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REQUIRED SURPLUS FOR THE INSURANCE RISK FOR CERTAIN LINES OF GROUP INSURANCE

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ABSTRACT

This paper studies the minimum surplus required to protect a group insurer from the insurance, or C2, risk. The sensitivity of this surplus figure to the security load, the retention limit, the distribution of amounts of insurance, and the general aging of the population is investigated. Detailed calculations are shown for group life insurance, accidental death and dismemberment insurance, and long term disability income insurance. Other types of group coverages also are discussed.

I. INTRODUCTION

The determination of the minimum surplus required by an insurer to provide a reasonable degree of protection from the risks it faces in the ordinary conduct of its business has received a great deal of attention recently ([7], [11], [15], [17]). The Society's Committee on Valuation and Related Problems has divided required surplus conceptually into three parts, C1, C2, and C3, which provide protection, respectively, against the asset value risk, the insurance risk, and the interest rate risk [17]. However, the question of how to determine the magnitudes of C1, C2, and C3 is largely unanswered. This paper deals with the determination of C2, the provision for the insurance risk, for several lines of group insurance.

The surplus required for a line of business depends on a great many characteristics including the distribution of claim amounts, the distribution of the lives insured by age and sex, the security loading in the premium, the insurer's retention limit for the line, and the method of experience rating employed. This paper investigates the sensitivity of required surplus to these and other characteristics. A sensitivity analysis of this type is useful not only as an aid to understanding the relative importance of various characteristics in determining required surplus, but also as a guide in the management of the portfolio. Insofar as required surplus is a measure of the risk inherent in a portfolio, an understanding of this sensitivity can be helpful in pricing decisions, in managing the growth and changing composition of the portfolio, and in associated risk management questions, such as reinsurance strategies.

Group insurance has grown tremendously over the past forty years. Since most group benefits are salary-related or otherwise sensitive to inflationary pressures in the economy, continued rapid growth in volume of business is to be expected even without extending coverage to a wider population or introducing any new products. As an insurer's book of business grows, its exposure to risk increases, thereby creating a need for additional surplus. If a line of business is to be self-sustaining, it must generate the additional surplus by gains from operations. However, the extremely competitive nature of the group insurance market often leads to reduced margins in the premiums and, all too often, to inadequate premium income. The gain from operations necessary to sustain growth is very difficult to achieve. Faced with diminishing capacity to write new business, a group insurer may have to take steps to limit new business or, as a strategic move, to shift large blocks of business to an administrative services only, a minimum premium, or some other noninsured basis. Clearly, there are intricate relationships among growth, profit goals, and surplus requirements, which must be understood. Some of these relationships are explored in this paper. The reader also should consult the excellent paper by Pike [11].

The primary tool used in this paper to study surplus requirements is ruin theory. The technical considerations involved in this approach are discussed in section III. However, there are a number of conceptual issues related to the use of ruin theory that must be aired first; these are taken up in section II.

Surplus requirements are considered in detail in sections IV, V, and VI for each of three group coverages: life, accidental death and dismemberment, and long term disability. The extension of these methods and results to other coverages is discussed in section VII. The final section contains a number of observations on general factors that influence group insurers' surplus requirements.

Most of the underlying work for this paper was done at Mutual Life of Canada while I was on leave from the University of Waterloo. I wish to express my gratitude to Mutual Life for the invitation to undertake the surplus project, and to Kurt von Schilling, Jim Chapman and Robert Williams for their counsel, good fellowship, patience, and instruction in the intricacies of group insurance.

II. THE RUIN-THEORETIC MODEL

Consider an insurance enterprise free of expenses, taxes, and commissions. Premiums received in any one year are equal to expected claims plus a relatively small load for risk. As they are received, premiums are added to a fund that initially is composed entirely of surplus. All claims are paid from this fund, which earns no interest. The enterprise will be considered ruined as soon as aggregate claims exceed the sum of original surplus and aggregate premiums received. Now, reminiscent of introductory calculus classes, let us pick a number ϵ between zero and one; in practice, ϵ will be close to, but greater than zero, for example, $\epsilon = 0.0001$. A basic goal of ruin theory is to find the smallest value for initial surplus, $U(\epsilon)$, so that the probability that the enterprise ever will be ruined is no more than ϵ. This is the basic calculation device used in this paper.

This setting is only a model of the pure risk situation. Many factors important to an actual group insurance enterprise are not recognized. Clearly, the values for required surplus produced by the model must be adjusted to reflect these factors. Nonetheless, the model is extremely useful, particularly for sensitivity analysis of the effects of changes in the insurer's portfolio on surplus requirements.

The model is based on one-year term insurance; reserves are ignored. It may be adapted to a portfolio containing permanent insurance by considering the amount insured to be the net amount at risk. Since most group insurance is term insurance, this aspect of the model is not really restrictive. The major exception is disability income insurance, where disabled-life reserves play an important role quite different from that of reserves for permanent life insurance. Disabled-life reserves are discussed further in section VI, which deals with surplus requirements for long term disability insurance.

A basic assumption in applying the model is that the premium received represents the true expected claims plus a known security load. In real life, unfortunately, we cannot always be sure we know the true expected claims. Inflationary effects, prospective experience rating, and the difficulty of conducting up-to-date and statistically reliable experience studies all contribute elements of uncertainty to the expected claims numbers. As will be seen in the next section, it is possible to make provision in the model for random changes in incidence-of-claim rates. However, systematic changes in these rates must be handled by explicit adjustment of the surplus figures produced by the model.

The use of retrospective experience rating raises another question, particular to group insurance, about the applicability of the model. Suppose the true, expected claim amount for a particular group is known and, with the security loading, forms the premium for the group. If, strictly due to random effects, the group has favorable experience and has no outstanding deficit, some portion of the correct premium charged will be returned to the policyholder through the experience rating mechanism. The model, however, assumes that cash outflow is limited to claims; it does not allow for the experience refund. On the other hand, if random effects produce higher claims than expected, the fund will suffer the same loss on this group as it would if there were no experience rating. Further, the insurer would expect to recover the deficit before paying future experience refunds.

In fact, it is precisely because we use expected claims as the premium that we expect the surpluses and deficits for an insured group to balance. Thus, retrospective experience rating causes practice to depart from the model in two ways. First, a group may terminate while in a deficit position, leaving the insurer no opportunity to recover the experience loss. The potential loss from this source can be recovered by establishing a deficit termination risk charge as described in [12]. Second, some portion of the dividends paid to the policyholder is due to random favorable experience; the model assumes that these funds remain with the insurer. The balance of dividends paid represents a retrospective adjustment to the true level of expected claims; payment of these funds is entirely consistent with the model. It is necessary, then, to estimate the amount of dividends paid each year that is attributable to random favorable experience; this estimate depends on both the variability of claims for the coverage in question and the particular experience rating method being used. This quantity must be incorporated into the annual surplus growth target in order to maintain surplus at its proper level. Of course, this contribution to surplus may be at least partially offset by the year's increase in claim fluctuation reserves.

In the initial description of the model, it was noted that interest and expenses are ignored. The importance of these items can be expected to vary widely from company to company. The expense risk is far more important to a company that has a large block of administrative services only (ASO) or minimum-premium business than it is to a company that writes mostly fully insured business. In any case, the magnitude of the risk depends upon the cost allocation methods and the expense formula built into the premium. Allocation of interest income among lines of business also varies among companies. While the model does not assume that interest is earned on and credited to surplus, it does require annual increases in surplus to accommodate business growth. Clearly, interest on surplus is an important source of additional surplus. Therefore, we assume that interest on surplus is included in the gain from operations and eventually flows back into surplus.[1]

At first glance, it would appear that most companies would be concerned with avoiding ruin over the next twenty to fifty years. The present study, however, is based on the probability of ruin at any time in the future. Surprisingly, the infinitely long time span proves to be easier to work with mathematically than a finite time interval. Moreover, the use of an infinite span provides a measure of conservatism in the calculation of surplus requirements, allows for business growth, and also generates quite reasonable surplus figures. A modern discussion of methods used with a finite time span can be found in [13].

The ruin-theoretic approach to determining surplus requires the actuary to pick an acceptable probability of ruin, ϵ. The theory offers no guidance as to how to choose ϵ. Clearly, ϵ should be very small when considering ruin over a relatively short time interval (one to ten years) and somewhat larger for longer intervals. For the infinitely long interval, I prefer to use ϵ of the order of 0.0001. The choice of ϵ is parallel to the selection of the worst possible set of circumstances a company might be expected to experience and survive under a scenario approach. Thus, the fact that ϵ must be chosen by the user is not a reason to prefer a deterministic scenario approach to a stochastic ruin-theoretic approach.

Many actuaries prefer to relate surplus to a measure of liabilities or business in force; for group insurance, this measure is usually premium income. From a theoretical point of view, no such relationship is obvious or necessary. Moreover, in a dynamic environment, surplus requirements often will increase slower than the liabilities. Although the calculations described in this paper produce dollar figures for required surplus, the results are presented as percentages of expected claims. This protects the confidentiality of the data and better displays certain relationships in the results. None of the percentages listed herein should be adopted as a surplus/premium target without detailed investigation of the particular business. Once determined, a required surplus figure may be related to premium in order to track business growth and set profit goals over the short and medium terms. Nonetheless, it is important to remember that a required-surplus/premium ratio will change over time as the distribution of business changes; it will be necessary periodically to recalculate required surplus *ab initio* (from the beginning).

[1] An exception to this assumption is found in section VI in the discussion of Company B's surplus requirements.

