TRANSACTIONS OF SOCIETY OF ACTUARIES
1992 VOL. 44

A PRACTICAL GUIDE TO INTEREST RATE GENERATORS FOR C-3 RISK ANALYSIS

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ABSTRACT

Models of possible future paths for interest rates are a key element of actuarial and other financial studies. This paper discusses such models, or interest rate generators, which vary by intended purpose.

The primary focus is on fairly elementary generators for studying C-3 risk, with major emphasis on parametric, mean reversionary generators. A comparison of the models available in the literature and two original models is included. All models were adjusted to meet selected reasonableness conditions.

A guide to the generators is presented to permit comparisons. Statistical tests show that each generator is unique.

I. INTRODUCTION

Interest rate generation is a process that produces scenarios of future interest rates. One major application is pricing of options on bonds and other securities. Interest rates generated for option-pricing should be arbitrage-free and display put-call parity* (see Bookstaber [2], Hogan and Breidbart [9]). That is, the interest rates produced by this generator should be consistent with the financial market's view of interest rates currently and its expectation of where they are likely to go (pure expectations theory). Toevs and Dyer, in *The Term Structure of Interest Rates and Its Use in Asset & Liability Management* [19], and Nelson [15] mention that term structures vary by market, and they describe different theories of the term structure of interest rates. Each of these theories leads to a different interest rate generator. Interest rate generators designed primarily for option-pricing tend to produce sequences of forward rates, which can be converted to spot yield curves. The preferred generators tend to be either binomial lattices (see Pedersen

* Put-call parity refers to the relationship between the price of the put option, the call option and the bond price. It is usually described in terms of the current rates, r_n, the price of an n-period zero-coupon bond, B_n, and the exercise price, E, of both the put and call options. If the price of the put as determined by the interest rate scenarios is P and the price of the call as determined by the same scenarios is C, then put-call parity exists if for each n, $C-P=B_n-E/(1+r_n)^n$.

[16], Dyer [4], Bookstaber [2], Ho and Lee [8], Hogan and Breidbart [9]) or stochastic diffusion models based on differential equations (see Beekman and Shiu [1], Boyle [3], Hogan and Breidbart [9], Sharp [17]). Some models are based on statistical methods used to describe time series, especially those which analyze rates as autoregressive integrated moving average (ARIMA) processes (see Giaccotto [6] and Dhane [5]).

A second major purpose for interest rate generators is C-3 risk analysis for pricing or valuation purposes. What is desired is a variety of scenarios, not necessarily restricted to those which are likely, but which meet "reasonable conditions." Typically, these generators are used to produce spot yield curves and can be deterministic or stochastic. Although forward rates can be derived from these yield curves, no attempt is made to avoid arbitrage opportunities or require put-call parity. It is assumed that the investment strategy being used in the C-3 testing either has a limited number of choices or is completely deterministic. Thus, the investment strategy does not search for arbitrage opportunities to exploit.

Interest rate generators designed for C-3 analysis need to produce a sequence of entire spot yield curves for each scenario, because at any time the interest rate on a given bond or commercial mortgage needs to be compared with the then-current rate for the same remaining term of the asset, to make appropriate prepayment decisions.

Arbitrage-free interest rate generators typically generate only a single sequence of short rates per scenario. A sequence of long rates can be determined and used for prepayment assumptions when it is appropriate to compare the interest rate with that for a fixed term. For example, this method can be used for residential mortgages. It is a complex task to ensure that long rates are arbitrage-free and have an appropriate relationship to short rates because the arbitrage-free condition requires the average (in a present value sense) of all the scenarios 1 month forward 1 year from now to be the current 1-year forward rate and the average (present value sense) 1 month forward 30 years from now to be the current 30-year rate. The problem is how to define the 5-year forward rate 26 years from now or the 30-year forward rate 5 years from now, because there is little information in the tail of the yield curve.

The single sequence of short-term forward rates produced by a scenario in an arbitrage-free interest rate generator can be converted to a single spot yield curve, which is used for discounting purposes only. Thus a single scenario produces one curve rather than a sequence of yield curves per scenario.

A number of different interest rate generators suitable for C-3 analysis are presented and compared in this paper. Because of the difficulties inherent in making sequences of future spot rate curves arbitrage-free, no arbitrage-free generators are considered. The following types of interest rate generators are discussed: parallel shifts, lognormal generators, mean reversionary processes, and a Markov chain process.

II. CRITERIA FOR INTEREST RATE GENERATORS

A. General Criteria

Projected interest rates should be consistent with the rates that a company expects to earn on its assets. In general, this would include a spread over Treasuries or other risk-free securities.

Rates should be projected for a period long enough to meet New York Regulation 126 requirements and should be "reasonable" throughout the period. Other considerations include whether the generator produces a variety of yield curve shapes and whether the proportion of inverted and sloping ("normal") yield curves is representative of anticipated results.

Within a scenario, volatility of the long-term rates should be less than that of short-term rates. Annual changes in yield curve within a scenario should meet reasonable criteria. Some requirements are subjective. Graphing of scenarios as in Figures 1–7 provides a quick check of reasonableness.

Political pressures and government action, while adding to interest rate volatility, also tend to keep interest rates bounded. Political actions in both the U.S. and Canada prevent hyperinflation, which has plagued other countries with less stable economies. Thus, an upper bound on interest rates is appropriate. Similarly, political actions are applied in a recession to avoid a depression. The U.S. national debt and the ever-rising cost of health care in both the U.S. and Canada suggest the need for a lower bound on interest rates. Homer [10], in his historical study of interest rates, provides much support for bounds of 3 percent and 25 percent.

B. New York Regulation 126 Requirements

New York Regulation 126 requires that all companies licensed to do business in New York State either submit an Actuarial Opinion and Memorandum for all annuity, GIC, and single-premium whole life business, or hold significantly higher reserves.

It further requires projections until the major portion of the liability cash flows are gone. Required horizons vary by product type. A horizon of 20 years or longer is suggested for annuities in payment. New York also requires the projected rates to be between 4 percent and 25 percent inclusive.* Regulation 126 strongly recommends including seven deterministic parallel shift scenarios as a minimum for scenario testing.

III. DESCRIPTION OF TYPES OF INTEREST RATE GENERATORS

A. Parallel Shift Interest Rate Generators

These generators produce a set of additions to the current rate, which are used to shift the curve up or down from its previous position. Amounts can be deterministic or stochastic. The New York 7, which are projected for 30 years and remain constant after the final change, include:

- No change
- Rising ½ percent per year for 10 years
- Falling ½ percent per year for 10 years
- Rising 1 percent per year for 5 years, falling 1 percent per year next 5 years
- Falling 1 percent per year for 5 years, rising 1 percent per year next 5 years
- Pop up 3 percent first year
- Pop down 3 percent first year.

Stochastically, additions can be generated as a random walk over a symmetric interval such as $[-7, 7]$ or as differences between successive rates determined by a lognormal or other process. However, it would be necessary to skip any addition that results in any rate on the curve violating preselected bounds, regardless of the generation method.

B. Nonparallel Shift Interest Rate Generators

These generators change the shape of the spot yield curve as well as rate levels. Many of them are based on a lognormal process, although other methods, such as Markov chains, are suitable and easily modified to produce reasonable results.

IV. INTEREST RATE GENERATION PROCESSES

Eight parametric nonparallel shift interest rate generators are compared in the following sections. The first six are mean reversionary lognormal processes;

*A revision in the law since this paper was originally prepared has reduced the minimum.

these are the generators developed by Jetton [11], Strommen [18], Gurski [7] (two), and Mereu [12] (two). The seventh mean reversionary generator is a composite of what appeared to be the best features of the first six; it is referred to as "composite." The eighth generator is parametric and based on a Markov chain process (MCP). To facilitate comparisons between models, all the models were modified, if necessary, to produce rates at ¼, ½, 1, 2, 3, 5, 7, 10 (11)*, 20, and 30 years. Details of the Jetton, Strommen and Gurski generators can be found in Appendix A. IGM (interest generating mechanism) is the method proposed by Mereu. It specifically calculates an inflation rate and includes inflation in the calculation of long-term rates. Details of the two IGM processes can be found in Appendix B; the composite generator in Appendix C; and the Markov chain generator in Appendix D.

A. Mean Reversionary Lognormal Process

1. Lognormal

According to Dyer, the lognormal process is the continuous analogue to the binomial lattice. It is easier and faster to program and does not require the computation of more paths than will be used. Whereas the binomial lattice requires the computation of 2^n paths for an n-period projection, even if only 100 or 1000 are required, the lognormal process calculates just the desired number.

For a lognormal process, an initial spot rate is determined from the current yield curve, and subsequent rates from the recursive relationship

$$i_{t+1} = i_t e^{VF \times Z}$$

where VF is a volatility factor often referred to as drift and Z is a value from a normal [0, 1] distribution.

2. Mean Reversion

When rates developed by a lognormal process get out of bounds, merely applying bounds causes "stickiness" (a sequence of constant rates). A mean reversionary process is one in which the rates being generated are constantly being pulled towards a preset goal (the expected mean rate) by the use of a correction factor. In the models studied this is an additive term that disrupts the addition of exponents in the simple lognormal generator (see Jetton [11])

*The Markov chain process produces an 11-year rate instead of the 10-year rate.

and keeps the rates in a more reasonable range and the volatility of the projected rates bounded. Also, if absolute bounds are imposed, there is much less "stickiness" with the use of a mean reversionary generator.

All mean reversionary processes require a goal for the rate being modeled. They differ in which rate they model first, the correction factor being applied to the rate and how the other rates on the curve are determined.

