OPERATIONAL RISK CAPITAL PROVISIONS FOR BANKS AND INSURANCE COMPANIES

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Abstract

Operational risk has become recognized as a major risk class because of huge operational losses experienced by many financial firms over the last past decade. Unlike market risk, credit risk, and insurance risk, for which firms and scholars have designed efficient methodologies, there are few tools to help analyze and quantify operational risk. The new Basel Revised Framework for International Convergence of Capital Measurement and Capital Standards (Basel II) gives substantial flexibility to internationally active banks to set up their own risk assessment models in the context of the Advanced Measurement Approaches (AMA). This paper investigates the implications for using the AMA as a method to assess operational risk capital charges for banks and insurance companies within Basel II paradigms and with regard to U.S. regulations. The AMA developed in the paper uses actuarial loss models complemented by the extreme value theory to determine the empirical probability distribution function of the aggregated capital charges in the context of various classes of copulas. Publicly available operational risk loss data set is used for the empirical exercise.

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Comments are welcome

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1 - Introduction

There exist four computational methodologies to determine the regulatory capital requirement for financial institutions. These include fixed ratios, risk-based capital, scenario-based approaches[1] and probabilistic approaches (IAIS, 2000).

In many views (see for example IAIS, 2000, KMPG, 2002), probabilistic approaches such as the Advanced Measurement Approach (AMA), provide the preferred greatest framework for a meaningful capital requirement characterization. These methodologies use simulations to determine the full probability distribution of possible outcomes from which the capital requirement is determined using ruin-probability, expected policyholder approaches (Butsic,1994) or other risk measures. As such, probabilistic methodologies are the most complex of the four approaches to assessing regulatory capital charge in terms of consistency, codification, and data requirements. Their complexity is also reflected in large costs associated with their application (KMPG, 2002).

As to the operational risk, it has been assumed that this specific risk will be more accurately captured under the AMA[2] and, therefore, incentives in terms of a lower capital charge granted to AMA applicant banks that refine and develop sound operational risk methodologies (Fitch, 2004). However, due to the specificity of this major risk class, there is no clear idea about the actual implications for using the AMA as a method to assess operational risk capital charge and, importantly, how its implementation would ultimately result in a lower capital charge for

1 Under the fixed ratio method, the capital requirement is expressed as a fixed proportion of a proxy for exposure to risk often an item from the insurer's balance sheet or profit and loss account
Under the risk-based capital model, sub-results are determined by applying factors to exposures proxies such as invested assets risks, reserving risks, just like in the fixed ratio model.
KMPG (2002) describes scenario-based model as a methodology that explores the impact of specific risk variables to company specific exposure for insurers

2 The two other approaches include the basic indicator approaches set according to the fixed ratio methodology and the standardized approaches established according to the risk-based capital approach.

financial institutions that adopt it. According to a survey carried out by Fitch in 2004, forty-two large banks around the world believe that the AMA may generate capital charges that are not lower than those under the standardized or basic indicator approaches (Fitch, 2004).

As of today, there is a small body of literature that focuses on how the AMA should be effectively implemented in financial institutions. AMA literature started in 2001 when the Basel Committee on Banking Supervision (the Committee) published its document in September 2001 "Working Paper on the Regulatory Treatment of Operational Risk". With regard to the AMA-related academic literature, Embrechts et al. (2003), Chavez-Demoulin and Embrechts (2004b), Embrechts et al. (2004) question the ability of the standard actuarial model[3] as well as the extreme value theory[4] to adequately address AMA issues because the assumptions behind these models are barely in line with the actual characteristics of operational risk losses. The authors consider models that include the particular case of the Cramer-Lundberg model[5], and general risk processes where the underlying intensity model follows a finite state Markov chain, allowing the modeling of underlying changes in the economy. In line with Embrechts et al., Chernobai and Rachev (2004) advocate for use of the compound Cox model[6] or the alpha-stable distribution model[7] (depending on the finiteness of the second moment of the loss severity random variable) instead of the simple compound Poisson process.

As it appears, nearly all of these models suggest approaches which are more appropriate for larger data sets. As a result, there is a need for more formal empirical research about

3 Klugman et al. (2004)
4 Embrects et al. (1997)
5 See Embrects et al. (1997)
6 See Bening et al. (2002).
7 Zolotarev (1994), Embrechts et al., (1997), Rachev, S. Mittnik, S. (2000), Nolan (2001),

operational risk capital requirements, taking into account various constraints in terms of data availability, data collection costs, limited computational resources, and limited decision time.

On the practitioners' side, Frachot et al (2001), Frachot et al (2002), and Baud et al (2002) describe the Loss Distribution Approach (LDA) for operational loss and provide a methodology that allows banks to pool internal data with external data to calibrate operational risk capital charge. Fontnouvelle et al (2003) use the aforementioned methodology to provide preliminary empirical evidence on how publicly available operational loss data could be used to calibrate large loss severity distribution functions and capital charges. In their model, the random truncation point used to report publicly available losses is assumed to be logistically distributed. This assumption highly impacts the severity distribution function and even though it is computationally convenient, it has been criticized on the account that it is not grounded on empirical evidence (Leandri, 2003). In addition, the dependency across risk categories is not accounted for. Di Clemente et al (2003) develop a model that considers dependence structure based on the Student's t-copula and historical rank correlations. The empirical exercise, however, is carried out using catastrophe insurance loss data of three different lines – namely, hurricane, wind-storm, and flood. As such the authors do not consider actual operational risk loss data issues.

This study is concerned with the issues raised in determining the capital charge in the context of the AMA that (1) models the loss severity probability distribution function of large losses based on external data, size and quality of internal risk control of organizations, (2) estimates the diversification effect by investigating the sensitivity of the capital charge to different types of dependence structures among risk types. It also provides information on the capital charges that banks and insurance companies would be required to hold in order to face operational risk losses within Basel II paradigms and with regard to US regulations.

Our key conclusions are as follows: First, in the context of large losses, the assumption that contributors' operational loss data are sampled from the same probability distribution provides a straightforward way to apply the loss distribution approach to operational risk. As a result, by properly accounting for the reporting bias, size and quality of internal risk control, the loss severity of a typical organization could be calibrated accordingly. Second, the loss severity significantly impacts the capital charge, much more than assumptions regarding the dependence structure among risk types. Third, for banks and insurers, the loss event type, clients, products and business practices (CPBP) appears as the main risk driver. For banks, internal fraud accounts for a large proportion of losses as well. Fourth, the level of capital charge estimates indicate that banks and insurers need to place an extensive focus on this specific risk, chiefly by improving the quality of their internal risk control.

The remainder of the paper proceeds as follows. Section 2 provides a concise background on operational risk management. Section 3 describes the data set and gives key descriptive statistics. The methodology is explained in section 4 and section 5 presents the main results. Section 6 concludes.

2 – Background on Operational Risk Management

A look inside the banking industry over the last decade clearly reveals two stylized facts. On the one hand, increasing complexity of financial technology combined with deregulation and globalization trends have made banking practices more sophisticated and challenging. As a result, the industry faced new multifaceted risks envisioned as part of 'other risks' and as such, different from market and credit risk. These include system security and fraud risks due to the expansion of e-commerce, system failure risks on account of the use of highly automated technology, and many other significant risks resulting from the increased use of outsourcing arrangements and new risk mitigation techniques such as credit derivatives, swaps, and asset securitization (BCBS, 2003c). On the other hand, the banking industry all over the world has witnessed a growing number of insolvencies and experienced high-profile 'other risks' losses. In 1998, the press has reported more than US$7 billion of 'other risks' losses in financial service firms, including the insurance industry. These combined facts brought supervisors as well as banking and insurance executives to view the management of these 'other risks' as a comprehensive practice comparable to the management of credit and market risk (BCBS, 2003c).

In quest of solutions to issues raised by these challenging 'other risks' faced by the banking industry, the Committee sets up in its June 1999 First Consultative Package, the principle of developing a Pillar One explicit capital charge for 'other risks', such as operational risk. Subsequent to the consultation process and its own analysis, the Committee adopted a definition of operational risk in its January 2001 Second Consultative Package and decided that only this specific risk should be subject to capital charges under Pillar One of the Framework (Minimum Regulatory Capital Requirements). Additional components of other risks such as interest rate risk and liquidity risk will be addressed only through Pillar Two (Supervisory Review Process) and Pillar Three (Market Discipline)[8].

The definition of operational risk, formulated by the British Bankers' Association (BBA) has been refined in the September 2001 Working Paper on the Regulatory Treatment of Operational Risk, as follows: "the risk of loss resulting from inadequate or failed internal

8 The Committee believes that, taken together, these three elements (Minimum Regulatory Capital Requirements, Supervisory Review, and Market Discipline) are the essential pillars of an effective capital framework (BCBS, 1999).

processes, people and systems, or from external events". The Committee specified that the aforementioned definition encompasses legal risk but excludes systemic, strategic, and reputational risks for the purpose of a minimum regulatory operational risk capital requirement.

With regard to the quantification methodologies, the Committee decided to stay consistent with its objective of moving away from the one-size-fits-all approach that prevailed in 1988 Basel Accord. As a result, in its January 2001 Second Consultative package, the Committee published three methods for measuring operational risk capital charges in a continuum of increasing sophistication and risk sensitivity. These approaches include the Basic Indicator Approach that relates the capital charge to the gross income envisioned as a proxy for the bank overall risk exposure, the Standardized Approach that builds on the Basic Indicator Approach by business lines and the AMA that builds on the bank's internal loss data.

Until recently, within the banking industry, banks have managed operational risk as a "silo" focused activity, at the business-line level, and did not take a firm-wide view of operational risks except to the extent that it was envisioned as part of 'other risks', different from credit and market risk (BCBS, 2003a). However, in recent years, the previously observed trend has changed as a growing number of banks have paid extensive attention to operational risk as a specific discipline. This shift resulted from the two aforementioned facts, notably the newly multifaceted risks were thought as being partly responsible for the recent increase in operational risk losses across a number of banks. As a result, operational risk management was envisioned as a tool to reduce volatility in earnings and as such, a driver of shareholder value (BCBS, 2003a).

With regard to the insurance industry, insurers have historically addressed the major part of their operational risk which they thought to be process risk, indirectly, through insurance risk[9]

9 For property/casualty insurer, insurance risk refers to loss reserve risk and premium risk. For life insurer, it represents mortality and morbidity risk. For health insurer, it denotes premium risk (AAA, 2002).

(BCBS, 2003a). Indeed, insurers focused very early on process risk due to the size of manual processing of information intrinsic to their business such as policy underwriting and claim processing. However, since process risk losses result in extra claims paid, these additional amounts were directly incorporated into premium charged to policyholders. As a result, straightforwardly assessing operational risk was not a key priority for insurers and no explicit capital charge was considered mandatory.

Recently, it has been recognized that a major component of operational risks for insurance companies is market conduct risk which results from the use of third parties by insurers to sell their products. A well-publicized case of market conduct risk losses is that of Prudential Insurance Company of America in the mid-1990s (Shah et al., 2001).

Shah et al. (2001) also point out that the use of Internet sales decreases market conduct risk exposure, but at the same time increases system security and fraud risk exposures and, as such, represents another major potential source of operational risk for insurers.

In the United States, the National Association of Insurance Commissioners (NAIC) adopted its document "Risk-Focused Surveillance Framework, (the Framework)" in June 2004. Essentially, the Framework is conceived as a structured comprehensive methodology that proactively envisions the insurer's risk profile and the quality of its risk management practices. As such, areas of greatest risk to insurers are efficiently addressed and state insurance regulators are in the position to better identify and take action against any existing and emerging risks that could jeopardize the stability of the insurance company (NAIC, 2004). Operational risk is recognized as a major risk class within the nine risk classifications[10] identified by the Framework. The next section proceeds with the presentation of key descriptive statistics.

NAIC (2004) distinguishes nine risk classifications: Credit risk, market risk, pricing & underwriting risk, reserving risk, liquidity risk, operational risk, legal risk, strategic risk, reputation risk.

3- The Data Set

The empirical investigation of the implications of employing the AMA is carried out by using publicly available operational losses from Fitch Risk Management. This firm captures financial and non-financial operational risk losses that are in excess of $1 million from public sources such as court filings and news reports. In addition to individual losses, the data set contains various organizations' exposure indicators such as number of employees, gross income, assets, physical assets, compensation, and deposits. Typically, these large operational losses are used to supplement banks' internal loss data in calibrating the tail of the loss severity distribution.

In the sequel, key descriptive statistics are provided for the contributors of losses and individual losses that occurred in the United States. Contributors of losses are referred to as bank and insurance organizations in the US market that incurred the losses captured by Fitch.

For the period ranging from 1980 to 2002, Table 1.1 indicates that operational losses were captured from 1245 bank organizations grouped in 998 parent banks and 381 insurers grouped in 303 parent insurance organizations. The total losses incurred by these organizations amount to $58,552 million for banks, and $ 22,535 million for insurers. In terms of total number of losses per contributor, Table 1.1 also shows that Fitch has captured only one loss in excess of 1$ million from nearly 80% of contributors. This is an important fact that impacts the calibration of the observed loss distribution.

The following subsection analyzes the distribution of contributors' truncation point above which Fitch captures operational losses. Fitch is supposed to capture and report all losses in excess of a threshold set to $1 Million and it is worth investigating the actual distribution of this threshold by contributor. For US banks, Table 1.2 indicates that the contributor's truncation point ranges from $1 million to $ 1979 million. Among business lines, Retail banking has the highest number of contributors, i.e. 599 and the highest contributor's truncation point while payment and settlement has the lowest number of contributors, i.e. 21, and at the same time, the lowest contributor's truncation point, i.e. $209 million. As to loss event types, CPBP has the highest number of contributors, i.e. 436 and the highest contributor's truncation point, i.e. $ 1979 million. Internal Fraud ranks second with 436 contributors and the highest contributor's truncation point i.e. $1836 million.

With regard to the insurance industry, CPBP has the highest number of contributors, i.e. 264 and the highest contributor's truncation point, i.e. $1094 million.

Both bank and insurer contributors' truncation point are significantly skewed to the right. According to Table 1.2, 1.3 and 1.4, the coefficient of skewness is 14.80 for banks and 5.25 for insurers. A log scale is thus used to represent the distribution of contributors' truncation point.

Figures 1.1 and 1.2 show the histogram of the contributor's log-truncation-point for US banks and insurers. For the first category, according to Table 1.5, the contributor' truncation point at the 25th, 50th, 75th, and 95th percentiles are $2 million, $4 million, $12 million, and $80 million, respectively. For the insurers, these percentiles are $2 million, $5 million, $19 million and $120 million.

The results of these preliminary analysis visibly suggest that it would not be appropriate to treat the contributor's truncation point as constant and known.

As to the size of these contributors, table 2.1 provides summary statistics for banks and insurers' exposure proxied by their total revenue. It is noticed that more than 50% of both organizations have no exposure reported. The revenue is clustered according to the euclidean distance into 3 categories based on the size of the organizations, i.e. small size, medium size and large size. According to this classification scheme, within the US bank contributors, 26 contributors could be considered as large bank while within the US insurer contributors, 10 could be deemed as large insurer. Table 2.2 shows that the median of the total revenue amounts to $7,793 million for banks and $9,241 million for insurers. The two aforementioned classifications are used to calibrate the loss severity distribution according to organization size.

Figures 3.1 and 3.2 display the yearly aggregate losses for US banks from 1980 to 2002. One notices the existence of a cycle with peaks in 1984, 1988, 1994, 1998 and 2002. The length of the cycle is approximately four years. The first figure splits the total yearly aggregate losses into eight business lines. In 1988 and since 2000, retail banking has become a major business line in terms of yearly aggregate losses. Trading and sales ranks second. Figure 3.2 breaks the total yearly aggregate losses into the seven event types. Clearly, CPBP is the main risk driver of operational risk for US banks. Internal Fraud also accounts for an important part of the total yearly aggregate losses. Figure 3.3 analyzes CPBP losses by splitting them into various components defined by Fitch. Deceptive sales practices and concealment followed by failure to disclose appear to be the main risk drivers of CPBP.

As to US insurers, Figure 3.4 indicates that insurers' operational losses started increasing from 1992 and that CPBP is also the main risk driver. It may be the case that insurers' operational losses are subject to more disclosure from 1992. Similar to US banks, Figure 3.5 shows that deceptive sales practices and concealment most account for insurance CPBP losses.

Figure 3.6 compares the US bank and insurer yearly aggregate losses and indicates that banks incurred more operational losses than insurers.

As to loss occurrences, Figure 3.7 displays the US bank yearly loss occurrences and indicates an upward trend. The same result holds true for the US insurer yearly loss occurrences as shown by Figure 3.8.

Tables 4.1 and 4.2 show total loss amounts and occurrences incurred by the US banks from 1960 to 2003. Total loss amounts are split into BCBS eight business lines and seven event types. Retail banking followed by trading and sales is the leading business line while CPBP and internal fraud are the two major loss event types.

As to loss occurrences, retail banking has the highest number of individual losses both overall and specifically for CPBP. Again CPBP among the seven event types shows the highest number of individual losses. Internal fraud ranks second. Likewise, for the US insurers, Table 4.3 indicates that CPBP is the main risk.

One notices that some business lines and event types such as agency service, payment and settlement, damage to physical assets and business disruption & system failure have few observations or no observations.

In view of these results, it seems appropriate to conduct the calibration of the loss severity as well as the calculation of the capital charge by dividing banks' activities as follows:

1- All business lines – CPBP (or relationship risk class according to Fitch classification).

2- All business lines – Internal fraud and employment practices and workplace safety (or people risk class according to Fitch).

3- All business lines – Other event types.

For the insurance industry, the following classification is used.

1- CPBP

2- Other event types

4- The Methodology

4.1 - The standard collective risk model

Typically, in actuarial science and in the quantitative operational risk management field, three building-block assumptions underpin the standard collective risk model, commonly referred to as LDA. These include: (1) loss occurrence is a random variable modeled by a counting process that is generally the Poisson process, (2) loss severities are independent and identically distributed, and (3) loss occurrence distribution is independent from that of the loss severity sequence.

