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MORTALITY EXPERIENCE AROUND AGE 100

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ABSTRACT

This paper presents mortality rates by sex at ages 90 and above for persons who received retired worker benefits from the social security program for the first month that such benefits were paid. This group provides a nearly ideal source of data at the extremely old ages, because good proof of age was required at the time of initial entitlement to benefits and all members of that group are now dead. Mortality rates for these charter beneficiaries are compared with mortality rates calculated from the records of the medicare program and also to rates calculated from the *Vital Statistics* volumes published by the National Center for Health Statistics. The relative closeness of the mortality rates from the three sources demonstrates that mortality is now fairly well known around age 100, as noted by Rosenwaike, Yaffe, and Sagi. All three sources show a progressive deviation in mortality at the extremely high ages from what would be extrapolated by a Gompertz formula, thus corroborating research done by Perks, Humphrey, Vincent, Depoid, and others. Mortality rates of the charter beneficiaries increase with age to the end of the life span rather than eventually leveling off, thus corroborating the research done in Europe and contradicting some of the medicare rates presented by Bayo and Wilkin. All three sources analyzed in this paper show a crossover in mortality of males and females near age 100, unlike most of the European data. Further research is needed to establish whether this observed crossover is real or an aberration.

I. INTRODUCTION

The typical mortality curve for human lives starts at a very high level at birth, declines rapidly to a low point near age 10, and increases thereafter to the end of the life span. Most modern life tables are refined to contain a bulge in the mortality curve for males in the early twenties to

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account for the high incidence of suicides, homicides, and fatal accidents during young adulthood. After the early thirties, the mortality curve increases exponentially in close agreement with the formulas of Gompertz and Makeham. Students of the subject never fail to be amazed at how well a Gompertz curve, with only two parameters, fits the actual data over such a wide range of the most important ages. (In fact, elaborate analyses have been conducted to trace through time the changes in these two parameters.) At the extremely high ages, however, it seems that the mortality curve behaves somewhat differently [11]. In a 1932 paper, Perks [10] observed that at the higher ages the theoretical curves ran too high relative to the data. A similar observation based on English life tables was made in 1970 by Humphrey [4], who used Gompertz curves with lower values of the parameter c at ages over 85 in the analysis of mortality rates obtained solely from death records. Studies done on the experience of other European countries by Vincent [17] and Depoid [3] also show that mortality drops off from an exponential growth, although it continues to increase to the end of the life span.

Mortality at ages 95 and over remains an enigma in the United States. The analyses by Bayo [1], Rosenwaike [14, 15], and Wilkin [18] corroborate the drop-off from an exponential as observed in Europe but do not draw clear conclusions as to whether mortality rates continue to increase indefinitely, level off, or eventually decline. This uncertainty is closely related to the difficulties in obtaining reliable data on centenarians in the United States [2, 8, 12]. Data involving a large number of observations have generally been unreliable [16], while any reliable data have generally encompassed too few observations to be conclusive [9]. The difficulty is not limited, however, to the United States [5, 6]. The need for continuing the analysis in this area and for developing other sources of accurate data is evident—particularly as more people survive to these extremely high ages. We may hope that the accumulation of evidence in the future not only will help to satisfy our curiosity but will eventually provide enough information on which to base useful modeling of mortality patterns at those ages.

Data from three different sources are analyzed here. The first source consists of the charter old-age insurance beneficiaries of the social security program, which is viewed as the most reliable because rigorous proof of age was required for entitlement to benefits when the program began. The second is the medicare program, which may be regarded as being moderately reliable at the ages of interest. The third is the national vital statistics system, which is suspected of being the least reliable. Mortality rates from all three sources of data were graduated by a Whittaker-Henderson Type B procedure, using the exposures as weights and using the same coefficient of smoothness throughout. This procedure does not require the graduated rates to follow any preconceived pattern or formula. Tests using significantly higher and lower coefficients of smoothness showed the same general pattern of mortality as shown by the coefficient of smoothness which was used in the tables presented here. Hence, the tendency for the graduated mortality curves to increase, bend, stay flat, or decrease represents what we consider the best underlying pattern that can be obtained from the data.

II. CHARTER BENEFICIARIES DATA

The social security program started paying monthly retirement benefits in January 1940. All workers who received retirement benefits for that first month (called charter old-age insurance beneficiaries) are now dead, the last survivor having died at age 107 in May 1981. This represents another example in which individuals on whom good proof of age exists fail to survive beyond age 110 [6, 7].

The charter old-age insurance beneficiaries are limited to individuals born in the period 1872–75, because they had to be at least aged 65 in January 1940 and also needed six quarters of social security coverage to be eligible. A person born before 1872 could not have been eligible in January 1940, because coverage could not be earned after the sixty-fifth birthday in calendar years 1937 and 1938. Such a person could have at most five quarters of coverage (four in 1939 and the first quarter in 1940). Those born in 1872 could be eligible, but with some difficulty if their sixty-fifth birthday was in early 1937. The 1875 cohort of eligible persons was limited to those whose sixty-fifth birthday was on or before February 1, 1940. As shown in Table 1, most of the charter beneficiaries were born in either 1873 or 1874.

TABLE 1

SOCIAL SECURITY CHARTER OLD-AGE INSURANCE BENEFICIARIES

Calendar Year of Birth	Male	Female	Total
1872	2,656	257	2,913
1873	11,006	1,447	12,453
1874	13,536	1,774	15,310
1875	751	130	881
Total	27,949	3,608	31,557

As a result of the requirements noted above, the charter old-age insurance beneficiaries must have become entitled to benefits at ages 65–67 (with the average age at entitlement close to exact age 66). Thus, any workers belonging to the 1872–75 cohorts who became entitled at ages above 67 are not included in this study. In other words, the charter old-age insurance beneficiaries include only a portion of the total number of individuals in those cohorts.

Table 2 shows estimated probable ages at death of the last survivor based on the average age at entitlement of 66 and selected mortality assumptions. The actual experience was close to what would be estimated from the decennial life tables for 1939–41, 1949–51, and 1959–61. This demonstrates again that life tables are acceptable tools for estimating probabilities of survival to extremely high ages [16].

