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AN INTRODUCTION TO BUSINESS CREDIT INSURANCE

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ABSTRACT

Business credit refers to the credit granted by one business to another; it is not related to consumer credit. Because there is always the risk of a default, business credit insurance can be used to reduce the impact of defaults. An overview of current business credit practices and terminology is given, and the types of policies offered are described. A mathematical model of this type of insurance is introduced and analyzed. Two approaches to calculating gross premiums are given: the pure premium method and the loss ratio method. A method based on percentiles is recommended as a means of checking the adequacy of gross premiums.

1. INTRODUCTION*

The objective of this paper is to introduce actuaries to business credit insurance. In North America, this nontraditional area of insurance has evolved without the active participation of actuaries. This is somewhat surprising given the obvious analogy between the life and death of an individual and a business. Because of the short-term nature of business credit policies, some insurers may not have perceived the need for actuarial expertise. Instead, they have traditionally used underwriters to assess and price their policies. Actuaries have been called in to help with reinsurers and to test the adequacy of the premium structure.

Another reason for the minimal actuarial involvement in business credit insurance is the fact that for many of the companies involved in this business, this line of insurance represents only a small part of their overall business. In spite of its tradition as an underwriting-driven line, I still think that the business credit insurance field needs actuarial expertise to assist it in many areas, including product design, pricing, setting loss reserves, and so on.

The largest writer of business credit insurance in the U.S. is American Credit Indemnity (ACI) in Baltimore, Maryland. According to the *Best's Insurance Reports: Property Casualty* (1990), ACI is not only the oldest

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carrier in the U.S. writing business credit insurance exclusively, but also the largest of its type in the world. ACI accounts for approximately 70 percent to 80 percent of the U.S. market, with about $60 million of net premiums written in 1990. This shows that the business credit insurance market is very small compared to the overall credit insurance market, which had approximately $1.3 billion in net premiums written. Another insurer in the business credit market is Fidelity and Deposit Co. of Maryland. However, this line is a very small part of the overall operation. I must point out that there is a lack of industry data on business credit insurance as an independent line of operation (rather than being lumped together with other types of credit insurance).

Section 2 gives an overview of the business credit field and defines some of the terminologies used. Section 3 introduces the types of policies and services behind business credit insurance. In Section 4, a basic model of business credit insurance is described. It is used to derive gross premiums and to check the adequacy of these premiums. Section 5 contains comments.

Since this paper is introductory, a major problem not discussed is the problem of modeling and estimating credit risk (default probabilities). This problem is extensively discussed by Fiedler [9, chapters 2–5], who documents the difficulties associated with it, such as the continuously changing credit stock, the lack of adequate and timely data, the validity of risk measure used, and the influence of business conditions.

2. BUSINESS CREDIT?

2.1 Definition

In any modern economy, business credit is vitally important in facilitating the movement of goods and services through the various successive stages of production and distribution. If used properly, business credit can increase sales and profits for all. If business credit is used improperly, the growth of existing businesses and the creation of new businesses can be hampered. Fiedler [9, p. 5] states:

> Too much and too risky credit can lead to serious losses, possibly to a spiraling general deflation; too little and too safe credit can lead to a level of production, employment and income significantly below what otherwise would be achieved.

Following Briggs and Edwards [5, chapter 2, p. 14], business credit can be defined as follows:

Definition 1: *Business credit (also known as commercial credit, trade credit or mercantile credit) is simply the value transferred by a seller on trust and the time interval before the buyer's return of that value in the form of payment. It is used only in connection with the operation of a business and is accepted by one business from another business.*

Mills [15, p. 2] estimates that in the U.S., as much as 99 percent of the sales made between businesses is on credit.

There are other kinds of credit: export credit, financial or cash credit, and consumer credit. Export credit refers to the credit extended to "foreign" businesses. Unfortunately, sometimes it is not obvious which countries are considered to be foreign. For example, in the proposed Europe 1992, trade between Common Market countries will be considered to be "domestic;" see Dawson [7]. Likewise, some U.S. sales to Canada are considered to be domestic for the purposes of business credit insurance. A distinguishing feature of export credit is the extent of the political risks involved. See Briggs and Edwards [5, chapter 9] and Funatsu [10] for more on export credit. Business credit differs from financial (cash) credit because in the former, the value transferred is limited to the value of the goods and services exchanged—cash transfers are not included. Business credit also is different from consumer credit because the latter relates to credit extended to consumers to facilitate their retail purchase of goods and services.