As to operational risk, the yearly aggregate loss for a specific cell i among the 56 cells set by BCBS can be expressed as:

$$AggL_i = \sum_{k=1}^{N} L_k$$

where $AggL_i$ denotes the yearly aggregate loss, N the yearly loss occurrences, and L_k the loss severity. BCBS sets the capital charge for this specific cell to:

$$Cap_i = F_i^{-1}(99.9\%)$$

where F_i^{-1} denotes the quantile function of the distribution of the cell i aggregate loss. For a bank as a whole, the aggregate loss $AggL$ is the sum of the aggregate loss for each cell, that is:

$$AggL = \sum_{i=1}^{56} AggL_i$$

Initially, BCBS suggested that the total capital charge for a bank should be expressed as the sum of the capital charge for each cell, that is:

$$Cap = \sum_{i=1}^{56} Cap_i$$

which means that aggregate losses across all cells are perfectly correlated, and therefore, the frequency and severity that drive aggregate losses are in turn driven by one source of uncertainty. Later on, this extreme assumption was revisited so that banks are now permitted to use internally generated correlations to account for the possible dependence structure among risk classes.

The application of LDA to operational risk has been criticized by many authors, (see for example Embrechts et al, 2003). According to these authors, operational losses display many features that are barely in line with the assumptions behind LDA. Unfortunately, an extension of the standard LDA, accounting for all operational loss's specificities is beyond what is currently practicable in view of data availability and resources devoted to this risk class.

As pointed out by Frachot et al (2004), taking into account correlations between loss occurrences of events is feasible and do not significantly change the standard LDA model. By contrast, accounting for correlation between loss severities of different classes is much more complicated and extensively changes the underpinnings of the standard LDA. The next subsection uses the standard LDA combined with the extreme value theory, the random truncation statistical paradigm as well as copulas to derive the loss severity distribution and estimate the capital charge within the Monte Carlo simulation framework.

4.2 - Loss severity distribution

This subsection focuses on the calibration of severity distributions using publicly available operational loss data set. It was already pointed out that this specific data set is plagued by many biases that impede one's ability to uncover the true underlying loss severity distribution. Frachot et al (2003) suggest two models to account for these biases. The first model deals with reporting biases and specifically assumes that contributors' operational loss data are sampled from the same probability distribution, and as such are not different from each other. However, losses are captured according to some unobserved truncation point that needs to be accounted for. Essentially, model 1 assumes that with regard to publicly available operational loss data set, all losses are not captured and reported and one would expect a positive correlation to exist between the loss amount and the probability that the loss is reported. As a result, the data set contains a disproportionate number of very large losses (Fontnouvelle et al , 2003). This phenomenon is referred to as the reporting bias.

The second model deals with scaling issues and assumes that contributors' operational loss data are essentially different by nature because they originate from different probability distributions and as such, they need to be re-scaled. Furthermore, the unobserved truncation point should be accounted for. In an attempt to solve this scaling issue, Shih et al (2001) suggest the following relationship between the size of a firm and its individual loss amount:

$$L = R^{\alpha} \times F(\theta)$$

where L is the actual loss, R the gross revenue, α the scaling factor, and θ the vector of risk factors not explained by revenue. The study reveals that the size of a firm is poorly correlated to its size of loss.

Allianz (2001) proposes the following formula to scale operational losses.

$$X_{Bank.X} = X_{Bank.A} \times \left(1 + a \left(\left(\frac{EI_{Bank.A}}{EI_{Bank.X}} \right)^{b} - 1 \right) \right)$$

where $X_{Bank.X}$ is the gross operational loss at bank X, $X_{Bank.A}$ the gross operational loss at bank A, EI an exposure indicator, a and b parameters obtained by a regression analysis.

In line with Frachot et al (2003), it is acknowledged that model 2 is the most accurate approach. It constrains each bank to derive its specific scaling formula not only for the external loss data but also for the internal loss data generated by different business lines within the bank. However, its reliable implementation requires the use of a large set of data that banks and insurers currently do not hold. As soon as operational loss data sets substantially grow, further investigation could be carried out along this line of reasoning. As of today, for large losses, especially those in the tail (in excess of $1 million), it is worth assuming that the reporting bias is the most important issue within the model 1 framework. The following lines build on Fontnouvelle et al (2003) and suggest a symbolic approach that easies the calibration of the loss severity. The random truncation modeling is described as follows:

Let us consider two independent random variables X and H and let $f_{X|H<X}(x \mid h<x)$ denote the probability density function of the observed values of X randomly truncated by H (that is, X is observed when it exceeds the unobserved truncation point H). Then $f_{X|H<X}(x \mid h<x)$ is given by

$$f_{X|H<X}(x \mid h<x) = \frac{f_X(x)F_H(x)}{\int_{\mathbb{R}} f_X(t)F_H(t)dt}$$

where $f_X(x)$ and $F_H(x)$ represent the probability density function and the cumulative distribution function of X and H, respectively. Random truncation modeling is generally used in economics, reliability, and astronomy. In this latter field, it is known as the Malmquist bias in the study of galaxies.

Now let X^* denote the random variable representing the reported loss, u the nominal threshold ($1 million) and the $X = \log(X^*) - \log(u) \mid X^* > u$ the conditional excess loss. If one assumes that the distribution of operational losses of a specific business line/event type belong to the maximum domain of attraction of either the Frechet distribution or the Gumbel distribution and if we consider u as a sufficient high threshold, results from EVT (Embrechts et al, 1997), indicate that the distribution function of X may be approximated by $G_{0,\beta}(x) = 1 - \exp\left(-\frac{x}{\beta}\right)$ which is the exponential distribution with density $g_{0,\beta}(x) = \frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right)$.

As to the random truncation point H^*, in addition to the known and constant case, this paper assumes two other distributions for $H = \log(H^*)$, namely the logistic distribution as in Fontnouvelle (2003) and the normal distribution. A fourth case, worth mentioning is that of the alpha stable non Gaussian exponentially truncated distributions. This class of distributions is gaining importance in empirical finance in that it provides better fit of the tails of distribution than normal distributions. As pointed by Nolan (2001), these distributions are now more computationally tractable and should be part of quantitative risk managers' toolkit. This will be examined in future work.

Let (X, H) denote the random vector representing the conditional excess log losses and the log random truncation point, let $(H < X)$ denote the event that characterizes publicly available operational risk loss data. For the random truncation point distribution, let σ and μ denote the scale and location parameters respectively.

For the normal distribution,

$$F_H(h) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{h} \exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma} \right)^2 \right) dt = \Phi\left(\frac{h-\mu}{\sigma} \right)$$

where Φ denote the standard normal cumulative distribution function.

For the logistic distribution,

$$F_H(h) = \frac{1}{1 + \exp\left(-\frac{h-\mu}{\sigma} \right)}$$

The expression of the severity is then described as follows:

$$f_{X|H<X}(x\,|\,h<x)=\begin{cases}\dfrac{\exp\left(-\dfrac{x}{\beta}\right)\Phi\left(\dfrac{x-\mu}{\sigma}\right)}{\int_{\mathbb{R}}\exp\left(-\dfrac{t}{\beta}\right)\Phi\left(\dfrac{t-\mu}{\sigma}\right)dt}, & \text{for the normal case}\\[2ex] \dfrac{\dfrac{\exp\left(-\dfrac{x}{\beta}\right)}{1+\exp\left(-\dfrac{x-\mu}{\sigma}\right)}}{\int_{\mathbb{R}}\dfrac{\exp\left(-\dfrac{t}{\beta}\right)}{1+\exp\left(-\dfrac{t-\mu}{\sigma}\right)}dt}, & \text{for the logistic case}\end{cases}$$

The computation is based on 2003 year data available. The loss amounts are expressed in real terms, over m years. The lower bound of the domain of integration is adjusted accordingly. In real terms, a loss with nominal value $x_0 \geq u$ that occurs in year $k \leq 2003$, amounts to $x = x_0 \times \dfrac{CPI_{2003}}{CPI_k}$ in 2003, where CPI_k denotes the year k Consumer Price Index. As a result, the lower bound of the conditional excess loss $x = \ln\left(x_0 \times \dfrac{CPI_{2003}}{CPI_k}\right) - \ln(u)$ for any loss $x_0 \geq u$ that occurred in year k is

$$\begin{aligned} x_k^{lb} &= \ln\left(u \times \frac{CPI_{2003}}{CPI_k}\right) - \ln(u) \\ &= \ln\left(u \times \frac{CPI_{2003}}{CPI_k}\right) \end{aligned}$$

Thus,

$$f_{X|H<X}(x \mid h<x)=\begin{cases} \dfrac{\exp\left(-\dfrac{x}{\beta}\right)\Phi\left(\dfrac{x-\mu}{\sigma}\right)}{\int\limits_{x_k^{lb}}\exp\left(-\dfrac{t}{\beta}\right)\Phi\left(\dfrac{t-\mu}{\sigma}\right)dt}, & \text{for the normal case} \\ \dfrac{\dfrac{\exp\left(-\dfrac{x}{\beta}\right)}{1+\exp\left(-\dfrac{x-\mu}{\sigma}\right)}}{\int\limits_{x_k^{lb}}\dfrac{\exp\left(-\dfrac{t}{\beta}\right)}{1+\exp\left(-\dfrac{t-\mu}{\sigma}\right)}dt}, & \text{for the logistic case} \end{cases}$$

This means that the support of the probability density function $f_{X|H<X}(x \mid h<x)$ describing the observed losses is the interval $\left[\ln\left(\dfrac{CPI_{2003}}{CPI_k}\right),+\infty\right)$

Application

The following subsection describes the maximum likelihood methodology used to estimate the parameters $\Theta=(\beta,\mu,\sigma)$ of the observed severity distribution, assuming a logistic distribution for the log of the random truncation variable.

Suppose that one is interested in estimating the parameters of the severity for a specific business unit/event type cell. The publicly available operational risk data loss consists of n losses beyond \$1 million over m years, that is $\left\{\left\{x_1,T_{k_1}\right\},...,\left\{x_n,T_{k_n}\right\}\right\}$ with T_{k_i} denoting the year of occurrence of loss $x_i, 1\le k_i\le m$ and $1\le i\le n$.

$\left\{\left\{x_1, T_{k_1}\right\}, \ldots, \left\{x_n, T_{k_n}\right\}\right\}$ is then transformed into $X = \left\{\left\{x_1, x_{k_1}^{lb}\right\}, \ldots, \left\{x_n, x_{k_n}^{lb}\right\}\right\}$ where $x_{k_i}^{lb} = \ln\left(\dfrac{CPI_{2003}}{CPI_{k_i}}\right)$

The maximum likelihood function based on the data $X = \left\{\left\{x_1, x_{k_1}^{lb}\right\}, \ldots, \left\{x_n, x_{k_n}^{lb}\right\}\right\}$ is given by

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{f_X(x_i \mid \beta) \times F_H(x_i \mid \mu, \sigma)}{\int_{x_{k_i}^{lb}}^{\infty} f_X(t \mid \beta) \times F_H(t \mid \mu, \sigma) dt}$$

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{\dfrac{\exp\left(-\dfrac{x_i}{\beta}\right)}{1 + \exp\left(-\dfrac{x_i - \mu}{\sigma}\right)}}{\displaystyle\int_{x_{k_i}^{lb}}^{\infty} \dfrac{\exp\left(-\dfrac{t}{\beta}\right)}{1 + \exp\left(-\dfrac{t - \mu}{\sigma}\right)} dt}$$

This maximum likelihood procedure is computationally intensive and considerably complicated to handle (Baud et al, 2002, Fontnouvelle et al, 2003, Frachot et al 2003) mainly because of the integral appearing in the denominator. One way to deal with this specific integral is to implement a symbolic computational framework so as to convert it into symbolic “numerics”.

Since

$$x_{k_i}^{lb} \in \left\{\ln\left(\frac{CPI_{2003}}{CPI_{2003}}\right), \ldots, \ln\left(\frac{CPI_{2003}}{CPI_m}\right)\right\}$$

and $CPI_m \le ... \le CPI_{2003}$ do not depend on the loss amount x_i it is feasible to symbolically compute beforehand the vector of integrals

$$I_{k_i} = \int_{x_{k_i}^{lb}}^{\infty} \frac{\exp\left(-\frac{t}{\beta}\right)}{1+\exp\left(-\frac{t-\mu}{\sigma}\right)} dt$$

with $k_i \in \{T_1, ..., T_m\}$.

Now, one can express the maximum likelihood function as

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{\exp\left(-\frac{x_i}{\beta}\right)}{y_i \times \left(1+\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right)}$$

where $y_i = I_{k_i}$ for some k_i.

The computational process is now based on the set of data $\{\{x_1, y_1\}, ..., \{x_n, y_n\}\}$.

Specifically, the code of the above algorithm can be implement in *Mathematica* as follows:

Step 1

Define a vector containing the $x_{k_i}^{lb} = \ln\left(\frac{CPI_{2003}}{CPI_{k_i}}\right)$ as cpiVector

Step 2

Compute the vector of integrals symbolically

$$\text{integralVector=Table[} \int_{\text{cpiVector[[i]]}}^{\infty} \frac{\exp\left[-\frac{t}{\beta}\right]}{1+\exp\left[-\frac{t-\mu}{\sigma}\right]} dt, \{\text{i,1,Length[cpi]}\}$$

This vector is computed once and saved on disk for future use.

For example, the expression of I_{k_1} the first component of integralVector, is expressed in *Mathematica* numerics as follows:

$$\frac{e^{-\left(\frac{1}{b}+\beta\right)\tau}\left(e^{\beta\tau}\pi\,(-1+b\beta)\,\text{Csc}\left[\frac{\pi}{b\beta}\right]-b\,e^{\frac{\tau}{b}}\beta\,\text{Hypergeometric2F1}\left[1,1-\frac{1}{b\beta},2-\frac{1}{b\beta},-e^{-\beta\tau}\right]\right)}{\beta\,(-1+b\beta)}$$

Step 3

Define the probability density function as

$$f=\frac{\exp\left(-\frac{x}{\beta}\right)}{y\times\left(1+\exp\left(-\frac{x-\mu}{\sigma}\right)\right)}$$

Step 4

Express the maximum likelihood function as

$$\text{LogL=Log}\left[\prod_{\text{i=1}}^{\text{n}}\left(f\,/.\{x\to x_i,y\to y_i\}\right)\right]$$

Step 5

Compute the observed log-likelihood from a matrix dataMat, containing individual losses with their ages.

$$\text{sampleLik=LogL/.}\{\text{n->Length[dataMatrix]}\},x_{i\_}:\to\text{dataMat[[i,1]]}$$
$$y_{i\_}:\to\text{integralVector[dataMat[[i,2]]]}\}$$

Step 6

The values of the parameters are then computed by maximizing the objective function sampleLik with an optimization program. These values are relevant to large internationally active banks.

The aforementioned approach is easy to implement, significantly reduces the computing time and as a result, facilitates the calibration of the loss severity which is a major issue in operational risk modeling.

Severity of a specific organization

The parameters of the severity as well as those of the random truncation point were derived assuming that all losses from the data set were incurred by a typical large internationally active bank (Fontnouvelle et al, 2003). In the rest of the paper such a large internationally active bank will be simply referred to as an "industry-wide organization". Furthermore, it is possible to envision different categories of industry-wide organizations, each with a specific yearly loss frequency distribution. Tables 5.1 and 5.2 present the maximum likelihood estimates of the loss severity distribution parameters for such an organization. This section investigates the extent to which Fitch data set could be appropriate to calibrate the severity of a specific bank. In other words, if all these losses, drawn from the same probability distribution, were incurred by a specific firm, how could one account for the positive relationship that exists between the loss amount and the probability of its disclosure? This is an important question left for future research in Fontnouvelle et al (2003).

The following subsection proposes an approach rooted in model 1 that uses the concept of Probable Maximum Loss – PML to account for firm size and quality of control environment.

The concept of Probable Maximum Loss stems from fire insurance where it has been noticed that total losses were very infrequent in categories where there are public fire protection and fire-resistive structures. Bennett (1992) defines the PML as "the largest possible loss that may occur, in regard to a particular risk, given the worst combination of circumstances". Wilkinson (1992) and Kremer (1990, 1994) suggest to express the PML as either $(1-\theta)E[M_n]$ or $E[M_n]+\theta\sqrt{VarM_n}$ where $M_n = \max(X_1, X_2, ..., X_n)$ is the maximum of n claims and θ a safety loading coefficient. Cebrian et al (2004) obtain the PML by solving the following equation

$$P[M_n \le PML_\varepsilon] = 1-\varepsilon,$$

for some $\varepsilon > 0$. In other words, the PML can be considered as a high quantile of the maximum of a random sample of size n, that is

$$\mathrm{PML}_\varepsilon = F_{M_n}^{-1}(1-\varepsilon),$$

This latter formula can be estimated using two different methodologies. Wilkinson (1992) advocates the use of order statistics, while Kremer (1990, 1994) and Cebrian et al (2004) suggest a methodology rooted in extreme value theory. Details of the suggested model are as follows:

It is assumed that for a specific organization, each business line/event type has an explicit random truncation point distribution. It is further assumed that a PML could be assigned to each business line/event type, and this PML will reflect the size and the quality of internal control of the firm. Thus, to derive the scale and location parameters of the distribution function of the truncation point, it suffices to match percentiles at two different losses.

Let us consider a business line/event type with its specific PML. Let F_s denote the distribution function of the truncation point of the specific organization and F_i that of an industry-wide organization. It is worth noting that F_i is derived from the initial maximum likelihood estimation using the industry-wide operational losses. Let $F_S(PML)$ and $F_i(PML)$ denote the probability of disclosure of the PML according to these two distributions. If the value of $F_S(PML)$ is set at a certain level, according to expert judgment (depending on the firm's size and quality of internal risk control), and if for example, it is further assumed that the median of the two distributions are similar, then one is in position to derive the parameters of the specific truncation point and therefore, compute the underlying loss severity parameter using the maximum likelihood estimation approach. If $F_S(PML) \geq F_i(PML)$, then the underlying loss severity parameter is lower than that of an industry-wide organization and if $F_S(PML) \leq F_i(PML)$, the underlying loss severity parameter is greater that that of an industry-wide organization.

4.2 Frequency distribution.

In this study, simplicity demands that one uses the Poisson distribution to describe loss occurrences. Indeed, above a high threshold, the Peak Over Threshold (POT) model assumes that loss occurrences are Poisson distributed. To calibrate this distribution, one uses the fact that large international active banks incur an average of 50 to 80 losses above $1 million each year. (Fontnouvelle et al , 2003). Further investigation of frequency distributions in the context of common Poisson shock model (Lindskog et al, 2001) is left for future work.

4-3 Modeling the dependence structure using copulas.

As a tool to model joint effects of multiples risks, the concept of copula has recently attracted extensive attention from the financial community. This subject is relevant to operational risk practitioners since within the BCBS framework, banks are required to calculate the capital charge for each of the 56 business lines/event types and use a dependence structure model to aggregate these values. Simply stated, a copula function links univariate marginal distributions to their joint distribution. A theorem due to Sklar (1959) states that if $X = (X_1, ..., X_d)$ is a random variable with joint distribution function F, then there exists a copula function C such that

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_n(x_d))$$

where F_i is the ith marginal distribution function, for $i = 1, 2, ..., d$.