The actual experience differs from what would be estimated from the 1969–71 Decennial Life Tables, because of the higher mortality experienced by the charter beneficiaries in the 1940s and 1950s, and also because of the low mortality rates (based on medicare experience) that were used in conjunction with a mathematical formula to end those life tables. Although the earlier tables also were ended in an artificial way, they were based on extrapolations that resulted in rapidly increasing mortality rates to the end of the table. The 1939–41 tables assumed a third-degree curve generally after pivotal age 92, while the 1949–51 and 1959–61 tables were based on the Union Civil War veterans' experience [9].

The charter old-age insurance beneficiary data used in this study were drawn from the Social Security Master Beneficiary Record file as of the end of October 1981. The data consisted of the 2,962 old-age insurance beneficiaries who were entitled for January 1940 and who were alive on January 1, 1963. Information on beneficiaries who died before 1963 was not available, because only active accounts were included when the Mas-

TABLE 2

ESTIMATED* AGE AT DEATH OF LAST SURVIVOR OF CHARTER OLD-AGE INSURANCE BENEFICIARIES

Mortality Assumptions	Male	Female
Actual experience	107	105
1939–41 U.S. Life Tables	108	105
1949–51 U.S. Life Tables	107	105
1959–61 U.S. Life Tables	107	105
1969–71 U.S. Life Tables	111	110

* Assuming that those alive on February 1, 1940, were at exact age 66 (the approximate average age of the group).

ter Beneficiary Record file was transferred from physical files to magnetic tape in 1962. This was not a significant loss, since our main interest was in the experience at ages 90 and over, which we were still able to observe. At ages below 90, enough information of an acceptable quality is available from other sources to provide us with a good indication of the mortality patterns.

For each one of the 2,962 beneficiaries under study, the calendar year and month of birth and the calendar year and month of the occurrence of the event causing benefit termination were obtained. For old-age insurance beneficiaries, this event can logically be presumed to be death. The age at death of each individual was computed as the calendar year of the terminating event less the calendar year of birth, plus one-twelfth of the difference between the calendar month of the terminating event and the calendar month of birth. All individuals then were grouped according to tabulated age last birthday at death. For example, all individuals who died at age 95 years and 0 months to 95 years and 11 months were grouped together and their number referred to as the number of deaths at age 95. For the charter old-age insurance beneficiaries under study, Table 3 shows the numbers of deaths by tabulated age last birthday at death, sex, and year of birth.

Exposures were computed using the extinct-cohort method developed by Vincent [17]. According to this method, the exposure at a particular age is equal to the sum of all deaths tabulated at that age or at an older age. However, special calculations were needed to obtain the initial fraction of the year of age that the beneficiaries were exposed in 1963 before their birthdays.

Table 4 presents mortality rates by sex and by age for the charter old-age insurance beneficiaries. The ages associated with the computed mortality rates are one-half month removed from integral ages, because the information used was by calendar month rather than by exact day of occurrence. This table shows that, according to the charter old-age insurance beneficiaries experience, mortality increases with age to the end of the available data (death of the last survivor) and that there is no flattening out after age 100. This corroborates the conclusions of Myers in his study of the Union Civil War veterans [9]; those of Perks [10], Redington [11], and Humphrey [4] based on English lives; and the data presented by Vincent [17] and Depoid [3], who analyzed death registrations for France, Sweden, Switzerland, and the Netherlands. However, it contradicts some of the rates based on medicare data presented by Bayo [1] and Wilkin [18].

TABLE 3

NUMBERS OF DEATHS OF CHARTER OLD-AGE INSURANCE BENEFICIARIES*
(COHORTS BORN 1872–75)

AGE†	TOTAL DEATHS	MALE					FEMALE				
		Total Deaths	Deaths in Cohort Born				Total Deaths	Deaths in Cohort Born			
			1872	1873	1874	1875		1872	1873	1874	1875
85	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	0
88	187	161	0	0	147	14	26	0	0	22	4
89	442	359	0	132	218	9	83	0	28	52	3
90	491	403	26	184	186	7	88	7	38	37	6
91	459	363	27	142	188	6	96	7	38	48	3
92	352	267	23	111	127	6	85	8	28	45	4
93	311	238	21	90	121	6	73	8	30	31	4
94	196	148	16	66	64	2	48	2	23	22	1

* From the Master Beneficiary Record of the Social Security Administration, excluding deaths that occurred before January 1, 1963.

† Year and month of death less year and month of birth.

TABLE 3—*Continued*

AGE†	TOTAL DEATHS	MALE					FEMALE				
		Total Deaths	Deaths in Cohort Born				Total Deaths	Deaths in Cohort Born			
			1872	1873	1874	1875		1872	1873	1874	1875
95	149	110	12	49	47	2	39	3	14	21	1
96	121	102	6	43	52	1	19	1	7	10	1
97	74	57	5	27	24	1	17	1	10	5	1
98	59	45	7	12	25	1	14	0	5	8	1
99	42	25	3	8	13	1	17	3	7	7	0
100	31	21	2	7	11	1	10	1	4	5	0
101	17	14	2	7	5	0	3	0	1	2	0
102	15	10	1	6	3	0	5	0	3	2	0
103	5	3	1	1	0	1	2	0	0	2	0
104	8	8	1	2	5	0	0	0	0	0	0
105	2	1	0	1	0	0	1	0	1	0	0
106	0	0	0	0	0	0	0	0	0	0	0
107	1	1	0	1	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0
109 and over	0	0	0	0	0	0	0	0	0	0	0
85 and over	2,962	2,336	153	889	1,236	58	626	41	237	319	29

†Year and month of death less year and month of birth.

