

## Generalized Gerber-Shiu Function in Piecewise-deterministic Markov Processes

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**Abstract:** Introduced by Gerber and Shiu (1998) in NAAJ, the Gerber-Shiu expected discounted penalty function has ever since become the standard technical tool in the actuarial literature to analyze a variety of ruin-related quantities such as the probability of ultimate ruin, the joint distribution of the surplus prior to ruin and the deficit at ruin. As ruin theory progresses, great efforts have been made in the literature to study the dividends paid to shareholders up to ruin, which is not a special case of the Gerber-Shiu function. It has been brought to our attention that most techniques applied to the dividends problem are basically parallel to those employed in the analysis of the Gerber-Shiu function. The similarity between the solution methods led us to propose a more general function that contains both the Gerber-Shiu expected discounted penalty function and dividends paid up to ruin, as well as many others that have not been taken into consideration in the same systematic way, such as the insurer's accumulative utility, expected total discounted claim expenses, etc. Not only does the generalized Gerber-Shiu function accommodate more quantities, it is also applicable with a more general class of underlying risk processes called the piecewise deterministic Markov processes, which includes most well-studied non-diffusion processes such as the compound Poisson, Sparre Andersen with phase-type inter-claim times and Markov-modulated risk processes. Our major result provides an unifying approach to obtain integro-differential equations for all ruin-related quantities that fall in the category of the generalized Gerber-Shiu function. In the end, we shall demonstrate the application of generalized Gerber-Shiu function by recovering many well-known results in the literature as well as producing solutions to other aforementioned new quantities of interests.