III. TECHNICALITIES

Suppose the ideal insurance enterprise described in the beginning of the previous section had surplus U. A classic result of ruin theory[2] shows that $\psi(U)$, the probability that the enterprise ever will be ruined, has the form

$$\psi(U) = C(U)e^{-RU} \tag{1}$$

In general, the function $C(U)$ is not known explicitly. However, it is known to be bounded above by one and to be very close to that upper limit. (See pages 142-44 of [1].) For the portfolios on which the present study is based, preliminary calculations consistently yielded estimates for $C(U)$ between 0.9 and 1.0. In this paper we shall assume $C(U)$ is identically 1.0. This is conservative in that the resulting surplus figures are 2 to 3 percent higher than those obtained by using these preliminary estimates for $C(U)$.

The surplus $U = U(\epsilon)$ necessary to keep the probability of ruin less than ϵ, the maximum probability of ruin we are willing to accept, may be found by setting $\psi(U) = \epsilon$ in equation (1) and solving to yield

$$U(\epsilon) = -\frac{1}{R} \ln \epsilon \tag{2}$$

where ln is the natural logarithm to base e. It may be seen that if surplus is doubled, the ruin probability is squared; relatively small increments in surplus bring about significant improvements in security in the ideal situation.

The value of surplus $U(\epsilon)$, depends on the adjustment coefficient, R. This coefficient carries all the known information about the insurer's portfolio. Its calculation is clearly the most crucial step in determining required surplus. Therefore several results from the theory of risk concerning the determination of R are reviewed here.

If x is any positive integer, $p(x)$ will denote the probability that a claim is for amount x, given that it has occurred. The function $p(x)$ is the probability density function of the so-called secondary or claim amount distribution. The associated cumulative distribution is $P(x)$ where

$$P(x) = \sum_{t=1}^{x} p(t) \tag{3}$$

Now, if n is also a positive integer and $f(n)$ is the probability, there will

[2]The reader should consult any of the standard works on risk theory such as [1], [3], [8], [12], [18].

be exactly n claims in a single year (the probability density function of the number-of-claims distribution). Then the probability that in a single year *total* claims will aggregate to no more than x is given by

$$F(x) = \sum_{n=0}^{\infty} f_n P^{n*}(x) \tag{4}$$

where $F^{n*}(x)$ is the n-fold convolution of $P(x)$.

The conventional assumption is that the number-of-claims distribution is a Poisson distribution. Therefore

$$f(n) = e^{-k} \cdot \frac{k^n}{n!} \tag{5}$$

where k is the expected number of claims in a single year. It can be shown that the adjustment coefficient R is the nonzero root of the equation

$$1 + (1+\lambda)mR - \sum_x e^{Rx}p(x) = 0 \tag{6}$$

where λ is the security loading factor contained in the premium and m is the average claim amount. In practice, R is positive but very small, and equation (6) can be solved for R using the Newton-Raphson method with a very small number of iterations.

Implicit in the choice of the Poisson distribution for the number of claims are the assumptions that individual claims are mutually independent and that the correct claim rates are known. Both of these assumptions are open to question, particularly in the case of group insurance. Since most insured groups present the risks of common accident or epidemic, the independence assumption for claims should be replaced with a contagion assumption. The simplest nontrivial contagion assumption is linear contagion (see [3], pages 52-53). This gives rise to a negative binomial number-of-claims distribution

$$f(n) = \binom{n+a-1}{n} \left(\frac{a}{a+k}\right)^a \left(\frac{k}{a+k}\right)^n \tag{7}$$

where a is a parameter whose inverse, a^{-1}, is a measure of the intensity of contagion.

As mentioned previously, it is difficult to be certain that the table of incidence-of-claim rates being used is exact. This uncertainty can be handled by assuming a Poisson distribution based on a rate-of-claim table developed by multiplying the table in use by a random factor r, with mean one, and averaging over all possible values of r. Thus, if $K(r)$ is the cumulative distribution function for r, the number-of-claims density is given by

$$f(n) = \int_0^\infty e^{-rk} \frac{(rk)^n}{n!} \, dK(r) \tag{8}$$

If $K(r)$ is a Gamma distribution with parameter a, f turns out to be the negative binomial density given by equation (7). The inverse of the parameter a is a measure of the dispersion or variability of r. As a tends to infinity, the distribution of r tends to the distribution concentrated at one, and the number-of-claims distribution tends to the Poisson.

The negative binomial distribution thus arises in two ways as the number-of-claims distribution. The actuary taking this approach must choose a value for the parameter a. The choice is complicated because the degree of contagion and the degree of uncertainty in the basic claim rates both are modeled by a. I am unaware of any systematic methods for choosing a. The most elementary approach would be to carry out a few trial calculations of distributions and required surplus using the Poisson and the negative binomial distributions for several values of a. Comparison of the results should enable one to pick an appropriate a that allows for a sufficient margin to cover contagion and uncertainty in the rates. Most results reported in this paper are based on the value $a = 10{,}000$.

If we assume the number-of-claims distribution is negative binomial with parameter a in equation (7), and k is the expected number of claims per year, then the adjustment coefficient R needed to calculate required surplus from equation (2) is the nonzero root of the equation

$$1 + \frac{a}{k}\{1 - \exp[-(1+\lambda)mkR/a]\} - \sum_x e^{Rx}p(x) = 0 \tag{9}$$

where λ and m are the security loading and the mean claim amount, respectively.

Equation (6) depends only on the distribution of the amount of a single claim; the number of certificates or policies in force and the expected number of claims do not enter. Thus, if an insurer were to double its portfolio without disturbing the distribution of amounts of insurance, its surplus requirements based on the classic, compound Poisson model would not change. This apparent anomaly can be explained by the use of operational time. Define an operational time period to be a period in which a fixed number of claims, say 100, is expected. Thus, a doubling of the number of certificates in force would lead to a doubling of the number of operational time units in a calendar year. Since we are concerned with the behavior of the portfolio over an infinitely long (operational) time span, this doubling of time has no effect. While this can be understood on a mathematical level, it is disconcerting. One expects surplus to be related to company size through the variable k, the expected number of claims in one year.

The approach to the ruin problem outlined above does not require calculation of the distribution of aggregate claims in a single year. Some approaches to the ruin problem do require knowledge of this distribution. Historically, these methods have not received much attention because of the great difficulty in calculating the distribution of aggregate claims. However, since recent advances ([8], [16]) permit this distribution to be calculated fairly easily, these methods can be expected to come into greater prominence in the future. One such approach is discussed for group life insurance.

IV. GROUP LIFE INSURANCE

The calculation of the risk reserve $U(\epsilon)$ is a simple matter once the distribution $P(x)$ of the amount x of a single claim is known. The construction of this distribution is described and the results of the calculations are presented in this section. The sensitivity of the results to the probability of ruin, ϵ, the security loading, λ, the expected number of claims, k, the retention limit, and the relative number of certificates for large amounts are discussed in detail. The effects of growth of business, inflation, and the general aging of the insured population also are considered. Finally, an alternate approach to the ruin problem based on a one-year time horizon is described and compared with our basic method.

A. *The Distribution of Amount of Claim*

Ideally, the distribution should be derived from all in force certificates. They are grouped into cells by amount at risk, and the forces of mortality for all certificates in each cell are summed. The probability that a claim will be in amount range x is the ratio of the sum of the forces of mortality for the cell representing amount range x to the sum of the forces of mortality for all certificates in the portfolio. There are two simplifying assumptions in order. First, the amounts of insurance may be rounded to the nearest thousand dollars. This assumption reduces the number of amount cells significantly, while causing minimal distortion. Second, it is assumed that group life is yearly renewable term business with monthly premiums so that the net amount at risk may be taken to be the face amount of insurance, and policy reserves may be ignored. In fact, the portfolio upon which this study is based contains a relatively small number of paid-up certificates on retired lives, and they are generally for small amounts of insurance. The reserves for these certificates also were ignored.

This procedure is not always practical. Many insurers' group life portfolios contain hundreds of thousands or even millions of certificates or include large self-administered groups for which the basic data is not available at the insurer's head office. The portfolio under study contains self-administered groups, necessitating a different procedure.

Claim data for 1974–78 were collected and separated by calendar year of incurral. Waiver of premium claims were treated as death claims for the initial reserve established when the claim was admitted. Five empirical distributions, one for each year, were constructed based on relative frequencies of claims for various amounts. Since there were at least 1,680 claims per year and more than 2,000 in some years, it was felt that the data were sufficient to provide dependable distributions. To test this, the first two moments of each distribution were calculated. The mean claim amount increased between 1974 and 1978 at an average annual rate of 12.15 percent, while the standard deviation of claim amount increased over the same period at an average annual rate of 17.58 percent.

Over this same period the Industrial Weekly Earnings as computed by Statistics Canada increased at an average annual rate of 10.48 percent. If the amounts of insurance for the majority of certificates are related to employee earnings, the means and standard deviations of claim amounts should follow, at least approximately, the Industrial Weekly Earnings. If most amounts of insurance do rise with earnings, the standard deviation can be expected to increase more rapidly than the mean since the amounts on paid-up certificates and those coming from contracts providing a flat schedule of benefits will not change, so that the difference between these amounts and the mean claim amount will increase more rapidly than the mean claim amount. In the portfolio under study, the data exhibited an increase in mean claim amount and standard deviation of claim amount in excess of wage inflation from 1974 and 1976. This is attributable in large measure to a general increase in employee benefits for females as a result of equal pay legislation which came into effect during this period. It was felt these distributions exhibited fairly stable trends in relation to inflation and provided a proper base for the study.