TABLE 4

ANALYSIS OF MORTALITY AMONG CHARTER OLD-AGE INSURANCE BENEFICIARIES
(COHORTS BORN 1872–75)

EXACT AGE	NUMBER OF DEATHS		EXPOSURE		UNGRADUATED DEATH RATE		GRADUATED				
							Death Rate		Ratio of Female to Male	Ratio to Preceding Age	
	Male	Female	Male	Female	Male	Female	Male	Female		Male	Female
88.96	359	83	1,609	455	.22312	.18228	.22222	.17985	.8093	....	....
89.96	403	88	1,768	505	.22801	.17437	.23548	.19834	.8423	1.0597	1.1028
90.96	363	96	1,413	429	.25690	.22378	.24872	.21673	.8714	1.0562	1.0927
91.96	267	85	1,050	333	.25429	.25526	.26194	.23501	.8972	1.0532	1.0843
92.96	238	73	783	248	.30396	.29435	.27516	.25319	.9202	1.0505	1.0774
93.96	148	48	545	175	.27156	.27429	.28842	.27133	.9408	1.0482	1.0717
94.96	110	39	397	127	.27708	.30709	.30179	.28953	.9594	1.0464	1.0670
95.96	102	19	287	88	.35540	.21591	.31536	.30788	.9763	1.0450	1.0634
96.96	57	17	185	69	.30811	.24638	.32921	.32651	.9918	1.0439	1.0605
97.96	45	14	128	52	.35156	.26923	.34343	.34550	1.0060	1.0432	1.0582
98.96	25	17	83	38	.30120	.44737	.35812	.36492	1.0190	1.0428	1.0562
99.96	21	10	58	21	.36207	.47619	.37334	.38480	1.0307	1.0425	1.0545
100.96	14	3	37	11	.37838	.27273	.38914	.40516	1.0412	1.0423	1.0529
101.96	10	5	23	8	.43478	.62500	.40558	.42601	1.0504	1.0422	1.0515
102.96	3	2	13	3	.23077	.66667	.42266	.44736	1.0584	1.0421	1.0501
103.96	8	0	10	1	.80000	.00000	.44040	.46920	1.0654	1.0420	1.0488
104.96	1	1	2	1	.50000	1.00000	.45881	.49154	1.0713	1.0418	1.0476
105.96	0	....	1	....	.00000	....	.47789	....	....	1.0416	....
106.96	1	....	1	....	1.00000	....	.49763	....	....	1.0413	....

A closer study of the last two columns in Table 4 shows that the falling-off from an exponential increase is progressive but very gradual. We have no categorical explanation for this particular pattern of mortality. It could be argued that the pattern is the result of gradual attrition to a group with a lower average rate of mortality increase. However, we do not believe this argument to be the principal explanation for the pattern. When we combined the male and female data (which show different rates of mortality increases), the observed deviations from the exponential increases were not accentuated.

The reader may observe from Table 4 that there is a crossover in the mortality rate of males and females around age 98. The authors had been previously reluctant to recognize the validity of a sex crossover, but are now of the opinion that it should not be ruled out. However, because of the small number of observations at these very high ages, further research is needed to establish whether or not the crossover in mortality near age 100 is real.

III. THE MEDICARE DATA

The medicare program covering hospital expenses (Part A) and physicians' fees (Part B) started operating in 1966. It is estimated that, since then, about 98 percent of the United States population aged 65 and over has been covered by the program at any given time. Mortality data from the program have been obtained on a routine basis since 1968, and their analyses have been published in previous papers [1, 15, 18]. These analyses have been based on the mortality of all the eligible aged population as observed on a cross-sectional basis over a short period of one or two calendar years. The data generally have been regarded as being of acceptable reliability up to around ages 90 or 95 and to gradually lose their reliability as age increases. For ages over 100, the data rarely have been published, and when published have been accompanied by clear cautions about their reliability.

There are two major possible reasons for the unreliability of the data: (1) consistent bias in the statement of age and (2) spurious data in the tape files. The latter is generally due either to duplication of information (that is, individuals who were originally recorded more than once as eligible for medicare) or to incomplete recording of deaths (that is, some deaths fail to be reported to the program and the deceased individuals continue to appear in the medicare files as being alive). For ages under 90 these two sources of error are relatively small and have only a minor effect on the exposure. At the extremely high ages, however, their effect is no longer negligible. Very few individuals actually survive to these ages, and a minor error in the original recording of eligibles or in the recording of deaths could greatly affect the exposure. An extensive effort has been made to edit the medicare data in order to improve their quality. Further efforts would be relatively expensive and might not be justifiable.

In order to attain comparability with the charter beneficiary data, we limited the medicare analysis to the experience of the four cohorts born in calendar years 1872–75. We traced the mortality of these four cohorts for the period of available medicare data, namely, calendar years 1968–79. To improve the reliability of the data, we limited our analysis to those individuals who were insured (that is, those who were receiving monthly cash benefits from social security or from the railroad retirement system based on covered earnings), because, in most cases, these individuals had to prove their ages. Age at death was calculated as the calendar year of death less the calendar year of birth. Table 5 shows the numbers of deaths of insured medicare enrollees, by tabulated calendar age at death, sex, and year of birth.

The exposure for a cohort at a given age was computed as the sum of the deaths in the cohort at that age and at all higher ages up to the last year of observation, plus an adjustment for future deaths to the cohort. The adjustment was estimated as the sum of all deaths that occurred in the last year of observation (1979) at ages higher than the age of the cohort in that year. Computing the exposures in this manner avoids the problem of spurious data discussed earlier but does not help in correcting for misstatement of age.

Table 6 presents mortality rates by sex and age for the insured persons eligible for medicare. The ages associated with the computed mortality rates are one-half year removed from integral ages, because the information used was by calendar year rather than by exact day of occurrence. This table shows that, according to the medicare experience, female mortality increases with age to the end of the available data (age in 1979) and that there is no flattening out after age 100. The falling-off from the exponential appears to be about the same as for the charter old-age insurance beneficiaries; however, the actual mortality rates are somewhat lower. For males the medicare exerience shows a progressive falling-off from the exponential, with actual declines in mortality after age 105. The authors have strong doubts about these declines and about the sharpness of the falling-off. The medicare data also show a sex crossover in mortality around age 101, but this is subject to question because it is largely the result of the sharpness with which the male mortality falls off from the exponential.

TABLE 5

NUMBERS OF DEATHS OF INSURED MEDICARE ENROLLEES*
(COHORTS BORN 1872–75)

AGE†	TOTAL DEATHS	MALE					FEMALE				
		Total Deaths	Deaths in Cohort Born				Total Deaths	Deaths in Cohort Born			
			1872	1873	1874	1875		1872	1873	1874	1875
85	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	0
88	0	0	0	0	0	0	0	0	0	0	0
89	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0
93	5,570	2,818	0	0	0	2,818	2,752	0	0	0	2,752
94	8,389	4,139	0	0	1,962	2,177	4,250	0	0	1,975	2,275

* From the Hospital Insurance Master File of the Health Care Financing Administration, excluding deaths that occurred before January 1, 1968.