For absolutely continuous univariate marginals, there is a unique copula C such that

$$C(u_1, ..., u_d) = F(F_1^{-1}(u_1), ..., F_d^{-1}(u_d))$$

where

$$F_i^{-1}(u_i) = \inf \{x : F_i(x) > u_i\}, \qquad i = 1, ..., d$$

are the marginal quantile functions.

Recent developments on copulas can be found in Embrechts et al (1999), Lindskog (2001) and Nelsen (1999). Following is a brief presentation of some useful families of copulas that are used in this paper.

Tang et al (2004) describe three classes of copulas that are generally used in finance and insurance. These are the copulas of extreme dependence, the Archimedean copulas and the elliptical copulas. The copulas of extreme dependence include the independence copula, the Frechet lower bound for copula and the Frechet upper bound for copula. The independence copula or product copula $\Pi(u)$ is expressed as

$$\Pi(u) = u_1 ... u_d .$$

while, the Frechet bounds for copulas are

$$M(u) = \min(u_1, ..., u_d)$$

and

$$W(u) = max(u_1 + ... + u_d - d + 1, 0)$$

with

$$W(u) \leq C(u) \leq M(u)$$

Note that for $d \geq 2$, $M(u)$ defines a copula, called the comonotonic copula that describes a perfect positive dependence structure, while for $d > 2$, $W(u)$ is no longer a copula. Archimedean copulas or explicit copulas constitute the second class of copulas. They are based on one generator function, and as such, have simple closed forms (Aas, 2004). This class of copulas allows for asymmetry, and as a result, exhibits greater dependence in the negative tail or in the positive tail. However these copulas generally fail to account for multivariate dependence structure as they have one single parameter to describe the dependence. Examples of Archimedean copulas include the Clayton copula and the Gumbel copula. Elliptical copulas or implicit copulas comprise the third class of copulas. Typically, elliptical copulas are copulas implied by elliptical distributions. Well-known examples of elliptical distributions include multivariate normal, t-student, and logistic distributions. Elliptical copulas allow for joint extreme events, but fail to account for asymmetries. In addition, they do not have a simple close form. Regardless of these shortcomings, they are becoming more and more popular for empirical exercises as they are remarkably easy to simulate. Tang et al (2004) also acknowledge the flexibility of this family of copulas to account for differences in pair-wise dependence structure by using a variance-covariance framework.

The expressions of the aforementioned copulas are as follows:

For the normal copula:

$$C(u) = \Phi_R^d\left(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)\right)$$

where Φ_R^d denotes the joint distribution function of the d-dimensional multivariate standard normal distribution function with linear correlation matrix R.

In the bivariate case, the copula expression is:

$$C_\rho(u,v) = \int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)}\exp\left\{-\frac{s^2-2\rho st+y^2}{2(1-\rho^2)}\right\}dsdt$$

where ρ denotes the linear correlation coefficient of the bivariate normal distribution.

The expression of the Student's t-copula is

$$C(u) = t_{\nu,R}^d\left(t_\nu^{-1}(u_1),...,t_\nu^{-1}(u_d)\right)$$

where $t_{\nu,R}^d$ denotes the joint distribution function of the d-dimensional multivariate Student's t-distribution function with linear correlation matrix R and ν degrees of freedom.

In the bivariate case, the copula expression is:

$$C_{\rho,\nu}(u,v) = \int_{-\infty}^{t_\nu^{-1}(u)}\int_{-\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}}\left\{1+\frac{s^2-2\rho st+y^2}{\nu(1-\rho^2)}\right\}^{-(\nu+2)/2} dsdt$$

where ρ denotes the linear correlation coefficient of the bivariate Student's t-distribution function with ν degrees of freedom.

The Clayton copula has the following expression:

$$C_\delta(u,v) = (u^{-\delta}+v^{-\delta}-1)^{-1/\delta}$$

where $0<\delta<\infty$ denotes a parameter controlling the degree of dependence.

The Gumbel copula can be expressed as:

$$C_\delta(u,v) = \exp\left(-((-\log u)^\delta + (-\log v)^\delta)^{1/\delta}\right)$$

where $1 \le \delta < \infty$ denotes a parameter controlling the degree of dependence.

Following is the summary of the algorithm that simulates a vector $(x_1,...,x_d)$ with marginal $F_{x_1},...,F_{x_d}$ and the associated elliptical copula C

For the normal copula, the i[th] simulated loss is

$$x_i = F_{x_i}^{-1}\left(\Phi\left(A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d))\right)\right)$$

where $\Phi(u)$ denotes the standard normal cumulative distribution, A the lower triangular matrix obtained from the Choleski decomposition of the covariance matrix of F, and u_i for $1 \le i \le d$, are d independent standard uniform variables.

For the Student's t-copula, the i[th] simulated loss is

$$x_i = F_{x_i}^{-1}\left(t_\nu\left(\sqrt{\frac{\nu}{S}}\left(A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d))\right)\right)\right)$$

where t_ν is the Student's t cumulative distribution function with ν degrees of freedom, S a random number generated from the chi-square distribution random variable $\chi^2(\nu)$ independent from each of the standard normal variables $\Phi^{-1}(u_i)$.

It is worth noticing that the normal copula transformation gives rise to the simulation of random variables under the Wang Transform.

By setting

$$A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)) = \Phi^{-1}(u_i) + \lambda$$

where $\lambda = \Phi^{-1}(\alpha)$, with α denoting the specified rating target or confidence level.

One gets

$$x_i = F_{x_i}^{-1}\left(\Phi\left(\Phi^{-1}(u_i)+\lambda\right)\right)$$

This paper will compute the capital charge in the context of Elliptical copulas. The case of Archimedean copulas will be investigated in future work.

With regard to operational risk, Frachot et al (2004) point out that the most convenient and cheapest way to model correlation between aggregate losses is to assume that aggregate loss correlation is essentially driven by the underlying correlation between loss occurrences. The authors show that in this specific case, the correlation between two aggregate losses connected to two classes i and j is such that

$$0 \le \mathrm{cor}(AggL_i, AggL_j) \le \mathrm{cor}(N_i, N_j) \le 1$$

where N_i and N_j denote loss frequencies of classes i and j.

For example, in the case where the loss severity distribution is log-normally distributed, the authors prove that the aggregate loss correlation is a decreasing function of the kurtosis or the heaviness of the tail of the distribution. Therefore, aggregate loss correlations for large losses may be very small even in the case where loss occurrence correlation is high. It was shown that in the models dealing with highly-correlated losses, correlation between aggregate losses might be less than 10%. As indicated in the same paper, the maximum aggregate loss correlation for Credit Lyonnais is less than 4%.

The following subsection develops a framework used to derive the resulting capital charge, taking into account all possible dependence structures. As a matter of fact, one may select the copula that minimizes the distance to the empirical copula of the data (Romano, 2002). It is argued that for each event type subclass, an accurate estimation of the degrees of freedom as well as of the empirical copulas is unfeasible due to lack of sufficient data. Therefore, one should account for the different types of copulas considered to get the empirical distribution of the "comprehensive" capital charge. In this setting, 13 dependences structures are explored. These include the comonotonic dependence, the Student's t-copula with t ranging from 1 to 10, the normal copula and the independence copula. The framework is that of the finite mixture distribution, especially the component-mix distribution in which the resulting capital charge is expressed as a mixing weighted capital charges. Recent literature dealing with distributions formed from component-mixes can be found in Rose et al. (2002) and Titterington et al. (1985).

Specifically, component mix distributions are generated from linear combinations of distributions. Following Rose et al (2002), in the case of a discrete random variable X_i, let $f_i(x) = P(X_i = x)$ for $i = 1, ..., n,$ denote the probability mass function and let π_i denote a parameter such that $0 < \pi_i < 1$ and $\sum_{i=1}^{n} \pi_i = 1$. Then, the n-component-mix random variable is defined as

$$X \sim \pi_1 X_1 + ... + \pi_n X_n$$

and its probability mass function is expressed as

$$f(x) = \sum_{i=1}^{n} \pi_i f_i(x)$$

The parameters π_i for $i = 1, ..., n,$ are defined as the mixing weights and the functions f_i for $i = 1, ..., n,$ are called the component densities.

The next section presents the results related to the estimation of the loss severity distribution for industry-wide banks and insurers and for specific banks and insurers. The sensitivity of the capital charge to the choice of copulas is investigated and finally, the empirical probability distribution function of the resulting capital charge is derived.

5 – Results

5-1 Loss severity

The period of study ranges from 1960 to 2002 and is conducted for US banks and insurers for all business lines and event types combined, for business units and event type subclasses, and finally for business lines and event type subclasses. Banks' activities are divided into 3 business units according to BCBS classification and each business unit is analyzed as a whole and according to 3 event type subclasses. These event type subclasses, as analyzed in the descriptive statistics section, include CPBP, internal fraud-EPWS and all other event types. For insurers, the calibration is performed for all event types combined and for each event type subclass.

Table 5.1 and Figure 5.1 give the loss severity and the truncation point distribution parameters for each business unit. The results indicate that the constant and known assumption regarding the truncation point, yields the highest level of the severity parameter while the logistic assumption gives rise to the lowest level. Within model 1, the constant and known assumption does not account for reporting bias and assigns a uniform weight to all losses. Further developments (Table 5.7) show that this line of reasoning leads to a higher level of capital charges and to the belief that operational risk is extremely risky.

The most risky business unit is investment banking that comprises two business lines, namely corporate finance and trading and sales. Tail coefficients (all above 1) are

2.550, 1.1199 and 1.232 for the constant and known assumption, the logistic assumption, and the normal assumption, respectively. The most risky event type is CPBP, especially for investment banking.

As to insurers, tail parameters except for the constant and known assumption are less than 0.6. This range of tails lead to the conclusion that insurers' operational risk is less risky than that of banks.

The log likelihood of the three models suggests that the logistic distributional assumption most accounts for the reporting bias. But since the log likelihood yields a bias in comparing different distributions, the Akaike information Criterion (AIC) is computed and the likelihood ratio test is performed to acknowledge the fit of the logistic distribution (Werneman, 2005). The AIC is defined as follows:

$$AIC = -2\ln L + 2q$$

where $\ln L$ is the log-likelihood function and q is the number of parameters of the distribution fitted. The smaller the AIC, the better the model fits the data. Carriere (1998) defines the test statistic T for the likelihood ratio test as

$$T = AIC_{F_1} - AIC_{F_2} - 2q_{F_1} + 2q_{F_2}$$

where, F_1 denotes the distribution of the null hypothesis H_0, and F_2 the distribution of the alternate hypothesis H_α, q_{F_1} and q_{F_2} the number of parameters of F_1 and F_2, respectively. H_0 is rejected in favor of H_α whenever $T > \chi^2(q_{F_2}, \alpha)$ where α is the confidence level.

Table 5.2 and Figure 5.2 provide the loss severity and the truncation point distribution parameters analyzed by business lines. The Trading and sales unit appears to be the most risky, followed by agency services.

Tables 5.3 and 5.4 provide the results of the severity calibration by firm size. They indicate that small firms, or firms with revenue below the median, have the highest level of the tail parameter. These results are in line with those obtained by Shih et al (2001).

The severity parameter of a specific organization is calibrated using the methodology previously described. Table 5.5 provides the results of this calibration. The PML along with its probability is set to $1000 and 0.99, respectively. The resulting tail parameter is 0.472.

For the most prominent business lines and event types, Figures 5.3 to 5.12 show the Quantile-Quantile plots, the observed severity distribution and the underlying severity distribution. CPBP QQ-plot shows a slight decline in fit towards the tail, while retail banking display a substantial decline in fit. As to insurers, the QQ-plot cannot be displayed since the acceptance-rejection algorithm used to simulate the observed loss severities fails to converge. The Kolmogorov-Smirnov and Anderson-Darling tests have not been performed because these tests are not appropriate for distributions of excesses over some thresholds (Moscadelli, 2004)

Figures 5.13 and 5.14 present the graph of the distribution function of the random truncation point for both the specific organization and the industry-wide organization.

5-2 Capital charge

Value at Risk at 99.9% rating target is the risk measure required by BCBS. This paper aims at deriving the empirical distribution of the aggregated capital charge so as to reflect the distribution of estimates of the underlying parameters and randomness of the Monte Carlo simulations. Specifically, 1 million of aggregate marginal losses and 150 000 aggregate dependent losses are simulated. Aggregate loss sample correlations are computed from historical data and adjusted according to expert judgment. Typically, when sample aggregate loss correlations are negative, they are adjusted to 4% and when they are greater than 10%, they are lowered to 10%. Table 5.6 shows the sample aggregate loss correlations with their adjustments. For the base scenario, the estimates are assumed to be non-random. Other scenarios reflecting estimate and correlation uncertainty will be examined in future work. The computer program has been designed accordingly[10]. Figures 5.15, 5.16, and 5.17 present the distribution of the capital charges of the three event type subclasses, while Figure 5.18 gives the distribution of the aggregated capital charge under Student's t-copula with one degree of freedom. In all cases, distributions are approximately normal. Figure 5.19 plots the aggregated capital charge in terms of the degrees of freedom for elliptical copulas. It is noticed that the level of capital charge is inversely related to the number of degrees of freedom. The Cauchy copula gives rise to the highest aggregated capital charge, while the normal copula yields the lowest aggregated capital charge. For banks and insurers, Tables 5.10 and 5.11 give the aggregated capital charge along with the capital saving for both an industry-wide organization and a specific organization. The yearly loss frequency is assumed to be

[10] A simulated data set accompanied by the *Mathematica* and C# programs will be made available upon request to the author.

equal to 50. The highest capital saving is achieved through the independence copula case. In terms of percentage, the saving ranges from 6% to 10% for banks, and from 3% to 4% for large insurers. For the specific insurer it ranges from 5% to 10%. For large organizations, the capital savings are less significant for insurers since, due to lack of sufficient data, two event type subclasses was considered compared to three for the banks. This result was expected since the diversification benefit increases with the number of business line/event types used. These levels of capital charge need to be compared with those obtained by combining all business lines/event types. Table 5.9 allows such a comparison. For a typical large bank, when all business lines and event types are combined, the capital charge amounts to $3,460 million. In the case where bank’s activities are divided into three lines, the capital charge for the normal copula amounts to $6,324 million.

Table 5.12 provides the descriptive statistics for the distribution of the aggregated capital charge. The amount obtained under the Cauchy copula ranks first for most locations, scales, and percentile measures. The skewness and kurtosis excess coefficients are close to those of normal distribution.

The resulting capital is then calculated by weighting the distributions of the capital charges derived by assuming various dependence structures. It is argued that mixing weights can be assigned to each organization, and that the quality of a firm’s risk management practices determined these weights. The illustrative case assumes that the weight for each dependence structure is 8% except for the comonotonic dependence. As to this latter case, the weight is 4%. Table 5.13 provides these weights. To get the empirical distribution of the weighted capital charges, 1 million of n-component mix random variables are simulated (n=13). Figure 5.20 and 5.21 show the histograms of the mixing weighted capital charge for both the industry-wide organization and the specific organization. Table 5.14 provides the descriptive statistics. It shows that for the industry-wide bank, the mixing weighted capital as measured by the mean of the distribution is $6,433 million while for the specific organization, it amounts to $443 million.

6 - Conclusion

This study clearly reveals that operational risk is a major risk class, as evidenced by the level of capital charge that banks and insurers need to hold. The results suggest that the level of operational risk capital charge could exceed $6 billion for large internationally active banks, and $600 million for large insurers. These amounts are in line with those disclosed by these institutions, that is, 2-7 billions for banks and 2% of gross premium for insurers. They are also consistent with the amounts estimated in Fontnouvelle et al (2003) for banks. Consequently, all of this above validate the random truncation assumption further. However, it must be accompanied by a sound approach that deals with scaling issues. Therefore, the appropriate methodology that banks and insurers need to use to rescale the severity distribution is a crucial and promising line for future research. So far, due to the lack of large loss data set, the scaling formulas that have been suggested are still in their infancy.

This study also indicates that operational risk is driven by CPBP and internal fraud and that the quality internal control environment highly impacts the loss severity which in turn significantly drives the capital charge. Thus, another crucial area for future research is the quantification of bank and insurer internal risk control environments. It is worth mentioning that BCBS through the second pillar (supervisory review) and the third pillar (market discipline) provides an appropriate framework for this exercise.

The capital charge is also driven by assumptions about the dependence structure and the number of business lines/event types involved. As a result, BCBS needs to provide incentives to banks that refine their operational risk classification scheme.

Throughout this paper various theories have been tested. These include the stochastic truncation model, the extreme value theory and copulas. All these theories are directly and significantly relevant to the management of operational risk envisioned as a tool to reduce volatility in earnings and thereby, increase shareholder value.

