2007 Advanced Portfolio Management Formulae Sheet
December 2006

Fabozzi, Handbook of Fixed Income Securities

$$Y_d = \frac{(F-P)}{F} * \frac{360}{t}$$

$$P = F - (F * Y_d * \frac{t}{360})$$

$$K - L * (reference \cdot rate)$$

$$CPR = 1 - (1 - SMM)^{12}$$

$$O/C \cdot ratio \cdot for \cdot a \cdot tranche = \frac{principal(par) \cdot value \cdot of \cdot collateral \cdot portfolio}{principal \cdot for \cdot tranche + principal \cdot for \cdot all \cdot tranches \cdot senior \cdot to \cdot it}$$

$$I/C \cdot ratio \cdot for \cdot a \cdot tranche = \frac{scheduled \cdot \mathrm{int} \cdot due \cdot on \cdot underlying \cdot collateral \cdot portfolio}{scheduled \cdot \mathrm{int} \cdot on \cdot that \cdot tranche + scheduled \cdot \mathrm{int} \cdot on \cdot all \cdot tranches \cdot senior}$$

$$class \cdot X \cdot OC \cdot ratio = \frac{cash \cdot collateral \cdot account \cdot balance}{notional \cdot amount \cdot class \cdot X \cdot notes \cdot and \cdot notes \cdot senior \cdot excluding \cdot \sup er \cdot senior}$$

$$dispersion = \frac{\sum (t_1 - D)^2 PV(CF_i)}{\sum PV(CF_i)}$$

Hardy, Investment Guarantees

$$\frac{S_{t+w}}{S_t} \sim \mathrm{LN}(w\mu, \sqrt{w}\sigma) \Rightarrow \log \frac{S_{t+w}}{S_t} \sim N(w\mu, w\sigma^2)$$

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi w}} \exp\left\{ -\frac{1}{2} \frac{\left(\log(x) - w\mu\right)^2}{w\sigma^2} \right\}$$

$$\mathrm{E}\left[\frac{S_{t+w}}{S_t}\right] = e^{w\mu + w\sigma^2/2}$$

$$\mathrm{V}\left[\frac{S_{t+w}}{S_t}\right] = e^{2w\mu + w\sigma^2}\left(e^{w\sigma^2} - 1\right)$$

$$Y_t = \mu + a\left(Y_{t-1} - \mu\right) + \sigma\varepsilon_t$$

$$Y_t = \mu + a\left(Y_{t-1} - \mu\right) + \sigma_t\varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\left(Y_{t-1} - \mu\right)^2 + \beta\sigma_{t-1}^2$$

$$\pi_1 = \frac{p_{2,1}}{p_{1,2} + p_{2,1}}, \quad \pi_2 = 1 - \pi_1$$

$$p_n(r) = \Pr\left[R_n(0) = r\right] = \pi_1 \Pr\left[R_n(0) = r \middle| \rho_{-1} = 1\right] + \pi_2 \Pr\left[R_n(0) = r \middle| \rho_{-1} = 2\right]$$

$$\sigma^*(R_n) = \sqrt{R_n\sigma_1^2 + (n - R_n)\sigma_2^2}$$

$$F_{S_n}(x) = \Pr(S_n \le x) = \sum_{r=0}^{n} \Pr(S_n \le x | R_n = r)\, p_n(r)$$

$$F_{S_n(x)} = \sum_{r=0}^{n} \Phi\left(\frac{\log x - \mu^*(r)}{\sigma^*(r)}\right) p_n(r)$$

$$f_{S_n}(x) = \sum_{r=0}^{n} \frac{1}{\sigma^*(r)x} \phi\left(\frac{\log x - \mu^*(r)}{\sigma^*(r)}\right) p_n(r)$$ (from errata sheet)

$$y(t) = \exp\left\{w_y\delta_q(t) + \mu_y + yn(t)\right\} \quad \text{where } yn(t) = a_y yn(t-1) + \sigma_y z_y(t)$$

$$\mathrm{E}\left[y(t)\right] = e^{\mu_y}\,\mathrm{E}\left[\exp(w_y\delta_q(t))\right]\mathrm{E}\left[\exp(yn(t))\right]$$

$$M_{\delta_q}(u) = \exp\left(u\mu_q + \frac{u^2(\sigma_q)^2}{2}\right)$$

$$\mathrm{E}\left[y(t)\right] = e^{\mu_y} M_q(w_y)\left[\exp\left(\mu_{yn} + \frac{\sigma_y^2}{2(1 - a_y^2)}\right)\right]$$

$$DM(t) = d_d\delta_q(t) + (1 - d_d)DM(t-1)$$

$$\frac{\partial l(\mu,\sigma)}{\partial \mu} = \frac{1}{\sigma}\left(\sum_{t=1}^{n} y_t - n\mu\right)$$

$$\frac{\partial l(\mu,\sigma)}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma^3}\sum_{t=1}^{n}(y_t - \mu)^2$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{\mu})^2}{n}} \qquad \text{where} \quad \hat{\mu} = \overline{y}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \mu^2} = -\frac{n}{\sigma}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \mu \partial \sigma} = \frac{-1}{\sigma^2}(\sum_{t=1}^{n} Y_t - n\mu)$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \sigma^2} = \frac{3}{-\sigma^4}\sum_{t=1}^{n}(Y_t - \mu)^2 + \frac{n}{\sigma^2}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \mu^2}\right] = \frac{n}{\sigma}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \mu \partial \sigma}\right] = 0$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \sigma^2}\right] = \frac{2n}{\sigma^2}$$

$$\Sigma \approx \begin{pmatrix} \frac{\hat{\sigma}^2}{n} & 0 \\ 0 & \frac{\hat{\sigma}^2}{2n} \end{pmatrix}$$

$$l(\mu,\sigma,a) = \ln(\sqrt{\frac{1-a^2}{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{(Y_1 - \mu)^2(1-a^2)}{\sigma^2}\right)\right\}) +$$

$$\sum_{t=2}^{n} \ln(\sqrt{\frac{1}{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{(Y_t - (1-a)\mu - aY_{t-1})^2}{\sigma^2}\right)\right\})$$

$$= \frac{-n}{2}\ln(2\pi) + \frac{1}{2}\ln(1-a^2) - n\ln\sigma -$$

$$\frac{1}{2}\left\{\frac{(Y_1 - \mu)^2(1-a^2)}{\sigma^2} + \sum_{t=2}^{n}\left(\frac{(Y_t - (1-a)\mu - aY_{t-1})^2}{\sigma^2}\right)\right\}$$

$$\ln S_n \sim N(n\mu, (\sigma h(a,n))^2) \quad \text{where} \;\; h(a,n) = \frac{1}{(1-a)}\sqrt{\sum_{i=1}^{n}(1-a^i)^2}$$

$$\ln S_n - n\mu = Z_1 + Z_2 + ... + Z_n = \frac{\sigma}{1-a}\left\{\sum_{i=1}^{n} \varepsilon_i (1-a^{n+1-i})\right\}$$

$$F_{S_n}(x) = Pr\left[S_n \le x\right] = \sum_{r=0}^{n} Pr\left[S_n \le x \middle| R_n = r\right] p_n(r) =$$

$$\sum_{r=0}^{n} \Phi\left(\frac{\ln x - \mu^*(r)}{\sigma^*(r)}\right) p_n(r)$$

$$P_0 = (K - S_d)\frac{S_u e^{-r} - S_0}{S_u - S_d} = (K - S_d)e^{-r} p^* \quad \text{where} \quad p^* = \frac{S_u - S_0 e^r}{S_u - S_d}$$