† Year of death less year of birth.

TABLE 5—*Continued*

Age†	Total Deaths	Male: Total Deaths	Male: Deaths in Cohort Born 1872	1873	1874	1875	Female: Total Deaths	Female: Deaths in Cohort Born 1872	1873	1874	1875
95	8,934	4,355	0	1,331	1,470	1,554	4,579	0	1,283	1,532	1,764
96	8,846	4,346	993	1,053	1,107	1,193	4,500	841	1,001	1,210	1,448
97	6,785	3,240	652	777	875	936	3,545	687	751	954	1,153
98	4,801	2,141	464	505	574	598	2,660	464	601	718	877
99	3,641	1,621	362	339	462	458	2,020	402	404	561	653
100	2,496	1,097	236	259	282	320	1,399	271	306	391	431
101	1,730	730	184	165	174	207	1,000	192	203	285	320
102	1,130	490	102	121	119	148	640	116	130	175	219
103	753	275	73	74	57	71	478	99	115	118	146
104	511	192	42	42	42	66	319	70	63	88	98
105‡	373	154	34	28	46	46	219	27	56	68	68
106‡	178	92	14	26	26	26	86	32	18	18	18
107‡	88	24	6	6	6	6	64	16	16	16	16
108‡	76	44	11	11	11	11	32	8	8	8	8
109‡ and over	100	48	12	12	12	12	52	13	13	13	13
85 and over‡	57,363	28,142	3,338	5,638	8,461	10,705	29,221	3,279	5,205	8,449	12,288

† Year of death less year of birth.

‡ Estimated wholly or partially from data for calendar year 1979.

TABLE 6

ANALYSIS OF MORTALITY AMONG INSURED MEDICARE ENROLLEES
(COHORTS BORN 1872–75)

Exact Age	Number of Deaths		Exposure		Ungraduated Death Rate		Graduated				
							Death Rate		Ratio of Female to Male	Ratio to Preceding Age	
	Male	Female	Male	Female	Male	Female	Male	Female		Male	Female
92.50	2,818	2,752	10,647	12,259	.26468	.22449	.26132	.22513	.8615	....	....
93.50	4,139	4,250	15,054	17,637	.27494	.24097	.27351	.23823	.8710	1.0467	1.0582
94.50	4,355	4,579	15,664	18,355	.27803	.24947	.28589	.25197	.8813	1.0452	1.0577
95.50	4,346	4,500	14,494	17,014	.29985	.26449	.29837	.26635	.8927	1.0437	1.0571
96.50	3,240	3,545	10,148	12,514	.31927	.28328	.31071	.28134	.9055	1.0413	1.0563
97.50	2,141	2,660	6,908	8,969	.30993	.29658	.32259	.29682	.9201	1.0382	1.0550
98.50	1,621	2,020	4,767	6,309	.34005	.32018	.33373	.31272	.9371	1.0345	1.0536
99.50	1,097	1,399	3,146	4,289	.34870	.32618	.34381	.32900	.9569	1.0302	1.0520
100.50	730	1,000	2,049	2,890	.35627	.34602	.35253	.34568	.9806	1.0254	1.0507
101.50	490	640	1,319	1,890	.37149	.33862	.35969	.36283	1.0087	1.0203	1.0496
102.50	275	478	829	1,250	.33172	.38240	.36513	.38045	1.0420	1.0151	1.0486
103.50	192	319	554	772	.34657	.41321	.36875	.39849	1.0807	1.0099	1.0474
104.50	154	219	362	453	.42541	.48344	.37045	.41688	1.1253	1.0046	1.0462
105.50	92	86	208	234	.44231	.36752	.37016	.43558	1.1767	.9992	1.0448
106.50	24	64	116	148	.20690	.43243	.36784	.45456	1.2358	.9937	1.0436

IV. THE VITAL STATISTICS DATA

Since 1951 the National Center for Health Statistics has been publishing in its annual volumes *Vital Statistics of the United States* the number of deaths registered at ages 85 and over, by single years of age. These data have been used to study mortality at the higher ages by the extinct-cohort method [1, 14].

The analysis in this paper is limited to the experience of the three synthetic cohorts born in 1873–75. The synthetic calendar years of birth were calculated as the difference between the calendar year of death and the age last birthday at death. Table 7 shows the number of deaths recorded in *Vital Statistics* by tabulated age last birthday at death, sex, and the estimated year of birth. In estimating the number of deaths in each one of the cohorts shown in this table, the number of deaths recorded for a specific synthetic year of birth was equally divided between that year and the preceding calendar year. We believe that this procedure yields a fairly accurate distribution of the three synthetic cohorts among the four possible actual years of birth.

The procedure used to compute the exposures is similar to that used with the medicare data. It should be noted, however, that the true date of birth for each synthetic cohort spans the two-calendar-year period ending with the calculated synthetic year of birth. Under assumptions of uniform distribution of births and deaths, the average date of birth for each cohort would be January 1 of the calculated synthetic year of birth. For the group of the three synthetic cohorts born in 1873–75 the average date of birth would be January 1, 1874, or approximately the same as for the charter old-age insurance beneficiaries and for the insured persons eligible for medicare.