Descriptive Statistics of Fitch Dataset

1- Analysis of Contributors' number of losses and Truncation Point

Table 1.1 US Bank and Insurers - Number of Losses per Contributor

Number of Losses Per contributor	Banks				Insurers			
	Parent Organization		Organization		Parent Organization		Organization	
	Number	Percentage %	Number	Percentage %	Number	Percentage %	Number	Percentage %
1	791	79	1026	82	217	72	305	80
2 - 9	187	19	202	16	81	27	76	20
>9	20	2	16	1	4	1	0	0
Total	998	100	1244	100	302	100	381	100

Table 1.2 US Bank Contributors' Truncation Point ($ million) by Business Lines

	Business Lines								
	COFI	TRSA	REBA	COBA	PASE	AGSE	ASMA	REBR	All
Number of Contributors	47	96	599	237	21	52	121	242	1,244
Minimum	1	1	1	1	1	1	1	1	1
Maximum	213	1,899	1,980	453	209	536	417	254	1,980
Mean	16	114	18	24	20	24	32	10	23
Standard Deviation	6	18	9	7	7	9	8	5	10
Skewness	4	5	18	5	3	6	4	7	15
KurtosisExcess	21	21	378	29	10	38	16	59	259

COFI: Corporate Finance-TRSA: Trading & Sales- REBA Retail Banking- COBA: Commercial Banking- PASE: Payment & Settlement
AGSE: Agency Services – ASMA: Asset management- REBR Retail Brokerage

Table 1.3 US Bank Contributors' Truncation Point ($ million) by Event Types

	Loss Event Types						
	DAPA	EXFR	EPWS	INFR	EDPM	CPBP	BDSF
Number of Contributors	6	272	53	436	79	598	7
Minimum	1	1	1	1	1	1	1
Maximum	89	242	52	1,899	417	1,980	363
Mean	23	13	9	27	15	30	61
Standard Deviation	6	5	3	12	7	11	12
Skewness	2	5	2	12	7	12	2
KurtosisExcess	1	32	6	153	49	175	2

DAPA: Damage to Physical Asset- EXFR: External Fraud- EPWS: Employment Practices & Workplace Safety- INFR: Internal Fraud- EDPM: Execution, Delivery & Process Management - CPBP: Clients, Products & Business Practice BDSF: Business Disruption & System Failure

Table 1.4 US Insurer Contributors' Truncation Point ($ million) by Event Types

	Loss Event Types							
	DAPA	EXFR	EPWS	INFR	EDPM	CPBP	BDSF	ALL
Number of Contributors	1	19	17	71	53	264	1	381
Minimum	208	1	1	1	1	1	341	1
Maximum	208	295	94	420	92	1,094	341	599
Mean	208	21	21	21	8	38	341	25
Standard Deviation		8	5	8	4	10		8
Skewness		4	2	5	4	7		5
KurtosisExcess		14	1	31	14	69		37

Table 1.5 US Bank & Insurer Contributors' Truncation Point All Event Types Summary Statistics

	Percentile($ million)			
	25%	50%	75%	95%
US Bank	2	4	12	80
US Insurer	2	6	19	120

Figure 1.1 US Banks - Histogram of Contributor's Log-Truncation-Point. - All Business Lines and All Event Types

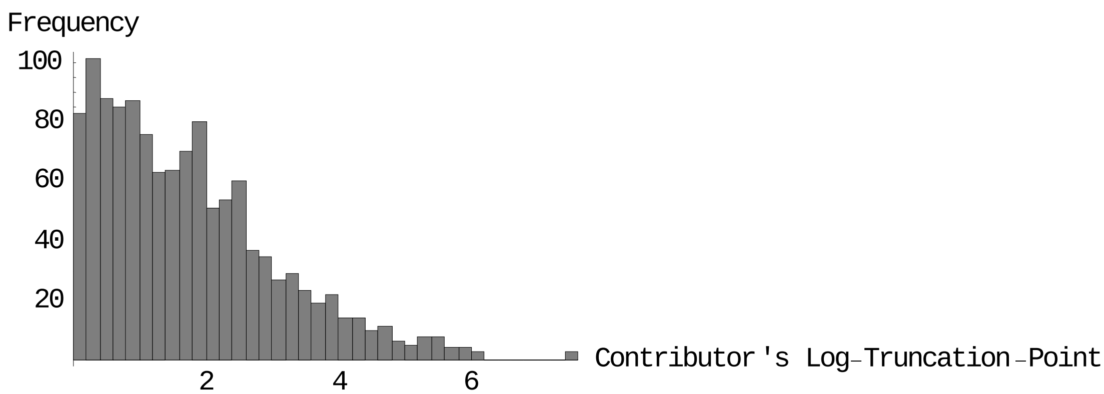
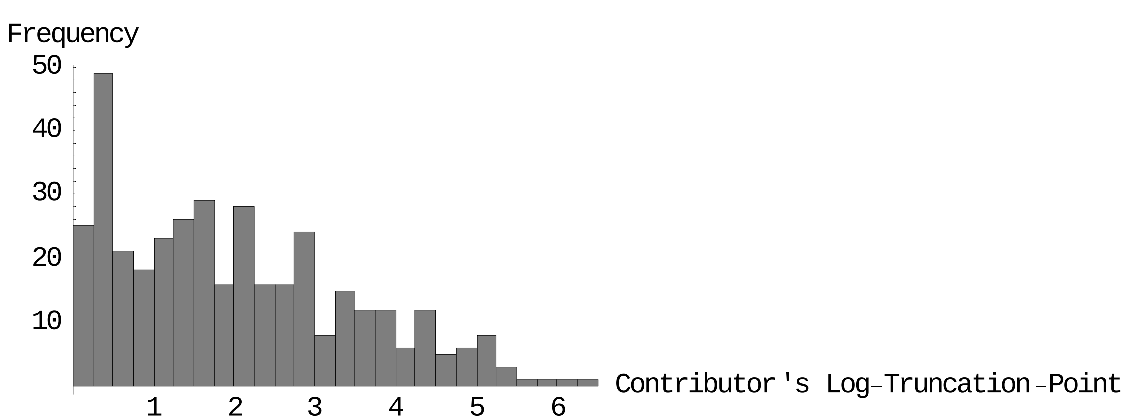


Figure 1.2 US Insurers - Histogram of Contributor's Log-Truncation-Point - All Business Lines All Event Types



2- Analysis of Exposure = Revenue ($Million)

Table 2.1 Distribution of Total Revenue
US Banks and insurers' total revenue classified into four clusters, three sizes: Small, Medium and Large

	US Banks				US Insurers			
	No Exposure Reported	Cluster 2 Small Size	Cluster 3 Medium Size	Cluster 4 Large Size	No Exposure Reported	Cluster 2 Small Size	Cluster 3 Medium Size	Cluster 4 Large Size
Number of Contributors	723	383	113	26	213	109	50	10
Mean		3,458	33,118	109,991		3,995	25,610	81,698
Min		1	18,631	72,772		17	14,978	60,391
Max		18,342	65,601	192,390		13,958	49,221	116,729
Std		4,547	12,749	33,603		3,907	7,774	18,824
Skewness		1	1	1		1	1	1
Kurtosis Excess		1	0	1		0	2	-1

Table 2.2 Distribution of Total Revenue
US Banks and insurers' total revenue by Quantile

	Total Number of Losses	Total With Revenue Reported	Min	25% Quantile	50% Quantile	75% Quantile	Max
US Banks	1989	891	1	927	7,793	24,695	192,390
US Insurers	530	250	17	3,055	9,241	26,158	116,729

3- Analysis of Aggregate loss amounts and Occurrences

Figure 3.1 US Banks Yearly Aggregate Losses By Business Lines & Settlement Year

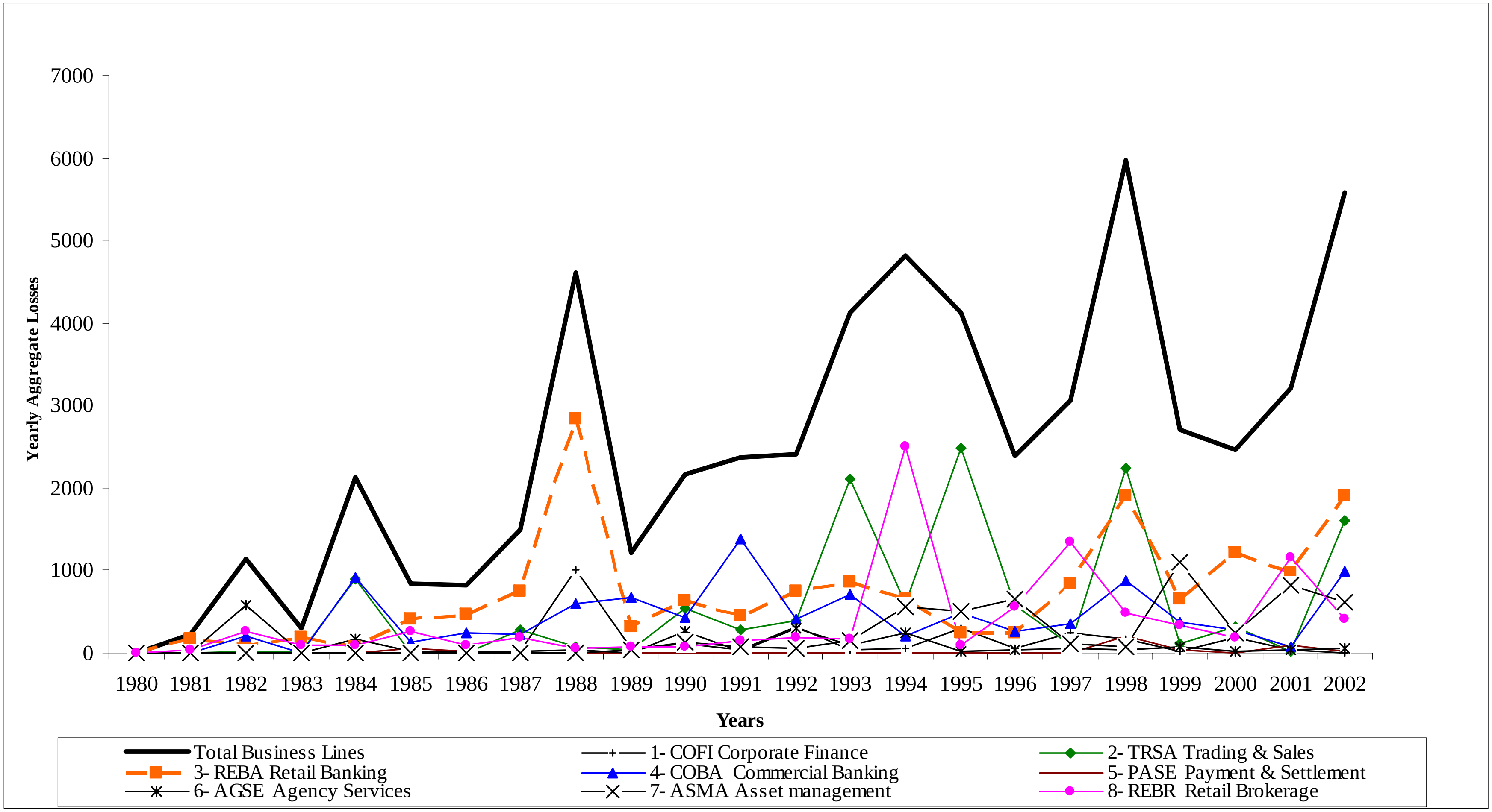


Figure 3.2 US Banks - Yearly Aggregate Losses By Event Types & Settlement Year

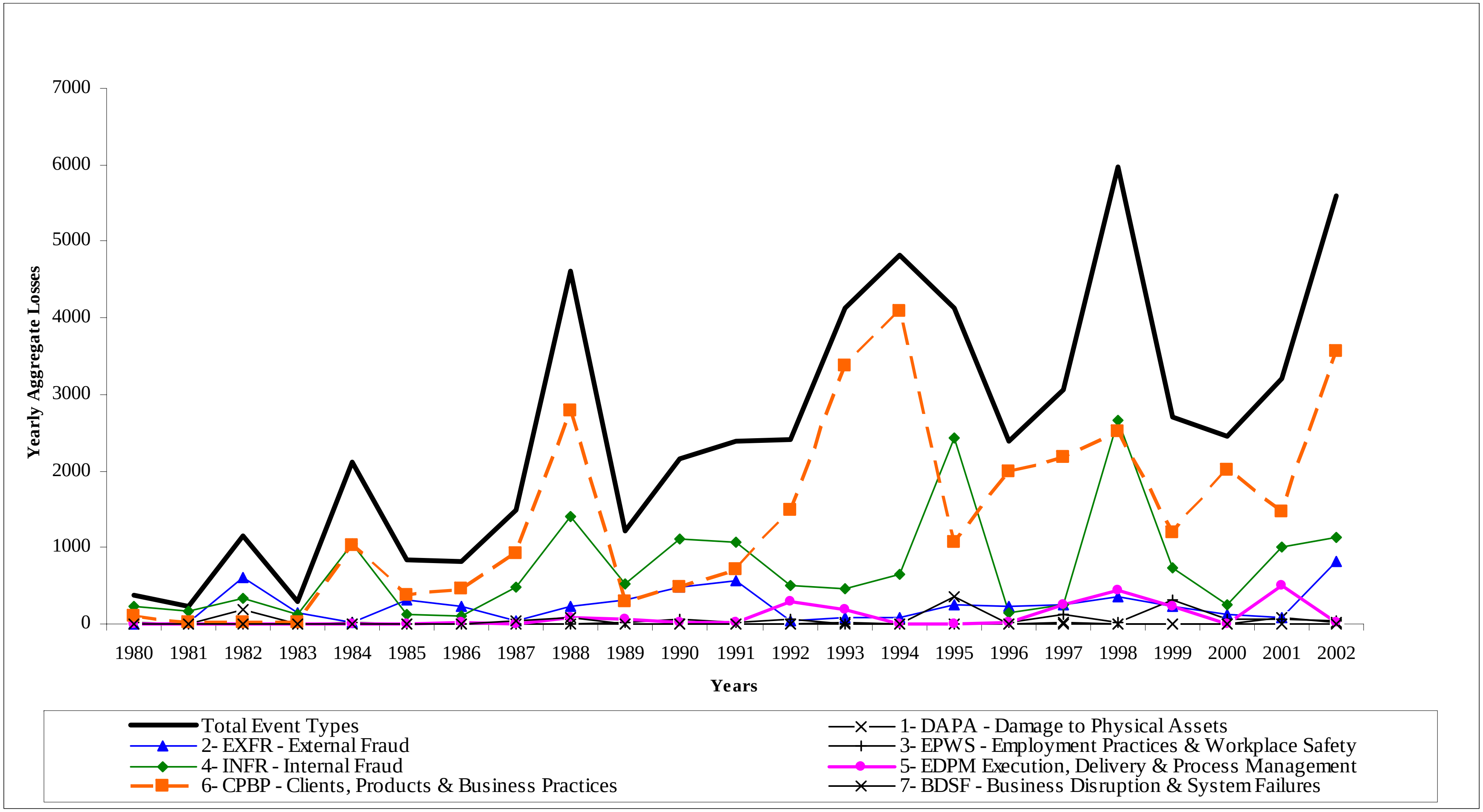


Figure 3.3 US Banks - Yearly Aggregate Losses By CPBP Sub Event Types & Settlement Year

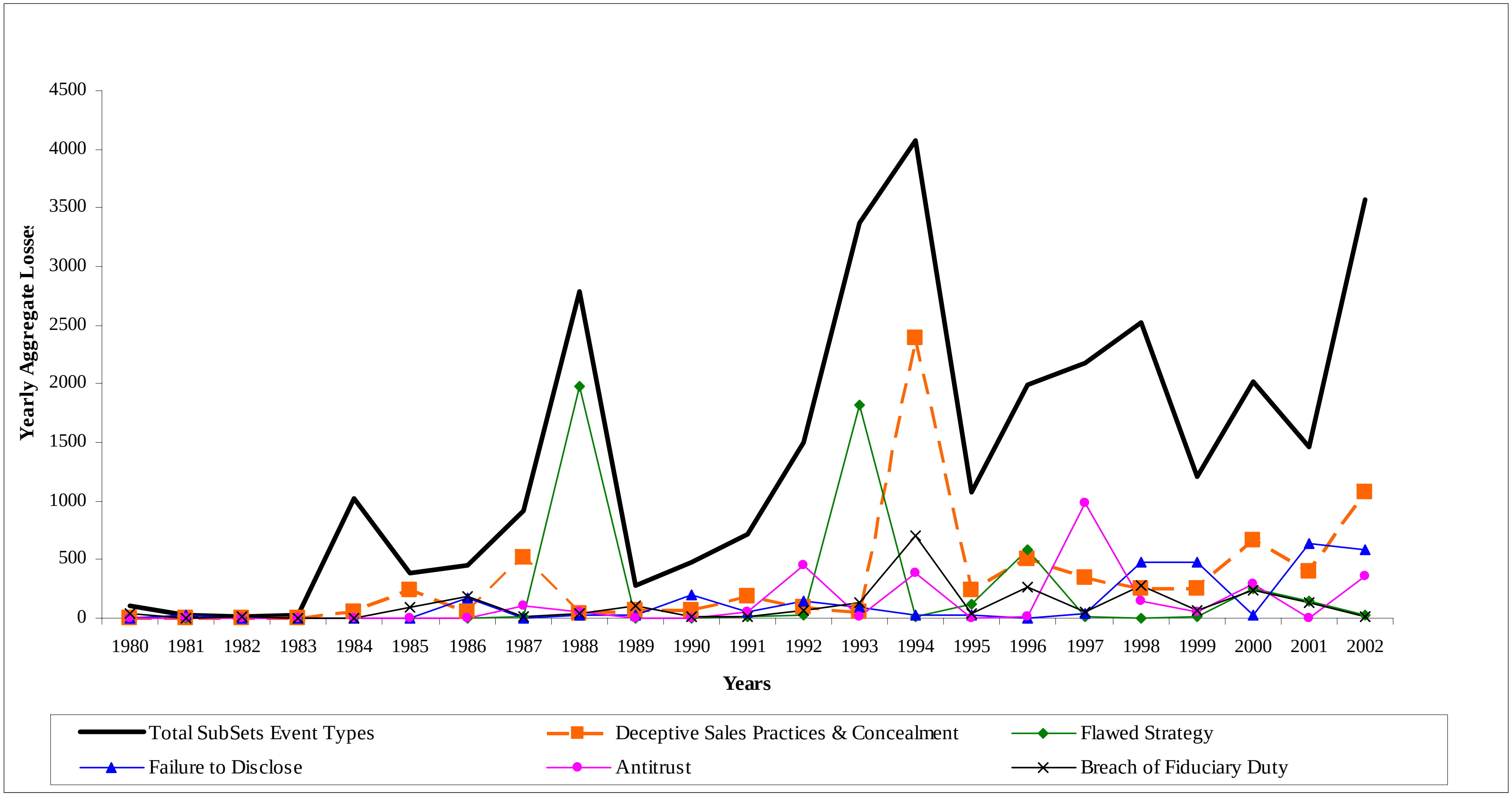


Figure 3.4 US Insurers - Yearly Aggregate Losses By Event Types & Settlement Year