3. Correction Factors

Correction factors (cf) differ in whether they are applied before or after the lognormal process. The simplest correction factor is a linear correction factor added to a tentative new rate, such as C [goal-tentative rate]; Mereu [12] and Gurski [7] use this method. Correction factors also can be determined as a function of the difference between the goal and the current rate (*diff*), as used by Jetton [11] and Strommen [18]. After comparing $C_1 \cdot diff$ with $C_2 \cdot diff^3$, we use whichever has the smaller absolute value. This correction factor is then added to i_t before the new rate is calculated, that is,

$$i_{i+1} = (i_t + cf)e^{VF \times Z}.$$

Although this method appears to be somewhat more effective than the simple linear correction, the relative efficiency is probably more dependent on choices of parameters than on the method used.

4. Secondary Rate Determination

The generators studied typically determine at least one other set of spot yield rates with some stochastic component prior to determining the remainder of the curve. This component can be based on a deterministic calculation of the other rate plus a random number times the assumed long-term volatility (Jetton) or on a deterministic ratio (Strommen). Mereu used a mean reversionary process on the ratio of the short- to long-term rates. Gurski used a lognormal process with correlation coefficients and recognized constraints on spreads between the rates involved. While Gurski determines three rates—long, intermediate, and short—all others generate only long and short. All models use a deterministic method to develop the rest of the curve. Jetton and Strommen use arbitrary fixed weights applied to the 1- and 20-year rates to determine the intermediate yields. All curves produced by their generators are monotonic, either "normal" or inverted. Jetton's curves are always "normal" at lower interest rates and inverted at higher rates. Strommen's curves tend to be either "normal" or inverted at any interest rate level (due to his use of a ratio). However, only "normal" shapes appear at very low interest rates. The others (Mereu and Gurski) use a formula for calculating the intermediate rates. Gurski actually has two methods for calculating intermediate rates using logarithmic or quadratic splines, both of which lead to curves that are either monotonic or piecewise monotonic (with the 10-year rate as the maximum or minimum if the curve is not monotonic).

Another possibility for determining the other rates when more than two points on each curve have been determined is Lagrange interpolation. With three sets of rates (1, 10 and 30 years), this may lead to strange relationships between 10- and 30-year rates due to the wide interpolation interval (see Figure 4, the Gurski-2 method).

B. Composite Process

A final mean reversionary interest rate generator was formed by a combination of the Jetton and Gurski methods, which uses Lagrange interpolation on five points (1-, 5-, 10-, 20-, and 30-year rates). With the 10-year as the focal point, Jetton's mean reversionary correction factor is applied before the lognormal process. The remaining rates are based upon correlations with the 10-year rate. The spread between the 10-year and other rates is reduced by 60 percent of the excess over a preset parameter. All spread allowances were taken to be symmetric.

Table 1 summarizes the features of each mean reversionary generator. Note that bounds were imposed upon all generators, and yield curves were extended to ¼–30 years.

Figures 1–7 show typical results from a single scenario; each generator shows yield curves for each of the first five years. Graphing permits observation of the shapes of the yield curves produced and aids in determining reasonableness quickly.

C. Markov Chain Process

The Markov chain process interest rate generator was developed based upon an analysis of Barra spot yield curves from quarter ends March 31, 1980 through December 31, 1983. Some of the Barra curves had recognizable shapes, while others did not. All shapes were determined as percentages of the 20-year rate after a parallel shift so that all 20-year rates were 12 percent. Then an envelope containing these shapes was determined, and seven shapes were identified and coded (see Figure 8):

1. Steep normal curve (upward sloping)
2. Early peak (rates climb and then drop off)

TABLE 1

SUMMARY OF CHARACTERISTICS OF THE INTEREST RATE GENERATORS

Generator Name	Correction Factor	Key Rate	Secondary and Other Rates Calculated	Remaining Rates	Other Comments
Gurski					
1	After	10-yr	1-yr, 30-yr	Log/Quad Splines	Constraints on spreads and correlation coefficient
2	After	10-yr	1-yr, 30-yr	Lagrange	
Jetton	Before	1-yr	20-yr	Arb. Weights	Monotonic
Mereu					
IGM (1)	After	INFL	1-yr, 10-yr	Formula	Monotonic or bowed
IGM (2)	After	INFL	1-yr, 10-yr	Formula	Monotonic or slightly humped
Strommen	Before	1-yr	20-yr ratio	Arb. Weights	Monotonic
Composite	Before	10-yr	1, 5, 20, 30	Lagrange	Constraints on spreads and correlation coefficients

3. Oscillating, starting up
4. Level
5. Oscillating, starting down (the mirror of 3)
6. Early valley (the mirror of 2)
7. Steep inverted (the mirror of 1).

This model determines the shape of the curve and the level of rates independently with each subject to certain constraints. The current yield curve is used as the initial curve.

Along with the current curve, the model requires the best approximation for its shape. Abrupt changes in shape are not permitted. Currently the matrix, which determines the result of the random walk on the shapes, does not permit changing shape code by more than two in a year. Because the steep normal yield curve is the most common shape, a deliberate attempt is made to revert toward that shape, curve code 1. An arbitrary matrix of probabilities was set up to reflect the probability of moving from one shape code to another (see Appendix E). This was translated to a table lookup for the code for the next shape based on the code for the current shape and a random number. The shapes thus far determined will be referred to as theoretical. See Appendix F for sample results.

FIGURE 1

INTEREST RATE CURVES GENERATED BY JETTON METHOD
(SCENARIO: FIRST FIVE YEARS)

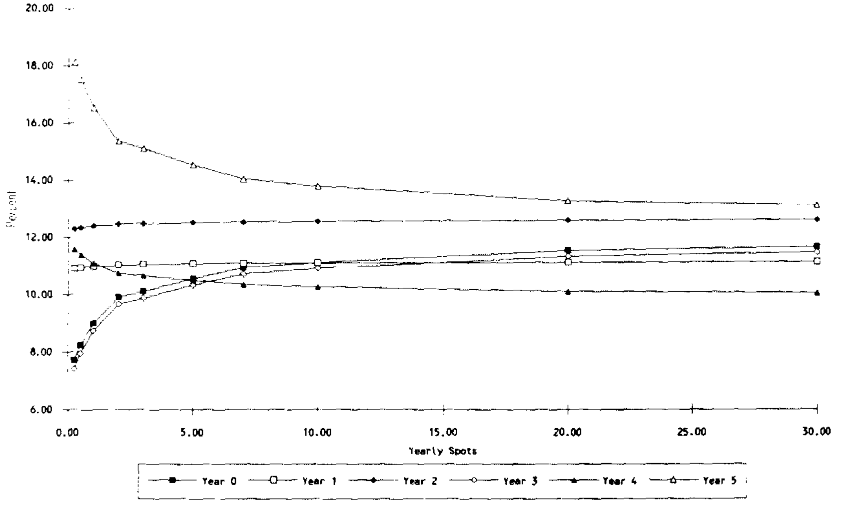


FIGURE 2

INTEREST RATE CURVES GENERATED BY STROMMEN METHOD
(SCENARIO: FIRST FIVE YEARS)

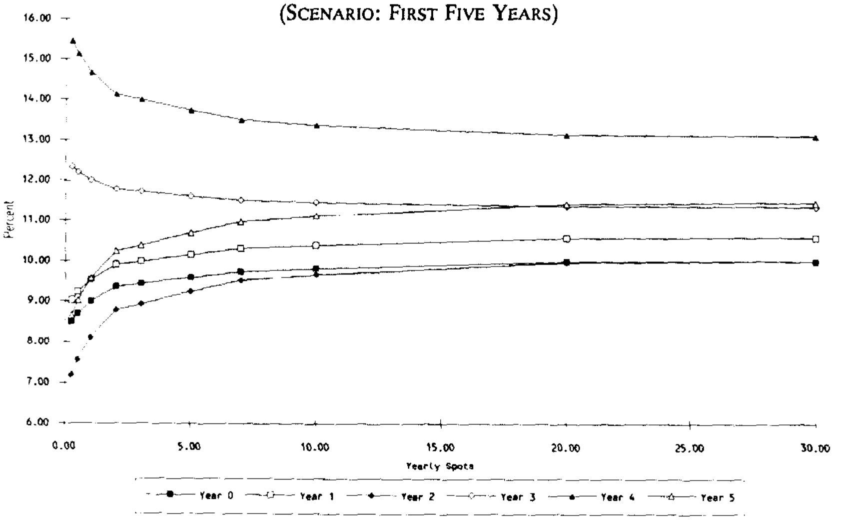


FIGURE 3

INTEREST RATE CURVES GENERATED BY GURSKI-1 METHOD
(SCENARIO: FIRST FIVE YEARS)

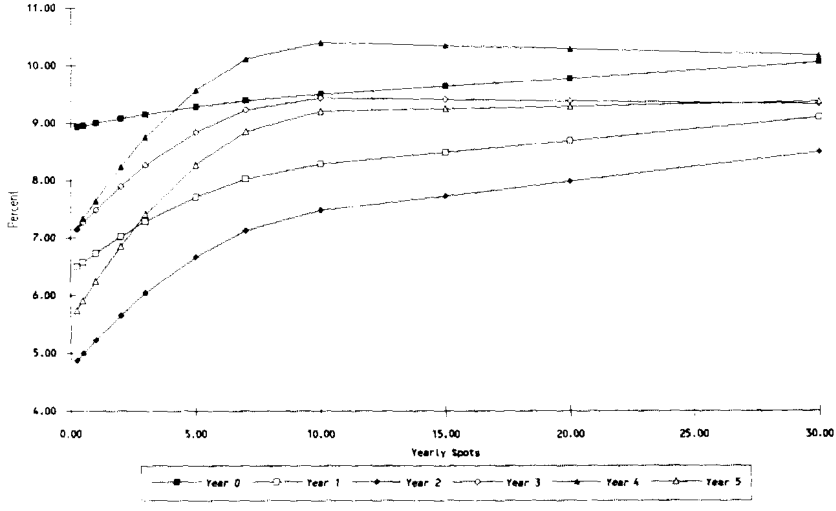


FIGURE 4

INTEREST RATE CURVES GENERATED BY GURSKI-2 METHOD
(SCENARIO: FIRST FIVE YEARS)