$$P_0 = e^{-rT} \mathrm{E}_Q\left[\left(G - S_T(1-m)^T\right)^+\right]$$

$$P_0 = (1-m)^T \left\{e^{-rT} E_Q\left[(G(1-m)^{-T} - S_T)^+\right]\right\}$$

$$P_0 = Ge^{-rT}\Phi(-d_2) - S_0(1-m)^T \Phi(-d_1) \quad \text{where}$$

$$d_1 = \frac{\log\left(S_0/G\right) + \left(r + \log(1-m) + \sigma^2/2\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$H(0) = \mathrm{E}_T\left[\mathrm{BSP}_0(T)\right] = \int_0^n \mathrm{BSP}_0(t)\, {}_t p_x^\tau \mu_{x,t}^{(d)} dt$$

$$H(0) = \int_0^n (Ge^{-rt}\Phi(-d_2))\, {}_t p_x^\tau \mu_{x,t}^{(d)} dt +$$

$$\int_0^n (-S_0(1-m)^t \Phi(-d_1))\, {}_t p_x^\tau \mu_{x,t}^{(d)} dt$$ (from errata sheet)

$$\alpha = \frac{B}{S_0 \ddot{a}^\tau_{x:\overline{n}|i'}}$$

$$H(t) = \sum_{w=t}^{n-1} {}_{w|}q_x^d P(t,w) + {}_n p_x^\tau P(t,n)$$

$$HE_t = H(t) + {}_{t-1|}q_x^d \left((G - F_t)^+\right) - H(t^-)$$

$$TC_t = \tau S_t \left|\Psi_t - \Psi_{t-1}\right|$$

$$A = \Phi^{-1}\left(\frac{1+\beta}{2}\right)\sqrt{N\alpha(1-\alpha)}$$

$$\xi = 1 - \Phi\left(\frac{\log G/S_0 - n(\mu + \log(1-m))}{\sqrt{n}\sigma} \right)$$

$$\Pr\left[F_n + V_\alpha e^{rn} > G \right] \geq \alpha$$

$$V_\alpha = (G - F_{F_n}^{-1}(1-\alpha))\, e^{-rn}$$

$$V_\alpha = (G - F_0 \exp(-z_\alpha \sqrt{n}\sigma + n(\mu + \ln(1-m))))e^{-rn}$$

$$\mathrm{CTE}_\alpha(L) = \frac{(1-\beta')\mathrm{E}\left[\mathrm{X} \middle| X > V_\alpha \right] + (\beta' - \alpha)V_\alpha}{1-\alpha}$$

$$\mathrm{CTE}_\alpha(L) = \mathrm{E}\left[(G - F_n)e^{-rn} \middle| F_n < (G - V_\alpha e^{rn}) \right]$$

$$\mathrm{CTE}_\alpha(L) = e^{-rn}\left\{ G - \frac{e^{n(\mu + \log(1-m) + \sigma^2/2)}}{1-\alpha}\Phi(-z_\alpha - \sqrt{n}\sigma) \right\}$$

$$\mathrm{CTE}_\alpha(X) = \frac{(1-\xi)}{(1-\alpha)}\mathrm{CTE}_\xi(X)$$

$$E\left[L\right] = e^{-rn}\left\{ G(1-\xi) - F_0 \exp(n(\mu + \ln(1-m) + \frac{\sigma^2}{2}))\Phi(A) \right\}$$

where $A = \dfrac{(\ln G_{F_0} - n(\mu + \ln(1-m)) - n\sigma^2)}{\sqrt{n}\sigma}$

$$\log(1+i_t)\middle|\rho_t^y = \mu_{\rho_t^y}^y + \phi_{\rho_t^y}^y \left(\log(1+i_{t-1}) - \mu_{\rho_t^y}^y \right) + \sigma_{\rho_t^y}^y \varepsilon_t$$

$$H_0 = B(0,n)\mathrm{E}_Q\left[F_n(ga_{65}(n) - 1)^+ \right]$$

$$H_0 = F_0 \mathrm{E}_Q\left[\left(\frac{ga_{65}^d(0,n)}{B(0,n)} - 1 \right)^+ \right]$$

$H_t = F_t\left\{ ga_{65}(t)\Phi(d_1(t)) - \Phi(d_2(t)) \right\}$ where

$d_1(t) = \dfrac{\log(ga_{65}(t)) + \sigma_y^2(n-t)/2}{\sigma_y\sqrt{n-t}}$ and $d_2(t) = d_1(t) - \sigma_y\sqrt{n-t}$

Babbel and Fabozzi , Investment Management for Insurers

$$P\left[D(t);k\right] = \max\left[0, k - D(t)\right]$$

$$C\left[spread(t);k\right] = (spread - k) * notional \cdot amount * risk \cdot factor$$

$$payment = \left[credit \cdot spread \cdot at \cdot maturity - contracted \cdot credit \cdot spread\right] * duration * notional \cdot value$$

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = \frac{1}{e^{\phi(T-t)\times(T-t)}} E\left[\frac{1}{e^{\int_t^T r_s ds}}\right]$$

$$r_s^* = r_s + \phi(s-t) + \phi'(s-t)\times(s-t)$$

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = E\left[\frac{1}{e^{\int_t^T (r_s + \phi(T-t))ds}}\right] = E\left[\frac{1}{e^{\int_t^T r_s^* ds}}\right]$$

$$dr = (k\theta - (k+\lambda)r)dt + \sigma\sqrt{r}dw^* \quad \text{where } w^*(t) = w(t) + \int_0^t \frac{\lambda}{\sigma}\sqrt{r(s)}ds$$

$$\frac{p(t,TB)}{B(t)} = E^*\left[\frac{1}{B(TB)}\right] \qquad p(t,TB) = E^*\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$p(t,TB) = A(t,TB)\exp(-r(t)G(t,TB))$$

$$A(t,TB) = \left[\frac{2\gamma\exp\left[(b+\gamma)\frac{TB-t}{2}\right]}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$$G(t,TB) = \frac{2(\exp(\gamma(TB-t))-1)}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma}$$

$$\text{where } b = k+\lambda \quad c = k\theta \quad \gamma = \sqrt{b^2 + 2\sigma^2}$$

$$C(t) = p(t,TB)\chi^2(2\gamma^*(\varphi+\psi+G(T,TB)), \frac{4c}{\sigma^2}, \frac{2\varphi^2 re^{\gamma(T-t)}}{(\varphi+\psi+G(T,TB))}) -$$

$$Xp(t,T)\chi^2\left[2r^*(\varphi+\psi), \frac{4c}{\sigma^2}, \frac{2\varphi^2 re^{\gamma(T-t)}}{(\varphi+\psi)}\right]$$

$$dr = (\phi(t) - \alpha(t)r)dt + \sigma(t)dw^{**} \quad \phi(t) = \theta(t) + \alpha(t)b - \lambda(t)\sigma(t)$$

$$\frac{x(t)}{B(t)} = E^{**}\left[\frac{x(\tau)}{B(\tau)}\right]$$

$$p(t,TB) = E^{**}\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$\alpha(t) = \frac{-\partial^2 G(0,t)\big/\partial t^2}{\partial G(0,t)\big/\partial t}$$

$$\phi(t) = -\alpha(t)\frac{\partial F(0,t)}{\partial t} - \frac{\partial^2 F(0,t)}{\partial t^2} + \left[\frac{\partial G(0,t)}{\partial t}\right]^2 \int_0^t \left[\frac{\sigma(\tau)}{\partial G(0,\tau)\big/\partial\tau}\right]^2 d\tau$$