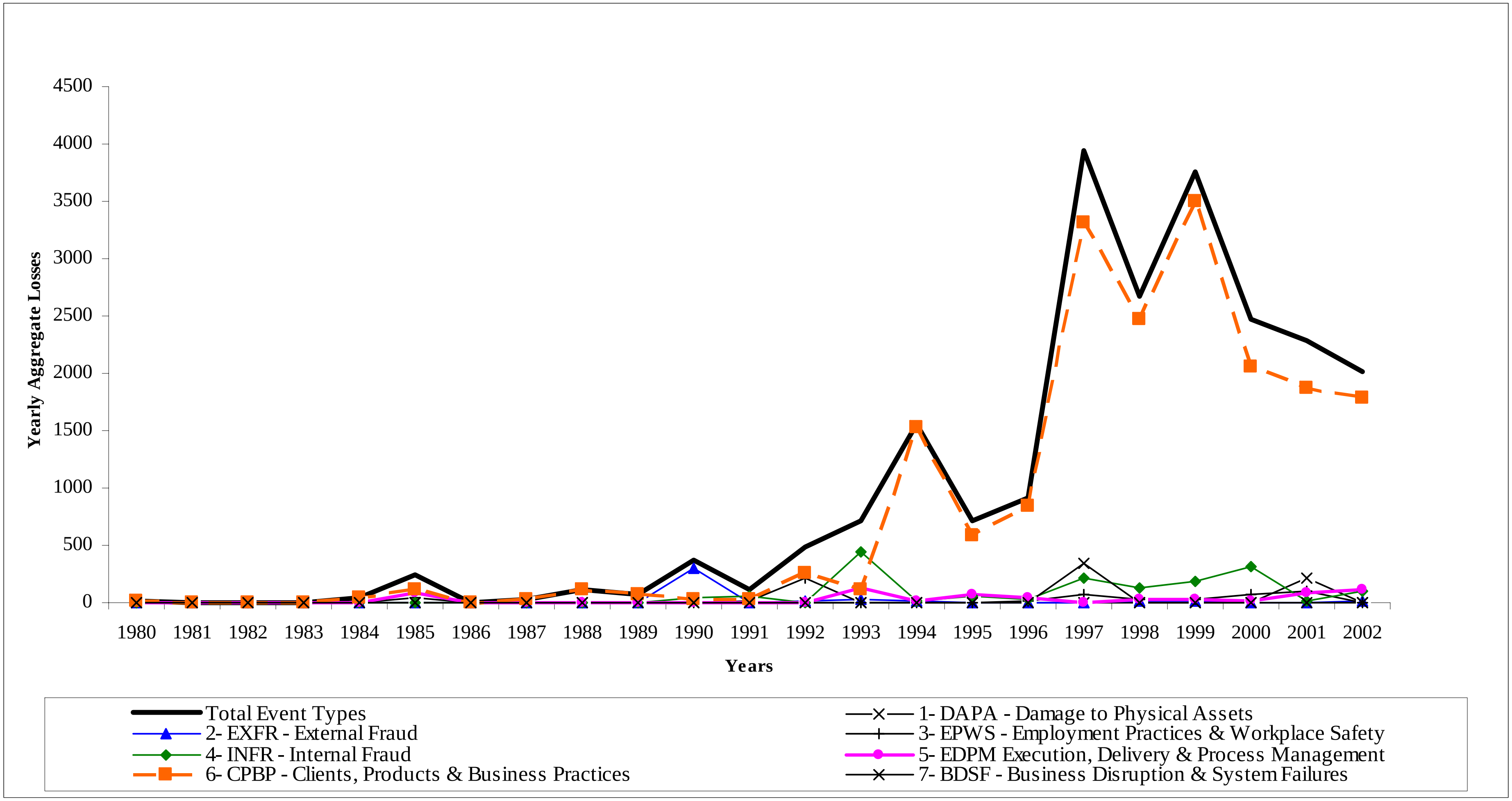


Figure 3.5 US Insurers - Yearly Aggregate Losses By CPBP Sub Event Types & Settlement Year

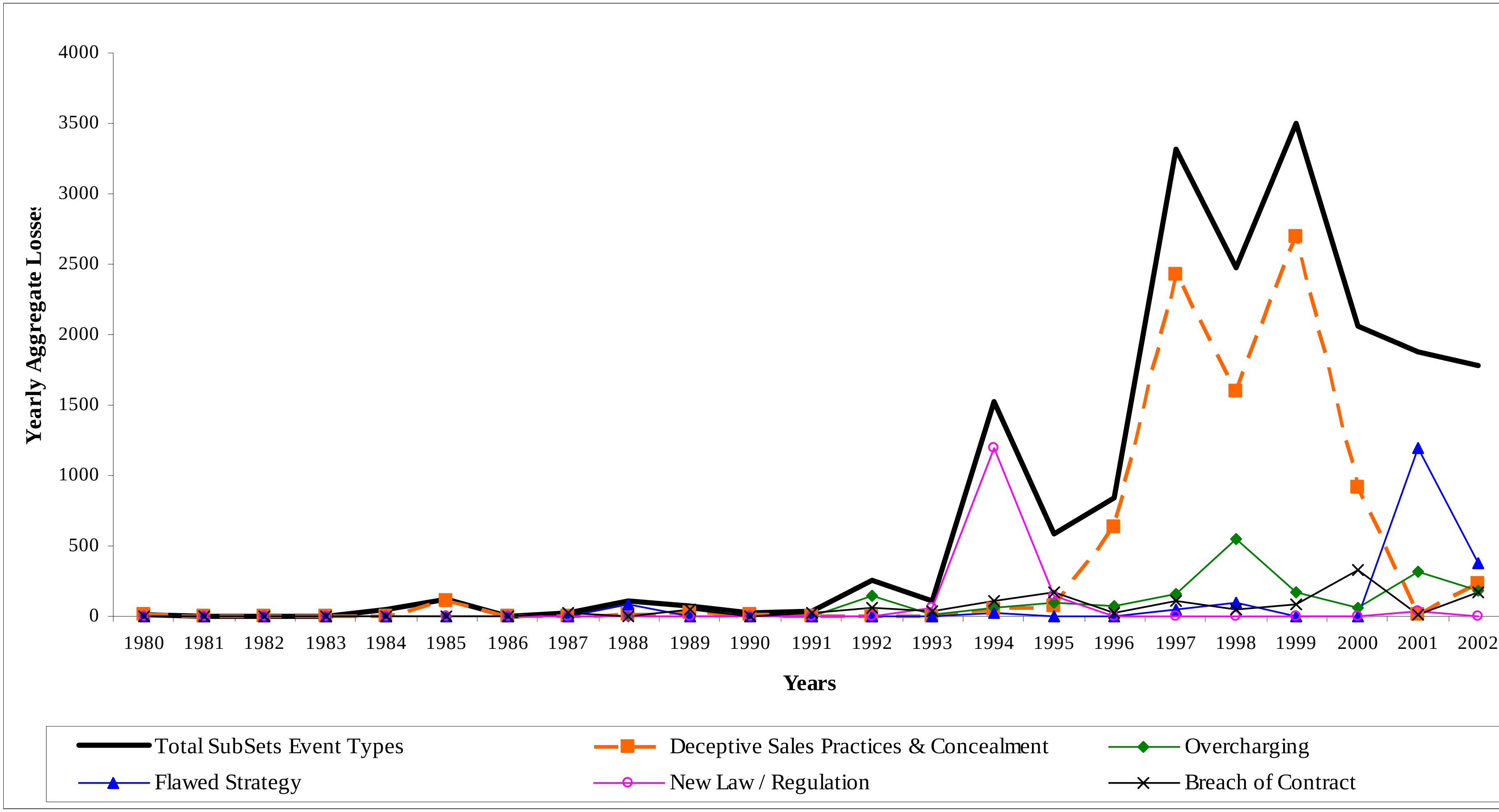


Figure 3.6 US Banks & Insurers - Yearly Aggregate Loss by Event Types & Settlement Year

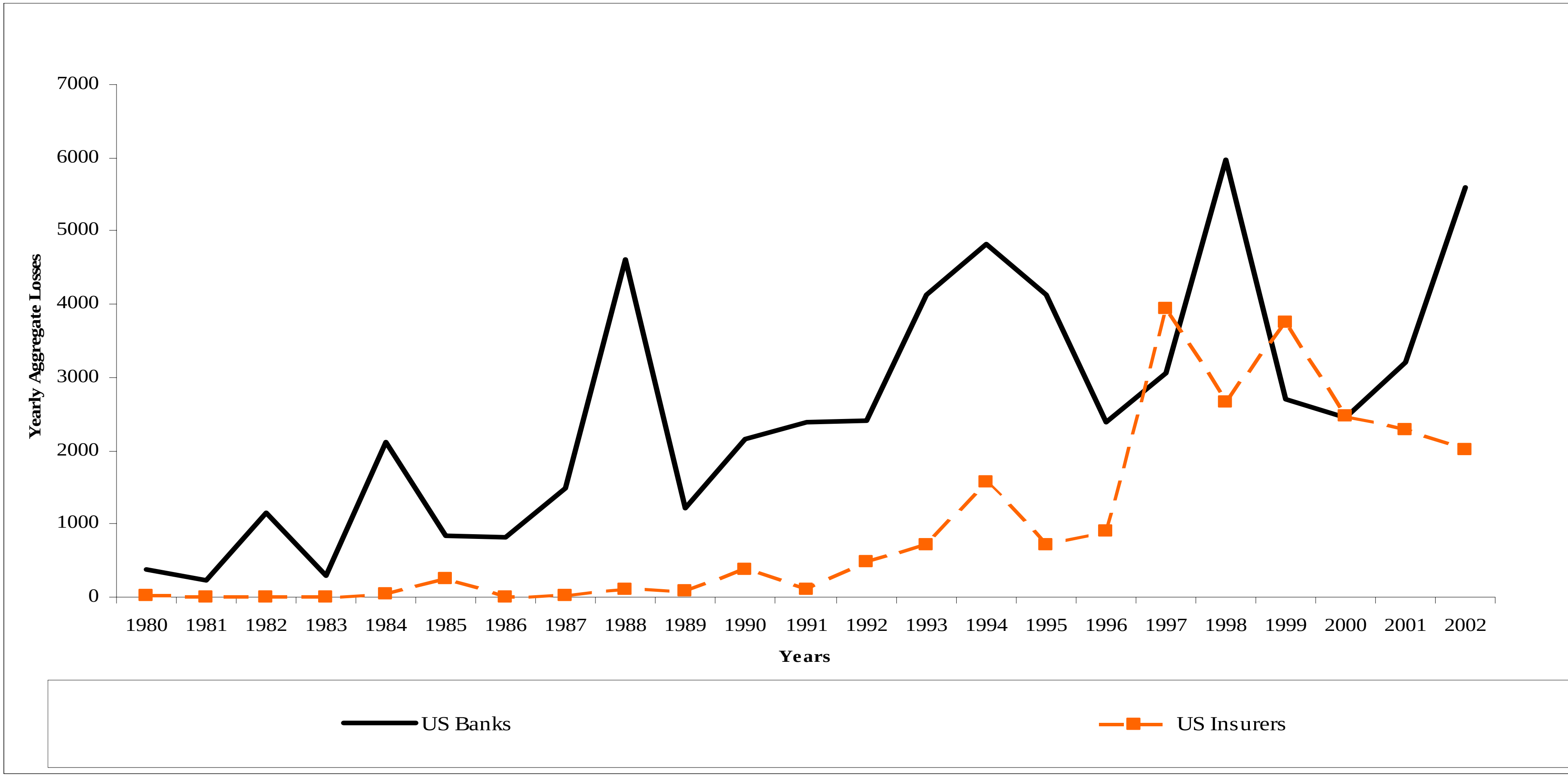


Figure 3.7 US Banks - Yearly Aggregate Loss Amounts & Occurrences by Settlement Year

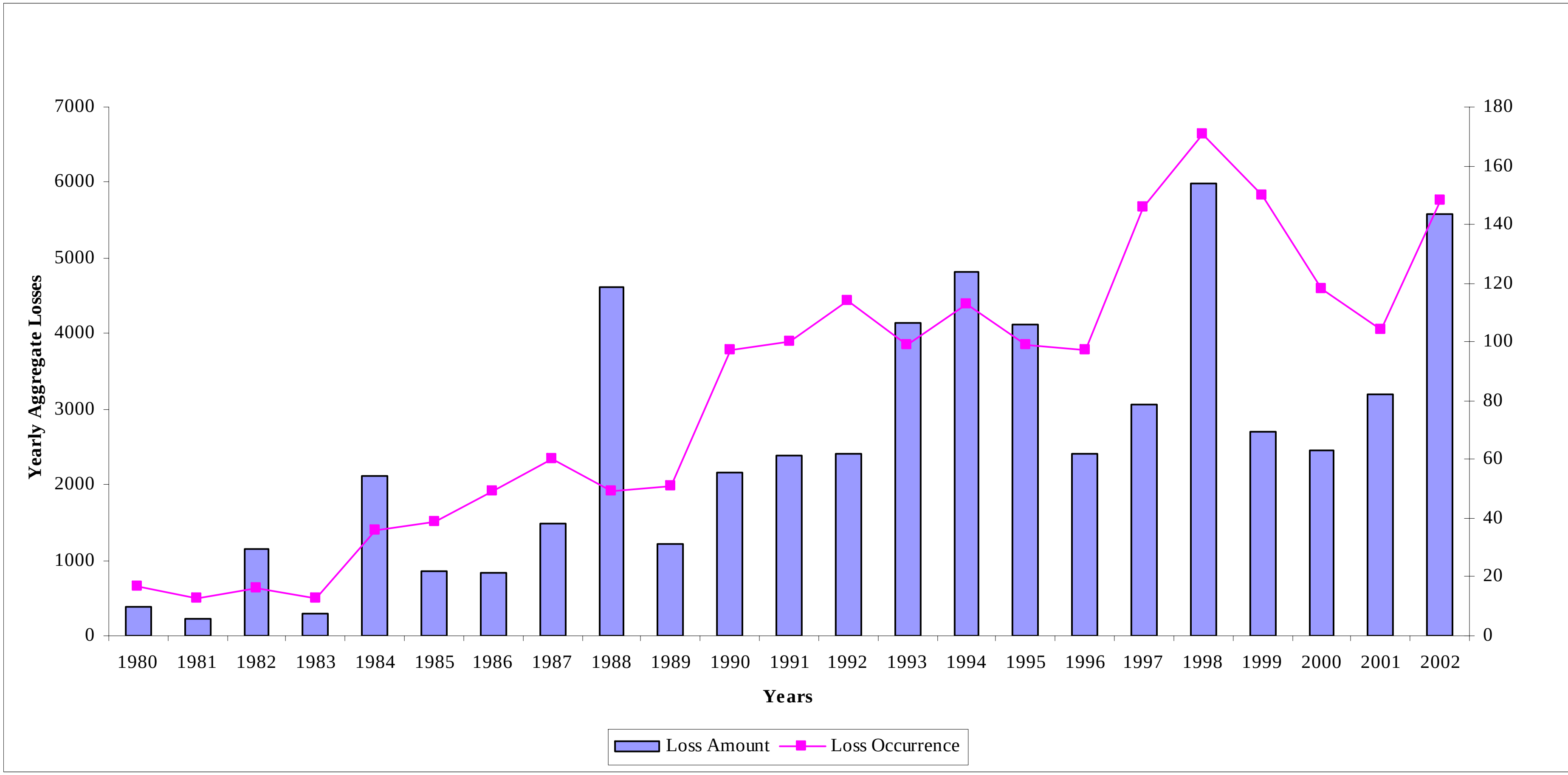
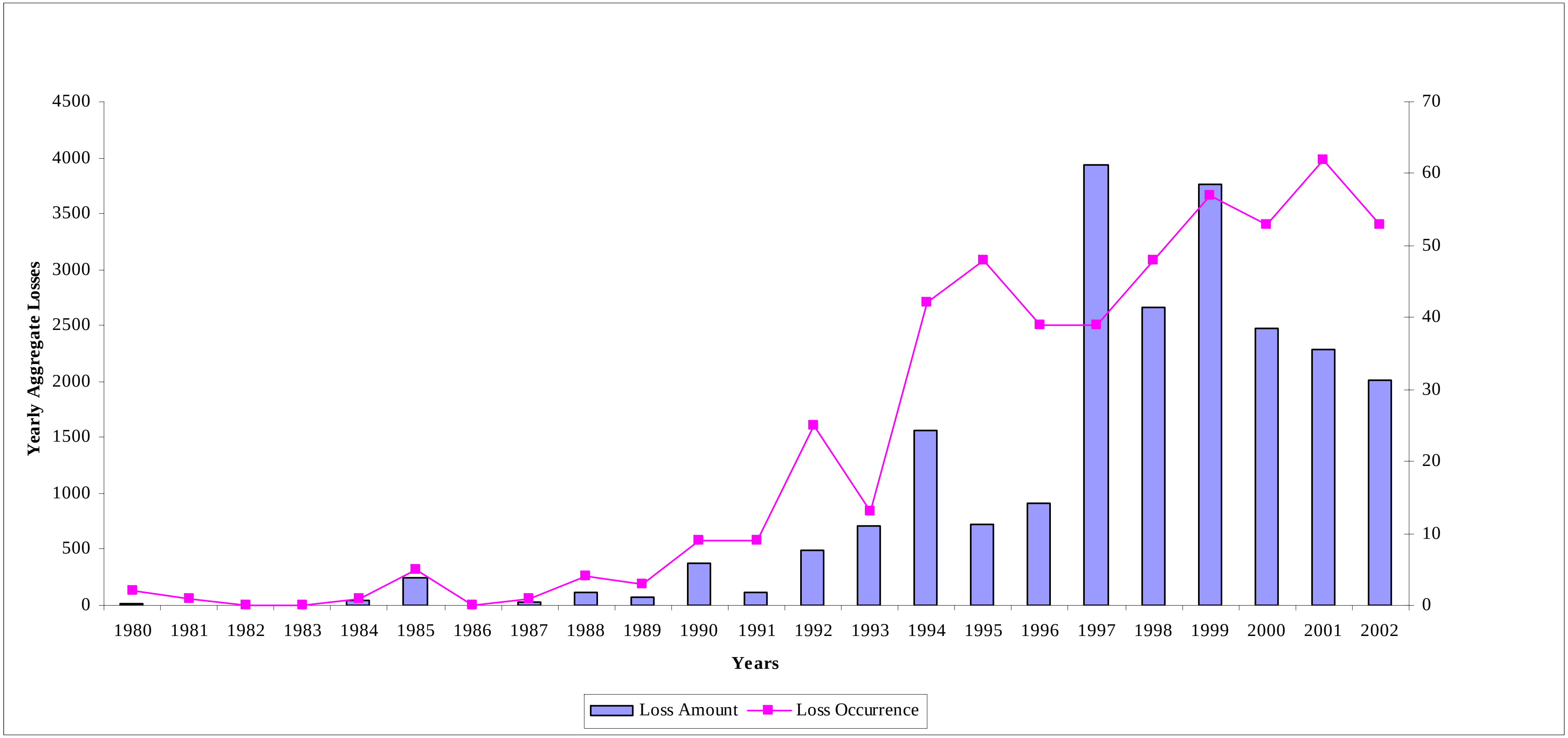


Figure 3.8 US Insurers - Yearly Aggregate Loss Amounts & Occurrences Settlement Year



4- Analysis of individual losses

Table 4.1 US Banks - Total Loss Amount by Business Lines & Event Types

Banking	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Corporate Finance	1,426	0	8	1,214	0	0	4	2,652
Trading & Sales	6,670	0	5	7,232	0	363	223	14,494
Retail Banking	3,623	1,830	292	13,409	22	3	990	20,169
Commercial Banking	3,605	2,843	327	3,491	213	128	42	10,649
Payment & Settlement	61	8	0	304	89	8	4	474
Agency Services	123	758	3	1,296	0	0	362	2,542
Asset management	2,046	204	111	3,249	0	0	532	6,143
Retail Brokerage	1,072	52	214	7,383	0	16	54	8,791
Total	18,626	5,695	961	37,579	324	519	2,212	65,915

Table 4.2 US Banks - Loss Occurrences by Business Lines & Event Types

Banking	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Corporate Finance	12	0	1	62	0	0	1	76
Trading & Sales	48	0	2	60	0	1	8	119
Retail Banking	272	191	20	271	3	1	51	809
Commercial Banking	74	127	14	101	3	2	8	329
Payment & Settlement	6	2	0	13	1	1	1	24
Agency Services	13	3	1	44	0	0	4	65
Asset management	40	9	3	82	0	0	5	139
Retail Brokerage	69	12	33	293	0	3	18	428
Total	534	344	74	926	7	8	96	1989

Table 4.3 US Insurers - Loss Occurrences by Business Lines & Event Types

	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Loss Amount	1,616	411	573	19,214	208	341	648	23,011
Loss Occurrences	74	21	20	344	1	1	68	529

Table 5.1 US Banks & Insurers – Industry-Wide Organization Severity Analysis by Business Units and Event Types
Impact of the Random Truncation Distribution on the Tail: non Random & Constant, Logistically Distributed and Normally Distributed.