The number and size of individual claims for large amounts are particularly important for a study of the variation in aggregate claims arising from an insurance portfolio. (See Section 3.5.3 of [1] for a discussion of this point.) There were no claims greater than $175,000 in this study. However, the portfolio contains a significant number of certificates for amounts in excess of this figure. Moreover, the derivation of information relating surplus requirements to the relative number of large-amount certificates and to the retention limit for the line was a particular aim of the study. Hence, shadow claims were added to the 1978 claims in proportion to the expected number of claims for large amounts from the portfolio, grouped in cells $25,000 wide. The resulting relative frequency distribution of claim amounts became the basis for the study.

B. *Required Surplus*

The basic numerical results for the study are found in Tables 1–5. Most of the calculations are based on the compound Poisson model. The effects of switching to the negative binomial model are discussed following presentation of the basic results. Since $U(\epsilon) = -(1R)\ ln\ \epsilon$, it suffices to display values for $U(0.001)$; the values of $U(\epsilon)$ for arbitrary ϵ can be obtained by multiplying these by $ln\ \epsilon / ln\ 0.001$.

Values of $U(0.001)$ as a function of the security load are shown in Table 1. This function is approximately of the form $f(\lambda) = d/\lambda$ for some constant d; in fact, our function is slightly less steep than this f. As noted above, $U(\epsilon)$ varies as $1/R$. On the other hand, since

$$\sum_x e^{Rx}\, p(x)$$

TABLE 1

$U(0.001)$ AS A PERCENT OF EXPECTED CLAIMS
COMPOUND POISSON DISTRIBUTION

Security Load λ	R (000,000)	$U(0.001)$
0.01	0.610	55.43
0.02	1.197	28.16
0.03	1.761	19.09
0.04	2.302	14.57
0.05	2.821	11.84
0.06	3.318	10.04
0.07	3.795	8.74
0.08	4.251	7.77
0.09	4.688	7.03
0.10	5.106	6.43

is the moment generating function of the distribution of claim amount, equation (6) reduces to

$$\lambda\mu_1 = \sum_{i=2}^{\infty} \frac{u_i}{i!} R^{i-1} \tag{10}$$

where μ_i is the ith moment about the origin of this distribution. Therefore, $\mu_1 = m$. Now, as can be seen from Table 1, R is less that 10^{-5}. Thus, the series on the right-hand side of (10) converges rapidly, and its behavior is dominated by that of the first term. Since all coefficients are positive, it follows that if λ is multiplied by a constant factor $c > 1$, R will increase approximately by a factor c but, in fact, by slightly less than c, due to the presence of the relatively small but positive higher order terms.

The basic distribution assumes a retention limit of $500,000 per certificate. To test the sensitivity of required surplus to the retention limit, two auxiliary distributions were constructed. If the retention limit is $L <$ \$500,000, the probability for all amounts greater than or equal to L in the original distribution is aggregated at L to form a new truncated amount-of-claim distribution. Results are shown in Table 2. In the present case, surplus requirements are not very sensitive to large changes in the retention limit if the limit is fairly high. Of course, the number of certificates for amounts near any of the limits considered is relatively very small. The proportion of certificates for large amounts certainly will have an effect on the sensitivity of required surplus to the retention limit, so these results are not necessarily indicative of those for other insurers' portfolios.

Increasing numbers of requests for large amounts of insurance emphasize the importance of considering the sensitivity of required surplus to the proportion of large-amount certificates in the portfolio. The results of one set of such calculations are shown in Table 3.

The shadow claims for amounts greater than $200,000 were increased

TABLE 2

$U(0.001)$ AS A PERCENT OF EXPECTED CLAIMS
COMPOUND POISSON DISTRIBUTION

Retention Limit	Security Load λ				
	0.01	0.02	0.03	0.04	0.05
$300,000	54.03	27.44	18.55	14.12	11.46
400,000	54.89	27.90	18.89	14.39	11.70
500,000	55.48	28.16	19.09	14.57	11.84

TABLE 3

$U(0.001)$ AS A PERCENT OF EXPECTED CLAIMS
COMPOUND POISSON DISTRIBUTION

Security Load λ	Retention Limit					
	$300,000			$500,000		
	Amount-of-Claims Distribution*					
	A	B	C	A	B	C
0.01	54.03	71.86	91.62	55.48	77.89	102.61
0.02	27.44	36.56	46.59	28.16	39.75	52.31
0.03	18.55	24.79	31.58	19.09	27.04	35.54
0.04	14.12	18.92	24.08	14.57	20.69	27.17
0.05	11.46	15.39	19.57	11.84	16.88	22.13
0.06	....	13.03	16.56	10.04	14.34	18.78
0.07	....	11.35	14.42	8.74	12.54	16.39
0.08	....	10.08	12.81	7.77	11.18	14.60
0.09	....	9.10	11.56	7.03	10.12	13.19
0.10	....	8.33	10.54	6.43	9.28	12.06

* A: standard distribution (cf: Table 2)
B: number of claims over $200,000 multiplied by five
C: number of claims over $200,000 multiplied by ten

five-fold and ten-fold, respectively, to construct new amount-of-claim distributions. As can be seen, required surplus is very sensitive to the proportion of certificates for large amounts. Table 3 also shows that the retention limit becomes more important as the number of certificates for amounts near or in excess of that limit increases. It should be noted that the amount-of-claim distribution was altered by adjusting the probabilities (but not necessarily the numbers) of claims for large amounts. Thus, an insurer's risk is increased either if it adds a disproportionate number of large-amount certificates to its portfolio, or if the group of lives insured for large amounts ages relative to all lives covered by the portfolio.

As discussed previously, uncertainty or random variation in the mortality rates and contagion can be accommodated by using a negative binomial number-of-claims process and calculating the adjustment coefficient R from equation (9). This depends upon a coefficient a whose inverse is a measure of the degree of uncertainty. The effects of using various values of a in the calculation of $U(0.001)$ are shown in Table 4. All entries in this table are based on a security loading $\lambda = 0.04$. It can be seen that the negative binomial figures are almost the same as the Poisson values for large values of a. On the other hand, the margins in the surplus figures associated with values of a below 1,000 probably are excessive. Repeated calculations suggest values for a between 1,000 and 10,000 are appropriate for life insurance. Table 5 compares the Poisson-based values of $U(0.001)$ with those derived

TABLE 4

U (0.001) AS A PERCENT OF EXPECTED CLAIMS
SECURITY LOADING λ = .04
COMPOUND NEGATIVE BINOMIAL DISTRIBUTION

Parameter a	U(0.001)
∞	14.57
1,000,000	14.57
100,000	14.66
10,000	15.64
1,000	25.71
500	37.08
200	71.43
100	128.84

TABLE 5

U(0.001) AS A PERCENT OF EXPECTED CLAIMS
COMPOUND NEGATIVE BINOMIAL DISTRIBUTION

Security Load λ	Parameter a		Margin	
	∞	10,000	Absolute	Relative
0.01	55.43	59.83	4.40	7.94%
0.02	28.16	30.38	2.22	7.76
0.03	19.09	20.57	1.48	7.75
0.04	14.57	15.64	1.07	7.34
0.05	11.84	12.70	0.86	7.26
0.06	10.04	10.75	0.71	7.07
0.07	8.74	9.34	0.60	6.86

from the negative binomial process using a = 10,000. The margin built into surplus by the use of the negative binomial process decreases both absolutely and as a percent of $U(\epsilon)$ as the security loading increases. Calculations similar to those shown in Tables 2 and 3 using the negative binomial process demonstrate that the relative margins introduced into U (ϵ) decrease as risk, due to other causes, increases; that is, as uncertainty with respect to mortality levels becomes a relatively less important source of risk.

The discussion has been based on the assumption that the insured portfolio is static over time. For most group life portfolios this assumption clearly is not valid. New lives are being insured and amounts on previously issued certificates are being updated all the time. Therefore, it is necessary to introduce dynamic elements into the discussion. We shall identify change in the portfolio with growth in business and attempt to study the changes in surplus required by a few types of systematic growth. The assumption is that if change occurs in the portfolio, it either leaves the amount-of-claim distribution unchanged or changes this distribution in a simple way. Such growth will increase required surplus and the increase normally would be funded by retained earnings. Accordingly, the factors considered in determining the need for increases in surplus also will affect profit goals.

The growth-related phenomenon under discussion should be distinguished from the surplus strain that arises from new business due to the skewed incidence of expenses and commissions over time. A growing insurer has need for surplus to cover this new-business strain over and above the surplus needs discussed here.

Most group life business is on employee groups with the benefit level related to earnings. Over time then, amounts of insurance can be expected to follow trends in the general wage level. Therefore, it is appropriate to consider the effect on required surplus of an inflation of all amounts of insurance by a factor j generally expressed in the form $1+i$. It can easily be seen that if $U = U(\epsilon)$ is the required surplus for a portfolio before the inflation, then $U' = U'(\epsilon)$, the required surplus after inflation, is given by $U' = jU$; that is, required surplus is inflated by the same factor j. Suppose $p(x)$ is the probability, before inflation, that a claim will be for an amount x while $p'(y)$ is the probability that a postinflationary claim is for an amount y. Then, if $y = jx$, we have $p'(y) = p(x)$ while $m' = jm$ where m and m' are the pre and postinflationary mean claim amounts. In the Poisson case, the adjustment coefficient S of the new primed distribution is determined from equation (6), which now appears as

$$1 + (1+\lambda)m'S - \sum_y e^{Sy} p'(y) = 0$$

This is the same as

$$1 + (1+\lambda)mjS - \sum_x e^{jSx} p(x) = 0$$

which is just equation (6) for the preinflationary distribution with jS in place of R. Since the adjustment coefficient is unique, $jS = R$. It follows that $U' = (-1/S)\ ln\ \epsilon = jU$. In the negative binomial case, the same proof holds using equation (9) in place of equation (6). So, as long as provision is made to increase surplus systematically as amounts at risk increase, the probability of ruin can be contained. On the other hand, G.C. Taylor [14] has shown that if inflation occurs at a constant rate and surplus is not systematically augmented at that rate, then ruin is certain, irrespective of the size of that rate, the magnitude of the initial surplus, and the security loading in the premium.

