The Valuation of a Hybrid Pension Plan

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Outline

- Introduction and Background
- Models and Assumptions
- Numerical Techniques and Results
- Equilibrium Pricing Model
- Comments and Future Work

Introduction and Background

- Two basic kinds of pension plans:

  Defined Benefit(DB) and Defined Contribution(DC)

- In Italy and Australia, the hybrid pension plan is used by some pension fund managers.

- During the year 2002, The State of Florida's 600,000 public employees were given the unprecedented ability to convert their DB pension plan into a DC pension plan.

Introduction and Background

- "Greater of" benefit is composed by DB and DC. It guarantees minimum pension benefits.
- The payoff is same as the exchange option's payoff.
- McGill University offers the hybrid pension plan with minimum DB guarantee.

Description of Benefits(DB)

- The DB benefits depend on the current salary at exit time t:

$$D(x,t,T) = \alpha \times S_t \times n \times \ddot{a}_{65}^{(12)} \times {}_{T-t}E_t$$

$D(x,t,T)$:The actuarial value at time t

α :The accrual rate

S_t :The salary at time t

n :The years of service

$\ddot{a}_{65}^{(12)}$:The annuity factor

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Description of Benefits(DC)

- The DC benefits depend on the monthly contribution:

$$A(x,t+h,T) = A(x,t,T)\cdot(1+hf_t)+c\cdot h\cdot S_t$$

Where

c :The contribution rate

h :The time interval

Assumptions

Two random variables:

$$ds = \mu(s,t)dt + \sigma(s,t)dZ_s$$

$$df = \mu(f,t)dt + \sigma(f,t)dZ_f$$

Let $dZ_s dZ_f = \rho_{sf} dt$, we get

$$df = \mu(f,t)dt + \sigma(f,t)(\rho_{sf} dZ_s + \sqrt{(1-\rho_{sf}{}^2)}dZ_f)$$

Simulation Process

- Use the risk-adjusted stochastic difference equations:

$$s_{t+h} = s_t + (\alpha_s + b_s f_t - \lambda_s \sigma_s s_t^{\gamma_s - 1.0}) h + \sigma_s s_t^{\gamma_s - 1.0} \sqrt{h} Z_s,$$

$$f_{t+h} = f_t + (\alpha_f + b_f f_t - \lambda_f \sigma_f f_t^{\gamma_f - 1.0}) h + \sigma_f f_t^{\gamma_f - 1.0} \sqrt{h} (\rho Z_s + \sqrt{1 - \rho^2} Z_f),$$

Numerical Results

- First use Sherris'(1995) scenarios and compare with Sherris' (1995) results.
- Generate 10,000 paths to simulate the rate of salary growth and the crediting rate. The contribution rate is 15%.
- Consider the difference between two accounts

$$Max(D(x,t,T)-A(x,t,T),0)$$

6 Scenarios

Table 1: Assumptions for Stochastic Simulation Valuations

Scenario	Parameter Values					Initial Values	
	α_s	b_s	σ_s	λ_s	γ_s	s_0	f_0
1	0.072	-1.2	0.08	0.0	1.0	0.06	0.10
2	0.072	-1.2	0.08	0.0	1.0	0.06	0.10
3	0.072	-1.2	0.08	0.0	1.0	0.06	0.10
4	0.072	-1.2	0.08	0.0	1.0	0.06	0.135
5	0.072	-1.2	0.08	-0.1	1.0	0.06	0.135
6	0.072	-1.2	0.08	0.0	1.0	0.06	0.135

Scenario	Parameter Values						Discount Rate
	α_f	b_f	σ_f	λ_f	γ_f	ρ	
1	0.1296	-0.96	0.2	0.168	1.0	0.00	0.1
2	0.1296	-0.96	0.2	0.168	1.0	0.5	0.1
3	0.1296	-0.96	0.2	0.168	1.0	-0.5	0.1
4	0.1296	-0.96	0.2	0.000	1.0	0.00	0.1
5	0.1296	-0.96	0.2	0.000	1.0	0.00	0.1
6	0.1296	-0.96	0.2	0.100	1.0	0.00	0.1

