SOCIETY OF ACTUARIES

Article from:

Risk Management

July 2005 – Issue 5

COTOR: Adventures With Risk Theory

by Louise A. Francis

The Committee on the Theory of Risk (COTOR) is one of the Casualty Actuarial Society's (CAS) research committees. As such, the committee sponsors research related to risk theory, promotes continuing education on the topic and in general tries to engage in thought-provoking activities that will expand the leading edge in actuarial science. Our projects have included:

- Awarding a grant to conduct research on methods for computing risk loads for discounted liabilities.
- Sponsoring a statistical estimation challenge. The objective was to estimate excess loss costs of a sample of claims from a heavy-tailed distribution.
- Initiating a training project to train actuaries in modeling methods that could be very useful to actuaries in the practice of some of their core functions, such as reserving and ratemaking.
- Awarding a prize to the best paper in an American Risk and Insurance Association (ARIA) publication.

In keeping with the committee's goal to promote continuing education on risk theory, we have sponsored sessions at the CAS's Spring and/or Fall meetings for several years. One of the most successful of these presentations was the "COTOR Challenge." The challenge addressed a problem in extreme value theory. It originated when a COTOR member challenged his colleagues to estimate the pure premium in the layer 500K xs 500K based on a listing of 250 claims. The challenge was later refined and distributed to the membership of the CAS. Stuart Klugman, our resident loss-distribution expert, picked the sample of 250 claims generated randomly from an inverse transformed gamma distribution. The challenge was to estimate the average severity and 95 percent confidence intervals for the $5 million xs $5 million layer. In total, eight different people responded to the challenge, submitting a total of 10 different responses. The results of this challenge were presented at the November 2004 CAS Annual Meeting in Montreal. Five of the eight responders and Phil Heckman (applying his round 1 technique to the round 2 data) presented their results and techniques to a standing-room-only crowd. The committee chairman, Louise Francis, presented an award to three challenge participants—Dave Clark, Glenn Meyers and Jonathan Evans—based on a number of factors considered together, including the accuracy and the and clarity of the solutions, as well as the creativity used and ability of the method to lend itself to practical application.

When analyzing the submitted results for the challenge, there was a nearly 13 to 1 spread between the lowest to highest mean. All responders recognized there was tremendous uncertainty in the results (the range from upper to lower confidence level went from a low of eight to a high of infinity). All but two of the responders relied on approaches commonly found in the literature on fitting distributions or modeling extreme values. Only one of the results came within 10 percent of the true mean. Interestingly enough, half the responses were below the true mean and half were above. When an obvious outlier response was eliminated, and the remaining responses were averaged, the resultant average was within 2 percent of the true mean. The panel discussed that potential implications were for an insurance company and should not rely on the results of only one model when making important decisions.

A few general summary comments about the solutions submitted are in order. First, a number of participants used some form of the Pareto distribution. This is not surprising, as the Pareto distribution is prominently represented in the extreme value literature. Both the single parameter Pareto, popularized by Stephen Philbrick (1985), and a version dubbed "the Generalized Pareto" in some of the extreme value literature (there is actually more than one Generalized Pareto in the statistical literature) were used by various responders. Many of the formulas used in the fitting of a Pareto are relatively simple to implement, and the Pareto has a much heavier tail than some more conventional distributions such as the lognormal. However, since the Pareto is a truncated distribution, i.e., it is fit

Louise A. Francis, FCAS, MAAA, is consulting principal of Francis Analytic & Actuarial Data Mining, Inc. in Philadelphia, Penn. She can be reached at *Louise_francis@msn. com.*

only to data that exceed a selected threshold, there are significant issues with how to select the threshold. Different choices typically result in different parameter estimates and the different parameter estimates can result in very different estimates for excess layers of insurance.

A number of authors fit a "ground-up" distribution to the data, rather than fitting a distribution just to tail claims. In this category was the mixture approach. Mixtures of distributions are known to have heavier tails than the individual distributions have. Another approach used transformed the data (e.g., applied a functional transform such as the log of the claims) until a distribution near to one of the more conventional densities, such as the Lognormal or Gamma, is obtained. Certain transforms, such as the inverse, log and multiple log transforms, often result in distributions with heavy tails. A third approach involved the use of kernels to approximate the distribution. The kernel approach has appeared in the statistical literature recently as a non-parametric technique for approximating densities.

More details of the challenge, including write-ups of the responses submitted, can be found on the CAS Web site at *www.casact.org/cotor/*. There will be another round to the challenge in 2005. We intend to make it more challenging by adding an additional random factor commonly encountered in reinsurance estimating applications.

One of COTOR's sponsored research projects was dubbed "The Risk Premium Project." It addressed an aspect of the estimation of risk load for liabilities: the estimation of rates of return using Capital Asset Pricing Model (CAPM).

Eugene Fama and Kenneth French (1992) sent shockwaves through the finance community when they published a paper suggesting that Beta (the covariance of the company's stock return with that of the market) was not the only relevant factor for predicting a company's stock return. The author's research questioned one of the cornerstones of financial theory, CAPM, which has often been used to compute rates of return on equity, particularly in a regulatory environment.

CAPM states that

$$r_c = r_M + ß_c(r_m - r_f)$$

where

r_c is the company's return

r_M is the return on the entire market of all investments

$ß_c$ is the company Beta

$(r_m - r_f)$ is the market risk premium.

CAPM may be familiar to those involved in rate filings, as it is often one of the key financial theories used in the regulation of insurance companies to determine a "fair rate of return." The use of CAPM is controversial among actuaries, as it has sometimes been used to "prove" that insurance companies are exposed to very low risk and, therefore, merit little or no return above that supplied by the risk-free rate of return. Usually the "proof" involves demonstrating that insurance industry Betas are low or, in some cases, negative.

The CAS funded a team of researchers to advance the state of the art in the insurance industry, with respect to the use of CAPM based approaches, to derive rates of return. The research team incorporated a number of the most recent findings into a model for CAPM and rates of return, which is much richer than the conventional approach, as it incorporates a number of factors into the estimates that have been demonstrated to impact rates of return. A summary of the research can be found at the COTOR Web site *www.casact.org/cotor/*. A paper based on the research is forthcoming in the *Journal of Risk and Insurance*. ✦

References

Fama, Eugene F., and Kenneth R. French. 1992. The Cross-Section of Expected Stock Returns. *Journal of Finance*. 47:427-66.

Philbrick, Stephen. 1985. "A Practical Guide to the Single Parameter Pareto Distribution." Proceedings at the Casualty Actuarial Society. Online at *www.casact.org/pub/proceed/proceed85/85044.pdf*.

> "Clearly, the decisions made by management can only be as good as the information they are fed by the myriad of upstream financial, legal, actuarial and information analysts."