Business Units	Event Types	Tail b			Scale sigma		Location mu		LogLikelihood			AIC			# Loss	Max
		Constant	Logistic	Normal	Logistic	Normal	Logistic	Normal	Constant	Logistic	Normal	Constant	Logistic	Normal		Loss
US Banks																
All Business Units	All Event Types	1.826	0.750	0.886	0.934	2.177	4.481	4.481	-3187	-3107	-3116	6376	6220	6235	1989	2243
		0.052	0.086	0.040	0.104	0.108	0.417	-								
	CPBP	2.031	0.848	0.935	0.890	1.835	3.807	3.807	-1582	-1518	-1522	3166	3042	3048	926	2243
		0.091	0.111	0.050	0.082	0.102	0.513	-								
	Internal Fraud - EPWS	1.648	0.778	0.908	1.054	2.429	4.581	4.581	-936	-919.74	-921.9	1873	1845	1848	608	1899
		0.083	0.185	0.080	0.257	0.266	1.040									
	Other Event Types	1.560	0.352	0.805	0.432	2.621	5.636	5.636	-657	-645.84	-650.11	1316	1298	1304	455	535.8
		0.085	0.194	0.118	0.287	0.446	0.357	-								
Investment Banking	All Event Types	2.550	1.199	1.232	0.887	1.611	3.105	3.105	-378	-356	-356	757	717	716	195	1899
		0.276	0.288	0.112	0.116	0.165	1.018	-								
	CPBP	2.535	1.041	1.099	0.763	1.464	3.204	3.204	-235	-217	-218	473	440	439	122	1825
		0.369	0.277	0.126	0.106	0.160	0.944	-								
Banking	All Event Types	1.755	0.665	0.838	0.841	2.176	4.682	4.682	-1917	-1869	-1875	3836	3743	3755	1227	2000
		0.062	0.104	0.052	0.140	0.144	0.444	-								
	CPBP	2.079	0.978	1.018	0.887	1.648	3.128	3.128	-743	-710	-710	1488	1425	1425	429	2000
		0.140	0.171	0.069	0.088	0.128	0.750	-								
Other Business Lines	All Event Types	1.733	0.674	0.840	0.848	2.146	4.455	4.455	-879	-857	-860	1759	1720	1724	567	2243
		0.091	0.136	0.075	0.175	0.210	0.666	-								
	CPBP	1.813	0.634	0.831	0.757	2.037	4.468	4.468	-598	-579	-582	1198	1164	1168	375	2243
		0.121	0.134	0.088	0.159	0.219	0.617	-								
US Insurers																
	All Event Types	2.184	0.479	0.896	0.535	2.237	5.471	5.471	-942	-900	-912	1887	1806	1828	529	2272
		0.129	0.084	0.091	0.098	0.200	0.294	-								
	CPBP	2.540	0.598	0.857	0.591	1.743	4.862	4.862	-665	-609	-616	1331	1225	1236	344	2272
		0.220	0.101	0.079	0.088	0.118	0.331	-								
	Other Event Types	1.522	0.127	1.064	0.138	4.639	6.293	6.293	-263	-259	-262	527	524	529	185	420

Investment Banking includes two business lines: Corporate Finance and Trading and Sales. Banking includes: Retail Banking, Commercial Banking, Payment & Settlement and Agency Services. Other business lines include Asset Management and Retail brokerage. EPWS: Employment Practices and Workplace Safety.

Table 5.2 US Banks & Insurers – Industry-Wide Organization Severity Analysis by Business Lines and Event Types
Impact of the Random Truncation Distribution on the Tail: non Random & Constant, Logistically Distributed and Normally Distributed.

Business Lines	Event Types	Tail b			Scale sigma		Location mu		LogLikelihood			AIC			# Loss	Max Loss
		Constant	Logistic	Normal	Logistic	Normal	Logistic	Normal	Constant	Logistic	Normal	Constant	Logistic	Normal		
Corporate Finance	All Event Types	2.106	0.930	0.961	0.644	1.191	2.507	2.507	-133	-122	-122	267	250	248	76	990
		0.378	0.249	0.13	0.125	0.166	0.857	-								
	CPBP	1.994	0.656	0.722	0.550	1.156	3.054	3.054	-105	-94	-94	212	194	192	62	299
		0.425	0.237	0.127	0.127	0.164	0.802	-								
Trading & Sales	All Event Types	2.834	0.982	1.084	0.914	1.930	4.813	4.813	-243	-227	-227	488	461	459	119	1899
		0.407	0.374	0.154	0.243	0.232	1.213	-								
	CPBP	3.092	1.040	1.149	0.760	1.580	4.048	4.048	-128	-114	-115	257	235	234	60	1825
		0.716	0.422	0.196	0.165	0.206	1.123	-								
Retail Banking	All Event Types	1.592	0.755	0.866	1.046	2.339	4.253	4.253	-1185	-1166	-1169	2372	2339	2342	809	2000
		0.066	0.149	0.066	0.210	0.230	0.910	-								
	CPBP	2.034	1.005	1.056	1.055	1.985	3.459	3.459	-463	-450	-450	929	906	905	271	2000
		0.162	0.264	0.099	0.165	0.234	1.388	-								
Commercial Banking	All Event Types	2.103	0.540	0.788	0.579	1.779	4.741	4.741	-574	-539	-544	1149	1084	1092	329	766
		0.167	0.144	0.086	0.150	0.158	0.419	-								
	CPBP	2.251	0.810	0.860	0.672	1.337	3.455	3.455	-183	-167	-166	368	339	337	101	415
		0.365	0.305	0.121	0.140	0.161	0.985	-								
Payment & Settlement	All Event Types	-	-	-	-	-	-	-							24	209
Agency Services	All Event Types	2.073	0.962	1.004	0.893	1.671	3.285	3.285	-112	-107	-107	227	221	219	65	536
		0.358	0.536	0.182	0.253	0.340	2.292	-								
Asset Management	All Event Types	2.248	0.782	0.918	0.829	1.911	4.490	4.490	-252	-239	-240	505	485	484	139	967
		0.274	0.349	0.138	0.310	0.258	1.220	-								
	CPBP	2.291	0.670	0.850	0.699	1.799	4.655	4.655	-150	-141	-142				82	440
		0.380	0.424	0.173	0.394	0.300	1.234	-								
Retail Brokerage	All Event Types	1.565	0.573	0.778	0.734	2.110	4.365	4.365	-620	-605	-608	1242	1217	1221	428	2243
		0.092	0.107	0.085	0.150	0.257	0.584	-								
	CPBP	1.679	0.590	0.805	0.723	2.063	4.369	4.369	-445	-432	-435	892	871	874	293	2243
		0.123	0.119	0.099	0.151	0.270	0.646	-								

Figure 5.1 - US Banks
Tail Parameter by Business Units & Random Truncation Point Distributional Assumption
Ind
ustry-wide organization

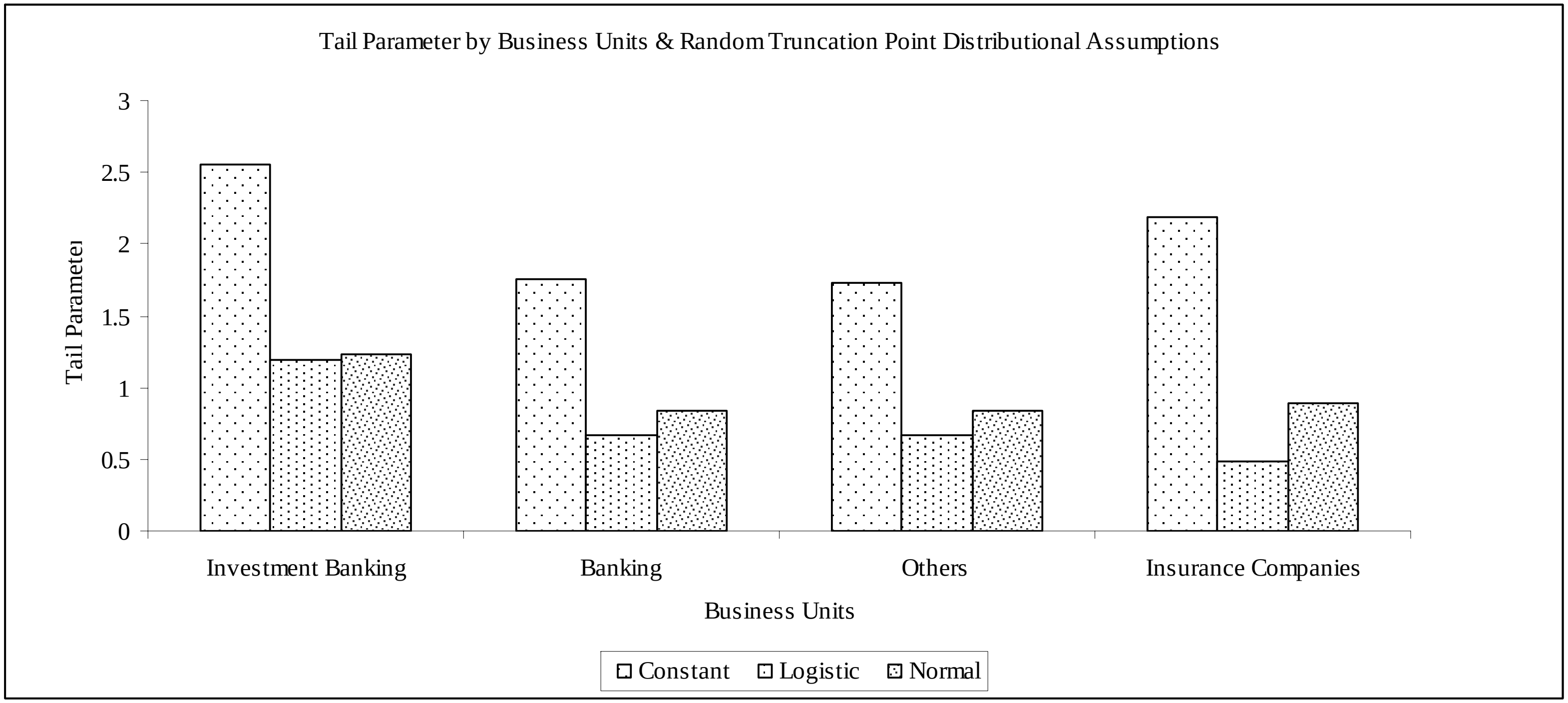


Figure 5.2 - US Banks
Tail Parameter by Business Lines & Random Truncation Point Distributional Assumption
Industry-wide organization

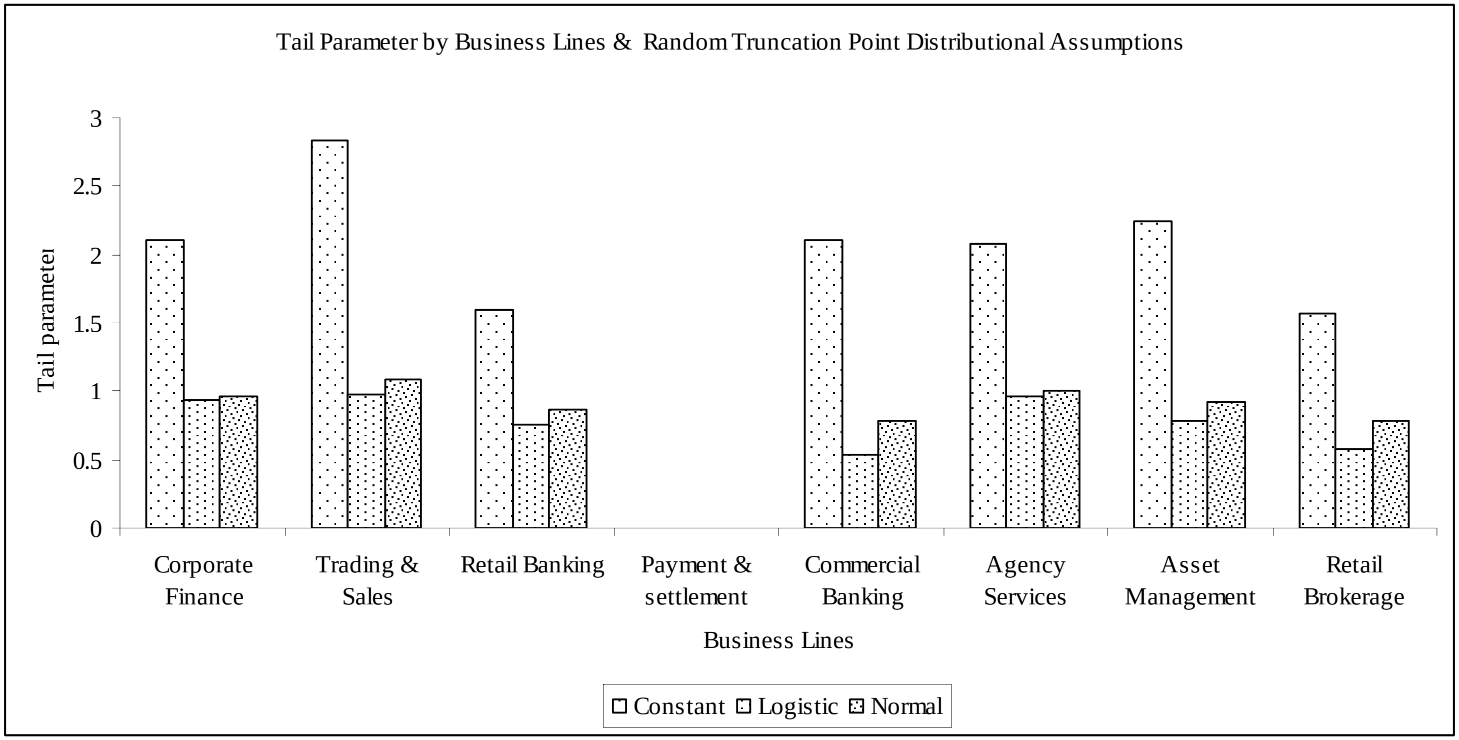
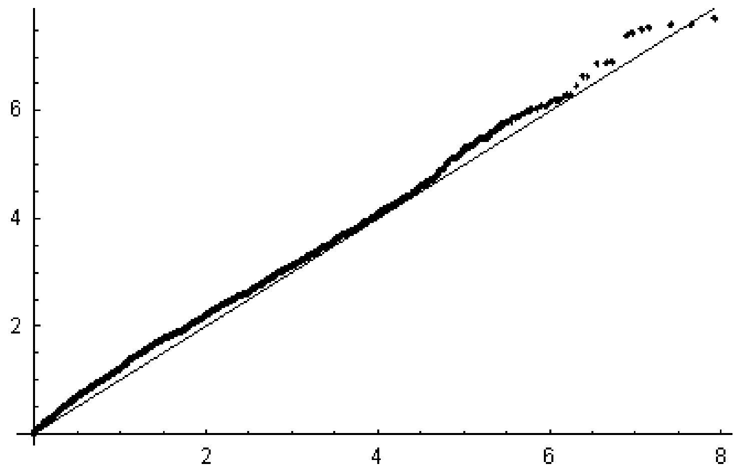


Figure 5.3 US Banks - Quantile-Quantile Plot All Business Lines All Event Types

Observed Quantile
Log[Loss]



Fitted Quantile, Log[Loss]

Figure 5.4 US Banks - Observed Severity Distribution and Underlying Severity Distribution. All Business Lines All Event Types.

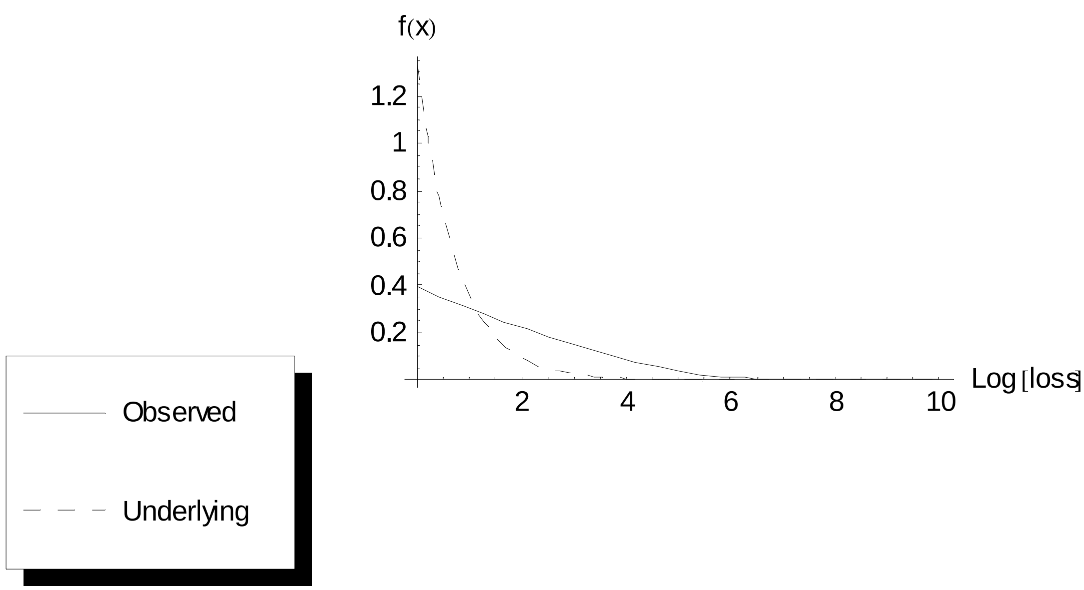
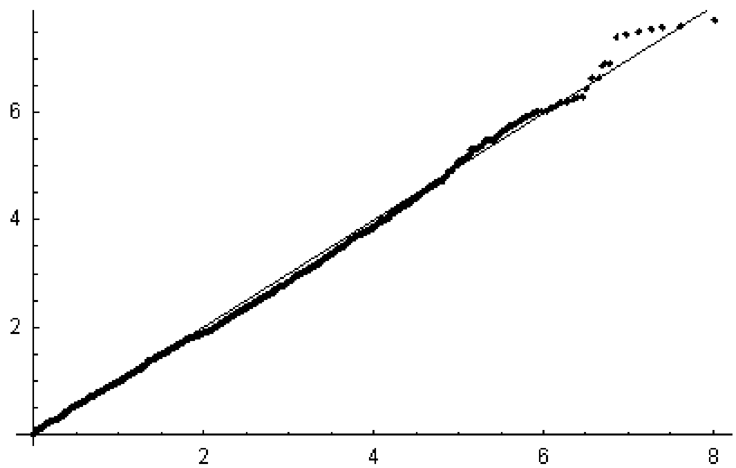


Figure 5.5 US Banks - QQ Plot CPBP

Observed Quantile
Log[Loss]



Fitted Quantile, Log[Loss]

Figure 5.6 US Banks - Observed Severity Distribution and Underlying Severity Distribution. CPBP.