Table 8 presents mortality rates by sex and age for the *Vital Statistics* recordees. The ages associated with the computed mortality rates are for the exact integral ages shown. This table shows that, according to *Vital Statistics* experience, female mortality increases with age to the end of the available data (age in 1978) and that there is no flattening out after age 100. However, the falling-off from the exponential appears to be sharper than for the charter old-age insurance beneficiaries or for the insured persons eligible for medicare. For males the *Vital Statistics* death experience shows a very fast falling-off from the exponential and actual declines in the mortality rates after age 101. The authors have very strong doubts about these patterns of mortality and believe that they result largely from misstatement of age. We tend to agree with Humphrey [4], Rosenwaike [13], and Siegel [16] that estimates of exposure derived from death

TABLE 7

NUMBERS OF DEATHS OF *Vital Statistics* RECORDEES*
(COHORTS BORN 1872–75)

AGE†	TOTAL DEATHS	MALE					FEMALE				
		Total Deaths	Deaths in Cohort Born				Total Deaths	Deaths in Cohort Born			
			1872	1873	1874	1875		1872	1873	1874	1875
85	87,873	39,679	6,498	12,901	13,341	6,939	48,194	7,732	15,644	16,364	8,454
86	81,079	35,840	5,803	11,742	12,116	6,179	45,239	7,199	14,623	15,420	7,997
87	75,365	32,505	5,277	10,519	10,975	5,734	42,860	6,840	13,900	14,590	7,530
88	67,412	28,338	4,484	9,099	9,685	5,070	39,074	6,014	12,511	13,523	7,026
89	60,180	24,710	4,071	8,205	8,284	4,150	35,470	5,725	11,631	12,010	6,104
90	53,039	20,899	3,469	6,805	6,980	3,645	32,140	5,206	10,445	10,864	5,625
91	42,530	16,187	2,582	5,194	5,511	2,900	26,343	4,204	8,451	8,968	4,720
92	36,470	13,501	2,181	4,383	4,569	2,368	22,969	3,672	7,474	7,812	4,011
93	30,270	10,670	1,738	3,464	3,596	1,872	19,600	3,114	6,274	6,686	3,526
94	24,353	8,295	1,366	2,732	2,781	1,416	16,058	2,527	5,173	5,501	2,857

* Estimated from publications of the National Center for Health Statistics, excluding deaths that occurred between the birthday and the following January 1 for the 1872-born cohort and excluding deaths that occurred between the birthday and the preceding December 31 for the 1875-born cohort.

† Age last birthday at death.

TABLE 7—*Continued*

Age†	Total Deaths	Male					Female				
		Total Deaths	Deaths in Cohort Born				Total Deaths	Deaths in Cohort Born			
			1872	1873	1874	1875		1872	1873	1874	1875
95	18,780	6,098	1,036	2,021	2,012	1,029	12,682	2,093	4,155	4,247	2,187
96	14,363	4,598	733	1,478	1,566	821	9,765	1,576	3,132	3,306	1,751
97	10,714	3,237	536	1,080	1,082	539	7,477	1,117	2,374	2,622	1,364
98	8,142	2,395	390	781	807	417	5,747	907	1,835	1,966	1,039
99	5,690	1,704	269	562	583	290	3,986	619	1,262	1,374	731
100	4,062	1,070	185	359	349	177	2,992	473	976	1,023	520
101	2,797	743	125	236	246	136	2,054	311	660	716	367
102	1,899	495	87	168	160	80	1,404	213	455	488	248
103	1,378	319	51	101	108	59	1,059	171	342	358	188
104‡	940	219	32	71	77	39	721	105	233	255	128
105‡	600	153	25	51	51	26	447	74	149	149	75
106‡	372	120	20	40	40	20	252	42	84	84	42
107‡	249	78	13	26	26	13	171	28	57	57	29
108‡	180	63	10	21	21	11	117	19	39	39	20
109 and over‡	339	114	19	38	38	19	225	37	75	75	38
85 and over‡	686,439	280,172	44,338	87,715	93,465	54,654	406,267	63,297	127,159	136,946	78,865

† Age last birthday at death.

‡ Estimated wholly or partially from data for calendar year 1978.

TABLE 8

Analysis of Mortality among *Vital Statistics* Recordees (Cohorts Born 1872–75)

Exact Age	Number of Deaths		Exposure		Ungraduated Death Rate		Graduated				
							Death Rate		Ratio of Female to Male	Ratio to Preceding Age	
	Male	Female	Male	Female	Male	Female	Male	Female		Male	Female
85.00	39,679	48,194	252,030	377,046	.15744	.12782	.15682	.12743	.8126	....	....
86.00	35,840	45,239	212,351	328,852	.16878	.13757	.17003	.13852	.8147	1.0842	1.0870
87.00	32,505	42,860	176,511	283,613	.18415	.15112	.18382	.15047	.8186	1.0811	1.0863
88.00	28,338	39,074	144,006	240,753	.19678	.16230	.19788	.16300	.8238	1.0765	1.0833
89.00	24,710	35,470	115,668	201,679	.21363	.17587	.21180	.17584	.8302	1.0704	1.0788
90.00	20,899	32,140	90,958	166,209	.22977	.19337	.22518	.18872	.8381	1.0632	1.0732
91.00	16,187	26,343	70,059	134,069	.23105	.19649	.23806	.20167	.8471	1.0572	1.0686
92.00	13,501	22,969	53,872	107,726	.25061	.21322	.25088	.21528	.8581	1.0538	1.0675
93.00	10,670	19,600	40,371	84,757	.26430	.23125	.26365	.22950	.8704	1.0509	1.0660
94.00	8,295	16,058	29,701	65,157	.27928	.24645	.27612	.24371	.8826	1.0473	1.0619
95.00	6,098	12,682	21,406	49,099	.28487	.25829	.28792	.25735	.8938	1.0427	1.0560
96.00	4,598	9,765	15,308	36,417	.30037	.26814	.29869	.27017	.9045	1.0374	1.0498
97.00	3,237	7,477	10,710	26,652	.30224	.28054	.30800	.28215	.9161	1.0312	1.0443
98.00	2,395	5,747	7,473	19,175	.32049	.29971	.31543	.29333	.9300	1.0241	1.0396
99.00	1,704	3,986	5,078	13,428	.33557	.29684	.32057	.30383	.9478	1.0163	1.0358
100.00	1,070	2,992	3,374	9,442	.31713	.31688	.32315	.31383	.9712	1.0081	1.0329
101.00	743	2,054	2,304	6,450	.32248	.31845	.32312	.32348	1.0011	.9999	1.0307
102.00	495	1,404	1,561	4,396	.31710	.31938	.32051	.33283	1.0384	.9919	1.0289
103.00	319	1,059	1,066	2,992	.29925	.35394	.31542	.34182	1.0837	.9841	1.0270
104.00	219	721	747	1,933	.29317	.37300	.30797	.35030	1.1375	.9764	1.0248
105.00	153	477	528	1,212	.28977	.36881	.29821	.35812	1.2009	.9683	1.0223
106.00	120	252	375	765	.32000	.32941	.28617	.36524	1.2763	.9596	1.0199

registration data are generally more reliable than those derived from census data, but that they should not be regarded as being completely reliable. The *Vital Statistics* data show a sex crossover in mortality around age 102. This is not too different from what was found from the charter old-age insurance beneficiaries data and from the medicare data. However, because this crossover is largely the result of the rapid falling-off of the male rates, we believe that further evidence is still needed to validate the existence of a crossover in mortality at extremely high ages.