2.2 Terms of Sale

Whenever business credit is extended, the seller and the buyer must agree on the conditions of payment, referred to as the terms of sale. The *terms of sale* are usually determined by the traditional or customary terms of sale prevailing in the market or trade. Some factors influencing credit terms are the effect of credit on sales and profit, the rate of stock turnover, the character of the goods, the nature of the credit risk, the seller's financial resources, general economic conditions, and so on; see Cole [6, pp. 312–15].

Terms of sale include many familiar terms such as "cash before delivery," "cash on delivery (COD)," and "cash in advance." However,

these do not constitute business credit terms; rather they are prepayment terms. For credit to exist, there must be a period (between the invoice date and a later date of payment) during which payments can be made without incurring penalties. Typically the terms of sale is written as "d/m net n," where d is the discount offered if payment is made within m days of the invoice date; otherwise the full amount (without any discount) is due in n days of the invoice date. For example, suppose a customer is issued an invoice for \$1000 dated July 7 with terms 2/14, net 30; the customer can either pay \$980 (a 2 percent discount on \$1000) on or before July 21, or pay \$1000 on or before August 6. It is better to use the discount if the interest rate on borrowed funds is less than $360d/n \times 100\%$. In the above example, the discount should be accepted if the borrowing rate is less than 45 percent. It is common now simply to use terms of sale without discounts, for example, "net 30 days."

2.3 The Credit Risk

As Cole [6, chapter 12, p. 305] points out, business credit is the most important type of short-term credit used and accepted by businesses. This fact is evident by virtue of the vast number of transactions involving business credit and the many different policies associated with accepting business credit.

Each time one business extends credit to another, the seller is accepting a certain degree of risk. This risk includes the risk of being paid late or of not being paid at all (in default). The risk of default is, of course, the most serious risk.

It is important to realize that, when selling on credit, the profit on the sale is not realized until the money is collected. This is true even though the value of the sale may be recorded as an asset in the balance sheet. Some companies recognize this by reducing the value of receivables according to the number of months they are overdue.

If payment is made beyond the due date, the resulting interest and collections expense can adversely affect profits. For example, if the cost of borrowing is i and the net profit rate on sales is p, then any debt that is overdue for more than $12p/i$ months will result in a loss (negative profit). Briggs and Edwards [5] provide some sample calculations showing the effects of overdue debts on profits. The following tables are patterned after their calculations. Table 1 shows the number of months ($12p/i$) after which a loss occurs. For example, if the interest rate is 8

percent and the net profit is 15 percent on the sale, then it takes 22.5 months ($12 \times 0.15/0.08$) before the net profit is reduced to zero.

TABLE 1

Interest Rate (i)	Net Profit on Sales (p)				
	20%	15%	10%	5%	1%
6%	40	30	20	10	2
8	30	22.5	15	7.5	1.5
10	24	18	12	6	1.2
12	20	15	10	5	1
15	16	12	8	4	0.8

Another source of concern is that each bad debt requires a certain amount of new sales to cover the loss it produces. So for a given profit margin of p per sale, each bad debt will require a $1/p$-fold increase in sales just to cover the bad debt losses. Table 2 shows the amount of new sales required to recover a $1,000 loss if new sales are paid immediately.

TABLE 2

Net Profit (p)	New Sales Required per $1000 Loss
1%	$100,000
5	20,000
10	10,000
15	6,667
20	5,000

A popular measure of the extent of late payments is the average collection period or the *days sales outstanding (DSO)*. For example, a DSO of 50 days means that it takes an average 50 days to convert sales to cash. Unfortunately, the DSO figure masks the different payment patterns that may exist: small debts tend to be paid off earlier than large debts, and the longer a debt remains outstanding, the less likely it is to be repaid. These realities suggest that a more sophisticated measure of the average collection period may be a weighted average. I propose a

new measure called the weighted days sales outstanding. The *weighted days sales outstanding (WDSO)* can be defined as

$$WDSO = \frac{\Sigma(\text{days until collection} \times \text{debt})}{\Sigma \text{ debt}}.$$

As an example, suppose the XYZ company had 3 customers. In particular, customer 1 had a debt of \$1000, which was repaid in 40 days; customer 2 a debt of \$10,000, repaid in 60 days; and customer 3 a debt of \$100,000, repaid in 80 days. The average collection period (DSO) is (40+60+80)/3=60 days. The weighted DSO is (ignoring the trailing 000's)

$$WDSO = \frac{40 \times 1 + 60 \times 10 + 80 \times 100}{111}$$
$$= 77.84,$$

a figure that more accurately reflects the importance of the \$100,000 outstanding debt.