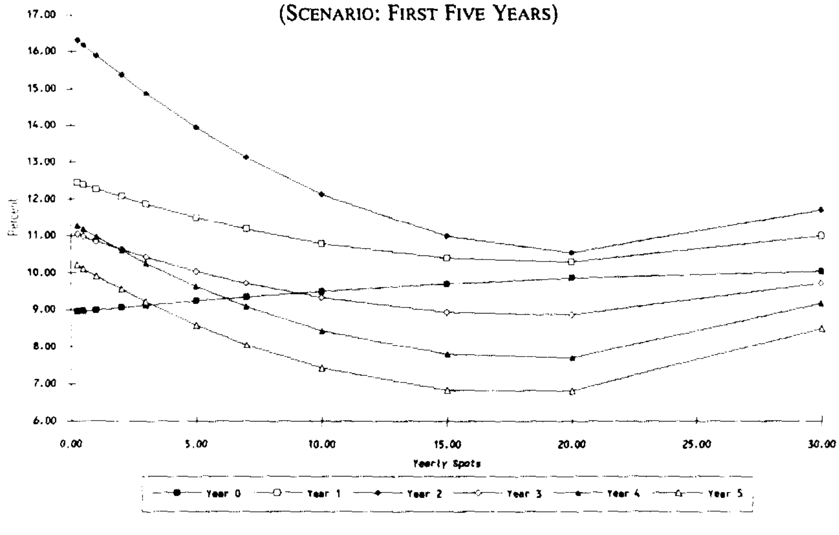


FIGURE 5

INTEREST RATE CURVES GENERATED BY IGM-1 METHOD
(SCENARIO: FIRST FIVE YEARS)

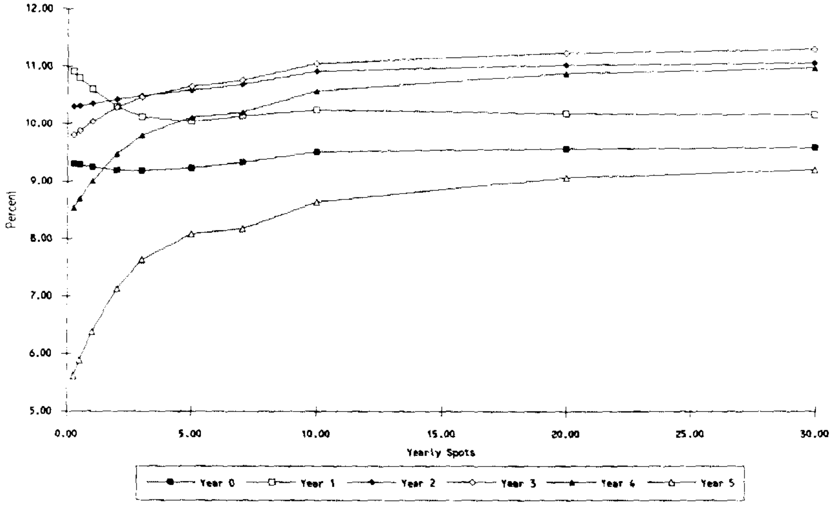


FIGURE 6

INTEREST RATE CURVES GENERATED BY IGM-2 METHOD
(SCENARIO: FIRST FIVE YEARS)

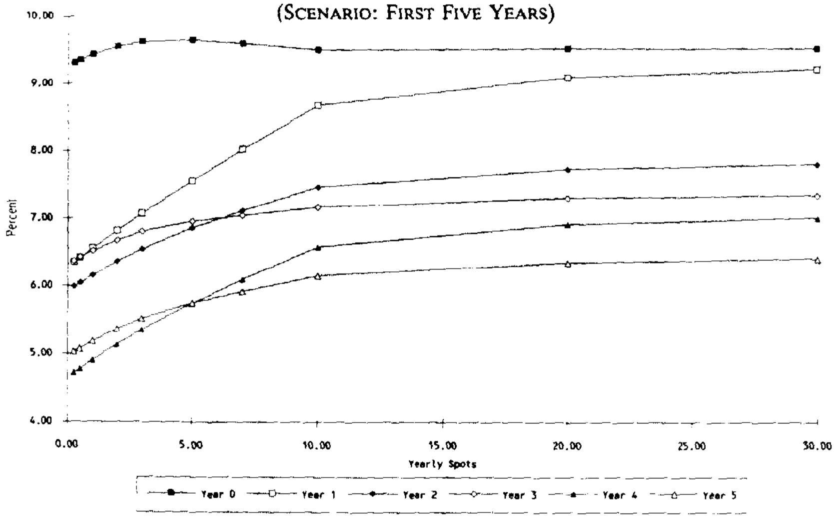


FIGURE 7

INTEREST RATE CURVES GENERATED BY COMPOSITE METHOD
(SCENARIO: FIRST FIVE YEARS)

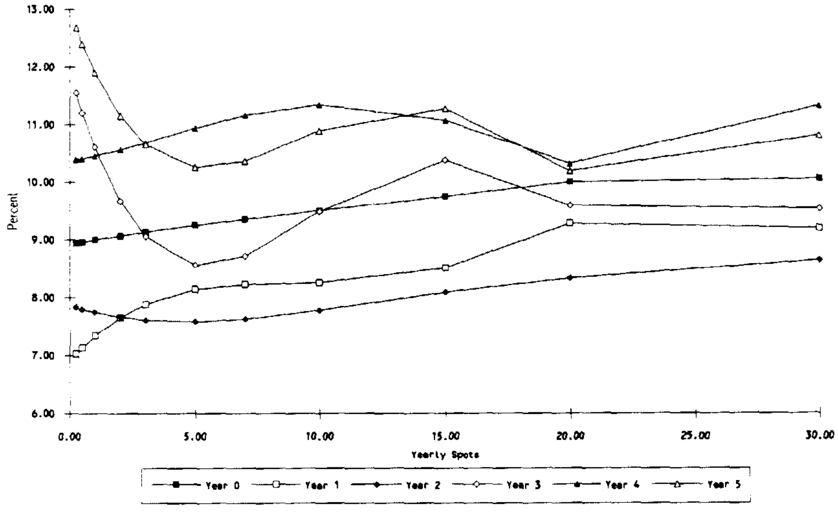
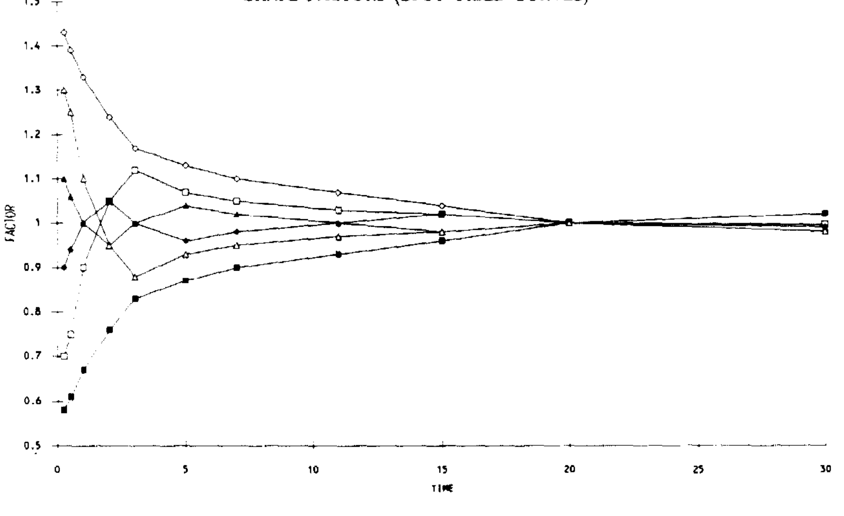


FIGURE 8

SHAPE FACTORS (SPOT YIELD CURVES)



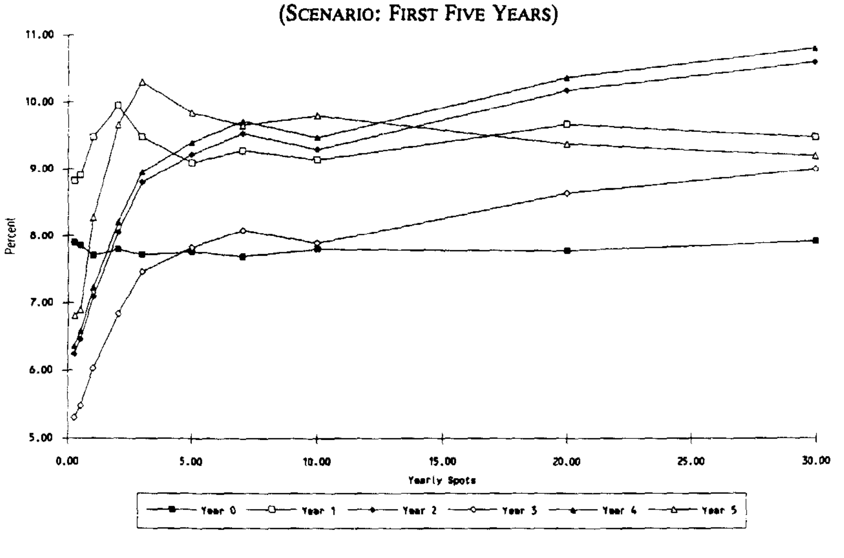
Tentative rates for the 20-year spot curve are determined based on a uniform distribution (see Appendix D) or a mean reversionary lognormal process (a later version not included in this study). These are adjusted so that the yearly change is no more than a preset percentage (*ADJRMAX*) of the current year's rate.