Growth in a group life insurance portfolio also can come about by the addition of new certificates, which will increase the expected number of claims. As discussed in section III, the Poisson-based approach is insensitive to the size of a portfolio as measured by the number of certificates in force or the expected number of claims. However, the expected number of claims, k, enters directly into equation (9) which is based on the negative binomial, number-of-claims process. The calculations in Tables 4 and 5 used a value for k of 2,700. In Table 6, the degree of dependence of the results on k is indicated; calculations are presented for k values of 3,000 and 3,300, representing increases of 11 and 22 percent, respectively. Required surplus is moderately sensitive to k if the negative binomial parameter a is in an acceptable range, say, a is at least 1,000. For example, if a is 1,000 and the security loading λ is 4 percent, a 22 percent increase in the number of expected claims requires surplus to be increased by less than 10 percent.

C. *The Single Year Approach*

The problem of required surplus also can be treated by consideration of aggregate claims in a one-year period. Using an algorithm described by several authors ([8], [16]) it is possible to construct the distribution of aggregate claims over a one-year period from a file of in force certificates and an appropriate mortality table (with the loadings removed). In the present instance, a distribution was constructed using all certificates under head

TABLE 6

$U(0.001)$ AS A PERCENT OF EXPECTED CLAIMS
SECURITY LOADING λ = .04
COMPOUND NEGATIVE BINOMIAL DISTRIBUTION

Negative Binomial Parameter a	Expected claims k		
	2,700	3,000	3,300
∞	14.57	14.57	14.57
1,000,000	14.57	14.57	14.57
100,000	14.66	14.68	14.69
10,000	15.64	15.77	15.90
1,000	25.71	26.96	28.23
500	37.08	39.61	42.16
200	71.43	77.80	84.18
100	128.84	141.60	154.41

office administered groups. For convenience, all amounts of insurance were rounded to the nearer thousand dollars, and the annual number of claims was assumed to have a Poisson distribution. As might be expected, the resulting distribution is very close to the normal distribution but has a slightly thicker tail. Details of this comparison are shown in Table 7. Most certificates for very large amounts in the portfolio were in the head office administered groups. It follows that the ratio of the standard deviation to the mean for the distribution of aggregate claims for the entire portfolio should be less than 0.053, the value of this ratio for the distribution of head office administered groups (this is illustrated by Table 3). Moreover, because the number of certificates in head office administered groups was less than half the number of certificates in the total portfolio, the total portfolio distribution of aggregate claims was more closely approximated by a normal distribution than was the head office administered distribution. Of course, the approximation for the latter distribution was very good indeed as shown in Table 7.

Therefore, to test the security afforded by available surplus over a one-year period, we assume the distribution of aggregate claims for the total portfolio is normal with mean E and standard deviation $0.050E$. Suppose

TABLE 7

COMPARISON OF NORMAL DISTRIBUTION TO ACTUAL DISTRIBUTION OF HOME OFFICE ADMINISTERED GROUP LIFE BUSINESS

PROBABILITY THAT CLAIMS EXCEED SPECIFIED CLAIM LEVEL					
Specified Claim Level	Normal Distribution	Actual Home Office Administered Distribution	Specified Claim Level	Normal Distribution	Actual Home Office Administered Distribution
E	.5000	.4943	$1.08E$	.0656	.0679
$1.01E$	.4251	.4194	$1.09E$	.0448	.0473
$1.02E$	.3531	.3481	$1.10E$	.0296	.0320
$1.03E$	.2857	.2822	$1.11E$	.0190	.0211
$1.04E$	.2251	.2233	$1.12E$	.0118	.0136
$1.05E$	.1727	.1724	$1.13E$	.0070	.0086
$1.06E$	.1288	.1294	$1.14E$	.0041	.0052
$1.07E$	.0932	.0949	$1.15E$	.0023	.0031

Distribution	Mean	Standard Deviation	Skewness	Kurtosis
Normal	E	$.053E$	0	3.0
Home Office Administered	E	$.053E$	0.08809	3.01028

available surplus is U and that the mortality charge built into the premium is $(1 + \lambda)E$. Then the enterprise will be ruined in a single year if claims exceed $(1 + \lambda)E + U$. For example, if $\lambda = 0.04$ and U is $U(0.001) = 0.1564E$, the value derived from the compound negative binomial with $a = 10{,}000$. Then, based upon the normal distribution described above, the probability of ruin in one year is 0.00004[3]. Thus, we can see the degree of protection afforded by a given amount of surplus is highly dependent on the time span under consideration. Pages 149-51 of [1] provide a discussion of the one-year probability of ruin and the effect of company size on this probability. The one-year approach also offers some guidance in interpreting an insurer's annual operating results. For example, if the distribution of aggregate claims can be represented by the distribution for home office administered business (HOA) of Table 7, then a security load of 3 percent will produce an operating loss with probability 0.2822 or approximately twice every seven years. Clearly, the distribution of aggregate claims is useful in relating pricing strategy to operating gain and loss and in explaining operating results to management.

The effects of the population bulge resulting from the post World War II baby boom upon a group insurer's portfolio can be studied, in part, by means of the distribution of aggregate claims. Various studies indicate that the average age of the population will increase approximately five years by the early part of the next century. The average age of the working population covered by group insurance will increase somewhat less. The working population is assumed to contain only those between ages 15 and 65. Moreover, the composition of the working force is expected to change not only because of changing birth and mortality rates but also because of changes in participation rates. Additional information and projections on the labor force of the Province of Ontario are given in the article by D.K. Foot [5]. During the current decade, a slight shift in the portfolio of insureds in favor of those with lower mortality rates can be expected. The situation for the 1990s should be quite different, since the major population bulge will be well into the working ages and the aging effect will come into play. The overall aging of the working population by one year implies a mortality increase of approximately ten percent, based on mortality tables used for group life. In the distribution arising from head office administered business discussed already, mortality rates were increased by 10 and 20 percent, respectively,

[3] It should be noted that this calculation is highly dependent on the exact value of the ratio of the standard deviation to E. This ratio, as discussed above, is certainly less than 0.053. The value 0.050 employed in the calculation is probably too large, and, therefore, too conservative. But, this argument depends on the appropriateness of the use of the normal distribution. In part, the use of a slightly larger than necessary standard deviation is intended to allow for the error introduced by the assumption of a normal distribution.

to simulate one and two years of overall aging. It was found that the standard deviation increased by 4.9 and 9.6 percent, respectively. Thus, while the risk increased absolutely, it decreased relative to expected claims. If, as is traditional, surplus is maintained in a fixed relationship to expected claims, relative safety actually is increased as the insured population ages.

V. ACCIDENTAL DEATH AND DISMEMBERMENT INSURANCE

The determination of surplus requirements for group accidental death and dismemberment (AD&D) insurance is very much like the determination already described for group life insurance. Accordingly, the discussion in this section is confined to those aspects of the problem in which the coverages differ.

A principal characteristic of AD&D coverage is its low frequency of claim. For most insurers, the secondary distribution of amount of claim cannot be constructed from claims data because of the small number of claims experienced in a single year. Of course, the preferred method for constructing this distribution is based upon complete knowledge of all certificates in force. Since this knowledge often is not available, other methods must be used. For the study on which this paper is based, the secondary distribution was derived from the distribution used for group life insurance. The original distribution was modified in two ways:

1. All claims for less than $5,000 were eliminated. AD&D claims for very small amounts are very rare and generally are for partial dismemberment. The group life claims for amounts under $5,000 all arose from paid-up policies that did not have matching AD&D average. In the present instance, their elimination had little effect, was conservative and was consistent with the nature of AD&D coverage.
2. The relative probabilities of a claim being for amounts above $100,000 were increased slightly since most policies containing certificates for large amounts did have matching AD&D coverage.

The use of the group life distribution as a starting point is based on the assumption that AD&D coverage is sold with and in amounts matching group life coverage. To the extent that the assumption does not hold for a particular insurer, adjustments of the type discussed above are necessary.

A second problem is assessing required AD&D surplus revolves around the accidental death rates. Since these rates are so low, a small change in the absolute rate represents a much greater percentage change than for ordinary mortality rates. Moreover, accidental death rates are more apt to be influenced by external forces than are mortality rates. The Metropolitan Life *Statistical Bulletin* indicates that motor vehicle accidents account for approximately 45 percent of all accidental deaths in the United States. Motor vehicle accidents dropped sharply in the United States in 1973-74 when the fuel shortage first was felt and a national speed limit of 55 miles per hour was imposed. Since then, the accident rate has been rising, probably due to consumer adjustment to rising fuel prices and to less stringent enforcement of the speed limit in some jurisdictions. Although Canada has fewer motor vehicles per capita than does the United States, the Canadian motor vehicle accident death rate is 11 percent higher. The difference can be attributed to higher speed limits, poorer roads and, for a time, lower fuel prices. In general, these changes in a major component of accidental death rates have not been fully reflected in published experience data.

Accordingly, a great deal of caution is necessary with respect to rates when calculating required surplus for AD&D. It is necessary to use a negative binomial model with the value for a in equation (4) significantly smaller than the value used for the group life calculation. Also, some care should be paid to the selection of the value for k, the expected number of claims in equation (9). It is the author's experience that this number may be very difficult to determine in real life.

The general pattern of the results for AD&D surplus with respect to security loading and retention limits is similar to the pattern for group life. Consequently, no further results are given here.