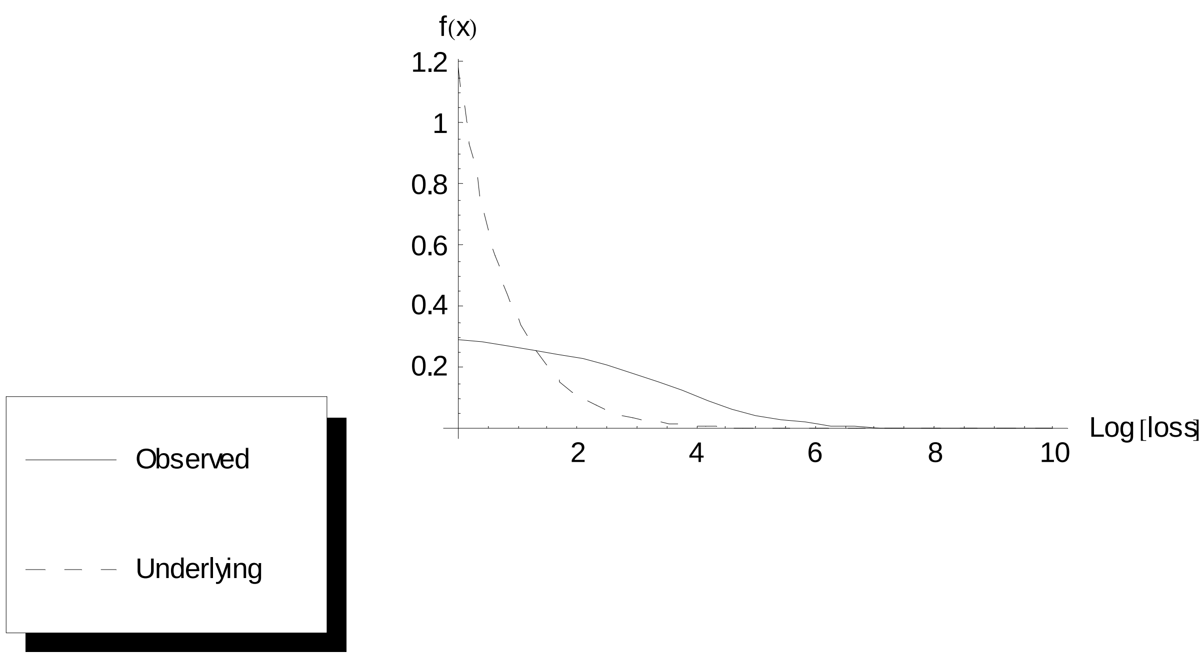
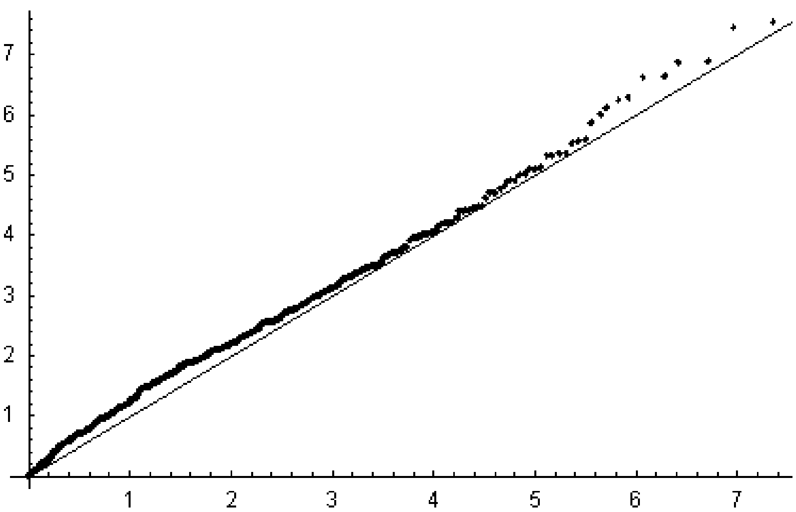


Figure 5.7 US Banks - QQ Plot Internal Fraud

Observed Quantile
Log[Loss]



Fitted Quantile, Log[Loss]

Figure 5.8 US Banks - Observed Severity Distribution and Underlying Severity Distribution. Internal Fraud - EPWS.

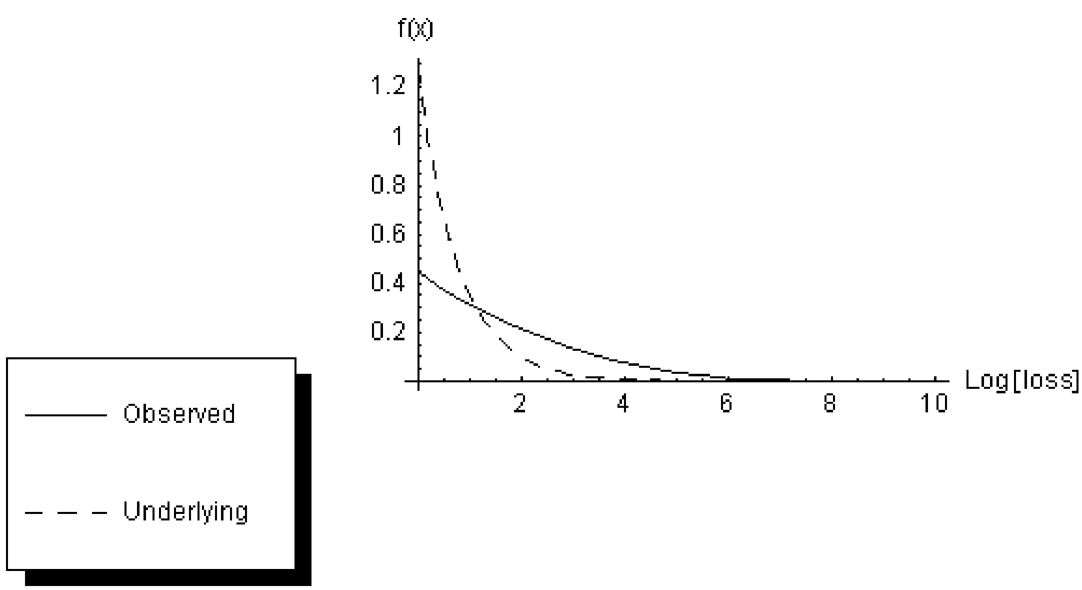


Figure 5.9 US Banks- Observed Severity Distribution and Underlying Severity Distribution. Retail Banking.

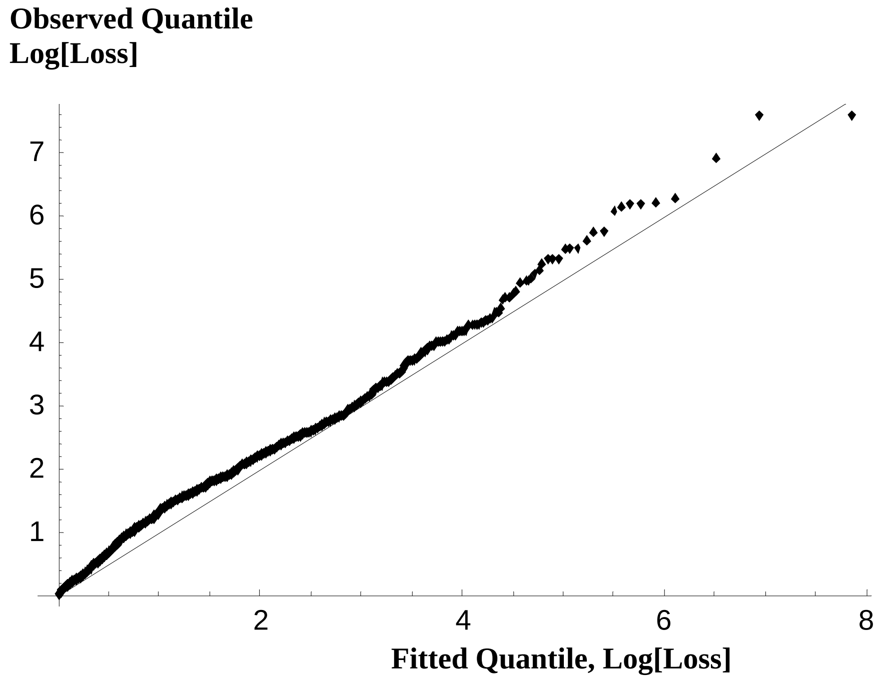


Figure 5.10 US Banks- Observed Severity Distribution and Underlying Severity Distribution Retail Banking.

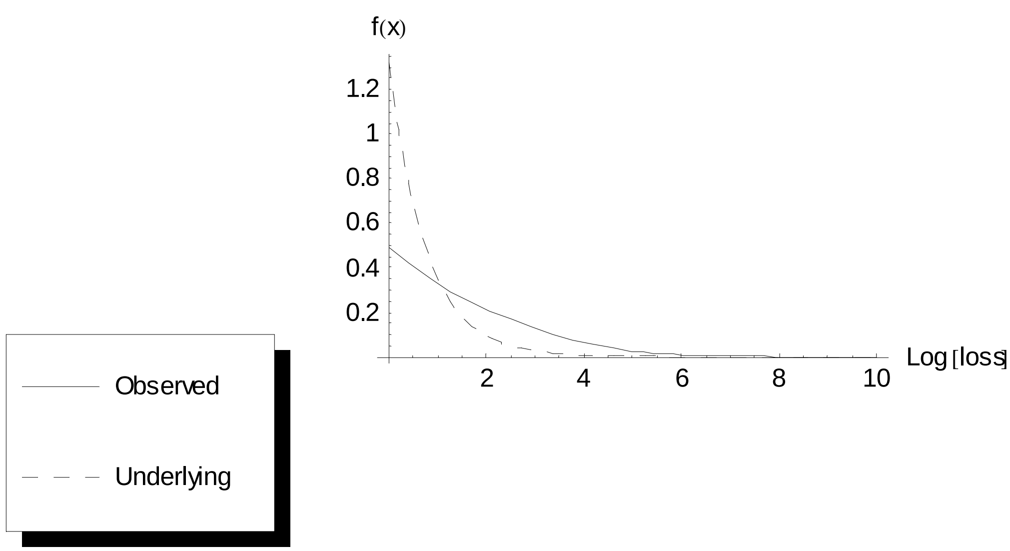


Figure 5.11 US Banks- Observed Severity Distribution and Underlying Severity Distribution. Retail Brokerage

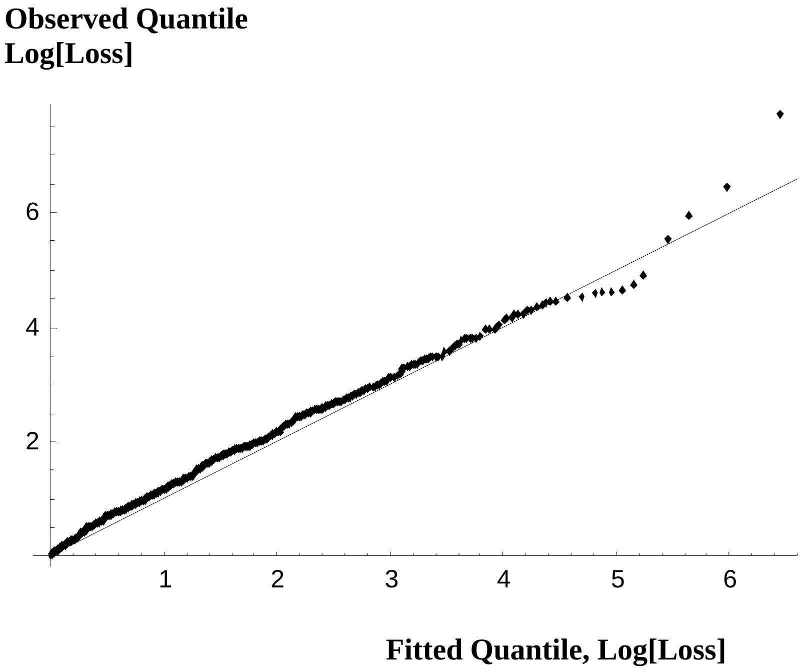


Figure 5.12 US Banks- Observed Severity Distribution and Underlying Severity Distribution Retail Brokerage.

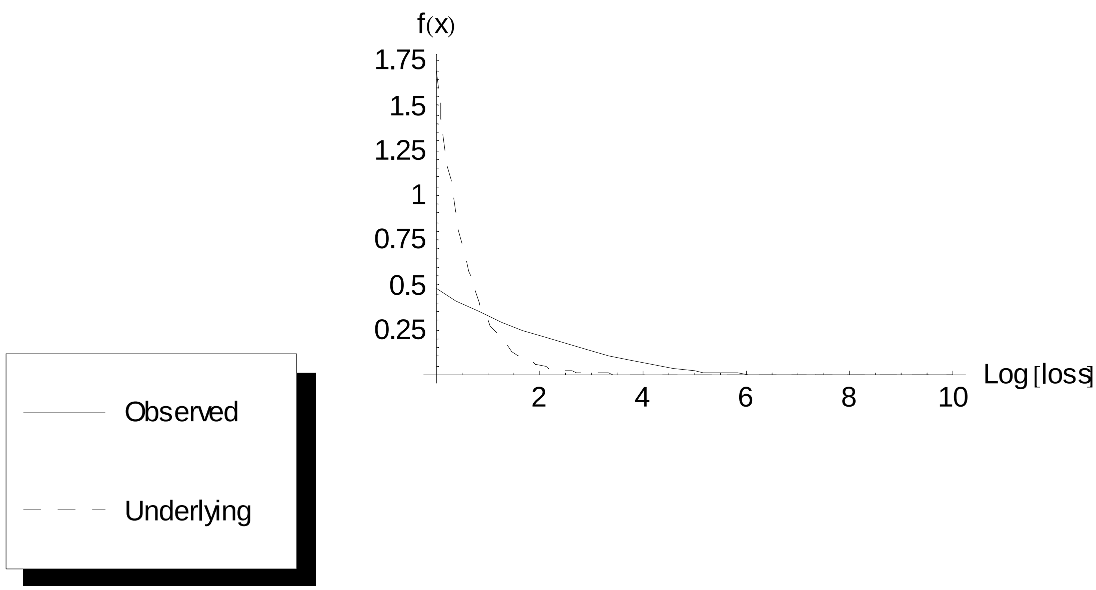


Table 5.3 – US Banks & Insurers: Severity by Size (1)
All Business Lines and Event Types

	Bellow Median Revenue				Above Median Revenue			
	Number of Losses	Severity Parameter	Maximum Loss($M)	99.95 Quantile of the Underlying Severity ($M)	Number of Losses	Severity Parameter	Maximum Loss ($M)	99.95 Quantile of the Underlying Severity ($M)
US Banks	445	0.759	2,243	320	446	0.6794	1,824	175
US Insurers	125	0.6108	2,272	104	125	0.512	1,852	49

Table 5.4 – US Banks & Insurers: Severity by Size (2)
All Business Lines and Event Types

	US Banks			US Insurers		
	Small Size	Medium Size	Large Size	Small Size	Medium Size	Large Size
Number of Losses	582	243	64	144	88	11
Severity Parameter	0.878	0.497	0.322	0.571	0.598	
Maximum Loss ($M)	2,243	631	363	2,272	1,852	198
99.99% Quantile of Underlying Severity ($M)	790	44	12	76	94	

Table 5.5 – Determination of the Severity of a Specific Firm

				Severity Distribution Parameter		
	PML ($M)	Median ($M)	Prob[Truncation Point <= Specific PML]	Tail	Location	Parameter
Industry-Wide Organization	2,243	88	0.931	0.750	4.481	0.934
Specific Firm	1,000	88	0.99	0.472	4.481	0.528