Cost of Max of DB and DC Accounts

New Entrant Expected Cost (as a Percentage of Salary) and Standard Errors of Retirement Benefit Option Features for Stochastic Valuation Simulation Scenarios

Stochastic Scenario	*Age at Entry*				
	20	*30*	*40*	*50*	*55*
1	16.3	17.2	16.8	15.6	16.1
	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
2	16.4	17.2	16.6	15.4	16.0
	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
3	16.8	17.3	16.9	15.8	16.4
	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
4	32.0	32.5	26.6	19.9	17.3
	(0.4)	(0.2)	(0.2)	(0.1)	(0.0)
5	31.9	32.2	26.6	19.8	17.3
	(0.4)	(0.4)	(0.2)	(0.1)	(0.0)
6	20.9	21.8	19.9	17.2	15.8
	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)

Note: Percentages equal the monthly expected cost of the benefits as a percentage of monthly salary using 10,000 simulations. The standard error of the expected cost is shown in parentheses.

Cost of Guarantee

Table 3.3: The Mean Cost and Standard Error as a percentage of salary

Entry Age	20	30	40	50
Scenario	Mean(StErr)	Mean(StErr)	Mean (StErr)	Mean (StErr)
1	0.2116 (0.04312)	0.8700 (0.1661)	1.795 (0.2946)	1.809 (0.2446)
2	0.1190 (0.02454)	0.5345 (0.1035)	1.184 (0.1951)	1.378 (0.1838)
3	0.3063 (0.06256)	1.239 (0.2349)	2.096 (0.2903)	2.325 (0.3816)
4	0.0486 (0.01688)	0.2267 (0.07474)	0.6541 (0.1728)	0.8801 (0.1644)
5	0.0767 (0.02627)	0.3536 (0.1098)	0.8939 (0.2178)	1.060 (0.1903)
6	0.1159 (0.03061)	0.5315 (0.1279)	1.168 (0.2331)	1.354 (0.2110)

Remarks

- Need more information to choose the scenario
- Parameters look outdated
- The market is incomplete. DC and DB accounts are not traded.
- Mixture of P-Measure and Q-Measure
- Consider the equilibrium model

Equilibrium Pricing Model

- The equilibrium approach plays a very important role in modern finance. It can be used in the incomplete market.
- Many major pricing models can be derived by the equilibrium approach, such as CAPM, B-S option pricing formula and CIR model.
- The equilibrium approach considers the utility function and agent's consumptions.

Equilibrium Model

- Use power utility function:

$$u(x) = x^{\beta} / \beta, \beta < 1, \beta \neq 0$$

- The price of the option:

$$P = \frac{E^{p}[u'(S_t) \cdot Payoff_t]}{E^{p}[u'(S_0)]}$$

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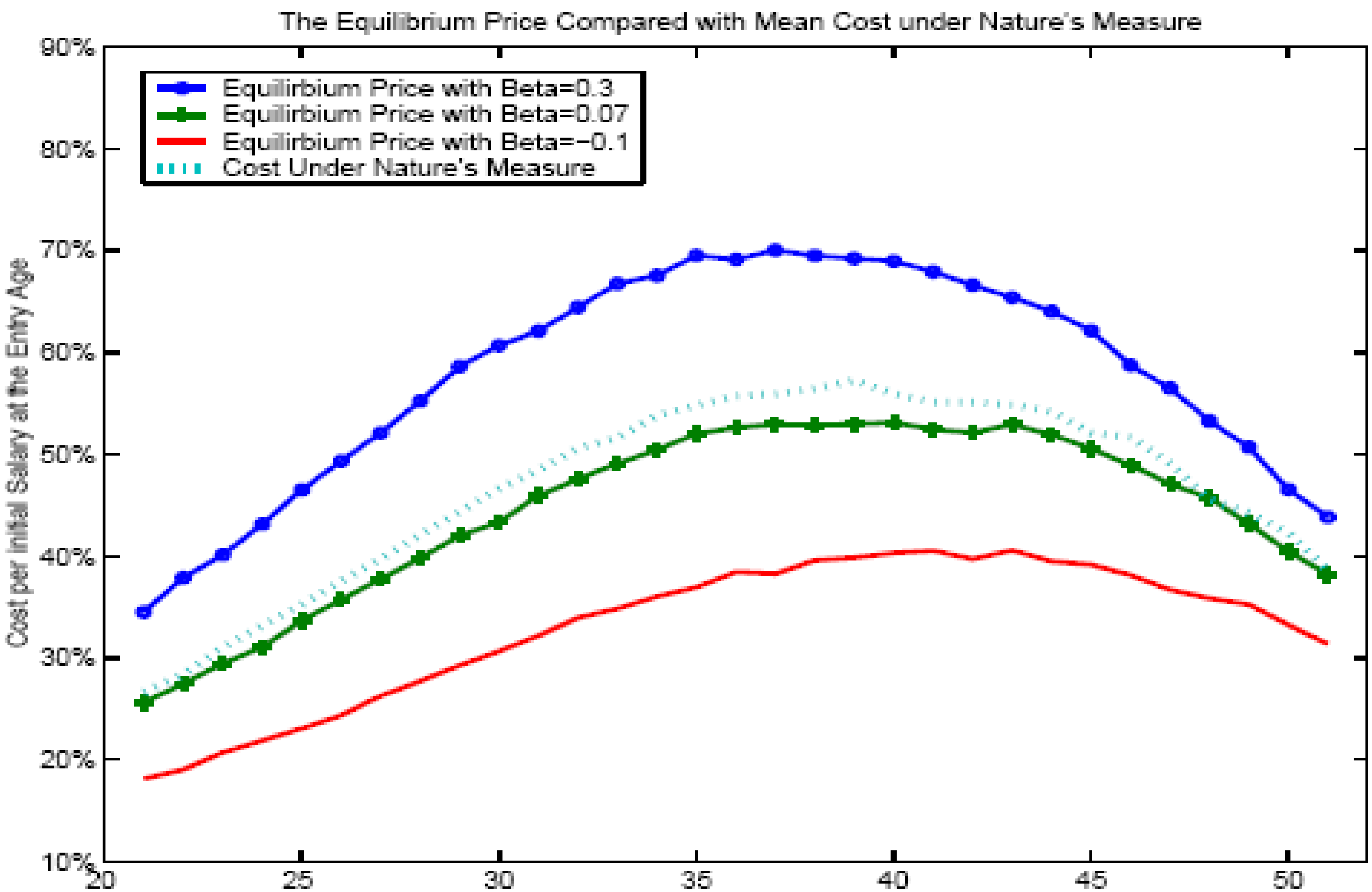
Updated Parameters

- Based on last 50 years Canadian data from *Report on Canadian Economic Statistics 1924-2003*