VI. GROUP LONG TERM DISABILITY INCOME INSURANCE

As contrasted with group life or AD&D, group long term disability income insurance (LTD) is a very complicated coverage. Three factors complicate the determination of required surplus for LTD:

1. LTD involves two sets of contingencies, claim incidence and claim continuance. The value of an LTD claim is not usually known at the time the claim is admitted.
2. Rates of incidence of claim and termination of claim for LTD are subject to systematic changes due to external factors, primarily economic.
3. Although LTD is sold on a yearly renewable term basis, the associated disabled-life reserves must cover the long term, so that interest is an important factor. Consequently, this coverage is exposed to asset depreciation (C1) and interest rate (C3) risks. Although these risks are significant and an important part of the total surplus problem for LTD, they are not the focus of this paper and are not discussed further.

In order to separate the risk arising from incurrals of disability from the risk arising through the claim continuation assumptions, an LTD insurer may be divided conceptually into two parts, Company A and Company B. Company A receives the full premium and insures against incurrals of LTD claims. Company B sells disabled life annuities. When a claim is admitted, Company A pays to Company B a lump sum equal to the initial reserve established for the claim, plus a security load. As a first step, the required surplus for each of these companies is studied separately.

The basis chosen to calculate the disabled life annuity reserves is of extreme importance to the calculations. The valuation assumptions are assumed to be the actuary's best estimate of future experience with respect to claim incidence and continuation. The reserve interest rate, however, is not assumed to be a new money rate. Rather, it is assumed to be a rate that the insurer is almost certain to earn over the near and medium terms. The experience rating process may contain an excess interest credit so that the total interest credited to the client on reserves held for the case closely approximates the insurer's actual investment earnings on these funds.

The surplus requirements for Company B are considered first. Since its annuitants are Company A's claimants, it will have far fewer contracts in force than Company A has active certificates. An approach to ruin based on the individual annuitants is feasible. Indeed, in this situation, the use of individual risk theory is preferable. The collective risk theory model that we have been using up to this point requires the assumption that any annuitant whose claim terminates is replaced at once by a similar one; this does not seem appropriate for Company B because Company A is the source of its annuitants.

Assume for convenience that Company B pays benefits continuously. As shown in Chapter 5 of [19], if money is discounted at a force of interest δ, the present value of an annual benefit of one dollar to a disabled life (x), $\bar{a}^{\delta}_{x}$, can be written as the expected value of an annuity certain, $\bar{a}^{\delta}_{\overline{T|}}$, where T, a random variable, is the total future time during which (x) will continue to receive the benefit. The variance of the present value of (x)'s benefit is shown to be equal to

$$(2/\delta)\{\bar{a}^{\delta}_{x} - \bar{a}^{2\delta}_{x}\} - \{\bar{a}^{\delta}_{x}\}^2$$

If we assume the values of benefits payable to distinct annuitants are mutually independent, the variance of the present value of all future benefits payable to the closed group of Company B's current annuitants is the sum of the variances of their individual benefits calculated as above. According to Lindeberg's Theorem ([4], page 262), the distribution of Company B's liability for its current claimants is approximately normal, which mean equal to the claim reserve, and variance calculated as above. The surplus required to ensure that the probability of ruin is less than ϵ is an amount S such that the probability that the present value of all future benefits does not exceed the reserves plus S is at least $1 - \epsilon$. For a portfolio with a sufficiently large number of disabled lives, this can be calculated from the normal distribution.

Since the distribution involves present values, interest earnings on surplus S at force of interest δ must be credited directly to surplus.

Company B is exposed to the risk that in times of unfavorable economic conditions the duration of claims will lengthen systematically. The magnitude of this risk can be measured by a recalculation of claim reserves, reducing claim termination rates to some fixed fraction of their previous values. Results of such calculations are shown in Table 8. This table also demonstrates the dependence of claim reserves on the valuation interest rate.

Table 9 compares the changes in financial values due to the deterioration in claim termination rates to surplus values calculated, as described before, based on the normal distribution and $\epsilon = 0.001$. (The critical region extends beyond 3.1 standard deviations.) Surplus calculated this way may not be sufficient if the economic environment worsens significantly. Table 8 demonstrates that an increase in the valuation interest rate, when justified by the investment climate, can provide proper margins for unfavorable claim termination experience.

In order to arrive at a proper surplus level for Company B, it is suggested

TABLE 8

LONG TERM DISABILITY RESERVES UNDER DIFFERING CLAIM TERMINATION RATES

Interest Rate	Claim Termination Rates: Normal (A)	75 Percent of Normal (B)	50 Percent of Normal (C)
0.05	100.00	114.15	132.20
0.06	95.77	108.82	125.30
0.07	91.92	104.00	119.15
0.08	88.41	99.64	113.61

TABLE 9

MARGINS FOR CHANGES IN CLAIM TERMINATION RATES AS A PERCENT OF THE 5 PERCENT INTEREST RESERVE WITH NORMAL TERMINATIONS

Interest Rate	Change in Claim Termination Rates: 3.1 Standard Deviations*	From Normal to 75 Percent of Normal	From Normal to 50 Percent of Normal
0.05	6.71	14.15	32.20
0.06	6.22	13.05	29.53
0.07	5.79	12.08	27.23
0.08	5.42	11.23	25.20

* Normal claim termination assumptions; for the normal distribution, $\epsilon = 0.001$ corresponds to 3.1 standard deviations.

the actuary first determine a worst possible set of claim termination rates that the company might expect to experience and survive intact. The required surplus might be taken as the sum of (1) the difference between reserves for active claims on the "worst possible" and the "usual" claim termination bases, and (2) two standard deviations calculated on the lower termination basis. It can be seen from Table 10 that while claim reserves are quite sensitive to the claim termination basis, the standard deviation is not nearly so sensitive to this change in assumptions.

The margin contained in Company A's claim payments to Company B might properly be calculated in relation to Company B's need for surplus to cover the C2 risk. Company A's gross payment might be the initial disabled-life reserve to be established for the claimant, calculated using the conservative claim termination assumptions that are used in the determination of Company B's required surplus.

The procedure for determining Company A's surplus requirements is similar to that used for life and AD&D insurances. The pattern of results is much like that observed in section IV and will not be discussed in detail here. The major difficulty is the construction of the amount-of-claim distribution. Since the amount of a claim is a reserve amount which is not usually on file for active lives, it normally will not be possible to construct the secondary distribution from in force data. It may be possible to use initial reserves established under recent claims experience if they are representative of the insurer's current active-life exposure. If it is decided to construct the secondary distribution by calculating the individual reserve that would be established if each active life were to become disabled, some recognition should be given to differing incidence rates according to the length of the waiting period. Also, if rates are scaled to recognize prospective experience rating, the factor used must be split as a product of two factors, the first being an adjustment for incidence and the second an adjustment for claim

TABLE 10

STANDARD DEVIATION OF THE PRESENT VALUE
OF BENEFITS FOR DISABLED LIVES
AS A PERCENT OF THE 5 PERCENT INTEREST RESERVE WITH NORMAL TERMINATION

Interest Rate	Claim Termination Rates		
	Normal	75 Percent of Normal	50 Percent of Normal
0.05	2.16	2.27	2.30
0.06	2.01	2.09	2.10
0.07	1.87	1.93	1.93
0.08	1.75	1.80	1.80

termination or persistency. The first factor will apply to incidence rates used in constructing the secondary distribution while the second factor will affect the amount class of the potential claim.

There is considerable uncertainty about the correct level of LTD claim incidence rates both because the construction of the secondary distribution may require some rough approximations and because incidence rates are subject to change by outside influences. Consequently, it is advisable to use the negative binomial model with a relatively small value for the parameter a. Table 11 gives an indication of how the results of our surplus calculations vary with a; required surplus is very sensitive to reductions in a below 1,000. For LTD, a suggested value for a is 1,000. Using this value, required surplus is increased by 45 percent over that required by a value of a of 10,000, the value used in the group life calculations.

Since Company A receives policyholders' premiums and insures the active certificates, considerations of growth of business apply directly to Company A in the same manner as they apply to a group life insurer. Consequently, if surplus is tracked separately for Companies A and B, then the bulk of retained earnings should be held in Company A's surplus. An exception is that interest earned on Company B's surplus should be credited directly to that surplus.

Assuming that required surplus has been calculated for each of Companies A and B, a total surplus figure for the LTD line still must be produced. If the risks presented by Companies A and B are assumed to be mutually independent, the proper figure is the sum of the surplus requirements for each of the companies. The author prefers the independence assumption to the argument that a higher than normal incidence of claims induced by an economic downturn will, to some extent, be offset by higher rates of recovery, with concomitant greater release of disabled-life reserves, at the time of the ensuing economic recovery. Unfortunately, the offset is dependent on

TABLE 11

RELATIVE VALUES OF $U(0.001)$ FOR COMPANY A
SECURITY LOADING $\lambda = .04$
COMPOUND NEGATIVE BINOMIAL DISTRIBUTION

Parameter a	Surplus $U(.001)$
∞	100.00
10,000	109.79
2,000	131.91
1,000	159.59
500	215.12

the length of time necessary for an economic recovery to take hold. Moreover, recovery occurs unevenly among industries and firms. It is difficult to see how such an offset prudently could be incorporated into the calculation of a reserve whose purpose is to ensure the solvency of the LTD line.

VII. SURPLUS FOR OTHER GROUP COVERAGES

The study on which this paper is based considered only group life, AD&D, and LTD. Other coverages were not studied due to a lack of time and of data in a form that could readily be used for this type of investigation. Based on the experience gained in the study, several points related to the extension of the study to other coverages are given below.