V. GENERAL OBSERVATIONS

We would like to caution the reader that the data presented here trace a few cohorts through a period of many years. This is significantly different from the more usual studies of mortality involving a cross-section of many different cohorts over a few years of observation. The reader should be aware that if the usual cross-sectional data were to show, for example, that mortality increases at a rate of 9 percent per year of age and if mortality in general were decreasing by 2 percent per calendar year, the tracing of the cohorts would show mortality increasing at a rate of about 7 percent per year of age. It therefore could be argued that some of the observed pattern of mortality could be due to the time element involved in the tracing of the cohorts rather than solely to the age element. For example, during the early 1960s, when the charter beneficiaries were in their late eighties and early nineties, mortality in the United States was not improving significantly, and the rates obtained from tracing a cohort would be similar to those of a cross-section. In the late 1960s, mortality in the United States began to decrease gradually, and this would be reflected in a progressive deviation from the Gompertz curve at ages 95–100 for the cohorts. A continuous decline in mortality in the United States in the mid- and late 1970s would be reflected in a lower, but almost fixed, rate of increment for the cohorts at ages over 100. Although this argument is somewhat appealing, it cannot be accepted at this stage because of the difficulty in proving it, since data on improvements in mortality at the extremely high ages are not reliable enough for that purpose. Further, we have some doubts about this argument because it implies that mortality has been improving recently at excessively high rates for the extremely aged.

Table 9 compares the graduated mortality rates from each of the three sources of data. The relative closeness of the mortality rates from the three sources demonstrates that, as was noted by Rosenwaike, Yaffe, and Sagi [15], mortality at the extremely high ages is known in the United

TABLE 9

Comparison of Graduated Death Rates*
(Cohorts Born 1872–75)

Exact Age	Male					Female				
	Death Rate			Ratio to Charter OAIBS		Death Rate			Ratio to Charter OAIBS	
	Charter OAIBS	Insured Medicare	*Vital Statistics*	Insured Medicare	*Vital Statistics*	Charter OAIBS	Insured Medicare	*Vital Statistics*	Insured Medicare	*Vital Statistics*
89	.22276	....	.21180	....	.9508	.18058	....	.17584	....	.9738
90	.23601	....	.22518	....	.9541	.19908	....	.18872	....	.9480
91	.24925	....	.23806	....	.9551	.21746	....	.20167	....	.9274
92	.26248	....	.25088	....	.9558	.23574	....	.21528	....	.9132
93	.27570	.26735	.26365	.9697	.9563	.25392	.23159	.22950	.9120	.9038
94	.28896	.27963	.27612	.9677	.9556	.27207	.24500	.24371	.9005	.8958
95	.30234	.29206	.28792	.9660	.9523	.29027	.25906	.25735	.8925	.8866
96	.31592	.30448	.29869	.9638	.9454	.30863	.27374	.27017	.8869	.8754
97	.32979	.31659	.30800	.9600	.9339	.32728	.28898	.28215	.8830	.8621
98	.34403	.32811	.31543	.9537	.9169	.34629	.30467	.29333	.8798	.8471
99	.35874	.33873	.32057	.9442	.8936	.36573	.32076	.30383	.8770	.8307
100	.37398	.34814	.32315	.9309	.8641	.38563	.33724	.31383	.8745	.8138
101	.38981	.35609	.32312	.9135	.8289	.40601	.35415	.32348	.8723	.7967
102	.40627	.36240	.32051	.8920	.7889	.42688	.37153	.33283	.8703	.7797
103	.42338	.36693	.31542	.8667	.7450	.44825	.38936	.34182	.8686	.7626
104	.44115	.36960	.30797	.8378	.6981	.47011	.40758	.35030	.8670	.7451
105	.45959	.37031	.29821	.8057	.6488	....	.42613	.35812	....	....
106	.47869	.36900	.28617	.7708	.5978	....	.44497	.36524	....	....

* Interpolated from calculated rates to obtain rates associated with integral ages.

States with greater accuracy than has been thought—although perhaps not with the same high level of accuracy that prevails in European countries that have had birth registration for over a century. At age 90, all three sets of rates are within 5 percent for males and within 6 percent for females (assuming that the mortality of the insured medicare enrollees lies between the mortality of the two other groups, as it does for all observed ages). At age 95, all three sets of rates are within 5 percent for males and within 12 percent for females. At age 100, all three sets of rates are within 14 percent for males and 19 percent for females. These larger differentials could be attributed to misstatement of age among the *Vital Statistics* recordees, while the greater differential for females could be due to the paucity of the charter old-age insurance beneficiary data.

Figure 1 shows the log of the colog of the graduated survival rates of each of the three groups of males. Under this transform, a mortality curve following a Gompertz formula would become a straight line. The curve for male charter old-age insurance beneficiaries is straight after about age 95, but with a smaller slope than for the lower ages. It appears that male mortality follows one Gompertz curve up to about age 85 and then grad-