Figure 5.13
Industry –Wide vs Specific Firm Truncation Distribution

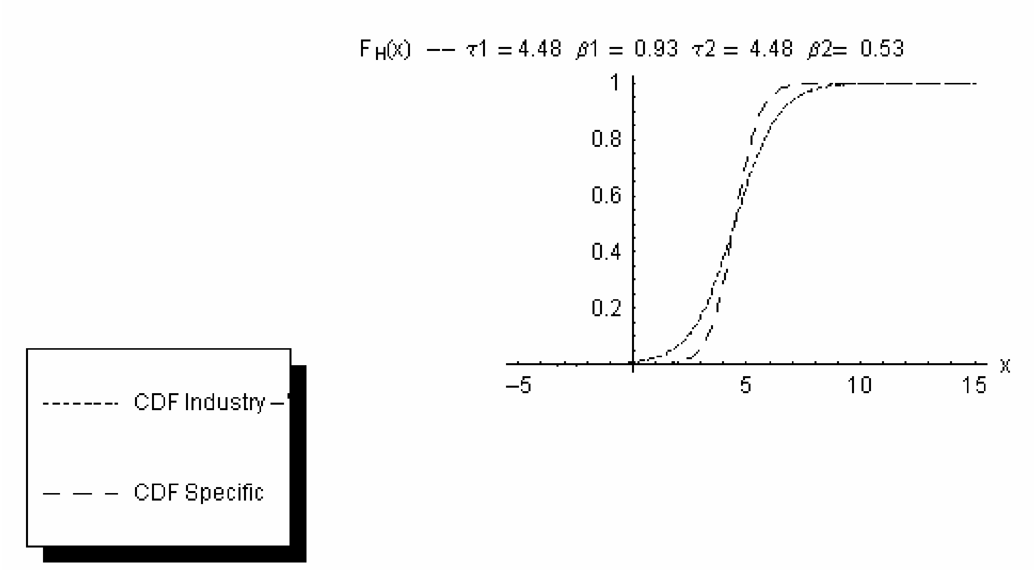


Figure 5.14
Industry –Wide vs Specific Firm Truncation Distribution

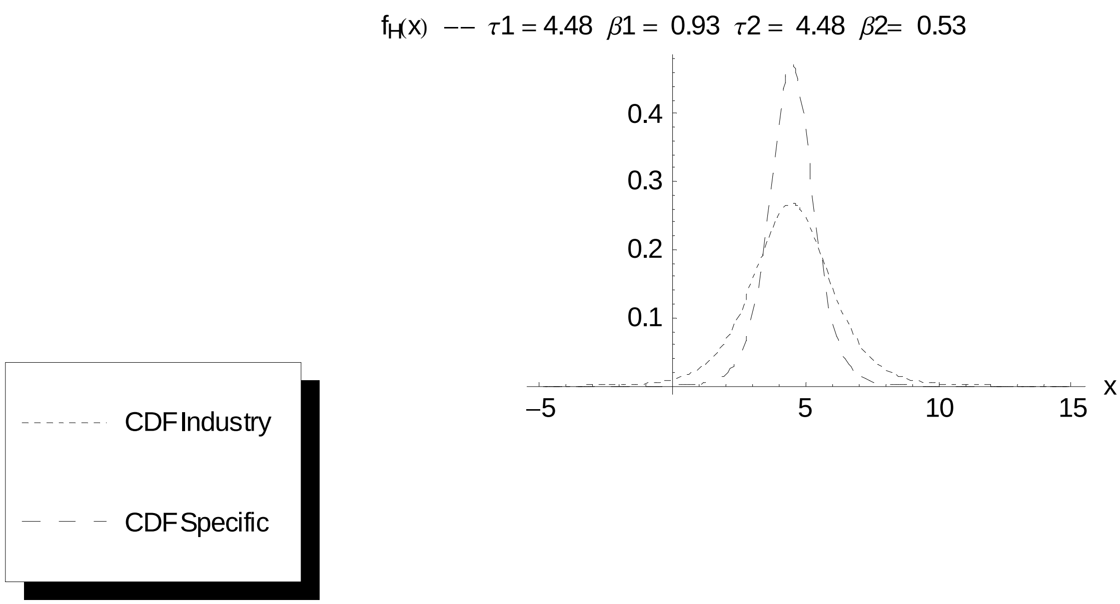


Table 5.6 US Banks and Insurers.
Industry-Wide Organization Loss Severity Parameter, Loss Occurrences and Aggregate Loss Sample Correlation

Class	Business Units	Event Types	Number of Observations	Amount ($M)	Severity Parameter	Weight	Sample Correlation			Adjusted Sample Correlation			# Yearly Claims
1	All Business Units	CPBP	926	37,579	0.848	0.466	1	0.319		1	0.100		50
	All Business Units	Other Event Types	1063	28,336	0.659	0.534	0.319	1		0.100	1		
2	All Business Units	CPBP	926	37,579	0.848	0.466	1	0.232	0.528	1	0.100	0.100	50
	All Business Units	Internal Fraud	608	19,587	0.778	0.306	0.232	1	0.664	0.100	1	0.100	
	All Business Units	Other Event Types	455	8,749	0.352	0.229	0.528	0.664	1	0.100	0.100	1	
3	Insurance	CPBP	344	19,214	0.598	0.650	1	0.307		1	0.100		50
	Insurance	Other Event Types	185	3,797	0.127	0.350	0.307	1		0.100	1		

Table 5.7 US Banks and Insurers
Capital Charge's Sensitivity to the Truncation Point Distributional Assumption
All Business Lines and All Event Types

	Constant		Logistic		Normal	
	Severity	VaR ($M)	Severity	VaR ($M)	Severity	VaR ($M)
US Banks	1.826	>100,000	0.750	2,089	0.886	8,220
US Insurers	2.184	>100,000	0.479	180	0.896	8,633

Assuming yearly number of loss occurrences exceeding $1M equal to 25

Table 5.8 Capital Charge - All Business Lines and All Event Types
($M)

	Severity	Yearly Number of Loss Occurrences in Excess of $1M				
	parameter	5	10	25	50	70
US Banks	0.750	599	1,041	2,106	3,562	4,596
US Insurers	0.479	70	104	179	278	350

Table 5.9 Capital Charge for Three Business Line and Event Type Combinations

	All Business Lines & Event Types		CPBP - Other Event Types		CPBP- Internal Fraud- Other Event Types	
	VaR ($M)	% Increase	VaR ($M)	% Increase	VaR ($M)	% Increase
US Banks	3,460	0	5,653	63	6,324	83
US Insurers	285	0	611	115		

Assuming yearly number of loss occurrences exceeding $1M equal to 50

Figure 5.15 CPBP Capital Charge Distribution
US Banks

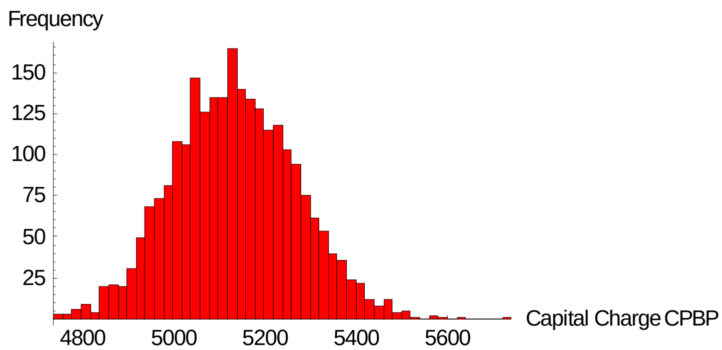


Figure 5.16 Internal Fraud Capital Charge Distribution
US Banks

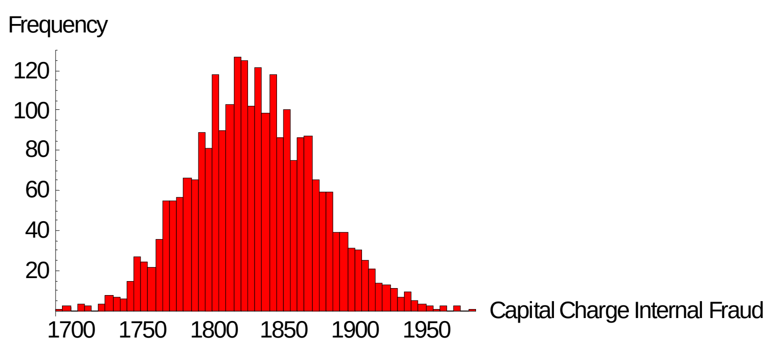


Figure 5.17 Other Event Types Capital Charge Distribution
US Banks

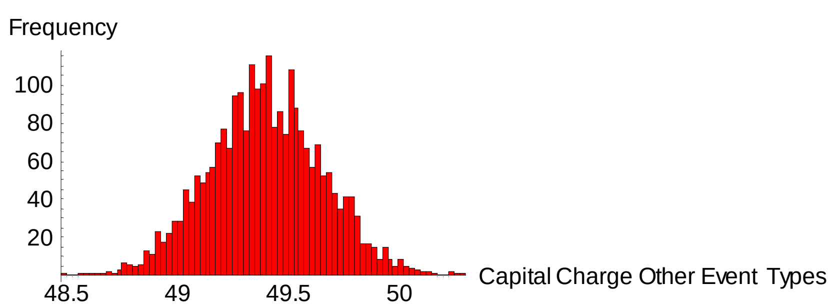


Figure 5.18 Aggregated Capital Charge Distribution Using Cauchy Copula
US Banks

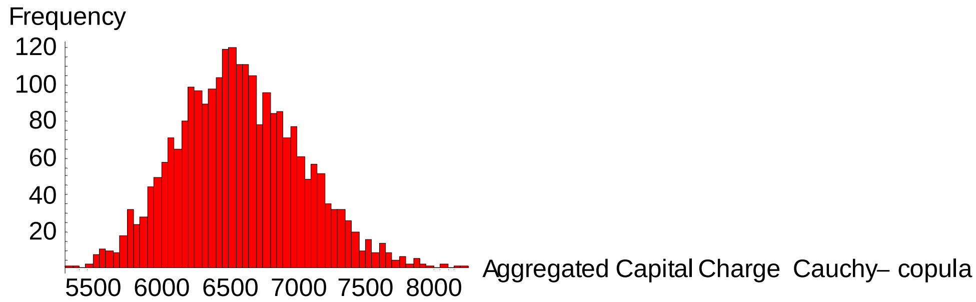


Figure 5.19
Aggregated Capital Charge for Student't -Copula
US Banks

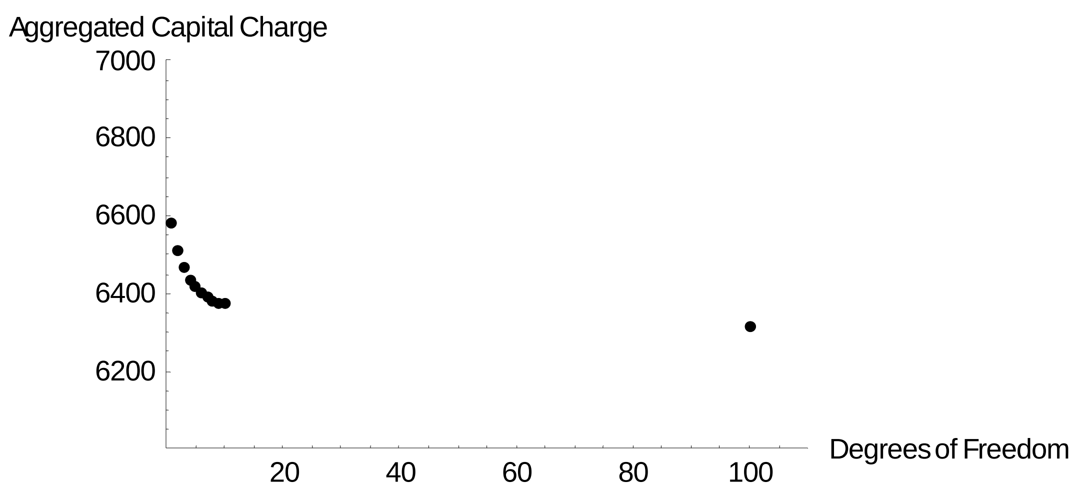


Table 5.10 Capital Charge and Capital Saving by Types of Copulas.
US Banks
($Million)

		Industry-Wide Bank			Specific Bank		
		VaR	Capital Saving	Percentage	VaR	Capital Saving	Percentage
	Comonotonic	7,015	0	0.00	481	0	0.00
Degrees of freedom	1 - Cauchy	6,582	433	6.58	448	33	7.26
	2	6,509	506	7.77	446	35	7.81
	3	6,468	547	8.45	444	36	8.17
	4	6,435	580	9.01	443	37	8.41
	5	6,416	599	9.33	442	38	8.68
	6	6,400	615	9.61	441	39	8.88
	7	6,391	624	9.77	441	40	9.00
	8	6,381	634	9.93	440	40	9.17
	9	6,374	641	10.06	440	41	9.26
	10	6,373	642	10.07	440	41	9.30
	Infinite- Normal	6,317	698	11.05	437	44	10.02
	Independent	6,290	724	11.52	433	47	10.92

Banks' activities are classified into three event types: CPBP – Internal Fraud & EPWS and Other Event Types.

Table 5.11 Capital Charge and Capital Saving by Types of Copulas.
US Insurers
($Million)

		Industry-Wide Insurer			Specific Insurer		
		VaR	Capital Saving	Percentage	VaR	Capital Saving	Percentage
	Comonotonic	625	0	0.00	125	0	0.00
Degrees of freedom	1 - Cauchy	612	14	2.237	119	6	4.91
	2	611	15	2.427	118	7	5.92
	3	611	15	2.424	117	8	6.84
	4	610	15	2.502	117	9	7.53
	5	611	15	2.412	116	9	8.04
	6	611	15	2.403	116	10	8.38
	7	611	14	2.370	115	10	8.65
	8	611	15	2.385	115	10	8.84
	9	610	15	2.444	115	10	9.01
	10	610	15	2.443	115	10	9.14
	Infinite- Normal	608	18	2.908	113	12	10.43
	Independent	606	19	3.159	112	13	11.78

Insurers' activities are classified into two event types: CPBP and Other Event Types

Table 5.12 US Industry-wide Bank.
Descriptive statistics of the Capital Charge
($million)

		Mean	St-Dev	Median	95-Perc	99-Perc	Min Confidence Interval	Max Confidence Interval	Skewness	Kurtosis Excess
Degrees of freedom	**1 - Cauchy**	6,582	455	6,560	7,353	7,717	6,552	6,612	0.260	-0.074
	2	6,509	456	6,485	7,284	7,650	6,479	6,539	0.294	0.040
	3	6,468	457	6,436	7,262	7,610	6,438	6,498	0.304	0.091
	4	6,435	454	6,404	7,205	7,584	6,405	6,465	0.304	0.129
	5	6,416	456	6,392	7,204	7,572	6,386	6,446	0.318	0.179
	6	6,400	451	6,376	7,160	7,575	6,370	6,430	0.300	0.225
	7	6,391	451	6,371	7,149	7,576	6,361	6,420	0.309	0.177
	8	6,381	448	6,358	7,142	7,528	6,351	6,410	0.325	0.328
	9	6,374	448	6,354	7,140	7,536	6,344	6,403	0.296	0.153
	10	6,373	453	6,349	7,142	7,516	6,343	6,403	0.336	0.242
	Infinite- Normal	6,317	436	6,298	7,083	7,384	6,288	6,345	0.293	0.217
	Independent	6,290	436	6,279	7,037	7,342	6,262	6,319	0.274	0.163

Table 5.13 Mixing Weights by types of copulas.

	Comonotonic	4%
Degrees of freedom	1 - Cauchy	8%
	2	8%
	3	8%
	4	8%
	5	8%
	6	8%
	7	8%
	8	8%
	9	8%
	10	8%
	Infinite- Normal	8%
	Independent	8%

Table 5.14 US Industry-wide Bank & Specific Bank Descriptive statistics of the mixing weighted Capital Charges ($million)

	Industry-Wide Bank	Specific Bank
Minimum	5,892	410
Maximum	7,007	481
Mean	6,433	443
Median	6,431	443
1% Quantile	6,071	422
5% Quantile	6,234	431
95% Quantile	6,639	455
99.9% Quantile	6,830	467
Confidence Interval 1	6,433	443
Confidence Interval 2	6,434	443
Skewness	0.1030	0.0786
Kurtiosis Excess	0.0037	0.0029

Figure 5-20 US Industry-wide Bank
Base Scenario
Histogram of the mixing weighted Capital Charges

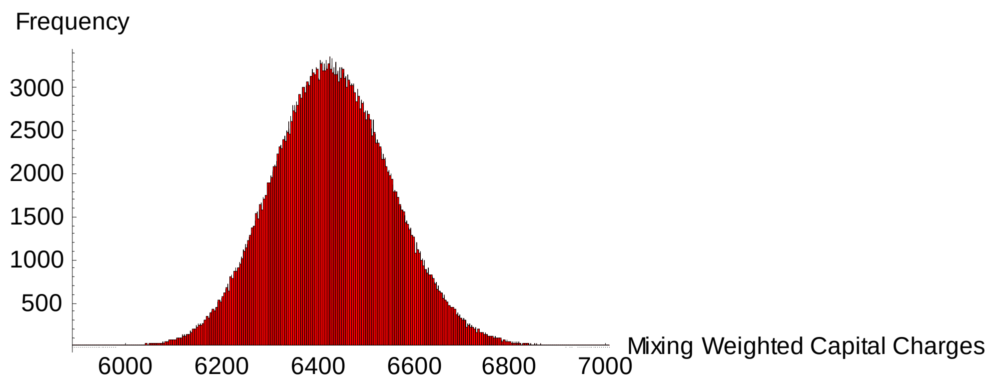
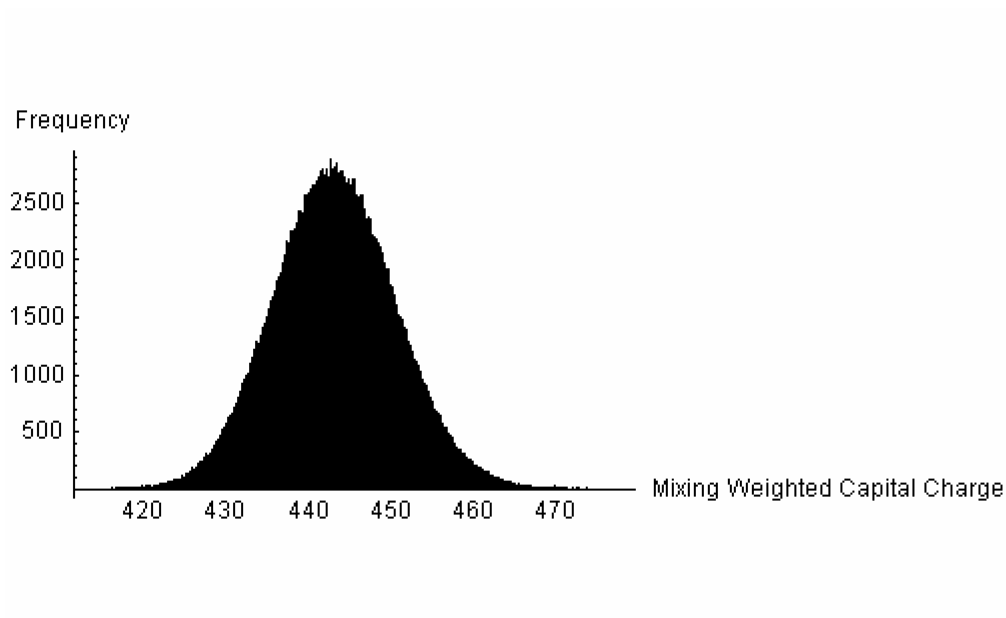


Figure 5-21 US Specific Bank
Base Scenario
Histogram of the mixing weighted Capital Charges



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