The construction of the amount-of-claim distribution is a fundamental problem. The exact definition of what constitutes a single claim is crucial. In preparing the distribution, presumably from recent claims data, it is necessary to add together all amounts paid that are considered to form part of the same claim. For high claim frequency lines, this can be a formidable task. The insurer's ability to construct a distribution in this way will be greatly dependent on the nature of its claim administration systems.

For medical coverages, the primary risks come from price inflation and changes in benefit utilization. These can be handled in a manner similar to that discussed in section IV with respect to the effects of wage inflation on life insurance. Actually, this approach will cover only anticipated inflation. A major risk is that pricing will be inadequate due to unexpectedly high rates of inflation. Lacking any clear guide as to how high inflation rates might go, it is difficult to suggest any systematic way of providing for this risk. Ultimately, we must rely on a company's free surplus.

VIII. OTHER CONSIDERATIONS

To this point, our discussion has been concerned with the risks of unfavorable claims experience. The insurance risk includes other potential sources of loss for the insurer. Rate guarantees, whether implicit or explicit, restrict the insurer's ability to take corrective action in the event premiums prove to be insufficient. Thereby, the resulting financial losses are prolonged and multiplied. A guarantee may be implicit if it is the practical result of the insurer's methods of administration. If a company cannot react quickly to changing patterns of experience by doing frequent experience studies and making adjustments to its premium structure, it should have additional surplus in recognition of the increased financial risk. The amount of this additional surplus should be related to the threshold level of change in experience that would prompt an experience study, the speed with which new premium rates can be implemented and administered, and the extent of rate guarantees in the current in force business.

Expenses are always a major concern. With the rise of administrative services only business, they play an even more important role for the group insurer. An extra surplus provision to cover unforseen increases in expenses is particularly important if the insurer has ASO contracts in force. It would be appropriate to relate this provision to ASO fees earned.

The extra risk introduced by experience rating, and the deficit termination risk ([2], [10]) in particular, has been widely discussed in recent years. If an adequate risk charge is not being made to cover this risk, a further increase in surplus is indicated. This component of surplus properly should be related to the total risk charges that would be collected in a year if the company's pricing and the marketplace allowed such a charge.

The total required surplus figure for a group insurer is a composite of requirements to cover a variety of risks. Should the total be the sum of each of these requirements or are there offsets that allow for a smaller total surplus? Certain group insurance mechanisms provide limited offsets. Among these are claim fluctuation reserves accumulated from experience refunds that remain with the insurer. These do not provide a dollar-for-dollar offset to surplus since the funds can be applied only to losses from the particular case for which they are being held. A similar situation applies to contract provisions allowing for cross rating between different coverages for the same group. An excellent discussion of these matters can be found in [11]. I am inclined toward holding the sum of the requirements for each of the coverages unless it can be demonstrated that experience on two or more coverages are in some manner mutually dependent. Independence should be assumed, unless there is evidence to the contrary. Such independence leads to a surplus requirement that is truly the sum of the parts.

REFERENCES

1. BEARD, R.E., PENTIKAINEN, T., AND PESONEN, E. *Risk Theory, The Stochastic Basis of Insurance*, (2nd Edition) London: Chapman and Hall, 1977.
2. BOLNICK, HOWARD J. "Experience-Rating Group Life Insurance," *TSA*, XXVI (1974), 123–223.
3. BOWERS, N., GERBER, H., HICKMAN, J., JONES, D., AND NESBITT, C. *Study Note on Risk Theory*, Chicago: Society of Actuaries, 1982.
4. ———. *Actuarial Mathematics*, Study note of the Society of Actuaries, Itasca, Ill., 1984.
5. BUHLMANN, HANS. *Mathematical Models in Risk Theory*, New York: Springer-Verlag, 1970.

6. FELLER, W. *An Introduction to Probability Theory and its Applications*, (Volume 2, 2nd Edition) New York: Wiley, 1971.
7. FOOT, D.K., PESANDO, J.E., SAWYER, J.A., AND WINDER, J.W.L. *The Ontario Economy, 1977–1987*, Toronto: Ontario Economic Council, 1977.
8. GERBER, H.U. *An Introduction to Mathematical Risk Theory*, Philadelphia: Huebner Foundation, 1979.
9. HICKMAN, J.C., CODY, D.D., MAYNARD, J.C., TROWBRIDGE, C.L., AND TURNER, S.H. Discussion of the "Preliminary Report of the Committee on Valuation and Related Problems," *RSA*, V (1979), 241–84.
10. LECKIE, ROBIN B. "Some Actuarial Considerations for Mutual Companies," *TSA*, XXXI (1979), 187–259.
11. PANJER, H.H. "The Aggregate Claims Distribution and Stop-Loss Reinsurance," *TSA*, XXXII (1980), 523–45.
12. ———. "Recursive Evaluation of a Family of Compound Distributions," *ASTIN Bulletin*, XII (1981), 22–26.
13. PANJER, H.H., AND MEREU, J.A. "Analysis of the Deficit Risk in Group Insurance," *TSA*, XXXII (1980), 305–47.
14. PIKE, B.N. "Financial Planning and Control for Group Insurance," *TSA*, XXIX (1977), 243–67.
15. SEAL, H.L. *Stochastic Theory of a Risk Business*, New York: Wiley, 1969.
16. ———. *Survival Probabilities, the Goal of Risk Theory*, New York: Wiley, 1978.
17. TAYLOR, G.C. "Probability of Ruin Under Inflationary Conditions or Under Experience Rating," *ASTIN Bulletin*, X (1979), 149–62.
18. TROWBRIDGE, C.L. "Theory of Surplus in a Mutual Insurance Organization," *TSA*, XIX (1967), 216–32.
19. WILLIAMS, R.E. "Computing the Probability Density Function for Aggregate Claims," *Proceedings, Canadian Institute of Actuaries*, XI (1979–80), 38–47.

DISCUSSION OF PRECEDING PAPER

JAMES RAMENDA:

Any actuary who has worked with the concept of required surplus can appreciate Mr. Brender's excellent paper. The application of risk-theory calculations to this problem is extremely complicated, and he is to be congratulated for his efforts.

As Mr. Brender indicates in his final paragraph, the total required surplus for a group insurer is a composite amount covering a variety of risks. He notes that when correlations among the various risks can be demonstrated, the required composite surplus may be less than the sum of the requirements for each part. Awareness of the impact of correlated risks is growing as the theory underlying required surplus becomes more refined. In fact, the work of the Combination of Risks Task Force indicates that the sum of the requirements may not be the maximum composite requirement, at least in the presence of the C-3 risk (Cody, D.D.; Mateja, M.E.; McCarthy, D.J.; and Vanderhoof, I.T. Discussion on "Society Research Affecting the Valuation Actuary," *RSA*, 9 (1983), 1657-84). Clearly, there is more technical analysis to be done in this area.

As a specific example, Mr. Brender identifies the cross-rating of group coverages at the case level as a mechanism that creates "mutually dependent" experience. This cross-rating typically produces group life results that respond inversely to accident and health (A&H) results, the latter being cyclically driven. This correlation suggests not only a composite required surplus for the two lines, but a composite profit center for management purposes, since earnings in one line are dependent upon experience in the other.

Expanding this logic to an entire company raises some interesting management issues. Suppose the interaction of risks between two lines suggests a combined required surplus less than the sum of the parts. Further, suppose that these lines are not closely related, so that a combined profit center is not feasible. How should the composite surplus be allocated among the lines for the purpose of computing returns on equity at the line-of-business level? One approach would be to charge each line its full "stand-alone" surplus. The difference between the sum of such amounts and the composite would be allocated as a credit against the corporate line's surplus position. Presumably, this allocation and the resulting boost to the corporate line's return on equity (ROE) would reward the corporate line for orchestrating a synergistic combination of businesses. Financial officers in operating lines, of course, probably would not be enthusiastic over this approach.

Naturally, the words "corporate" and "allocation," used in close proximity, portend enormous complications. Nonetheless, allocation of surplus relief due to related risks is a logical extension of the basic surplus allocation question. Actuaries, following Mr. Brender's excellent technical lead, should keep the management perspective on required surplus in mind since every technical refinement affecting surplus allocation creates the need for management interpretation.

JOHN K. AHRENS:

Mr. Brender is to be commended for his efforts in applying theory to actual experience in order to develop the base of "science" available to group actuaries in their work. It is only from such mathematical models that the "art" which underlies so much of a group actuary's work in the constantly changing group insurance field can be developed. The purpose of my comments is to provide a rationale and illustrative figures to help actuaries and senior management deal with all group lines of business, especially medical lines, upon which Mr. Brender briefly comments in sections VII and VIII. My comments and approach are more generalized and strategic whereas Mr. Brender's comments are case specific and tactical.

Once we move away from the mathematical models, we are able to incorporate more elements into defining required surplus. I will mention each of the major ones with some brief comments.

Catastrophes. This includes those of a natural, man-made, and environmental nature. Natural catastrophes include earthquakes and weather-related occurrences. Man-made catastrophes include occupational risks like mining, chemical disasters, such as gas leaks, and nuclear incidents from meltdowns or conflict. Nuclear conflict should be addressed since it could be limited or widespread. One could try to exclude it from policies or assume that, if widespread, the company's solvency is a very minor concern, and, if limited, the government would aid insurance companies. Environmental catastrophes include epidemics, toxic poisoning, and significant medical risks, such as AIDS or the release of a virus from bioengineering efforts. Exposure to localized catastrophes is a much larger risk for the Blue Cross Blue Shield Companies or small regional or statewide companies, as compared to national insurance companies.