The DSO and WDSO figures can be used to provide a crude estimate of the reduction in profits caused by delayed payments. If $p\%$ is the profit expected, $c\%$ is the collection expenses per \$1 outstanding, and interest cost is $i\%$, the resulting adjusted profit due to the late payment is

$$\text{adjusted profit} = p - c - i \times \frac{\text{DSO}}{360}.$$

If XYZ had a profit margin of 15 percent and a collections expense of 5 percent and borrowed funds at 9 percent, then its adjusted profits would be 8.5 percent ($0.15-0.05-0.09\times60/360$) using DSO or 8.05 percent ($0.15-0.05-0.09\times77.84/360$) using WDSO.

Zuckerman [22] and Marshall [13] discuss ways in which companies can reduce their DSO and WDSO figures. In particular, Zuckerman recommends that companies charge interest on overdue accounts, while Marshall suggests a judicious use of credit discounts to enhance cash flow.

2.4 Bad Debts

Most business credit sales are eventually paid, while a very small proportion of debt is never paid. A debt that is long overdue is called a *bad debt*. Typically on a 30-day net invoice, 60 days past due may be considered a bad debt. When accounts are more than 90 days overdue, they are called *slow pay* and are usually referred to a collection agency. Biggar [2, p. 83] suggests a rule of thumb for deciding when to send an overdue account to a collection agency: delay sending the account until the gross profit (that was anticipated by the sale) has been exhausted.

Bad debts occur for a variety of reasons, including: (1) disputed billings due to a wrong price charged, defective goods delivered, late delivery, and so on; (2) the creditor has an ineffective credit department; (3) the debtor has a lax attitude toward paying off debts; (4) the debtor is experiencing temporary cash-flow problems; (5) the debtor is in the process of declaring financial insolvency or bankruptcy; or (6) fraud, that is, the debtor has no intention to repay the debt.

Not all bankruptcies are due to genuine financial hardships. Sometimes businesses may use bankruptcy courts for purposes other than protection from creditors, for example, to obtain wage cuts from strong labor unions or to protect them from potential liability lawsuits; see Platt [18] and Wise [21]. Paris [16], Kaiser [11] and Lawson [12] describe the many ways that bankruptcy laws can be used to the disadvantage of some or all creditors. It is not unusual for it to take several years to recover any outstandings through the bankruptcy courts.

The incidence of bad debts can be reduced if the creditor has a well-run credit department. Such a department must have clearly defined credit policies and goals with respect to assessing credit risks, setting credit limits and collecting outstanding debts. Credit information can be obtained through business credit-reporting agencies such as the Dun & Bradstreet Business Credit Services, the Business Credit Reporting Service of the National Association of Credit Management or TRW. Credit limits must be set at levels that, based on the customer's credit report, the customer can comfortably repay. Setting too high a credit limit inevitably leads to bad-debt situations, while setting limits that are too low can stifle sales and economic growth. Recall Fiedler's comments in the Introduction to this paper.

2.5 Collections

When an account is overdue, the creditor has the legal right to attempt to collect the overdue debts. In pursuing these debts, the creditor must employ professional, moral and ethical tactics. The creditor should always keep in mind the objectives of collections: (1) to recover the company's overdue receivables as quickly and as economically as possible, and (2) while pursuing (1), to not alienate the delinquent customers to such an extent that they stop doing business with the creditor. There are numerous books on the subject of credit and collections, including Pierce [17], Biggar [2], Mills [15], Bancroft [1], and Bond [3]. An excellent description of the services provided by credit reporting agencies is given by Cole [6]. In addition, Cole's text provides an excellent overview of the entire credit industry.