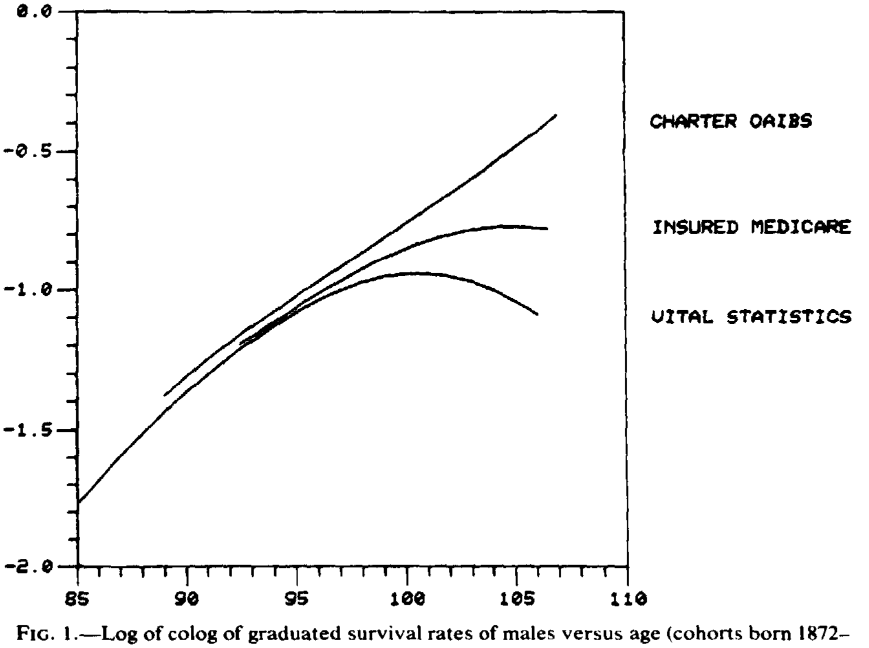

FIG. 1.—Log of colog of graduated survival rates of males versus age (cohorts born 1872–75).

ually bends to a second Gompertz curve at about age 95, which has a lower value for the parameter c. This is not too different from the conclusion of Humphrey [4] on the basis of death records in England and Wales.

Figure 2 shows the log of the colog of the graduated survival rates of each of the three groups of females. After age 95, both the charter old-age insurance beneficiaries and the insured medicare enrollees follow straight lines having very nearly the same slopes, which are somewhat less than the slope at the lower ages. It appears that female mortality also follows one Gompertz curve up to about age 85 and then gradually bends to a second Gompertz curve at about age 95, which has a lower value for the parameter c.

VI. CONCLUSIONS

The number of observations of charter old-age insurance beneficiaries is not sufficient for the results to be conclusive. However, these results are significant when viewed in the context of previously reported data

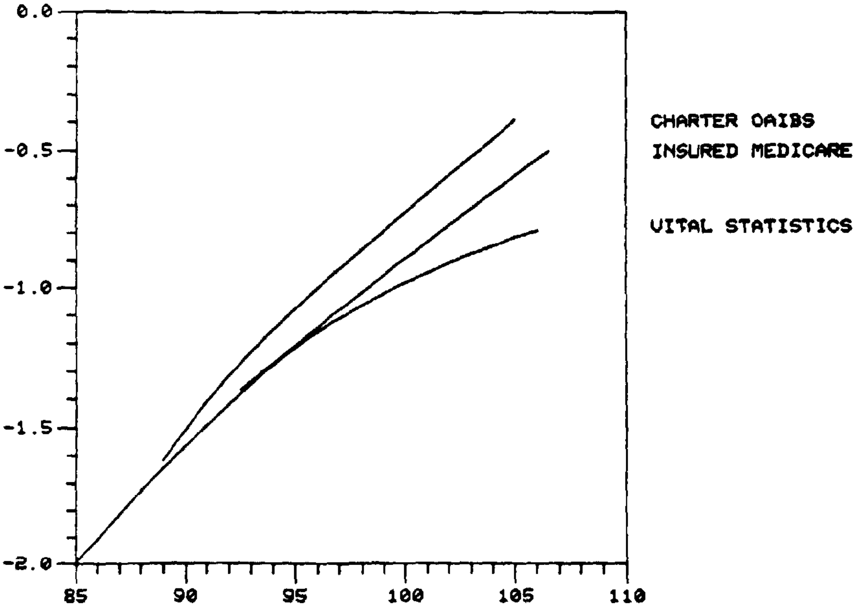

FIG. 2.—Log of colog of graduated survival rates of females versus age (cohorts born 1872–75).

drawn from other sources. With this premise, the authors draw the following conclusions:

1. Mortality at the extremely high ages is known in the United States with a higher degree of accuracy than was previously believed.
2. After around age 85, mortality in the United States, like mortality in European countries, falls off from an exponential increase with age.
3. In the United States, as in many European countries, mortality at the higher ages increases continuously to the end of the life span.
4. Further studies are needed to establish whether the pattern of mortality at the extremely high ages (95 and over) can be represented, without much loss of accuracy, by a Gompertz curve with a lower value for the parameter c than estimated from ages under 85.
5. There is a possibility that a sex crossover in mortality occurs around age 100, but significant doubts remain about its existence.

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DISCUSSION OF PRECEDING PAPER

BRUCE D. SCHOBEL:

The authors have made a significant contribution to the actuarial literature by collecting and analyzing the rather limited data available on mortality rates at very advanced ages. However, their second major conclusion, that increases in mortality rates fall off from the exponential at very advanced ages, may be at least partially a reflection of their methodology, rather than just an indication of the underlying mortality curve.

Mortality rates from all three sources considered in the paper were graduated by a Whittaker-Henderson Type B process, using exposures as the weights. This graduation method allows the user essentially to choose the relative emphasis between "smoothness" and "fit," where smoothness is generally measured by the sums of the squares of the differences (usually third-order) in the graduated values. At the highest ages, where the exposures are smallest, fit becomes increasingly less important, and the graduated values will tend to take on the characteristics of points on a polynomial, because such values will minimize the squares of the differences. No polynomial increases exponentially; thus, the increases in the values will demonstrate the falling-off from the exponential that the authors observed.

To demonstrate this with an example, consider the exposures and ungraduated values shown in Table 4 for male charter old-age insurance beneficiaries. Assume that the rate shown for males at age 88.96 is correct and that mortality rates at subsequent ages can be generated through repeated multiplication by 1.05. These hypothetical rates are close to those shown in Table 4, but they are truly exponential.

If a Whittaker-Henderson Type B graduation method is applied to these hypothetical rates, the graduated values show the expected falling-off from the exponential. Graduated values are shown in Table 1 of this discussion for smoothing coefficients of 1, 100, and 10,000. In every case, even with the smallest smoothing coefficient, some falling-off from the exponential can be seen, with the larger smoothing coefficients producing larger deviations.

The declines in the ratios of these graduated mortality rates to the rates at the preceding age are not as large, even with a smoothing coefficient of 10,000, as are the declines shown by Bayo and Faber. Thus, some (or even most) of the falling-off from the exponential may be characteristic of the underlying "true" mortality curve. At least part of this effect, however, must be attributed to the graduation method chosen, which produces a similar effect even on truly exponential mortality rates.