$$C(t) = P(t,TB)N(h) - XP(t,T)N(h - \sigma_p)$$

where $h = (\frac{\sigma_p}{2}) + (\frac{1}{\sigma_p})\ln\left[\frac{P(t,TB)}{(XP(t,T))}\right]$

$$\sigma_p^2 = \left[G(0,TB) - G(0,T)\right]^2 \int_t^T \left[\frac{\sigma(\tau)}{\partial G(0,\tau)/\partial\tau}\right]^2 d\tau$$

$$A(0,t) = \left[\frac{2\gamma\exp\left[(b+\gamma)\frac{t}{2}\right]}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$G(0,t) = \dfrac{2(\exp(\gamma t)-1)}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}$ Where $b = k + \lambda \quad c = k\theta \quad \gamma = \sqrt{b^2 + 2\sigma^2}$

$$P\max(P(0,TB), M(0), 0, T) = E^*\left[\frac{M(T)}{B(T)}\right] - P(0,TB) =$$

$$E^*\left[M(T)(\exp(-\int_0^T r(s)ds))\right] - P(0,TB)$$

$$r_{t_i} = r_{t_{i-1}} + (k\theta - (k+\lambda)r_{t_{i-1}})(t_i - t_{i-1}) + \sigma\sqrt{r_{t_{i-1}}}\sqrt{(t_i - t_{i-1})}\tilde{\varepsilon}$$

$$PM\widehat{AX}(P(0,TB),M(0),0,T) = (\left\{\frac{1}{N}\sum_{n=1}^{N} M_n(T)\exp\left[-\sum_{i=1}^{m} r_n(t_{i-1})(t_i - t_{i-1})\right]\right\} - P(0,TB))$$

$$PMAX(P(0,TB),M(0),0,T) = E^{**}\left[\frac{M(T)}{B(T)}\right] - P(0,TB) =$$

$$E^{**}\left[M(T)(\exp(-\int_0^T r(s)ds))\right] - P(0,TB)$$

$$r_{t_i} = r_{t_{i-1}} + (\phi(t_{i-1}) - \alpha(t_{i-1})r_{t_{i-1}})(t_i - t_{i-1}) + \sigma\sqrt{r(0)}\sqrt{t_i - t_{i-1}}\tilde{\varepsilon}$$

$$DVBP = \frac{Par \cdot amount \times (price + accrued) \times \text{mod}ified \cdot duration}{1{,}000{,}000}$$

$$DVBP = \frac{dollar \cdot par \cdot amount \times (chnage \cdot in \cdot cons\tan t - OAS \cdot price)}{yield \cdot curve \cdot shift \cdot in \cdot bps * 100}$$

$$P_{j0} = \frac{1}{|S_0|}\sum_{s=1}^{|S_0|}\sum_{t=1}^{T}\frac{C_{jt}^s}{\prod_{i=1}^{t}(1 + r_i^s)}$$

$$P_{j0} = \frac{1}{|S_0|}\sum_{s=1}^{|S_0|}\sum_{t=1}^{T}\frac{C_{jt}^s}{\prod_{i=1}^{t}(1 + \rho_j r_i^s)}$$

$$P_{j\tau}^{\sigma} = \frac{1}{|S_0|}\sum_{s=1}^{|S_0|}\sum_{t=1}^{T}\frac{C_{jt}^s}{\prod_{i=\tau}^{t}(1 + \rho_j r_i^s)}$$

$$P_{j\tau}^{\sigma} = \frac{1}{|S_{0,\sigma}|}\sum_{s(\sigma)\in S_{0,\sigma}} p_{j\tau}^{s(\sigma)}$$

$$P_j^- = \frac{1}{|S_0|}\sum_{s=1}^{|S_0|}\sum_{t=1}^{T}\frac{C_{jt}^s}{\prod_{i=1}^{t}(1 + \rho_j r_i^{-s})}$$

$$P_j^+ = \frac{1}{|S_0|}\sum_{s=1}^{|S_0|}\sum_{t=1}^{T}\frac{C_{jt}^s}{\prod_{i=1}^{t}(1 + \rho_j r_i^{+s})}$$

$$\Gamma_j = \frac{P_j^+ - 2P_{j0} + P_j^-}{50^2}$$

$$R_{j\tau}^{s} = \frac{F_{j\tau}^{s} + V_{j\tau}^{s}}{P_{j0}}$$

$$h = \frac{\Delta V - \Delta S}{\Delta F}$$

$$h = \frac{-\Delta S}{\Delta F} = \frac{-\beta_S}{\beta_F}$$

$$dS_f P_T \left(S_f / S_n \right) = \frac{dS_f}{S_f \sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{\left(\ln\left(\frac{S_f}{S_0} \right) - \mu T \right)^2}{2\sigma^2 T} \right)$$

$$\int dS_f P_T \left(\frac{S_f}{S_0} \right) \ln\left(\frac{W_f\left(S_f \right)}{W_0} \right)$$

Litterman, Modern Investment Management: An Equilibrium Approach

$$R_{L,t} - R_{f,t} = \beta(R_{B,t} - R_{f,t}) + \varepsilon_t$$

$$SR_i = \frac{\mu_i - R_f}{\sigma_i}$$

$$RACS_t = \frac{E_t\left[S_{t+1} - S_t(1+R_f) \right]}{\sigma_t\left[S_{t+1} \right]}$$

$$RACS_t = \frac{E_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1}) - (A_t - L_t)(1+R_f) \right]}{\sigma_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1}) \right]}$$

$$RACS_t = \frac{E_t\left[A_t(R_{A,t+1} - R_f) \right]}{\sigma_t\left[A_t(1+R_{A,t+1}) \right]} = \frac{E_t\left[R_{A,t+1} \right] - R_f}{\sigma_t\left[R_{A,t+1} \right]}$$

$$E_t\left[F_{t+1} \right] = F_t E_t\left[\frac{1+R_{A,t+1}}{1+R_{L,t+a}} \right] \frac{1}{1-p} - \frac{p}{1-p}$$

$$E_0\left[F_t \right] = \left[\frac{1+E\left[R_x \right]}{1-p} \right]^t F_0 + p \frac{1 - \left[\frac{1+E\left[R_x \right]}{1-p} \right]^t}{E\left[R_x \right] + p}$$

$$w_0 = \frac{\alpha + (\lambda\sigma_e^2 - \mu_e)(1-\beta)}{\lambda\left[\sigma_n^2 + (1-\beta)^2\sigma_e^2\right]}$$

$$w_{\min\cdot vol} = \frac{1-\beta}{(\frac{\sigma_n}{\sigma_e})^2 + (1-\beta)^2}$$

$$(\frac{\sigma_\tau}{\sigma_e})^2 = (1-\omega)^2 + 2\beta w(w-1) + w^2\left[\beta^2 + (\frac{\sigma_n}{\sigma_e})^2\right]$$

Hull, Options, Futures and Other Derivatives

$$h^* = \rho\frac{\sigma_S}{\sigma_F}$$

$$N^* = \frac{h^* N_A}{Q_F}$$

$$c + D + Ke^{-rT} = p + S_0$$

$$\ln S_T \sim \phi\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)$$

$$E(S_T) = S_o e^{\mu T}$$

$$\operatorname{var}(S_T) = S_0^2 e^{2\mu T}\left[e^{\sigma^2 T} - 1\right]$$

$$x = \frac{1}{T}\ln\frac{S_T}{S_0}$$

$$x \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right)$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \qquad \text{where} \quad u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2}$$

$$dS = \mu S dt + \sigma S dz$$

$$df = (\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial f}{\partial S}\sigma S dz$$

$$\Delta f = (\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)\Delta t + \frac{\partial f}{\partial S}\sigma S \Delta z$$