Fluctuation. This is the risk of a significant statistical fluctuation in a particular year. The fluctuation risk is primarily a function of the size of the block of business and the maximum exposure per risk. Analysis of this risk lends itself well to statistical methods.

Pricing. In group insurance, both actuaries and underwriters generally are the major determiners of the pricing risk. A significant pricing error (in determining expected claims) can dramatically increase the probability of poor results. Therefore, the knowledge of these persons and the management information systems that support their efforts determine the magnitude of this risk. For companies without actuaries or underwriters, consulting actuaries and senior management should be evaluated. Policy provisions and rating margins also have an important impact on this risk.

Management. There are other management risks not directly related to pricing. Key areas that can have the most impact are claims—because of possible coordination of benefits (COB), audits, and case-management savings—and marketing—since it has a significant impact on the pressure that is put on pricing. Another area to examine is incentives and incentive compensation. Look at each area, including senior management, and determine their true incentives (are claims people measured only on production, marketing people on new business or increase in premium, and senior management on maintaining a low profile?).

Lawsuits. The potential risk of lawsuits should be evaluated with regard to every line of group business. Exposure may be significantly higher in certain lines of business, such as medical, especially multiple employer trust (MET) business. Certain states, such as California, carry significantly higher risks both in frequency and in the amount of awards. This would also cover the risk of nuisance lawsuits (where there are no contractual grounds).

Governmental Policy. This should cover unexpected or unpredictable state and federal regulations affecting companies directly (state mandated benefits, taxation of employee benefits) and indirectly diagnostic related group (DRG) reimbursement for medicare, wage and price controls). In the most severe instance, such action could even force the termination of a line of business.

In developing required surplus guidelines, which are based on "art" rather than "science," it is important that the underlying premises be explained (just as mathematical theories are stated in "science"), so that anyone who disagrees with the underlying premise, will question the results.

Return on Surplus or Return on Equity (ROS or ROE)

1. Return on surplus is a sound basis for evaluating risks and opportunities,
   a. at the product level;
   b. at the corporate level;
   c. at the insurance industry level; and
   d. at the national financial markets level.

Since significant work has already gone into the numerator, which is earnings, the denominator, which is required surplus, needs more attention.

2. Return on surplus of 15 percent or some similar number is not the magic level that group writers should be judged against at a corporate level. It is probable that group writers may have an average ROS of 20–25 percent.
3. The formula must be simple to calculate and not vary significantly over the years.
4. Historical profit and the ROS record of the product line and management talent are important indicators of future abilities and should be factored into the formula.
5. With growth, required surplus as a percentage of premium generally goes down, provided the growth is not excessive, and the results are reasonable.
6. The capability of a company to manage a line depends upon many talents and systems; size helps in affording necessary functions.
7. There are many costs (often hidden) associated with dabbling in too many products, which can be considered as surplus.
8. Group writers must be segmented by size in the market, since various forces impact each differently. Small writers have some advantages such as service, captive agents, or long-term relationships, that serve to encapsulate them somewhat from market competition. Thus, they are able to earn higher profit margins, although those margins are almost impossible to maintain if the writer grows. Competitive pressures significantly reduce profit margins so that the largest (or jumbo) writers usually must operate with slim margins.

The results of these premises are reflected in an extensive table that provides possible formulas for determining required surplus for most group lines of business. The table is shown at the end of this discussion for easier reference. An explanation of each of the components follows.

Premium Class

Premium class is used as a means of separating business into four categories, which actually represent a myriad of factors—only a few of which will be described here. Only with a great deal of effort is a company in a position to change its business significantly enough to move up in a category.

The jumbo writers are the industry and market leaders in a product line. There are usually ten to fifteen companies in this category for any product line. They tend to have the line set up internally as a profit center with its own manager and a full-time FSA plus support underwriters and possibly its own claim department. The line has marketing specialists and very good management information systems. Strategic planning is an important function, and senior management is informed about the results and direction.

The large writers represent the rest of the significant players in the marketplace for that product line. Capabilities in the key functions will vary more by company than is true of the jumbo writers. There is actuarial support, but it may be project oriented and not full time. Underwriters do not deal exclusively with the product. The claims unit is good and marketing persons sell other products as well. Information flow is adequate, but results are not scrutinized more often than quarterly. The product line is not given special attention in planning but is incorporated in total group results, and senior management is not always informed of its progress. On the whole, it is reasonably well managed. A few key persons can make a significant difference.

Medium writers are in a more difficult position. They are large enough to compete with the large and jumbo writers, especially in regional markets. However, they are relatively understaffed in the key areas, especially actuarial. Management information systems are not as good as they should be, and the claims areas worry first about backlogs and turnaround time. Overall quality of the marketing force is not high, but several key sales persons do very well. Turnover in key positions can be a problem. Strategic planning is given a back seat to fighting fires.

Small group writers usually have access to some captive business which may support their other endeavors. Senior management's deep involvement in many decisions can be advantageous. There is little actuarial support; consultants may be used for projects. Underwriting may consist of one or two persons at most. The claims area is very dependent on one or two key persons. The information systems are relatively good, but the database is small. Marketing generally is not controlled except for captive business. Reinsurance is a significant issue and is used extensively. Turnover is not a big problem since the management team is close-knit. Strategic planning consists of looking for niches or unusual opportunities.

Minimum Surplus

This flat-dollar amount reflects the need in any line of business for some investment of resources (including management "opportunity cost") in areas such as marketing, product development, policy filing, administrative systems, claims systems, and management reporting. This includes not only equipment investment but also acquisition and training of human resources. Additionally, it reflects the greater risk inherent in having only a small presence in many product lines. This can be reduced significantly through bulk reinsurance or utilizing other companies' products and systems. The surplus for a line is the greater of the figure determined from the required surplus (percentage of premium) figure and the minimum surplus figure.

Required Surplus

To ease the determination of required surplus and to allow for comparisons between companies, surplus is determined as a percentage of premium. This does not cause many matching problems since group insurance is primarily one-year term insurance. An exception to this is long-term disability where a factor also is applied to open claim reserves. To put this on a premium basis for return-on-surplus figures, open claim reserves are assumed to equal 2.5 times annual premium.

The factors shown have been chosen with greatest emphasis placed on fluctuation, pricing, and management risks. Catastrophies, lawsuits, and government policy risks have been included only superficially since exposure to them can vary significantly by market; thus, an average risk exposure was assumed. These factors are intended to represent C-2 risks only. C-1 and C-4 risks can be approximated by loading the C-2 figures by 15–20 percent.

These assumptions regarding technical abilities, management, and systems by premium class have been built in to the required surplus figures. Additionally, the target pretax profit for the premium class is assumed to have been earned on the average over the past five years (the approximate length of an underwriting cycle). Finally, the fluctuation risk, which is primarily a function of the size of the product line, is factored in, although the yearly fluctuations in historical results should be considered also. The pricing and management risks are the most significant (about 65 percent of the required surplus factor), so an appraisal of those abilities is the most critical for internal analysis. However, since most persons outside of the company are unable to evaluate this information, premium size will be the major determinant and should be followed unless it is definitely inappropriate.

Required surplus always reduces as a percentage of premium as premium increases. This is a result of the assumption that management knowledge and control increase along with the premium, so that the fluctuation risk becomes less severe and profit margins are more likely to be met. Finally, to the extent that actual profit margins have been less than target, the difference in percentage points should be added to the required surplus factor.

Growth Adjustment Factor

This factor reflects concerns that management should have toward a rapid increase or decrease over the previous year in a line of business. It is based on the assumption that too rapid growth in a competitive market implies inappropriate pricing and, thus, a very significant rating and management risk. The factor should vary by product line to reflect differences in competitive pressures, difficulty in ratings, and the potential of problems being difficult to detect. With declining blocks of business, the major risk is from an assessment spiral, retention of poorer risks, and lack of management attention.

Required surplus (as a percentage of premium) is increased by the growth adjustment factor after adjusting the growth for underlying trend and rating adjustments. The growth adjustment factor is applied to the required surplus factor, and that result is then applied to the premium increase in excess of trend or rate adjustments. The normal required surplus factor is applied to the prior year's premium, adjusted for trend and rate adjustment.

This process complicates the determination of required surplus and can be ignored except for certain lines of business (MET, nonrefunding medical, specific and aggregate stop-loss, and special risk pools), which are higher risk, and if there has been significant growth. For declining business, an allowable "real" decline of the 10–15 percent should be permitted without any adjustment. Then the growth adjustment factor is applied to the required surplus factor and multiplied by the amount of premium reduction below the allowable decline in the premium. This figure is then added to the normal required surplus determined for the current reduced premium. This has the impact of significantly increasing required surplus on declining blocks of business. Some people may consider this adustment too severe and may not want to follow this approach since movement to a lower premium class and profits below the target pretax profit tend to increase the required surplus already.

Target Pretax Profit and Target Return on Surplus

Generally, a decreasing target profit ratio was assumed as size increased because of the increased impact of competition. Usually, required surplus also decreases as a percentage of premium but not as dramatically. The result is usually a decreasing target return on surplus. However, this is not necessarily ominous because the lower required surplus should increase the likelihood that actual results will meet the target ROS at those levels for large or jumbo writers. The large and jumbo writers show a higher target ROS than small writers for several product lines where competition is not so intense, such as disability, dental, other ancillary benefits, and special risk pools. However, very few writers are in a position to change their posture in those markets. In the larger-case medical (refunding) product line, increases in systems capabilities and in sophistication—assumed to be inherent with large and jumbo writers—produce significant improvements in ROS.

The target-return-on-surplus figures shown in the table should not be used automatically to determine markets to enter or to set profit objectives. They are given primarily to show the relationships and how the process ties in together. However, it appears that a well-managed group operation should expect to be able to achieve after-tax return on surplus considerably in excess of 15 percent, provided it is able to protect itself during the downside of the underwriting cycle.