The final results are determined by applying a maximum percentage change to all rates (*ADJMAX*) (that is, next year's rate must be between $(1-ADJMAX) \times$ current rate, and $(1+ADJMAX) \times$ current rate). Afterwards, they are adjusted for overall maximum and minimum bounds. *ADJMAX* is generally larger than *ADJRMAX*, because short rates tend to be more volatile than long rates (see Milgrom [13]). Murphy [14] also supports the conclusion that yields of long bonds are less volatile than those of short bonds. He also concludes that volatility is higher when rates are higher, giving indirect support for the imposition of relative bounds. The imposition of relative bounds often has the effect of changing the shape of the curve away from the theoretical. This is desirable because not all curves studied had perfect shapes. Sample parameters are found in Appendix D.

Figure 9 shows the first five yearly changes in yield curves of a scenario.

FIGURE 9

INTEREST RATE CURVES GENERATED BY MARKOV CHAIN PROCESS
(SCENARIO: FIRST FIVE YEARS)



V. EVALUATION OF INTEREST RATE GENERATORS

The generators were run by using the parameters that were suggested by their authors and then modified, if necessary, to obtain "reasonable" results (see Appendixes). Each of the eight generators (including the Markov chain process) was run for 100 30-year scenarios. Statistical information was created by using the statistical software package SAS.

Appendixes F and G give the mean, median, standard deviation, and minimum and maximum values for each scenario for both the 1-year and the 20-year rates. A count of the number of "inverted" curves is also provided. An "inverted" curve is herein considered to be one in which the 1-year rate is larger than the 20-year rate by at least 0.25 percent.

The generators are all different. Each one has unique features, strengths and weaknesses. In addition to the different curve shapes that result from these generators, every pair of generators was significantly ($\alpha=0.05$) distinct for at least one variable included on the statistical summary. The Ryan-Einot-Gabriel-Welsch Multiple F tests were run for each variable included in the statistical guide to scenarios; the results are included in Appendix G.

Mereu's IGM method (either version 1 with the original formula or version 2 with the new formula) is the only method that separately considers inflation and a real rate of return.

The methods that apply the correction factors before rather than after the normal process are more successful in keeping the rates in bounds (that is, the imposition of arbitrary bounds is required less often). These generators rarely ever reached the upper bound. The lower bound was used somewhat more often, depending upon how close the goal was to the bounds. When the goal of the mean reversionary process was 8 percent, the 25 percent upper bound was never reached in 100 30-year scenarios for Jetton, Strommen or the composite generators. In the same situation a lower bound of 3 percent was reached only occasionally. (The other generators essentially had higher goals for the 1-year rate.) When a run was made with a minimum of 4 percent or a goal of 7 percent, the lower bound was reached more often. The standard deviation of rates about their mean within a scenario was also smaller when the correction factor was applied before the lognormal process, than when the correction factor was applied after the lognormal process.

The RGW Multiple F test procedure for the 1-year standard deviation showed the following groups (marked from highest to lowest standard deviation):

- IGM-2, Gurski-2
- Gurski-2, Gurski-1, IGM-1

- MCP
- Composite, Strommen, and Jetton.

(Differences between generators in the same group are not significant; other differences are statistically significant.)

For the standard deviation in 20-year rates the RGW Multiple F groups are:

- IGM-2
- IGM-1, Gurski-2, Gurski-1, MCP
- Jetton, Strommen
- Composite.

Note that the Markov chain process does about as good a job of controlling variation as do mean reversionary generators.

The number of "inverted curves" (no data here for MCP) produced the following groupings (from high to low):

- Gurski-2
- Gurski-1
- Jetton, IGM-2, Composite
- IGM-2, Composite, IGM-1
- IGM-1, Strommen.

Thus each of the generators has been separated from the others by a measure that is independent of the goal (if any) that was set for the 1- or 20-year rates.

Because the composite, Strommen, and Jetton generators have the smallest standard deviations for both 1- and 20-year rates, it appears that the "before" method of mean reversion is somewhat more successful than the "after" method. However, both methods would be very sensitive to changes in parameters.

Figures 10–17 show a sample of the yearly changes in the 1-year and 20-year rate for each of the eight generators for the same scenario used in the previous section.

VI. CONCLUSIONS

Clearly a variety of interest rate generators with mean reversionary properties can be developed from the general methods presented here. By experimenting with parameter settings, any of these generators can be adapted to a variety of requirements.

There are some guidelines for choosing parameters. Historical data can be used as a starting point. Murphy [14] suggests that volatility depends on