3. BUSINESS CREDIT INSURANCE

3.1 What Is It?

As I pointed out in the Introduction, a very large proportion of trade among businesses is done on the basis of credit. Even though most firms are careful in choosing the customers to whom credit is to be extended, they know that there will be normal bad-debt losses. Businesses make allowances for this by factoring their expected losses into the cost of doing business. However, in some cases this may not be adequate. Some bad debts or potential bad debts may be so large that they pose a risk to the viability of the creditor. When the credit risk is great, the firm may opt to transfer some of the risk through business credit insurance. As Simms [19] points out, this is especially true in tough economic times.

Cole [6, p. 469] defines business credit insurance as follows:

Definition 2: *Business credit insurance is an arrangement between an insurance company and a business under which the insured firm is guaranteed payments against abnormal credit losses arising from the failure of business debtors to pay their obligations to the insured.*

Note that business credit insurance differs from customer credit insurance because, in the case of the latter, the event insured against is the disability or death of the debtor; see Fagg [8].

Business credit insurance is indemnity insurance; that is, it compensates the insured for a covered loss. The amount of compensation depends on (but never exceeds) the size of the loss. This is in contrast to life insurance, which is contingency insurance, that is, it pays a predetermined amount on the occurrence of the insured event (for example, death). Another feature that distinguishes indemnity insurance from contingency insurance is subrogation. After paying a claim under an indemnity policy, the insurer has the legal right to receive all rights of the insured against third parties; that is, the insurer has the right to salvage any values that may accrue after paying the claim. This prevents the insured from recovering more than was lost. Note that subrogation is different from collections, the latter being a *service* provided by insurers for a fee.

3.2 Types of Policies

Typically, a business credit insurance policy is a short-term contract lasting 12 months. Vaughn [20, p. 558] divided these policies into two basic types: forward and back cover. Policies that are written to cover losses arising from defaults occurring during the policy term, regardless of the time of sale, are called *back coverage* policies. On the other hand, policies issued to cover losses due to sales occurring during the policy term, regardless of when default occurs, are called *forward coverage* policies.

Policies can further be distinguished by the type of accounts insured. Three types of coverage are common: "specific" coverage, "spread" or "general" coverage, and "datum line" coverage. *Specific coverage* pays for losses involving specific customers of the insured and is usually limited to a few accounts. Creditors use this coverage when the outstanding balances of only a few customers represent a serious risk to their financial health. It is generally used by businesses that deal with only a limited number of buyers. Under this type of coverage, the insurer can cancel coverage on future shipments to any debtor if the insurer has information that the debtor's financial situation has deteriorated. Because the creditor selects the accounts to be insured, there is a distinct possibility for adverse selection against the insurer.

When businesses choose *spread* or *general coverage,* all their customers with a credit rating at or above the level specified in the policy will be insured. Often it is not necessary to investigate individual

customers because the amount of coverage and the quality of accounts covered are determined by reference to business credit ratings, such as those published by Dun & Bradstreet. Most policies are in fact written on the Dun & Bradstreet ratings. Each account offered for insurance must have a capital rating (which shows the net worth of the company, for example, 5A, 4A, ..., BA, BB, etc.) and a credit rating (high, good, fair, limited). Coverage is then determined by a table of business ratings selected by the insured and placed into the contract. Only debtors whose credit ratings fall within the specifications of the contract are covered. As an example, suppose a policy specifies coverage on accounts with a capital rating of BB or higher and a credit rating of "good" or "high" on the Dun & Bradstreet Business Information Reports. If the company subsequently sells to a debtor with ratings "CB, high," the account will not be insured. Cole [6, 373–80] contains an explanation of the Dun & Bradstreet Business Information Reports.

Under *datum line cover*, every individual account must be insured when its outstanding debt reaches a certain level called the datum line. At the start of the policy, every account that exceeds the datum line during the previous 12 months must be offered for insurance for the first year. During the year any other account that passes the datum line must be offered for insurance within 30 days. If accepted, the account remains covered for the remainder of the current policy and for the next policy year. Note that if the account returns below the datum line, it still remains insured.