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

$$\Delta \Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 \right)\Delta t$$

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

$$f = e^{-rT}\hat{E}\left(S_T\right) - Ke^{-rT}$$

$$\hat{E}\left(S_T\right) = S_0 e^{rT}$$

$$f = S_0 - Ke^{-rT}$$

$$c = S_0 N\left(d_1\right) - Ke^{-rT} N\left(d_2\right)$$

$$p = Ke^{-rT} N\left(-d_2\right) - S_0 N\left(-d_1\right)$$

where $d_1 = \frac{ln\left(S_0 / K\right) + \left(r + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{ln\left(S_0 / K\right) + \left(r - \sigma^2 / 2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$c = e^{-rT}\left[S_0 N\left(d_1\right)e^{rT} - KN\left(d_2\right)\right]$$

$$S\left(t_n\right) - D_n - Ke^{-r\left(T - t_n\right)} \geq S\left(t_n\right) - K$$

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

$$c = S_0 e^{-qT} N\left(d_1\right) - Ke^{-rT} N\left(d_2\right)$$

$$p = Ke^{-rT} N\left(-d_2\right) - S_0 e^{-qT} N\left(-d_1\right)$$

$$d_1 = \frac{ln\left(S_0 / K\right) + \left(r - q + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln\left(S_0 / K\right) + \left(r - q - \sigma^2 / 2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$dS = (r - q)Sdt + \sigma Sdz$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

$$c = e^{-rT}\left[F_0 N\left(d_1\right) - KN\left(d_2\right)\right]$$

$$p = e^{-rT}\left[KN\left(-d_2\right) - F_0 N\left(-d_1\right)\right]$$

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

$$f = e^{-rT}\left[pf_\mu + (1 - p) f_d\right]$$

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf$$

$$p + S_0 e^{-qT} = c + Ke^{-rT}$$

$$Se^{(r-q)\Delta t} = pSu + \left(1 - p\right)Sd$$

$$p = \frac{a - d}{u - d}$$

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}}$$

$$a = e^{(r-q)\Delta t}$$

$$f_{N,j} = \max\left(K - S_0 u^j d^{N-j}, 0\right)$$

$$f_{i,j} = e^{-r\Delta t}\left[pf_{i+1,j+1} + \left(1 - p\right) f_{i+1,j}\right]$$

$$f_{i,j} = \max\left\{K - S_0 u^j d^{i-j}, e^{-r\Delta t}\left[pf_{i+1,j+1} + \left(1 - p\right) f_{i+1,j}\right]\right\}$$

$$\Delta = \frac{f_{11} - f_{10}}{S_0 u - S_0 d}$$

$$\Gamma = \frac{\left[\left(f_{22} - f_{21}\right) / \left(S_0 u^2 - S_0\right)\right] - \left[\left(f_{21} - f_{20}\right) / \left(S_0 - S_0 d^2\right)\right]}{h} \quad \text{where} \quad \text{h} = 0.5\left(\text{S}_0 u^2 - S_0 d^2\right)$$

$$\Theta = \frac{f_{21} - f_{00}}{2\Delta t}$$

$$\nu = \frac{f^* - f}{\Delta\sigma}$$

$$u = e^{\left(r - q - \sigma^2/2\right)\Delta t + \sigma\sqrt{\Delta t}}$$

$$d = e^{\left(r - q - \sigma^2/2\right)\Delta t - \sigma\sqrt{\Delta t}}$$

$$u = e^{\sigma\sqrt{3\Delta t}}, \quad d = \frac{1}{u}$$

$$p_d = -\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}, \quad p_m = \frac{2}{3}$$

$$p_u = \sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}$$

$$a = e^{\left[f(t) - g(t)\right]\Delta t}$$

$$p = \frac{e^{\left[f(t) - g(t)\right]\Delta t} - d}{u - d}$$

$$S\left(t + \Delta t\right) - S\left(t\right) = \hat{\mu} S\left(t\right)\Delta t + \sigma S\left(t\right)\varepsilon\sqrt{\Delta t}$$

$$S\left(t + \Delta t\right) = S\left(t\right)\exp\left[\left(\hat{\mu} - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right]$$

$$\theta_i\left(t + \Delta t\right) - \theta_i\left(t\right) = \hat{m}_i\theta_i\left(t\right)\Delta t + s_i\theta_i\left(t\right)\varepsilon_i\sqrt{\Delta t}$$

$$\frac{\partial f}{\partial t} + (r - q)S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j}}{\Delta S}$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j} - f_{i,j-1}}{\Delta S}$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S}$$

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\Delta S^2}$$

$$a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j} \quad \text{where}$$

$$a_j = \frac{1}{2}(r-q) j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t \text{ , } b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t \text{ ,}$$
$$c_j = -\frac{1}{2}(r-q) j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

$$\frac{\partial f}{\partial S} = \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S}$$

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i+1,j+1} + f_{i+1,j-1} - 2 f_{i+1,j}}{\Delta S^2}$$

$$f_{i,j} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1} \quad \text{where}$$

$$a_j^* = \frac{1}{1+r\Delta t}\left(-\frac{1}{2}(r-q) j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right), \; b_j^* = \frac{1}{1+r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$$

$$c_j^* = \frac{1}{1+r\Delta t}\left(\frac{1}{2}(r-q) j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

$$\alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j}$$

$$\text{where} \quad \alpha_j = \frac{\Delta t}{2\Delta Z}(r - q - \frac{\sigma^2}{2}) - \frac{\Delta t}{2\Delta Z^2}\sigma^2$$

$$\beta_j = 1 + \frac{\Delta t}{\Delta Z^2}\sigma^2 + r\Delta t$$

$$\gamma_j = \frac{-\Delta t}{2\Delta Z}(r - q - \frac{\sigma^2}{2}) - \frac{\Delta t}{2\Delta Z^2}\sigma^2$$

$$\alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} = f_{i,j}$$

Where $\alpha_j^* = \frac{1}{1+r\Delta t}\left[-\frac{\Delta t}{2\Delta Z}(r-q-\frac{\sigma^2}{2})+\frac{\Delta t}{2\Delta Z^2}\sigma^2\right]$

$$\beta_j^* = \frac{1}{1+r\Delta t}(1-\frac{\Delta t}{\Delta Z^2}\sigma^2)$$

$$\gamma_j^* = \frac{1}{1+r\Delta t}\left[\frac{\Delta t}{2\Delta Z}(r-q-\frac{\sigma^2}{2})+\frac{\Delta t}{2\Delta Z^2}\sigma^2\right]$$

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(u_{n-i}-\bar{u}\right)^2$$

$$\sigma_n^2 = \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}u_{n-i}^2 + \lambda^m\sigma_{n-m}^2$$

