four suggested asset allocation bases for a participant is *less* than $PVFB_j$ for that participant, we have the following:

$$PVFB_j < AAB_j \frac{\text{Net assets}}{\Sigma AAB_j} < PVFB_j \frac{\text{Net assets}}{\Sigma AAB_j} .$$

We see from this equation that a necessary condition for the problem to occur is that $1 <$ Net assets/ΣAAB_j. In other words, the net assets must exceed the sum of the individual amounts forming the basis of the asset allocation (net assets $> \Sigma AAB_j$).

Because we are actually concerned with AAB_j(Net assets/ΣAAB_j), the excess of the net assets over ΣAAB_j may have to be such that the ratio

significantly exceeds one. The smaller the difference between AAB_j and $PVFB_j$, the smaller the ratio need be.

For $AAB_j = EA_j$ the necessary condition (net assets $> \Sigma EA_j$) means that there have been experience gains (from investment sources, liability sources, or both). Unless there have been significant gains, this basis would not have the problem requiring reallocation until participants are close to the assumed retirement age.

For $AAB_j = PVAB_j$, the necessary condition (net assets $> \Sigma PVAB_j$) may occur early in the life of a plan as the result of the funding for projected benefits. This basis appears to present a greater potential for allocating assets in excess of *PVFB* for a participant.

For $AAB_j = AL_j$, assuming that the accrued liability used is the entry-age normal accrued liability, the necessary condition (net assets $> \Sigma AL_j$) indicates that the plan may very well be fully funded within the meaning of section 412(c) of the Internal Revenue Code; that is, the full-funding limitation may be zero or close to zero. Absent significant experience gains this seems less likely to occur than under the other bases.

For $AAB_j = APVFB_j$, the assets may very well exceed the sum of pro rata shares of *PVFB* for the participants. Furthermore, it appears that this is more likely to occur in the early years of a plan with a greater number of future years to retirement.

(AUTHOR'S REVIEW OF DISCUSSION)

ARNOLD F. SHAPIRO:

The advantage of having discussion of a paper is in providing a forum for expressing different points of view. Mr. Christensen discusses the advantages of the individual level premium method (ILPM) over the modified methods, while Mrs. Tino and Mr. Holland discuss the constraints the IRS imposes on modified cost methods.

Mr. Christensen is correct that the modified cost methods are not universally applicable. Actually, the same may be said of any class of cost methods.

Mr. Christensen also points out that the ILPM often is used to fund small plans. This is consistent with the Summary of Results (Table 3), wherein the ILPM shows good adequacy. It is important that the amortization period be modified to recognize the retirement date of the plan sponsor(s); otherwise there may be insufficient time to properly amortize gains and losses. Any cost method that does not explicitly coordinate the amortization of gains and losses with the pension planning horizon of the firm may lead to problems.

Mrs. Tino and Mr. Holland make some welcome clarifying comments

pertaining to sections 404 and 412 of the Code. There are a few points where there appears to be some misunderstanding or where further consideration is required.

Mrs. Tino and Mr. Holland demonstrate that using the attained age as of each valuation date as the weighting factor for each valuation would not lead to level costs under the modified aggregate cost method. However, my suggestion was to use instead the attained age at the date the actuary first does a valuation for the plan.

The rationale for the IRS position regarding acceptable approaches to asset allocation under the individual aggregate cost method (IACM) is a worthwhile supplement to the paper. The argument presented has a limitation, since it is predicated on the assumption that the method is initially applied in the first year of a plan. Quite often, the IACM is first applied in a takeover situation. In the first year of the takeover, the asset allocation probably will not be done on the basis of expected assets, since this information generally is not available. In these and similar situations, the argument given in the discussion would not stand.

Mrs. Tino and Mr. Holland suggest that the allocation basis under the IACM would be a primary reason for an asset reallocation. In fact, the two primary cases in which a reallocation may be necessary are instances of significant experience gains and takeover cases where the IACM is being applied for the first time. As noted, the closer participants are to their normal retirement age, the more likely the problem is to occur. Additionally, since the full-funding limitation affects only the total contribution, in a great many cases it will not alleviate the problem where excessive funds are allocated for only one or two participants.

I would like to extend my thanks to the discussants. While I did not always agree, their input is stimulating.