TABLE 1

ANALYSIS OF HYPOTHETICAL MORTALITY RATES

Exact Age	Exposure	Ungraduated Death Rate	Smoothing Coefficient 1		Smoothing Coefficient 100		Smoothing Coefficient 10,000	
			Graduated Death Rate	Ratio to Preceding Age	Graduated Death Rate	Ratio to Preceding Age	Graduated Death Rate	Ratio to Preceding Age
88.96	1,609	.22312	.22312	....	.22312	....	.22316	....
89.96	1,768	.23428	.23428	1.0500	.23428	1.0500	.23426	1.0498
90.96	1,413	.24599	.24599	1.0500	.24599	1.0500	.24596	1.0499
91.96	1,050	.25829	.25829	1.0500	.25829	1.0500	.25827	1.0500
92.96	783	.27120	.27120	1.0500	.27120	1.0500	.27119	1.0500
93.96	545	.28476	.28476	1.0500	.28476	1.0500	.28476	1.0500
94.96	397	.29900	.29900	1.0500	.29900	1.0500	.29901	1.0500
95.96	287	.31395	.31395	1.0500	.31395	1.0500	.31397	1.0500
96.96	185	.32965	.32965	1.0500	.32965	1.0500	.32969	1.0501
97.96	128	.34613	.34613	1.0500	.34613	1.0500	.34620	1.0501
98.96	83	.36344	.36344	1.0500	.36344	1.0500	.36354	1.0501
99.96	58	.38161	.38161	1.0500	.38161	1.0500	.38173	1.0500
100.96	37	.40069	.40069	1.0500	.40069	1.0500	.40080	1.0499
101.96	23	.42073	.42073	1.0500	.42074	1.0500	.42075	1.0498
102.96	13	.44176	.44176	1.0500	.44179	1.0500	.44160	1.0496
103.96	10	.46385	.46385	1.0500	.46389	1.0500	.46335	1.0493
104.96	2	.48704	.48705	1.0500	.48705	1.0499	.48601	1.0489
105.96	1	.51140	.51141	1.0500	.51129	1.0498	.50957	1.0485
106.96	1	.53696	.53694	1.0499	.53662	1.0495	.53404	1.0480

(AUTHORS' REVIEW OF DISCUSSION)

FRANCISCO R. BAYO AND JOSEPH F. FABER:

We regret that some readers may have been misled by our casual use of the expression "exponential increase in mortality with age." In using this expression, we do not mean a geometric growth in the death rates but rather a geometric growth in the force of mortality (which is characteristic of the Gompertz family of curves). It can be shown that a geometric growth in the force of mortality implies a less than geometric growth in the death rates.

Mr. Schobel has pointed out in his discussion that the use of a Whittaker-Henderson Type B graduation formula weighted by exposures causes some distortion when applied to hypothetical rates constructed according to a strictly geometric growth pattern in combination with declining exposures. In particular, under these circumstances, the graduated rates tend to show a systematic bending from the geometric pattern at the extremely high ages, as shown in the left portion of Table 1 of this review. This is an interesting phenomenon and should be kept in mind when choosing an appropriate graduation technique for rates thought to exhibit a geometric pattern of growth.

When graduating rates (such as death rates) that come from an underlying Gompertz growth pattern, the Whittaker-Henderson Type B formula appears to us to be quite good. We tested this by first fitting a Gompertz curve to the observed data, then computing the implied death rates, and finally graduating the implied death rates. As shown in the right-hand portion of Table 1, the graduated rates are very close to the implied ungraduated rates throughout the entire range of ages, although, in this case, there is a tendency at the extremely high ages for the graduated rates to be slightly higher than the ungraduated ones. This means that deviation in mortality rates graduated by a Whittaker-Henderson Type B formula that are on the low side of Gompertz growth at the very highest ages, such as the actual rates for charter old-age insurance beneficiaries published in our paper, should be attributed to the characteristics of the data and not to the method of graduation.

TABLE 1

ANALYSIS OF HYPOTHETICAL MORTALITY RATES BASED ON MALE CHARTER OLD-AGE INSURANCE BENEFICIARIES

Exact Age	Exposure	Geometric Death Rate			Gompertz Death Rate		
		Ungraduated*	Graduated†	Ratio of Graduated to Ungraduated	Ungraduated‡	Graduated†	Ratio of Graduated to Ungraduated
88.96	1,609	.22312	.22335	1.0010	.22373	.22375	1.0001
89.96	1,768	.23428	.23423	.9998	.23520	.23519	1.0000
90.96	1,413	.24599	.24581	.9993	.24716	.24714	.9999
91.96	1,050	.25829	.25809	.9992	.25961	.25960	1.0000
92.96	783	.27120	.27107	.9995	.27258	.27258	1.0000
93.96	545	.28476	.28476	1.0000	.28606	.28608	1.0001
94.96	397	.29900	.29915	1.0005	.30005	.30008	1.0001
95.96	287	.31395	.31426	1.0010	.31458	.31461	1.0001
96.96	185	.32965	.33009	1.0013	.32963	.32964	1.0000
97.96	128	.34613	.34664	1.0015	.34520	.34519	1.0000
98.96	83	.36344	.36391	1.0013	.36129	.36126	.9999
99.96	58	.38161	.38192	1.0008	.37790	.37784	.9998
100.96	37	.40069	.40065	.9999	.39503	.39493	.9997
101.96	23	.42073	.42012	.9986	.41262	.41254	.9998
102.96	13	.44176	.44032	.9967	.43071	.43066	.9999
103.96	10	.46385	.46125	.9944	.44925	.44929	1.0001
104.96	2	.48704	.48292	.9915	.46822	.46844	1.0005
105.96	1	.51140	.50532	.9881	.48759	.48811	1.0011
106.96	1	.53696	.52845	.9842	.50734	.50829	1.0019

* Calculated assuming geometric growth of 5 percent per year of age from age 88.96.

† Calculated using a Whittaker-Henderson Type B formula.

‡ Calculated from the Gompertz curve fitted to ages 88.96 through 94.96 using the method of least squares.