The reason for this discussion is simply to emphasize that practicing group actuaries and senior management cannot afford to ignore the use of required surplus for any group product line simply because good empirical data or mathematical models do not exist. Even though the major product lines have been mentioned, a new area has emerged, which can be called preferred provider arrangements (PPA). This can be used as an example of how to apply this approach to a product until other methods become available.

Required surplus for preferred provider arrangements could be determined as a factor to be applied to the required surplus for the line of business with which it is used. Currently, that factor should be greater than 1.00, since it is so new that the potential risks are largely unknown and still in the process of being identified. In the longer term (three to five years), factors below 1.00 are expected for the better PPA approaches. Required surplus also could be factored by putting the block into the line where it will be used as a "new line," i.e., a jumbo medical writer administrative services only (ASO) would treat PPA as a new premium block and, thus, apply minimum surplus (five million) and growth adjustment factors to the higher required surplus.

My comments serve to acknowledge Mr. Brender's work, encourage him and others to address some of the more difficult group product lines, and help us continue such efforts which are embodied in the motto of the Society.

REQUIRED SURPLUS FOR GROUP PRODUCT LINES

Components	Group Insurer Classification			
	Small	Medium	Large	Jumbo
	Life—Nonrefunding			
Premium Class (millions)	<\$3	\$3–\$15	\$15–\$50	\$50+
Minimum Surplus (millions)	\$.5	\$1.0	\$0.0	\$0.0
Required Surplus (% of Prem.)	30%	22%	15%	10%
Growth Adjustment Factor	1.20	1.20	1.10	1.00
Target Pretax Profit (% of Prem.)	15%	11%	7.5%	5%
Target Return on Surplus	50%	50%	50%	50%
	Life—Refunding			
Premium Class (millions)	<\$5	\$5–\$20	\$20–\$50	\$50+
Minimum Surplus (millions)	\$1.0	\$2.0	\$0.0	\$0.0
Required Surplus (% of Prem.)	33%	25%	18%	13%
Growth Adjustment Factor	1.15	1.10	1.05	1.00
Target Pretax Profit (% of Prem.)	7%	5%	3.5%	2.5%
Target Return on Surplus	21%	20%	19%	19%

REQUIRED SURPLUS FOR GROUP PRODUCT LINES—*Continued*

Components	Group Insurer Classification			
	Small	Medium	Large	Jumbo
	Medical—MET (Under 25 Lives)			
Premium Class (millions)	<$10	$10–$25	$25–$100	$100+
Minimum Surplus (millions)	$2.0	$3.0	$0.0	$0.0
Required Surplus (% of Prem.)	25%	20%	15%	10%
Growth Adjustment Factor	2.00	1.50	1.25	1.10
Target Pretax Profit (% of Prem.)	10%	7%	5%	3%
Target Return on Surplus	40%	35%	33%	30%
	Medical Regular (25–100 Lives)—Nonrefunding			
Premium Class (millions)	<$5	$5–$15	$15–$50	$50+
Minimum Surplus (millions)	$1.0	$2.0	$0.0	$0.0
Required Surplus (% of Prem.)	15%	11%	9%	6%
Growth Adjustment Factor	1.50	1.20	1.10	1.00
Target Pretax Profit (% of Prem.)	8%	5%	3.5%	2%
Target Return on Surplus	53%	45%	39%	33%
	Medical Regular (Premium Equivalents)—Refunding			
Premium Class (millions)	<$75	$75–$200	$200–$500	$500+
Minimum Surplus (millions)	$5.0	$8.0	$0.0	$0.0
Required Surplus (% of Prem.)	10%	8%	6%	4%
Growth Adjustment Factor	1.20	1.10	1.00	1.00
Target Pretax Profit (% of Prem.)	3%	3%	2.5%	2%
Target Return on Surplus	30%	38%	42%	50%
	Medical—Administrative Services Only or Cost Plus (Premium Equivalents)			
Premium Class (millions)	<$20	$20–$75	$75–$300	$300+
Minimum Surplus (millions)	$3.0	$4.0	$0.0	$0.0
Required Surplus (% of Prem.)	8%	5%	3%	2%
Growth Adjustment Factor	1.10	1.05	1.00	1.00
Target Pretax Profit (% of Prem.)	2%	1.25%	0.75%	0.5%
Target Return on Surplus	25%	25%	25%	25%
	Medical—Specific and Aggregate Stop Loss			
Premium Class (millions)	<$3	$3–$10	$10–$35	$35+
Minimum Surplus (millions)	$1.0	$2.0	$0.0	$0.0
Required Surplus (% of Prem.)	50%	40%	33%	25%
Growth Adjustment Factor	1.50	1.35	1.25	1.20
Target Pretax Profit (% of Prem.)	20%	16%	13%	10%
Target Return on Surplus	40%	40%	40%	40%
	Disability—Weekly Indemnity			
Premium Class (millions)	<$1	$1–$5	$5–$20	$20+
Minimum Surplus (millions)	$0.2	$0.5	$0.0	$0.0
Required Surplus (% of Prem.)	30%	22%	17%	15%
Growth Adjustment Factor	1.20	1.10	1.00	1.00
Target Pretax Profit (% of Prem.)	10%	8%	7%	7%
Target Return on Surplus	33%	36%	41%	47%
	Long-Term Disability			
Premium Class (millions)	<$3	$3–$15	$15–$40	$40+
Minimum Surplus (millions)	$1.0	$2.0	$0.0	$0.0
Required Surplus (% of Prem.)	30%	20%	15%	10%
(% of Claim Reserves)	20%	20%	15%	15%
Growth Adjustment Factor	1.25	1.10	1	1
Target Pretax Profit (% of Prem.)	20%	18%	15%	15%
Target Return on Surplus	25%	26%	29%	32%

REQUIRED SURPLUS FOR GROUP PRODUCT LINES—*Continued*

	Dental			
Premium Class (millions)	<$2	$2–$10	$10–$50	$50+
Minimum Surplus (millions)	$.5	$1.0	$0.0	$0.0
Required Surplus (% of Prem.)	30%	22%	15%	9%
Growth Adjustment Factor	1.30	1.20	1.00	1.00
Target Pretax Profit (% of Prem.)	10%	8%	6%	4%
Target Return on Surplus	33%	36%	40%	44%
	Accidental Death and Disablement			
Premium Class (millions)	<$.5	$.5–$3	$3–$10	$10+
Minimum Surplus (millions)	$0.2	$0.4	$1.0	$0.0
Required Surplus (% of Prem.)	40%	30%	25%	20%
Growth Adjustment Factor	1.10	1.10	1.00	1.00
Target Pretax Profit (% of Prem.)	16%	13%	10%	8%
Target Return on Surplus	40%	40%	40%	40%
	Credit, Mortgage (Life and Health)			
Premium Class* (millions)	<$5	$5–$20	$20–$50	$50+
Minimum Surplus (millions)	$1.5	$2.5	$0.0	$0.0
Required Surplus (% of Prem.)	40%	30%	25%	20%
Growth Adjustment Factor	1.25	1.20	1.00	1.00
Target Pretax Profit (% of Prem.)	16%	13%	10%	8%
Target Return on Surplus	40%	40%	40%	40%

*Earned Premium (less unearned premium reserve)

	Special Risk Pools (CATCO, Special Accident, High Limit Excess)			
Premium Class (millions)	<$10	$10–$35	$35–$100	$100+
Minimum Surplus (millions)	$5	$7.5	$0.0	$0.0
Required Surplus (% of Prem.)	75%	50%	33%	25%
Growth Adjustment Factor	1.50	1.33	1.25	1.10
Target Pretax Profit (% of Prem.)	20%	15%	15%	15%
Target Return on Surplus	27%	30%	45%	60%
	Ancillary Benefits (Individually; Medical Conversions, Vision, Audio, Rx Drugs)			
Premium Class (millions)	<$2	$2–$5	$5–$15	$15+
Minimum Surplus (millions)	$.5	$1.0	$0.0	$0.0
Required Surplus (% of Prem.)	35%	25%	20%	20%
Growth Adjustment Factor	1.25	1.15	1.00	1.00
Target Pretax Profit (% of Prem.)	20%	18%	15%	15%
Target Return on Surplus	57%	72%	75%	75%

(AUTHOR'S REVIEW OF DISCUSSION)

ALLAN BRENDER:

Many thanks to Messrs. Ahrens and Ramenda for adding to the paper with their valuable discussions. While their comments relate more to the larger question of total surplus required for all risks than to the particular questions addressed in the paper, a few comments are in order.

Mr. Ramenda discusses a major difficulty in any surplus policy—the fact that a company's total surplus requirement is not necessarily the sum of its parts. His suggestion of a corporate surplus component has been implemented by several companies with a definite surplus policy. A description of the approach taken in Mr. Ahren's company can be found in a paper by Richard Robertson and Richard Kischuk, which is a Part 10 Study Note.

Mr. Ahrens has provided us with a well thought-out approach to fixing a total surplus requirement for a group insurance division. His approach is meant to cover a much wider class of risks than was discussed in the paper and is the sort of formula the practicing actuary needs. The main thrust of the paper is to describe ways to determine the minimum surplus, required surplus, and growth adjustment factor used by Mr. Ahrens. Such items as target pretax profit, target return on surplus, and the division of cases by premium amount are management decisions which will vary from company to company. The numbers cited in Mr. Ahren's discussion may be appropriate for his company or may have been provided for purposes of illustration only. In either case, the reader should take care not to adopt any numbers cited in the paper or discussions without investigating their appropriateness for the particular application.