FIGURE 10

INTEREST RATE CURVES GENERATED BY JETTON METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

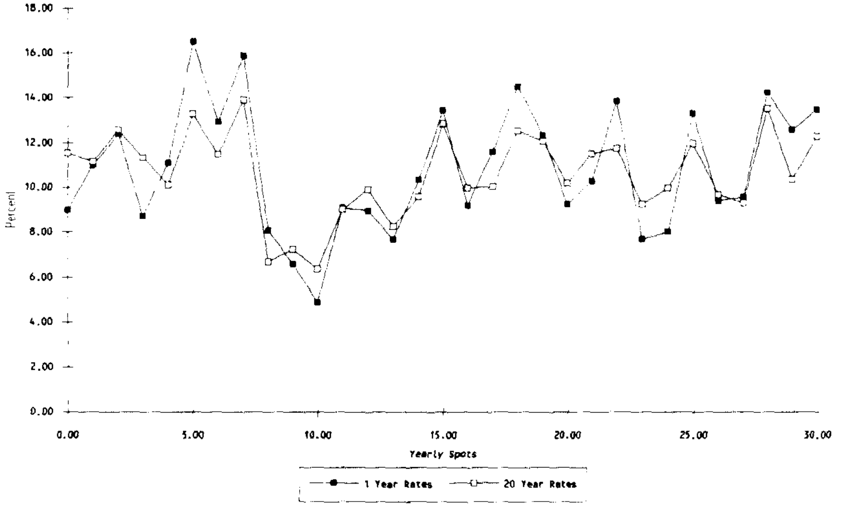


FIGURE 11

INTEREST RATE CURVES GENERATED BY STROMMEN METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

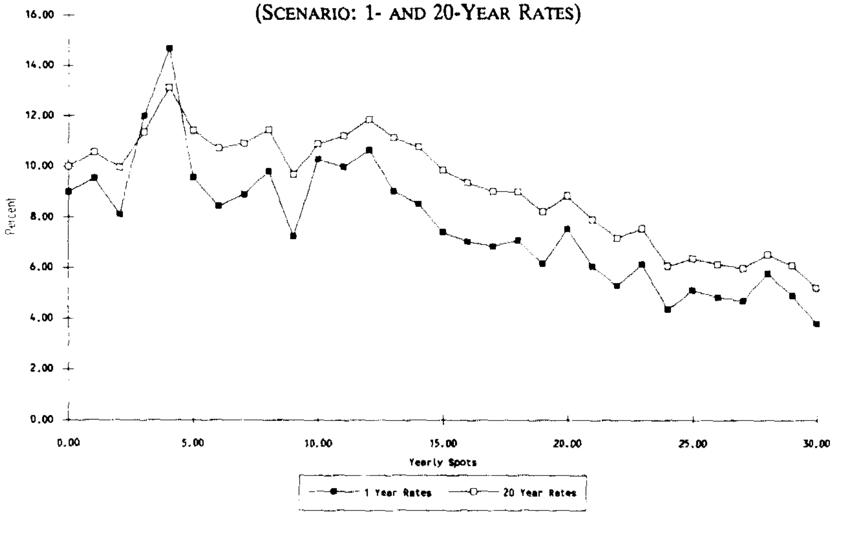


FIGURE 12

INTEREST RATE CURVES GENERATED BY GURSKI-1 METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

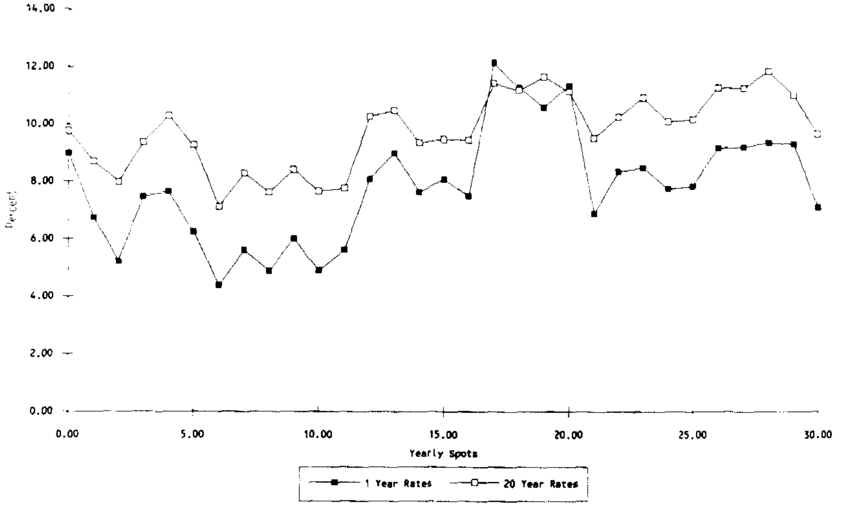


FIGURE 13

INTEREST RATE CURVES GENERATED BY GURSKI-2 METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

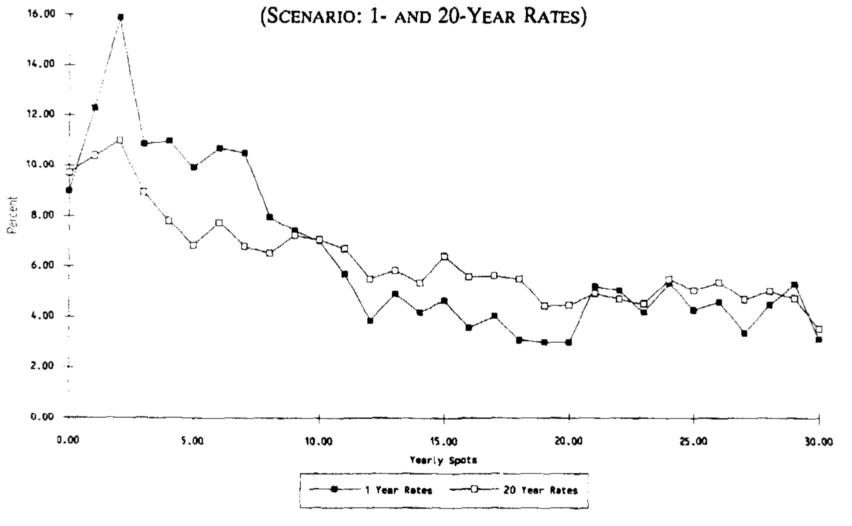


FIGURE 14

INTEREST RATE CURVES GENERATED BY IGM-1 METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

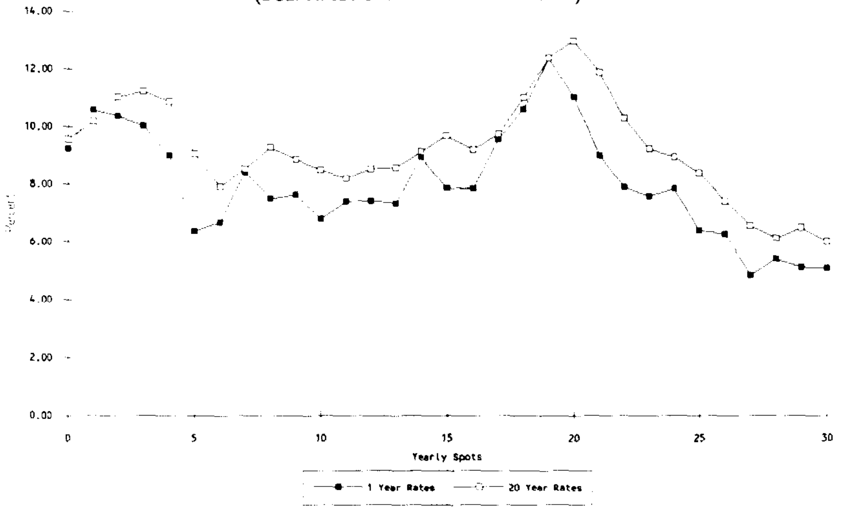


FIGURE 15

INTEREST RATE CURVES GENERATED BY IGM-2 METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

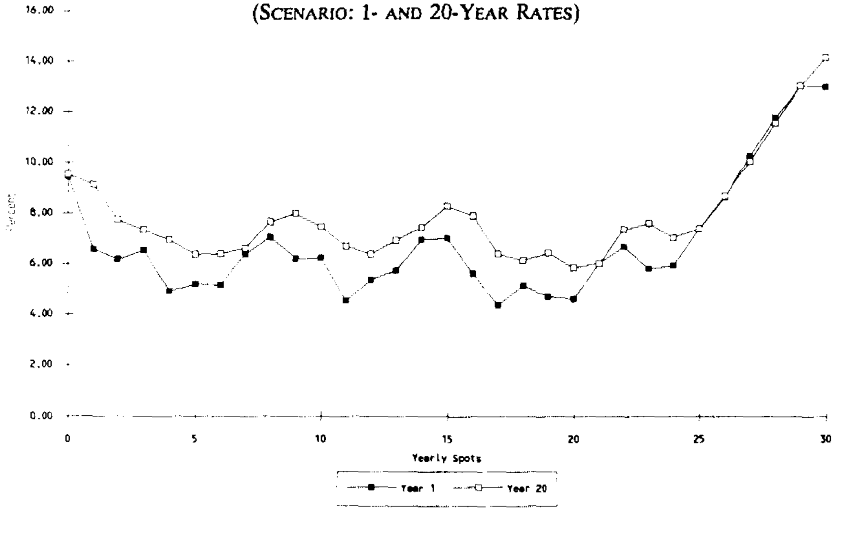


FIGURE 16

INTEREST RATE CURVES GENERATED BY COMPOSITE METHOD
(SCENARIO: 1- AND 20-YEAR RATES)

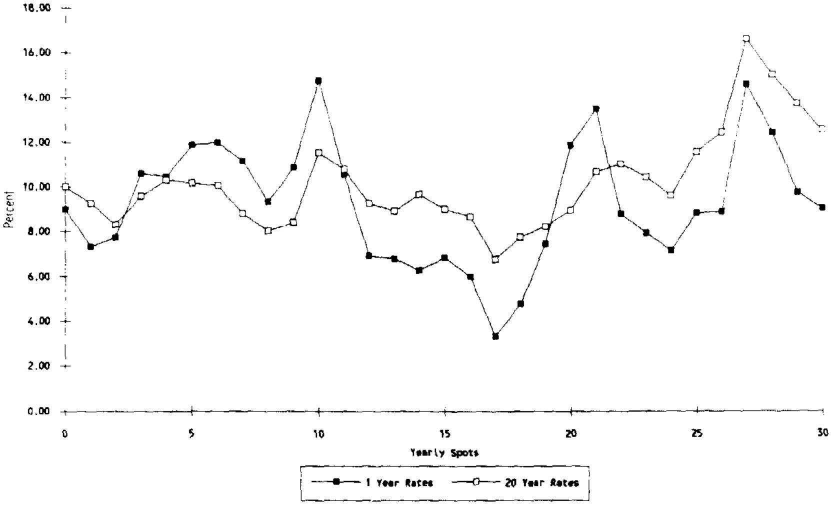
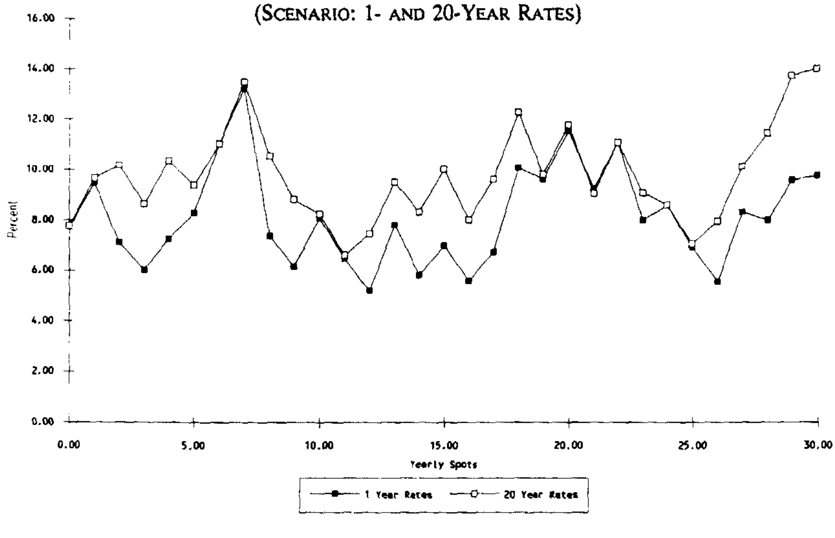


FIGURE 17

INTEREST RATE CURVES GENERATED BY MARKOV CHAIN PROCESS METHOD
(SCENARIO: 1- AND 20-YEAR RATES)



both the maturity of the bond (in years) and the length of the time interval being considered. He fit the volatility for U.S. corporate bonds from 1900 to 1965 to:

$$0.23\ [\text{interval length (yr)}]^{0.61}/[\text{maturity (yr)}]^{0.40}$$

and the volatility of U.S. government bonds from 1950 to 1986 to:

$$0.27\ (\text{interval length})^{0.40}/(\text{maturity})^{0.28}.$$

However, he also notes that volatility of different types of bonds in like periods is similar and that historical era has a greater influence on volatility than does bond type. The main considerations should be the ability of the generator to regularly produce reasonable scenarios that do not need culling.

There is no simple answer about which generator is best for such tasks as pricing, valuations, or testing to satisfy New York Regulation 126. One suggestion is to use some scenarios from each generator.

A more significant question is whether scenarios resulting from these generators can be made arbitrage-free. Can they also be adapted to display put-call parity? If so, they would provide an effective mean reversionary interest rate generator for option-pricing models.