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \beta\omega + \beta^2\omega + \alpha u_{n-1}^2 + \alpha\beta u_{n-2}^2 + \alpha\beta^2 u_{n-3}^2 + B^3\sigma_{n-3}^2$$

$$\prod_{i=1}^{m}\left[\frac{1}{\sqrt{2\pi v}}\exp\left(\frac{-u_i^2}{2v}\right)\right]$$

$$\frac{1}{m}\sum_{i=1}^{m}u_i^2$$

$$\sum_{i=1}^{m}\left[-ln\left(v_i\right)-\frac{u_i^2}{v_i}\right]$$

$$m\sum_{k=1}^{K}w_k\eta_k^2$$

$$w_k = \frac{m+2}{m-k}$$

$$\sigma_n^2 = (1-\alpha-\beta)V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 - V_L = \alpha\left(u_{n-1}^2 - V_L\right) + \beta\left(\sigma_{n-1}^2 - V_L\right)$$

$$E\left[\sigma_{n+t}^2\right] = V_L + (\alpha+\beta)^t\left(\sigma_n^2 - V_L\right)$$

$$\text{cov}_n = \frac{1}{m}\sum_{i=1}^{m} x_{n-i}y_{n-i}$$

$$\text{cov}_n = \lambda\,\text{cov}_{n-1} + (1-\lambda)\,x_{n-1}y_{n-1}$$

$$\text{cov}_n = \omega + \alpha x_{n-1}y_{n-1} + \beta\,\text{cov}_{n-1}$$

$$Q(t) = 1 - e^{\bar{\lambda}(t)t}$$

$$E_0 = V_0 N(d_1) - De^{-rT}N(d_2) \text{ where}$$

$$d_1 = \frac{lnV_0 / D + \left(r + \sigma_v^2/2\right)T}{\sigma_V\sqrt{T}}$$

$$d_2 = d_1 - \sigma_V\sqrt{T}$$

$$\sigma_E E_0 = N(d_1)\sigma_V V_0$$

$$x_i = a_i M + \sqrt{1-a_i^2}\,Z_i$$

$$Q_i\left(T \mid M\right) = N\left(\frac{N^{-1}\left[Q_i(T)\right] - a_i M}{\sqrt{1-a_i^2}}\right)$$

$$Q_i\left(T \mid M\right) = N\left(\frac{N^{-1}\left[Q_i(T)\right] - \sqrt{\rho}M}{\sqrt{1-\rho}}\right)$$

$$\beta_{AB}(T) = \frac{P_{AB}(T) - Q_A(T)Q_B(T)}{\sqrt{\left[Q_A(T) - Q_A(T)^2\right]\left[Q_B(T) - Q_B(T)^2\right]}}$$

$$\beta_{AB}(T) = \frac{M\left(x_A(T), x_B(T); \rho_{AB}\right) - Q_A(T)Q_B(T)}{\sqrt{\left[Q_A(T) - Q_A(T)^2\right]\left[Q_B(T) - Q_B(T)^2\right]}}$$

$$V(X,T) = N\left(\frac{N^{-1}[Q(T)] + \sqrt{\rho} N^{-1}(X)}{\sqrt{1-\rho}} \right)$$

$$Q_i(T|M) = N\left(\frac{N^{-1}[Q_i(T)] - a_i M}{\sqrt{1-a_i^2}} \right)$$

$$Q(T|M) = N\left(\frac{N^{-1}[Q(T)] - \sqrt{p} M}{\sqrt{1-\rho}} \right)$$

$$p(k,T|M) = \frac{N!}{(N-k)!k!} Q(T|M)^k \left[1 - Q(T|M)\right]^{N-k}$$

$$e^{-rT_1} \hat{E}\left[c \frac{S_1}{S_0} \right]$$

$$S_0 e^{-qT_2} M\left(a_1, b_1; \sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, b_2; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(a_2)$$

$$a_1 = \frac{ln\left(S_0/S^*\right) + \left(r - q + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}} \qquad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{ln\left(S_0/K_2\right) + \left(r - q + \sigma^2/2\right)T_2}{\sigma\sqrt{T_2}} \qquad b_2 = b_1 - \sigma\sqrt{T_2}$$

$$K_2 e^{-rT_2} M\left(-a_2, b_2; -\sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, b_1; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(-a_2)$$

$$K_2 e^{-rT_2} M\left(-a_2, -b_2; \sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, -b_1; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(-a_2)$$

$$S_o e^{-qT_2} M\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(a_2)$$

$$\max(c, p) = c + e^{-q(T_2-T_1)} \max\left(0, Ke^{-(r-q)(T_2-T_1)} - S_1\right)$$

$$H \le K: \; c_{di} = S_0 e^{-qT} (H/S_0)^{2\lambda} N(y) - Ke^{-rT} (H/S_0)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right)$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{ln\left[H^2/\left(S_0K\right)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$c_{do} = c - c_{di}$$

$$H \geq K:\ c_{do} = S_0N\left(x_1\right)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N\left(y_1\right) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(y_1 - \sigma\sqrt{T}\right)$$

$$c_{di} = c - c_{do}$$

$$x_1 = \frac{\ln\left(S_0/H\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{\ln\left(H/S_0\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$H > K:\ c_{ui} = S_0N\left(x_1\right)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0e^{-qT}\left(H/S_o\right)^{2\lambda}\left[N\left(-y\right) - N\left(-y_1\right)\right]$$
$$+Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right)\right]$$

$$c_{uo} = c - c_{ui}$$

$$H \geq K:\ p_{ui} = -S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N\left(-y\right) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(-y + \sigma\sqrt{T}\right)$$

$$p_{uo} = p - p_{ui}$$

$$H \leq K:\ p_{uo} = -S_0N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N\left(-y_1\right) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(-y_1 + \sigma\sqrt{T}\right)$$

$$p_{ui} = p - p_{uo}$$

$$H < K:\ p_{di} = -S_0N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + S_0e^{-qT}\left(H/S_0\right)^{2\lambda}\left[N\left(y\right) - N\left(y_1\right)\right]$$
$$-Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right]$$

$$p_{do} = p - p_{di}$$

$$c_{ELB} = S_0e^{-qT}N\left(a_1\right) - S_0e^{-qT}\frac{\sigma^2}{2\left(r-q\right)}N\left(-a_1\right) - S_{\min}e^{-rT}\left(N\left(a_2\right) - \frac{\sigma^2}{2\left(r-q\right)}e^{Y_1}N\left(-a_3\right)\right)$$

$$a_1 = \frac{ln\left(S_0 / S_{\min}\right) + \left(r - q + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$a_2 = a_1 - \sigma\sqrt{T}$$

$$a_3 = \frac{ln\left(S_0 / S_{\min}\right) + \left(-r + q + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$Y_1 = -\frac{2\left(r - q - \sigma^2 / 2\right) ln\left(S_0 / S_{\min}\right)}{\sigma^2}$$

$$p_{ELB} = S_{\max} e^{-rT}\left(N\left(b_1\right) - \frac{\sigma^2}{2\left(r-q\right)} e^{Y_2} N\left(-b_3\right)\right) + S_0 e^{-qT}\,\frac{\sigma^2}{2\left(r-q\right)} N\left(-b_2\right) - S_o e^{-qT} N\left(b_2\right)$$

$$b_1 = \frac{ln\left(S_{\max} / S_o\right) + \left(-r + q + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$b_2 = b_1 - \sigma\sqrt{T}$$

$$b_3 = \frac{ln\left(S_{\max} / S_0\right) + \left(r - q - \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$Y_2 = \frac{2\left(r - q - \sigma^2 / 2\right) ln\left(S_{\max} / S_0\right)}{\sigma^2}$$

$$\max\left(0, S_T - S_\tau\right) + \left(S_\tau - K\right)$$

$$r - \frac{1}{2}\left(r - q - \frac{\sigma^2}{6}\right) = \frac{1}{2}\left(r + q + \frac{\sigma^2}{6}\right)$$