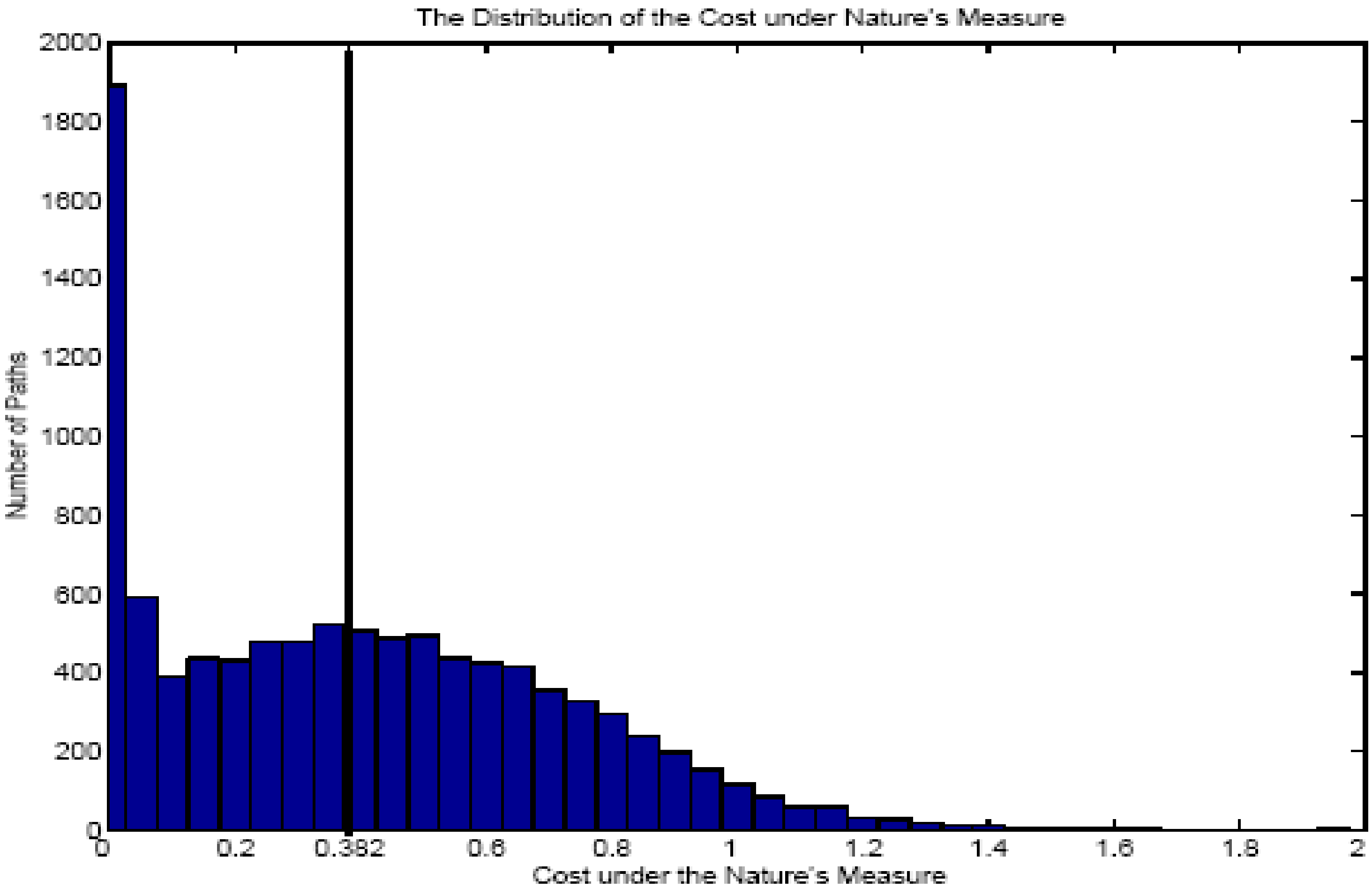
- We use updated parameters

$$\mu_s = 0.055, \sigma_s = 0.033,$$
$$\mu_f = 0.07, \sigma_f = 0.1,$$
$$\rho_{sf} = 0, r = 0.05, c = 0.15.$$

Equilibrium Model



The distribution of the mean cost under Nature's Measure



Comments

- The cost under Nature's measure can be expressed as the equilibrium price with appropriate parameter selection.
- The highest cost of guarantee occurs for employees with entry age 40.
- The cost under Nature's measure should be lower than the equilibrium price.

Future Work

- The relationship between Nature's measure, equilibrium approach and Q-measure.
- Risk management on this hybrid pension with minimum DB guarantee.

Thank You!