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APPENDIX A

Jetton

Original Generator

$$T_1(t + 1) = [T_1(t) + f(t)]e^{Z \times VF}$$

Z_1 = a value from $N(0,1)$

VF = 0.27

$f(t) = \min\{0.015[T_1(\infty) - T_1(t)]^3, 0.5[T_1(\infty) - T_1(t)]\}$
for $T_1(t) < T_1(\infty)$ = "goal"

$= \max\{0.015[T_1(\infty) - T_1(t)]^3, 0.5[T_1(\infty) - T_1(t)]\}$
if $T_1(t) > T_1(\infty)$

$T_{20}(t+1) = [aT_1(t+1) + b] + Z\sigma_{20}$

Z = a value from $N(0,1)$ not necessarily the same as above

$a = 0.8$ $b = 2.5$ if $T_1 \leq 10\%$

$a = 0.6$ $b = 4.5$ if $T_1 > 10\%$

$$\sigma_{20} = \begin{cases} 0.2+0.1\ T'_{20} \text{ if } T'_{20} \leq 10\%, \text{ where } T'_{20} = \text{anticipated 20-year rate (before random term)} \\ 1.2 \qquad \text{if } T'_{20} > 10\% \end{cases}$$

Other Rates

$$T_m(t+1) = [W_1(m)T_1(t+1)] + W_{20}(m)T_{20}(t+1)$$

Weights were given for $m = 2, 5, 7,$ and 10 years and other rates determined by linear interpolation.

m	$w_1(m)$	$w_{20}(m)$
2	0.64	0.36
5	0.39	0.61
7	0.24	0.76
10	0.16	0.84

Modifications to Method: Added Weights

¼	1.5	−0.5
½	1.3	−0.3
30	−0.05	1.05

Added a minimum rate of 3% and a maximum rate of 25%
Parameter choices $T_1[\infty] = 8\%$, $T_1(0) = 9\%$

Strommen: Original Generator

T_1 = the same as Jetton

$$T_{20}(t) = T_1(t)[1 + S(t)]$$

where S is defined as follows:

$$S(t+1) = aS(t) + b[T_1(t+1) - T_1(t)]/T_1(t) + (1-a)S(\infty)$$

$S(0)$ is determined from the two original rates $T_1(0)$, $T_{20}(0)$. Parameters are: $S(\infty) = 0.227$ (goal), $a = 0.718$, and $b = 0.587$. Intermediate rates are determined the same way as for Jetton.

Modifications

The same weights for ¼ and ½ year as used for Jetton and 0.02 and -1.02 for the 30-year rate. $T_1(0) = 9\%$, $T_{20}(0) = 10\%$ and $T_1(\infty) = 8\%$.

Gurski: Original Generator

10-year rate: $T_{10}(t+1) = T_{10}(t)e^{S_{10}Z_{10}(t)}$
1-year rate: $T_1(t+1) = T_1(t)e^{S_1[Z_1(t)(1-R_{1,10}^2)^{1/2}+Z_{10}(t)R_{1,10}]}$
30-year rates: $T_{30}(t+1) = T_{30}(t)e^{S_{30}[Z_{30}(t)(1-R_{30,10}^2)^{1/2}+Z_{10}(t)R_{30,10}]}$
The rates are then adjusted by using the formula

$$T^{adj}(t + 1) = T_i(t + 1) + C[T_i(\infty) - T_i(t + 1)]$$

The following parameter settings are used: C = 0.01, $S_1 = 0.220$, $S_{10} = 0.185$, $S_{30} = 0.15$, $R_{1.10} = 0.85$, and $R_{30,10} = 0.95$. Furthermore, spreads between the ten-year projected rate at time t and the 1-year and 30-year rate at time t are calculated. For the 1-year rate, whenever $|T_{10}(t) - T_1(t)| > 2.25\%$ (225 basis points), the 1-year value is adjusted so that the excess spread is reduced by 60%.

For the 30-year rate the spread is reduced by 60% of the excess over 90 basis points when $T_{30} > T_{10}$ and by 60% of the excess over 70 basis points when $T_{30} < T_{10}$. Again the 30-year rate is the one which is adjusted.

Gurski-1

The following parameter settings are used: $S_1 = 0.25$, $S_{10} = 0.15$, $S_{30} = 0.10$, $C = 0.01$, $T_1(\infty) = 8\%$, $T_{10}(\infty) = 10\%$, $T_{30}(\infty) = 10.5\%$. Intermediate rates are determined by quadratic splines from

$$T_{(t)} = a + bt + ct^2.$$

For values of $t < 10$, the following system of equations was solved for a, b, and c.

$$a + b + c = T_1(t)$$
$$a + 10b + 100c = T_{10}(t)$$
$$b + 20c = [T_{30}(t) - T_{10}(t)]/20 = M$$

(where M is the desired slope of the curve at 10). For values of t between 10 and 30, the following system was solved.

$$a + 10b + 100c = T_{10}(t)$$
$$a + 30b + 900c = T_{30}(t)$$
$$b + 20c = M$$

The original generator only used the first spline for values between 1 and 10. The modifications included using it to extrapolate to ¼ and ½ year.

Gurski-2

The only difference between this generator and the (modified) Gurski-1 is that the other points on the yield curve are determined by Lagrange interpolation, instead of quadratic splines, that is,

$$T_j = T_i\frac{(j-10)(j-30)}{(1-10)(1-30)} + T_{10}\frac{(j-1)(j-30)}{(10-1)(10-30)} + T_{30}\frac{(j-1)(j-10)}{(30-1)(30-10)}$$

APPENDIX B

IGM-1

This generator uses the inflation rate as its key rate.

$$INFL(t) = INFL(t-1) + INFDRIFT \times Z$$

subject to a minimum and maximum value.

Parameters Settings

Parameter	Original	Modified
INFDRIFT	3.58	3.58
MAX-INFLATION	15%	13%
MIN-INFLATION	−4%	−2%

Long-Term Rates

$B = LT(t-1)e^{(LDRIFT)Z}$

$LT(t) = B + LTREND[LGOAL + INFL(t) - B]$

Modification

Subject all rates to bounds of 3% and 25%

Parameters	Original	Modified
LDRIFT	0.108	0.108
LTREND	0.073	0.33
LGOAL (net of inflation)	4%	4%

Short-Term Rates (¼ year)

Determine ratio of short-term to long-term rates and calculate short-term = ratio × *LT*. Ratio is determined recursively as c = [ratio $(t-1)]e^{\text{SHRTDRIFT (Z)}}$. Ratio ($t$) = min [short ratio max, c + strend (short goal − c)].

Modification

Subject the ratio to an absolute minimum and to a relative minimum of (overall rate min)/*LT* rate and to relative maximum of (overall rate max)/*LT* rate.

Parameters

	Original	Modified
SHRTDRIFT	0.275	0.2
STREND	0.144	0.35
Ratio-goal	0.726	0.85
Ratio-max	1.3	1.3
Ratio-min	None	0.65

Other rates:

$D = LT - \text{SHORT}$

$J(x) = SHORT + (X-0.25)\,D/9.75$ (J is linear interpolation)

$I(x) = J(x) + (x-0.25)\,(x-10)\,(ax+b)$

where $a = 0.002501 + 0.003611(D - 1.1606)$

$b = -[0.021536 + 0.03563(D-1.1606)]$

for rates between ¼-year and 10-year rates. The rates for years beyond 10 were held equal to the 10-year rate.

Modification included determining the 20-year rate as

$$T_{20} = 1.19\,LT - 0.19\,T_1$$

subject to the overall max and min and the 30-year rate as

$$1.05\,T_{20} - 0.05\,T_1$$

IGM-2

The same as IGM-1 except that the adjustment to get $I(x)$ from $J(x)$ was subtracted instead of added for those rates between ¼ and 10 years. This adjustment was suggested by looking at the graphs of IGM-1.

APPENDIX C

COMPOSITE INTEREST RATE GENERATOR

Key rate is the 10-year rate.
10-year rate determination.

$$T_{10}(t+1) = [T_{10}(t) + f(t)]e^{S_{10}Z_{10}}$$

where Z_{10} is a value from $N(0,1)$ and $f(t)$ is defined as follows:

$$\begin{aligned} f(t) &= \min\{0.015[T_{10}(\infty) - T_{10}(t)]^3, 0.5[T_{10}(\infty) - T_{10}(t)]\\ &\quad \text{if } T_{10}(t) < T_{10}(\infty) = \text{goal}\\ &= \max\{0.015[T_{10}(\infty) - T_{10}(t)]^3, 0.5[T_{10}(\infty) - T_{10}(t)]\\ &\quad \text{if } T_{10}(t) \geq T_{10}(\infty) \end{aligned}$$

Secondary rates: 1, 5, 20, and 30 years developed recursively as follows.
Exponent $= S_i[R_{i,10}Z_{10}(t-1) + Z_i(1-R_{i,10}^2)^{1/2}]$ for $=$ 1, 5, 20, and 30
$DIFF = GOAL - RES\,[Jt-1]$
$F =$ Min $\{0.015DIFF^3, 0.5DIFF\}$ $DIFF > 0$
Max $\{0.015DIFF^3, 0.5DIFF\}$ $DIFF < 0$
$TVAL = (RES[Jt-1] + F)e^{\text{EXPONENT}}$
$SPREAD = TVAL - T10[Jt]$

$$SVAL = \begin{cases} 1, \text{ if } |SPREAD| > SLIM \\ 0, \text{ otherwise} \end{cases}$$

$$\begin{aligned} RES[Jt] &= TVAL - 0.6 \times SVAL \times (SPREAD/|SPREAD|)\\ &\quad (\times\,(|SPREAD|) - SLIM) \end{aligned}$$

subject to maximum and minimum values of 25% and 3%, respectively.
The following are the parameter settings used.