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

$$M_2 = \frac{2e^{\left(2(r-q)+\sigma^2\right)T} S_0^2}{\left(r - q + \sigma^2\right)\left(2r - 2q + \sigma^2\right)T^2} + \frac{2S_0^2}{\left(r-q\right)T^2}\left(\frac{1}{2\left(r-q\right) + \sigma^2} - \frac{e^{(r-q)T}}{r - q + \sigma^2}\right)$$

$$\sigma^2 = \frac{1}{T}\ln\left(\frac{M_2}{M_1{}^2}\right)$$

$$V_o e^{-qv^T} N\left(d_1\right) - U_0 e^{-qu^T} N\left(d_2\right)$$

$$d_1 = \frac{ln\left(V_o / U_o\right) + \left(q_U - q_V + \hat{\sigma}^2 / 2\right)T}{\hat{\sigma}\sqrt{T}} \qquad d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

$$\hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$$

$$dS = (r - q)Sdt + \sigma S^{\alpha} dz$$

$$\frac{dS}{S} = (r - q - \lambda k)dt + \sigma dz + dp$$

$$dS = (r - q)Sdt + \sigma(t)Sdz$$

$$\frac{dy}{S} = (r - q)dt + \sqrt{V} dz_S$$

$$dV = a(V_L - V)dt + \xi V^{\alpha} dz_V$$

$$dS = (r(t) - q(t))Sdt + \sigma(S,t)Sdz$$

$$\left[\sigma(K,T)\right]^2 = 2\frac{\partial C_{mkt} / \partial T + q(T)C_{mkt} + K\left[r(T) - q(T)\right]\partial C_{mkt} / \partial K}{K^2\left(\partial^2 C_{mkt} / \partial K^2\right)}$$

$$\frac{d\theta}{\theta} = mdt + sdz$$

$$\Delta f_1 = \mu_1 f_1 \Delta t + \sigma_1 f_1 \Delta z$$

$$\Delta f_2 = \mu_2 f_2 \Delta t + \sigma_2 f_2 \Delta z$$

$$\prod = (\sigma_2 f_2) f_1 - (\sigma_1 f_1) f_2$$

$$\Delta \prod = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2)\Delta t$$

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2}$$

$$\frac{df}{f} = \mu dt + \sigma dz$$

$$\frac{\mu - r}{\sigma} = \lambda$$

$$\mu - r = \sum_{i=1}^{n} \lambda_i \sigma_i$$

$$d\theta = \sigma\, dz$$

$$d\left(\frac{f}{g}\right) = (\sigma_f - \sigma_g)\frac{f}{g}dz$$

$$f_0 = g_0 E_g\left(\frac{f_T}{g_T}\right)$$

$$dg = rgdt$$

$$f_o = g_0 \widehat{E}\left(\frac{f_T}{g_T}\right)$$

$$f_0 = \hat{E}(e^{-\bar{\gamma}T} f_T)$$

$$f_0 = P(0,T)E_T(f_T)$$

$$A(t) = \sum_{i=0}^{N-1}(T_{i+1} - T_i)P(t, T_{i+1})$$

$$s(t) = E_A\left[s(T)\right]$$

$$f_o = A(0)E_A\left[\frac{f_T}{A(T)}\right]$$

$$c = P(0,T)E_T\left[\max(S_T - K, 0)\right]$$

$$c = e^{-RT}E_T\left[\max(S_T - k, 0)\right]$$

$$E_T\left[\max(S_T - K, 0)\right] = E_T(S_T)N(d_1) - KN(d_2)$$

$$f_0 = U_0 E_U\left[\max\left(\frac{V_T}{U_T} - 1, 0\right)\right]$$

$$f_0 = V_0 N(d_1) - U_0 N(d_2)$$

$$F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

$$y_i - \tfrac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

$$V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i$$

$$\frac{QL}{n_2} P(0, s_i) N(d_2^*)$$

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$$P_0 = -N(-d_1) V_0 + F e^{-rT} N(-d_2)$$

$$d_1 = \frac{\ln(V_0 / F) + \left(r + \tfrac{1}{2}\sigma^2\right) T}{\sigma \sqrt{T}} = \frac{\ln\left(V_0 / F e^{-rT}\right) + \tfrac{1}{2}\sigma^2 T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$y_T = -\frac{\ln \frac{B_0}{F}}{T} = -\frac{\ln \frac{F e^{-rT} - P_0}{F}}{T}$$

$$\pi_T = y_T - r = -\frac{1}{T} \ln\left(N(d_2) + \frac{V_0}{F e^{-rT}} N(-d_1) \right)$$

$$P_0 = \left[-\tfrac{N(-d_1)}{N(-d_2)} V_0 + F e^{-rT} \right] N(-d_2)$$

$$EL_T = F\left(1 - N(d_2) - N(-d_1) \frac{1}{LR}\right)$$

$$\frac{1}{T} \ln\left(\tfrac{F}{F - EL_T}\right) = -\frac{1}{T} \ln\left(\frac{F\left(N(d_2) + N(-d_1) \frac{V_0}{F e^{-rT}} \right)}{F} \right) = \pi_T$$

$$DD = \frac{\ln \frac{V_0}{DPT_T} + \left(\mu - \frac{1}{2}\sigma^2\right) T}{\sigma \sqrt{T}}$$

$$Q_T = N\left[N^{-1}(EDF) + \frac{(\mu - r)}{\sigma} \sqrt{T} \right]$$

$$Q_T = N\left[N^{-1}(EDF_T) + \rho_{V,M} \frac{\pi}{\sigma_M} \sqrt{T} \right]$$

$$Q_T = N\left[N^{-1}(EDF_T) + \rho_{v,m}\, SR\ T^{\theta} \right]$$

$$e^{-r_{v,i}t_i} = \left[(1-LGD)+(1-Q_i)LGD\right]e^{-r_i t_i}$$

$$r_{v,i} - r_i = -\frac{1}{t_i}\ln\left[1-Q_i LGD\right]$$

$$r_{v,i} - r_i = -\frac{1}{t_i}\ln\left[1-N\left(N^{-1}\left(EDF_{t_i}\right)+\rho_{v,m}\ SR\ T^{\theta}\right)LGD\right]$$

$$PV = (1-LGD)\sum_{i=1}^{n}\frac{C_i}{(1+R_i)^{t_i}} + LGD\sum_{i=1}^{n}\frac{(1-Q_i)C_i}{(1+R_i)^{t_i}}$$

$$PV = (1-LGD)\sum_{i=1}^{n}C_i e^{-i^{t_i}} + LGD\sum_{i=1}^{n}(1-Q_i)C_i e^{-i^{t_i}}$$

$$dr = \beta(m-r)dt + \eta dZ_r$$
$$dV = \mu V dt + \sigma V dZ_V$$
$$corr(dZ_r, dZ_V) = \rho dt$$

$$G_j(z) = \sum_{n=0}^{\infty}\frac{e^{-\bar{n}_j}\bar{n}_j^{\ n}}{n!}z^{nL_j} = e^{-\bar{n}_j+\bar{n}_j z^{L_j}}$$

$$G(z) = \prod_{j=1}^{m} e^{-\bar{n}_j+\bar{n}_j z^{L_j}} = e^{-\sum_{j=1}^{m}\bar{n}_j + \sum_{j=1}^{m}\bar{n}_j z^{L_j}}$$

Note: on the right, the first sum in the exponent, text has n bar **times** j. Should be n bar **sub** j. Full credit for either.

$$\frac{1}{n!}\frac{d^n G(z)}{dz^n}\Big|_{z=0}$$

$$Y = \frac{R+\lambda LGD}{1-\lambda+\lambda(1-LGD)}$$

$$Y\Delta t = \frac{Y\Delta t + \lambda\Delta t LGD}{1-\lambda\Delta t+\lambda\Delta t(1-LGD)}$$

$$Y = r + \lambda LGD$$

$$V(t,T) = E^*\left[\exp\left(-\int_t^T Y(s)ds\right)CF\right]$$

$$Y(t) = r(t) + \lambda(t)LGD + l$$

$$\lambda(t) = \lambda_0 + \lambda_1 r(t) + \lambda_2 W_M(t)$$

$$dM(t) = \left[r(t)dt + \sigma_M dW_M(t)\right]M(t)$$

$$l(t) = l_0 + l_1 r(t) + l_2 M(t) + l_3\left[M_H(t) - M_L(t)\right]^2$$

$$dr = (\alpha - \beta r)dt + \sigma_r dZ_r$$
$$dU = (a - bX)dt + \sigma_u dZ_u$$
$$corr(dZ_r, dZ_u) = \rho$$

V-C104-07

$$\int_{\xi_\rho}^{\infty} \frac{wf(w)dw}{1-\Phi(\xi_\rho)} = CTE(\rho)$$

V-C105-07

$$\sigma_\rho = \sigma\sqrt{\frac{1}{N} + (1-\frac{1}{N})\rho}$$

$$\frac{\partial \sigma_p^2}{\partial w_i} = 2w_i\sigma_i^2 + 2\sum_{j=1,\, j\neq i}^{N} w_j\sigma_{ij} = 2\operatorname{cov}(R_i, R_p)$$

$$\Delta VAR_i = \frac{\partial VAR}{\partial w_i W} = \alpha\frac{COV(R_i, R_p)}{\sigma_p}$$

$$\beta_i = \frac{COV(R_i, R_p)}{\sigma_p^2} = \rho_{ip}\frac{\sigma_i}{\sigma_p}$$

$$\Delta VAR_i = \alpha(\beta_i * \sigma_p) = \frac{VAR}{W} * \beta_i$$

$$\frac{\partial \sigma_N^2 W_N^2}{\partial a} = 2W\sigma_{ip} + 2a\sigma_i^2$$

$$a^* = -W\frac{\sigma_{ip}}{\sigma_i^2} = -W\beta_i\frac{\sigma_p^2}{\sigma_i^2}$$

V-C106-07

$$S_t = S_0 e((\mu - \frac{\sigma^2}{2})t + \sigma W_t)$$

$$CTE_\alpha = \frac{(1-\beta^{'})E\left[loss \middle| loss > V_\alpha\right](\beta^{'} - \alpha)V_\alpha}{1-\alpha}$$

V-C110-07

$$KRD_j = \frac{1}{P_0}\frac{P_j(-) - P_j(+)}{2\Delta Y}$$

V-C113-07

$$LPM_n = \sum_{Rp=-\infty}^{\tau} p_p(\tau - R_p)^n$$

V-C-114-07

$$k = (1+p)(1+r'+r'') - 1$$

$$E(R_i) = R_f + B_i \times [E(R_m) - R_f] + e_i$$

$$R_i = b_{i,0} + b_{i,1} \times F_1 + b_{i,2} \times F_2 + \ldots + b_{i,n} \times F_n + e_i$$

V-C117-07

$$B^o_{Risky} = \left[\sum_{t=1}^{T} \frac{C_{Risky}}{(1+R)^t}\right] + \frac{M}{(1+R)^T} = M$$

$$B^o_{Riskless} = \left[\sum_{t=1}^{T} \frac{C_{Risky}}{(1+r)^t}\right] + \frac{M}{(1+r)^T}$$

$$G_o = B^o_{Riskless} - B^o_{risky} = \sum_{t=1}^{T} \frac{(R-r)M}{(1+r)^t}$$

V-C120-07

$$r = \frac{D}{p} + g$$

V-C123-07

$$u_t = r_t^{\tau f} + \omega_t - \tau_t(r_t^m + \omega_t) + \delta_t - g_{t+1} + \gamma_t$$

V-C125-07

$$dS = \mu S dt + \sigma S dZ$$

$$dr = \mu(r,t) r dt + r\sigma dZ$$

$$\sigma(t,T) = \frac{\sigma\left(\dfrac{\Delta r(t,T)}{r(t,T)}\right)}{\sqrt{\Delta t}}$$

$$\sigma(t,T) = \frac{\sigma(\Delta r(t,T))}{\sqrt{\Delta t}}$$

$$dr = a(b-r)dt + \sigma\sqrt{r}dZ$$

$dr = a(b-r)dt + \sigma dZ$, (a>0)

$$dr = a_1 + b_1(l-r)dt + r\sigma_1 dZ$$

$$dl = (a_2 + b_2 r + c_2 l)dt + l\sigma_2 dW$$

$$dV = M(t,r)dt + \Omega(t,r)dZ$$

$$M(t,r) = V_t + \mu(t,r)V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr}$$

$$\Omega(t,r) = \sigma(t,r)V_r$$

$$d\Pi = (M_1(t,r) - \Delta M_2(t,r))dt + (\Omega_1(t,r) - \Delta\Omega_2(t,r))dZ$$

$$d\Pi = r\Pi dt$$

$$V_t + (\mu(t,r) - \lambda(t,r)\sigma(t,r))V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr} - rV = 0$$

$$P_i^n(1) = 2\left[\frac{P(n+1)}{P(n)}\right]\frac{\delta^i}{(1+\delta^n)} \qquad \delta = e^{-2r(1)\sigma}$$

$$P_i^n(T) = \frac{1}{2}P_i^n(1)\left\{P_i^{n+1}(T-1) + P_{i+1}^{n+1}(T-1)\right\}$$

$$r_i^n(1) = \ln\frac{P(n)}{P(n+1)} + \ln\left(\frac{1}{2}\left(\delta^{\frac{-n}{2}} + \delta^{\frac{n}{2}}\right)\right) + \left(\frac{n}{2} - i\right)\ln\delta$$

Note: Typo in text $r_i^n(1)1 =$ either way will receive full credit.

$$dr = (f^{'}(0,t) + \sigma^2 t)dt + \sigma dz$$

$$r(n)\sigma^s(n) = \frac{-\frac{1}{2}\ln\left[\delta(n)\delta(n-1)...\delta(1)\right]}{n}$$

$$P_i^n(1) = \left[\frac{P(n+1)}{P(n)}\right]\left[\tfrac{(1+\delta_{n-1}\delta_{n-2}..\delta_1)..(1+\delta_{n-1})2}{(1+\delta_n..\delta_1)..(1+\delta_n)}\right]\delta_n^i$$

$$dr = (f^{'}(0,t) + \sigma^2(t)t + \tfrac{\sigma^{'}(t)}{\sigma(t)}\left[r(t) - f(0,t)\right])dt + \sigma(t)dZ$$

$$P_{i,j}^n(1) = \frac{P(n+1)}{P(n)}\frac{(1+\delta_{n-1}^1..\delta_1^1)(1+\delta_{n-1}^1..\delta_2^1)..(1+\delta_{n+1}^1)2}{(1+\delta_n^1..\delta_1^1)..(1+\delta_n^1\delta_{n-1}^1)(1+\delta_n^1)} \times$$

$$\frac{(1+\delta_{n-1}^2..\delta_1^2)(1+\delta_{n-1}^2..\delta_2^2)..(1+\delta_{n-1}^2)2}{(1+\delta_n^2..\delta_1^2)(1+\delta_n^2..\delta_2^2)..(1+\delta_n^2)}(\delta_n^1)^i(\delta_n^2)^j$$

$$dr = \left\{ f^{'}(t) + \left|\sigma(t)\right|^2 t + \frac{\left|\sigma^{'}(t)\right|\cos\phi(t)}{\left|\sigma(t)\right|\cos\theta(t)}\left[r - f(t)\right]\right\}dt + \sigma(t)dW$$