Rate (yr)	S	R	$SLIM$
1	0.25	0.78	2.25%
5	0.2	0.95	1.75
20	0.11	0.95	1.00
30	0.1	0.95	0.80

All other rates were determined by Lagrange interpolation, that is

$$\begin{aligned} I_x = {} & \frac{(x-5)(x-10)(x-20)(x-30)}{(1-5)(1-10)(1-20)(1-30)} I_1 \\ & + \frac{(x-1)(x-10)(x-20)(x-30)}{(5-1)(5-10)(5-20)(5-30)} I_5 \\ & + \frac{(x-1)(x-5)(x-20)(x-30)}{(10-1)(10-5)(10-20)(10-30)} I_{10} \\ & + \frac{(x-1)(x-5)(x-10)(x-30)}{(20-1)(20-5)(20-10)(20-30)} I_{20} \\ & + \frac{(x-1)(x-5)(x-10)(x-20)}{(30-1)(30-5)(30-10)(30-20)} I_{30} \end{aligned}$$

APPENDIX D
DETAILS OF THE MCP GENERATOR

Key rate: 20-year rate.
The 20-year rate is determined as

$$Max\ \{[T_{20}(t-1)](1 - adjrmax),\ min\{[T_{20}(t-1)](1 + adjrmax),\ 0.015 + 0.001U\}\}$$

where U is a random integer between 0 and $1000(MAX - (MIN + 0.03))$

Parameters

$min = 0.03$
$max = 0.25$
$adjrmax = 0.20$
and the current rate is $T_{20}(0)$.

Shapes

Shapes are determined first by determining a sequence of shape codes. The original curve is considered and the code for the shape that it most closely resembles is an input. A sequence of 30 random integers from 1 to 10 is determined for each scenario.

$$Code\ (j + 1) = RW\,[seq(j), res(j)]$$

where *RW* is matrix that translates the probabilities given in Appendix E into a table lookup depending on the shape of the previous curve and the random number.

Then each shape code is associated with a set of factors so that the yield curve is the product of the shape code factors and the 20-year rate.

RW: SHAPE CODE FOR CURVE

Random #	Previous Shape 1	2	3	4	5	6	7
1	1	1	1	2	3	4	5
2	1	1	1	2	3	4	5
3	1	1	2	3	3	4	5
4	1	1	2	3	4	5	6
5	1	2	2	3	4	5	6
6	2	2	3	4	4	5	6
7	2	2	3	4	5	6	6
8	2	3	4	5	5	6	6
9	3	3	4	5	6	7	7
10	3	4	5	6	7	7	7

SHAPE FACTORS FOR MCP

Shape Code	Point on Curve 0.25	0.5	1	2	3	5	7	11	15	20	30
1.....	0.58	0.61	0.67	0.76	0.83	0.87	0.9	0.93	0.96	1	1.02
2.....	0.7	0.75	0.9	1.05	1.12	1.07	1.05	1.03	1.02	1	0.995
3.....	0.9	0.94	1	1.05	1	0.96	0.98	1	1.02	1	0.99
4.....	1	1	1	1	1	1	1	1	1	1	1
5.....	1.1	1.06	1	0.95	1	1.04	1.02	1	0.98	1	0.99
6.....	1.3	1.25	1.1	0.95	0.88	0.93	0.95	0.97	0.98	1	0.98
7.....	1.43	1.39	1.33	1.24	1.17	1.13	1.1	1.07	1.04	1	0.98

Finally each rate is bounded by $(1-adjmax)$ × prev rate and $(1+adjmax)$ × prev rate (for same point on curve) and also by the absolute *max* and *min* and the original curve is used in all scenarios for time 0.

Parameters: *adjmax* = 0.4
min = 0.03
max = 0.25
original shape 4 (level)
original curve December 19, 1989, which is (in percent) 7.9, 7.86, 7.71, 7.8, 7.72, 7.77, 7.7, 7.81, 7.78, 7.92, and 7.90.

APPENDIX E

PROBABILITIES OF MOVING FROM ONE SHAPE CURVE TO ANOTHER IN THE MARKOV CHAIN PROCESS (SET ARBITRARILY)

Old Shape/New Shape	1	2	3	4	5	6	7
1	0.5	0.3	0.2	0	0	0	0
2	0.4	0.3	0.2	0.1	0	0	0
3	0.2	0.3	0.2	0.2	0.1	0	0
4	0	0.2	0.3	0.2	0.2	0.1	0
5	0	0	0.3	0.3	0.2	0.1	0.1
6	0	0	0	0.3	0.3	0.2	0.2
7	0	0	0	0	0.3	0.5	0.2

Sample parameters for the Markov Chain Process:
ADJRMAX = 0.2
ADJMAX = 0.4
MINRATE = 3%
MAXRATE = 25%
Original Shape = 4
Original Curve = December 19, 1989

APPENDIX F

GUIDE TO THEORETICAL SHAPES

1	2	1	2	2	2	1	1	2	2	4	3	3	4	2	1	2	2	1	1	1	1	2	2	4	5	4	2	2	2	3
2	6	6	4	2	3	2	2	3	5	4	3	5	5	5	5	3	3	4	3	2	3	2	1	1	1	1	1	3	5	3
3	6	5	4	3	5	4	3	1	3	1	2	1	1	3	3	2	4	2	3	4	5	3	1	2	3	3	4	5	7	7
4	3	2	1	2	2	3	2	3	2	2	2	2	1	1	1	2	3	1	2	2	4	5	4	5	5	6	4	3	4	2
5	2	1	1	1	1	2	4	3	1	1	3	3	1	2	1	1	1	1	2	3	3	5	4	2	4	3	1	2	1	1
6	3	5	3	1	1	2	1	2	2	3	2	2	1	1	1	3	2	1	3	2	1	3	1	1	1	1	1	1	2	2
7	3	2	3	5	5	3	3	5	3	4	3	3	3	4	5	4	6	6	5	6	6	6	7	7	7	5	4	4	2	2
8	4	5	4	5	6	6	7	6	6	4	5	4	3	1	2	2	2	2	1	3	4	5	4	2	2	4	3	2	1	2
9	5	4	3	4	3	2	4	2	2	2	1	1	2	3	2	2	2	2	1	3	4	5	3	1	2	2	1	1	1	1
10	6	7	6	4	4	3	4	5	4	3	1	1	1	3	2	3	1	1	2	1	3	1	3	2	1	1	1	2	2	3
11	4	5	5	5	3	3	2	1	2	1	2	2	2	2	1	2	1	1	1	3	4	4	5	4	2	2	1	2	2	2
12	3	2	1	1	1	1	1	1	1	1	2	2	2	3	4	6	5	7	6	5	4	4	3	3	5	3	3	2	2	2
13	5	3	2	2	2	4	3	5	3	4	4	3	4	2	1	2	3	2	2	3	5	5	3	4	2	1	1	1	1	3
14	5	5	4	2	2	2	1	2	1	1	1	2	2	3	3	2	3	2	2	1	1	1	2	3	2	1	1	1	2	1
15	2	2	2	2	2	1	1	3	3	3	3	2	1	3	3	1	3	1	1	1	2	3	3	3	1	1	3	1	1	2
16	4	2	2	4	3	4	6	4	3	2	3	4	3	4	4	3	2	2	1	1	1	2	1	1	2	2	1	2	1	3
17	2	4	2	1	1	2	4	3	2	3	2	3	3	3	4	2	1	1	2	2	2	1	1	1	2	1	2	1	1	2
18	2	4	3	3	5	3	3	4	3	1	1	2	3	1	1	3	2	1	1	2	1	1	1	3	5	3	1	1	2	1
19	4	2	3	2	2	1	2	2	3	4	5	5	4	6	6	5	4	3	2	1	1	1	1	1	1	1	3	3	3	4
20	3	4	3	2	1	1	1	2	2	1	1	2	2	1	3	1	2	4	2	2	1	1	1	2	3	4	5	4	5	5
21	2	1	1	2	1	2	4	3	2	1	2	3	2	1	2	1	1	1	1	1	3	5	4	2	2	1	2	2	3	2
22	3	3	2	2	4	6	7	7	6	4	6	6	5	7	6	7	6	5	3	5	6	4	2	1	1	1	2	1	2	3
23	4	4	5	5	3	1	2	1	1	1	3	2	1	2	1	1	2	3	4	2	2	4	6	5	4	5	7	6	6	5
24	3	1	3	3	2	2	2	2	1	2	2	2	2	2	2	2	1	2	3	1	1	2	1	1	1	1	1	2	2	2
25	6	6	5	4	4	4	3	2	2	3	3	2	2	1	2	2	4	3	2	2	3	5	5	6	5	3	1	3	2	2
26	2	4	3	4	3	5	4	2	2	3	2	1	2	1	1	1	1	2	1	1	1	2	2	1	1	3	4	3	2	3
27	2	1	1	1	1	1	3	2	2	2	1	1	3	4	6	7	6	4	4	2	3	3	4	3	4	4	3	4	4	6
28	3	3	4	5	4	2	3	4	5	3	4	5	4	2	1	1	1	2	2	3	4	5	5	6	7	6	4	3	2	2
29	2	1	1	1	2	1	1	3	3	3	5	4	5	5	4	2	2	2	2	2	3	5	4	2	1	1	3	2	1	3
30	5	3	1	1	1	2	2	2	2	1	1	2	2	1	2	4	4	3	4	5	5	4	2	2	2	2	2	3	2	1