$$d\ln r = (\theta(t) - \frac{\sigma^{'}(t)}{\sigma(t)}\ln r)dt + \sigma(t)dW$$

$$dr(t) = (\alpha(t) - \beta r(t))dt + \sigma dW(t)$$

where $\alpha(t) = \frac{\partial f(0,t)}{\partial T^*} + \beta f(0,t) + \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$

$$dr = \left[\theta(t) + \mu - ar\right]dt + \sigma_1 dW$$

$$du = -budt + \sigma_2 dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma^p(t,T^*)P(t,T^*)dZ$$

$$df(t,T^*) = \sigma^p(t,T^*)\sigma_{T^*}^p(t,T^*)dt - \sigma_{T^*}^p(t,T^*)dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma(T^* - t)P(t,T^*)dZ(t,T^*)$$

$$L(t,T^*) = \frac{1}{\Delta}\left(\frac{P(t,T^*)}{P(t,T^* + \Delta)} - 1\right)$$

$$dL(t,T^*) = L(t,T^*)\left[\sum_{j=t^*}^{N^*}\frac{L(t,j\Delta)\Delta}{1+L(t,j\Delta)\Delta}\Lambda(T^*-j\Delta)\Lambda(T^*-t)dt + \Lambda(T^*-t)dZ\right]$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k}\frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}\tilde{Z}\right]$$

where $\sigma_j^2 j = \sum_{i=1}^{j}\Lambda_{j-i}^2$

caplet $C_k = L\delta_k P(t_{k+1})\left[F_k N(d_1) - R_x N(d_2)\right]$

where $d_1 = \dfrac{\ln\left[\dfrac{F_k}{R_X}\right] + \sigma_k^2\dfrac{t_k}{2}}{\sigma_k\sqrt{t_k}}$ $\qquad d_2 = d_1 - \sigma_k\sqrt{t_k}$

$$\text{swaption} = \sum_{i=1}^{mn}\frac{L}{m}P(t_i)\left[R_F N(d_1) - R_X N(d_2)\right] = L * A\left[R_F N(d_1) - R_X N(d_2)\right]$$

where $A = \dfrac{1}{m}\sum_{i=1}^{mn}P(t_i) \quad 1 \le i \le mn$

$$P(k+1,j) = P(k,j)\exp\left[\left(r(k) - \frac{\sigma^2(j-k)}{2}\right)\Delta + \sigma(j-k)\sqrt{\Delta}Z(j-k)\right]$$

$$\sigma^*(T^*-t) = (a + b(T^*-t))\exp(-c(T^*-t)) + d$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k}\frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}Z\right]$$

$$P(T^*,i;T) = \frac{P(T^*+T)}{P(T^*)}\cdot 2\cdot\frac{\prod_{t=T}^{T+T^*-1}h(t)}{\prod_{t=1}^{T^*-1}h(t)}\delta^{Ti} \quad \text{where } h(t) = \frac{1}{1+\delta^t}$$

V-C127-07

$$L_o R_{s(L)} = A_o R_A - L_o R_{L'}$$

$$R_{S(L)} = \left(\frac{A_0}{L_0}R_A\right) - R_L$$

$$R_A = R_f + \beta_A r_Q + \alpha$$

$$\sigma_A^2 = \beta_A \sigma_Q^2 + \omega_A^2$$

$$R_L = R_f + \beta_L r_Q + \alpha_L$$

$$\max(U_S) = R_S - \lambda \sigma_S^2$$

$$\max(U_S) = \left(\frac{A_0}{L_0} - 1\right) R_F + \beta_S \mu_Q - \lambda_\beta \beta_S^2 \sigma_Q^2 + \left(\frac{A_0}{L_0}\alpha_A - \alpha_L\right) - \lambda_\omega \left[\left(\frac{A_0}{L_0}\right)^2 \omega_A^2 - 2\frac{A_0}{L_0}\omega_A \omega_L + \omega_L^2\right]$$

$$P_{TIPS} = \frac{F}{(1+r)^T}$$

$$PV_{liability} = \sum_{t=0}^{T} \frac{CF_{active}(1+i_{wage})^t + CF_{retired}(1+i_{COLA})^t}{(1+i)^t (1+r)^t}$$

$$P_{EQUITY} = \sum_{t=0}^{\infty} \frac{Dvd_0 (1+g_r)^t}{(1+r)^t}$$

V-C129-07

external cash flow at the beginning of the period $r_t = \dfrac{MV_1 - (MV_0 + CF)}{MV_0 + CF}$

external cash flow at the end of period $r_t = \dfrac{(MV_1 - CF) - MV_0}{MV_0}$

$$MV_1 = MV_0(1+R)^m + CF_1(1+R)^{m-L(1)} + ... + CF_n(1+R)^{m-L(n)}$$

$$R_p = a_p + \beta_p R_I + \varepsilon_p$$

$$r_V = \sum_{i=1}^{n}[w_{Vi} r_i] = \sum_{i=1}^{n}\left[(w_{pi} - w_{Bi}) r_i\right] = \sum_{i=1}^{n} w_{pi} r_i - \sum_{i=1}^{n} w_{Bi} r_i = r_p - r_B$$

$$r_{AC} = \sum_{i=1}^{A} w_i (r_{Ci} - r_f)$$

$$r_{IS} = \sum_{i=1}^{A} \sum_{j=1}^{M} w_i w_{ij} (r_{Bij} - r_{Ci})$$

$$r_{IM} = \sum_{i=1}^{A} \sum_{j=1}^{M} w_i w_{ij} (r_{Aij} - r_{Bij})$$

$$r_V = \sum_{i=1}^{n} \left[(w_{pi} - w_{Bi})(r_i - r_B) \right]$$

$$r_V = \sum_{j=1}^{S} (w_{pj} - w_{Bj})(r_{Bj} - r_B) + \sum_{j=1}^{S} (w_{pj} - w_{Bj})(r_{pj} - r_{Bj}) + \sum_{j=1}^{S} w_{Bj} (r_{pj} - r_{Bj})$$

$$R_{At} - r_{ft} = \alpha_A + \beta_A (R_{Mt} - r_{ft}) + \varepsilon_t$$

$$T_A = \frac{\bar{R}_A - \bar{r}_f}{\hat{\beta}_A}$$

$$S_A = \frac{\bar{R}_A - \bar{r}_f}{\hat{\sigma}_A}$$

$$M_A^2 = \bar{r}_f + \left[\frac{\bar{R}_A - \bar{r}_f}{\hat{\sigma}_A} \right] \hat{\sigma}_M$$

$$IR_A = \frac{\bar{R}_A - \bar{R}_B}{\hat{\sigma}_{A-B}}$$

V-C130-07

$$r = r_f + OAS - D_{OAS} \Delta OAS - \sum D(i) \Delta r(i)$$

$$r = r_f + OAS - D_{OAS} \Delta OAS - \sum D(i) \Delta r(i) + \frac{r}{c}$$

$$r = r_f + OAS - D_{oas} \Delta OAS - \sum D(i) \Delta r(i) + r / c + pa - e_a$$

$$r = r_f + ROAS - \sum D_l (i) \Delta r(i) + e_l$$

$$r_i = r_f + NOAS - D_{noas} \Delta NOAS - \sum D(i) \Delta r(i)$$

$$\begin{aligned} r_a - r_i &= OAS - NOAS - D_{oas} \Delta OAS + D_{noas} \Delta NOAS \\ &\quad - \sum D_a(i) \Delta r(i) + \sum D_l(i) \Delta r(i) + r / c + pa - e_a \end{aligned}$$

$$r_i - r_l = NOAS - D_{noas} \Delta NOAS - ROAS - e_l$$