GUIDE TO THEORETICAL SHAPES—*Continued*

31	3	2	2	1	1	2	3	2	1	2	1	3	2	2	1	1	1	3	1	2	1	1	1	1	2	2	1	2	2	1
32	3	2	2	3	4	3	2	1	1	1	1	2	3	2	3	2	1	1	3	3	2	2	2	4	3	1	1	2	3	4
33	2	1	2	3	3	3	5	3	5	3	5	4	3	4	4	5	6	6	6	6	5	4	4	3	4	3	5	3	5	3
34	3	4	6	5	4	4	5	7	6	4	6	5	3	2	3	1	1	3	3	3	4	4	3	4	4	2	1	2	2	2
35	4	2	1	3	2	2	2	1	2	1	1	1	1	3	5	4	2	2	2	2	3	2	2	1	1	1	3	2	4	3
36	2	4	5	3	1	2	1	3	3	3	1	1	3	3	3	3	3	1	3	3	2	1	1	3	2	1	2	2	1	3
37	2	1	1	1	2	3	3	3	2	1	1	1	1	2	2	3	3	2	2	2	3	4	5	3	2	1	2	1	3	4
38	2	1	1	2	1	1	1	3	1	1	1	2	1	1	1	1	1	3	4	4	4	3	5	3	1	2	2	1	2	1
39	5	5	4	3	2	1	1	3	4	3	4	5	6	4	3	5	4	3	1	2	3	4	3	4	4	3	2	1	1	3
40	4	5	3	4	3	3	4	3	3	1	1	1	1	1	1	2	1	1	3	2	1	2	1	3	1	1	3	5	3	1
41	5	3	4	3	2	2	2	1	3	4	3	1	1	3	4	4	5	3	4	4	3	2	1	1	1	3	1	2	1	3
42	5	5	4	4	4	4	3	1	3	2	2	2	1	1	1	1	2	2	2	3	1	1	1	1	3	5	6	7	7	6
43	5	4	4	4	5	4	2	2	4	3	2	2	1	3	5	3	2	2	1	1	2	1	1	1	3	2	3	2	3	4
44	4	4	5	3	3	2	2	2	2	1	3	1	1	1	3	1	1	2	1	3	5	7	6	5	5	3	3	3	2	1
45	4	4	5	5	7	6	5	3	1	2	1	2	1	2	1	3	4	3	1	2	2	2	2	2	2	2	4	4	3	1
46	4	3	1	1	2	2	2	3	2	1	1	1	2	2	1	1	2	2	3	3	5	3	2	1	3	3	3	1	1	1
47	3	5	4	5	7	6	4	5	3	3	3	5	4	2	1	1	3	3	4	2	4	6	5	3	2	2	3	1	1	2
48	5	5	5	5	4	2	2	4	2	1	1	1	1	2	2	2	1	1	1	1	3	2	1	2	4	5	5	3	1	1
49	2	1	1	2	4	3	1	1	3	4	6	5	3	2	3	5	4	4	3	4	3	1	2	1	3	5	3	2	2	3
50	5	3	4	5	4	5	4	4	4	2	2	3	2	1	1	2	2	2	1	1	1	1	2	1	3	2	1	1	2	4

Guide to Scenarios

Num	1-Year Rates					20-Year Rates				
	Mean	Mid	Std	Min	Max	Mean	Mid	Std	Min	Max
1	8.91	8.46	2.12	5.79	14.40	10.38	10.40	2.07	6.88	16.00
2	12.43	10.72	4.16	6.96	21.70	13.36	13.40	4.09	6.96	21.70
3	13.25	13.11	3.26	7.71	22.06	13.85	14.04	2.84	7.92	19.77
4	12.61	11.99	3.36	7.68	19.12	14.04	13.70	3.42	7.92	21.24
5	11.14	9.94	3.79	6.90	20.50	13.41	12.30	3.57	7.92	21.36
6	9.54	8.78	2.94	5.49	17.80	11.87	11.81	2.80	7.58	17.80
7	12.22	12.77	3.35	6.40	18.58	11.81	11.70	3.19	6.40	18.58
8	12.97	13.31	2.65	7.71	18.35	13.63	14.00	2.79	7.92	18.84
9	10.83	11.04	2.52	5.45	15.21	12.62	11.95	3.58	7.92	22.70
10	12.51	12.53	3.05	7.71	19.10	14.23	13.86	3.12	7.92	20.28
11	11.70	11.02	4.10	5.70	20.70	13.54	13.06	5.04	5.70	23.00
12	13.02	12.60	3.83	6.97	19.92	14.54	15.00	3.56	7.75	20.30
13	12.39	12.20	3.69	6.32	19.30	13.70	14.52	4.03	6.32	20.64
14	9.52	9.48	2.14	5.56	13.76	11.57	10.97	2.69	7.58	18.60
15	10.09	9.83	2.39	6.16	16.17	12.16	12.17	2.07	7.92	16.85
16	12.05	11.52	2.96	7.71	20.32	13.54	13.46	2.69	7.92	20.32
17	9.30	8.53	2.71	4.81	15.26	11.04	10.48	2.67	7.13	17.80
18	11.68	10.87	3.25	7.58	18.87	13.71	14.49	3.04	7.58	18.87
19	10.74	10.29	2.53	6.81	16.59	12.03	12.35	2.23	7.58	16.59
20	12.53	11.50	3.40	7.71	19.44	14.67	14.30	3.14	7.92	21.60
21	10.68	11.10	3.12	4.82	16.10	12.80	12.84	2.82	7.20	17.75
22	12.66	11.51	3.77	7.38	20.30	12.71	12.46	3.09	7.92	18.60
23	9.45	9.48	2.00	4.81	13.65	10.28	10.18	1.72	7.12	14.49
24	9.86	9.52	3.02	5.36	16.96	11.83	11.38	2.85	7.92	18.84
25	11.37	9.60	4.25	6.14	19.90	11.92	10.75	4.24	6.14	21.98
26	11.60	12.29	3.32	5.70	17.44	13.51	13.20	3.03	7.92	20.20
27	12.43	12.38	3.39	7.17	18.32	13.61	13.63	2.71	7.92	19.01
28	9.90	9.48	2.52	5.89	14.82	10.45	9.79	2.94	5.89	18.96
29	11.09	10.81	2.49	7.50	16.44	12.75	12.63	2.33	7.92	18.48
30	11.05	10.71	3.77	4.55	17.50	12.51	12.40	3.35	6.07	18.96

GUIDE TO SCENARIOS—*Continued*

	1-Year Rates					20-Year Rates				
Num	Mean	Mid	Std	Min	Max	Mean	Mid	Std	Min	Max
31	7.56	7.41	1.58	4.62	11.47	9.47	9.50	2.15	5.52	15.93
32	9.05	8.68	2.82	4.77	16.95	10.47	10.37	2.93	5.30	16.95
33	13.67	13.63	3.43	7.17	19.87	13.76	13.44	3.16	7.92	19.87
34	14.06	13.77	3.65	7.71	22.68	14.52	14.69	3.02	7.92	19.20
35	10.06	9.63	2.37	5.76	14.47	11.93	12.24	2.34	7.58	17.12
36	9.80	9.80	2.33	5.20	14.98	11.52	11.29	2.74	7.23	18.72
37	11.39	10.97	3.38	5.12	17.30	13.27	13.50	2.73	7.58	19.08
38	8.13	8.37	2.41	4.42	12.61	10.13	10.06	2.56	6.32	16.07
39	10.20	10.30	2.80	4.86	16.26	11.07	10.48	2.85	7.20	19.50
40	10.49	10.78	1.93	7.24	14.26	12.58	12.36	2.18	7.92	17.11
41	11.69	11.38	3.09	5.70	17.52	13.39	13.20	3.15	7.92	20.04
42	10.18	9.50	2.87	5.46	17.55	11.48	10.90	2.96	7.28	19.50
43	12.33	10.58	4.64	6.40	22.60	13.71	12.93	4.49	6.40	22.60
44	10.00	9.48	4.16	3.89	22.18	10.92	10.00	3.30	5.76	18.40
45	9.64	9.44	2.08	5.09	14.41	10.70	10.25	1.99	7.58	16.01
46	8.95	8.57	1.81	5.66	13.95	10.81	10.53	2.25	7.62	16.70
47	11.27	11.33	3.69	5.60	19.47	11.92	11.33	3.34	6.32	17.70
48	10.81	10.80	2.54	6.56	18.29	13.02	13.06	2.97	6.56	18.29
49	11.16	11.47	3.36	4.92	17.00	12.31	12.05	2.83	7.34	18.84
50	10.92	10.59	2.60	5.64	16.99	12.76	13.05	2.99	7.99	17.98

APPENDIX G
SUMMARY OF SAS RESULTS

ESTABLISHING SIGNIFICANT DIFFERENCES

	Variables		
	STD 1	STD 20	INVCNT
F value (Prob > F)	38.11 (0.0001)	52.18 (0.0001)	36.08 (0.0001)

PINPOINTING THE DIFFERENCES MEAN WITH REGWF GROUPING

	Variables		
Generator	STD 1	STD 20	INVCNT
Composite	2.560 D	1.647 D	5.65 C, D
Gurski-1	3.650 B	2.876 B	9.32 B
Gurski-2	3.753 A, B	2.885 B	10.96 A
IGM-1	3.574 B	3.147 B	4.66 D, E
IGM-2	4.107 A	3.586 A	6.35 C, D
Jetton	2.457 D	2.069 C	6.49 C
MCP	2.961 C	2.796 B	N/A
Strommen	2.519 D	1.998 C	3.48 E

For REGWF: Means with the same letter are not significantly different.
STD 1 is the standard deviation of the 1-year rates.
STD 20 is the standard deviation of the 20-year rates.
INVCNT is the count of inverted curves.