In business credit insurance, losses are first split into two parts: the nonqualified losses and the qualified losses. The *nonqualified loss* (also called the *primary loss* or *normal loss*) is the amount of normal bad-debt losses usually written off by the firm as a cost of doing business. It is calculated from the firm's prior loss experience or as a percentage of net annual sales using bad-debt ratios for the industry in which the firm operates. A firm with a loss experience that is less than the national average for that line of business is generally considered to be a better risk and as such will demand a lower nonqualified loss. The insurance company and the creditor must agree on the nonqualified loss figure to be written into the contract. The nonqualified loss is not insurable. The *qualified loss* is the difference between the actual loss and the nonqualified loss. If this difference is negative, the qualified loss is defined to be zero.

There are two types of deductibles applied to the qualified loss: the "first-loss deductible" and the "coinsurance percentage." The *first-loss deductible* is the flat amount the insurer deducts from individual or

aggregate qualified losses. This amount must be paid by the insured account. The *coinsurance percentage* (also called the *uninsured percentage*) is the percentage of the total qualified loss on each insured account that the insured must also pay. The percentage is usually applied after the first-loss deductible is deducted. Typically the coinsurance percentage is between 10 percent and 20 percent.

Dawson [7] and Briggs and Edwards [5, chapter 8] describe some common types of cover involving deductibles and coinsurance percentages. As will be seen, some of these are equivalent to the well-known types of cover in traditional insurance/reinsurance, namely, "proportional," "excess-of-loss" and "stop-loss" covers; see Bowers et al. [4, chapter 13.4, 13.5) for a description of these traditional forms of cover.

The following notation will be useful in describing the characteristics of the losses on an account. If the firm has n accounts insured, then for its i-th account, let X_i be the size of the loss, Q_i be the amount of the X_i that is qualified under the terms of the policy, and Y_i be the loss paid by the insurer, $i = 1, 2, \ldots, n$. Also, the coinsurance percentage is α and d is the first-loss deductible. The total losses (qualified and nonqualified) incurred on all accounts by the insured business is $X=\sum_{i=1}^{n} X_i$, the total qualified loss is $Q=\sum_{i=1}^{n} Q_i$ and the total loss paid by the insurer is $Y=\sum_{i=1}^{n} Y_i$.

Individual First-Loss Cover: The d is applied to each loss and the insurer pays amount Y_i on the i-th claim where

$$Y_i = \begin{cases} 0 & \text{if } Q_i \leq d \\ (1-\alpha)(Q_i - d) & \text{if } Q_i > d, \end{cases}$$

and d is called the first loss.

Aggregate First-Loss Cover: Unlike the individual case, d is applied to the aggregate qualified losses giving

$$Y = \begin{cases} 0 & \text{if } Q \leq d \\ (1-\alpha)(Q - d) & \text{if } Q \geq d. \end{cases}$$

This is a combination of a stop-loss and a coinsurance arrangement. Here d is called the aggregate first loss. Because the insurer's payments are based on the aggregate claims, the individual Y_i's are no longer defined with respect to the insurer.

Threshold Cover: All small losses are individually excluded, so

$$Y_i = \begin{cases} 0 & \text{if } Q_i \le d \\ (1 - \alpha)Q_i & \text{if } Q_i > d. \end{cases}$$

Because the firm's loss is reduced if Q_i is just greater than d, the firm may be tempted to inflate its losses beyond d. Here d is called the loss threshold.

Minimum Retention Cover: The policyholder retains the larger of d or the coinsurance amount on each loss. Thus,

$$Y_i = \begin{cases} 0 & \text{if } Q_i < d \\ Q_i - d & \text{if } d < Q_i \le d/\alpha \\ (1 - \alpha)Q_i & \text{if } Q_i > d/\alpha \end{cases}$$

Here d is called the minimum retention.

Whole Turnover: These are very similar to first-loss policies except there is no deductible; that is, $d=0$. When $d=0$, the first-loss covers are all identical.

Catastrophic Cover: As described in Briggs and Edwards [5, 125–27], this type of cover can be used by major companies with very large sums at risk on their business debtors. Specific cover may be time-consuming and expensive to arrange, so the company retains and pays the following losses:

1. All nonqualifying losses, that is, losses that are not appropriate for consideration under the policy
2. All qualifying losses until the aggregate retention limit has been reached
3. Any losses after the maximum liability limit has been exhausted.

The insurer pays the rest. To describe this mathematically, let R_1 be the aggregate retention limit and R_2 be maximum liability limit. Under catastrophic cover, the insurer pays Y, given by

$$Y = \begin{cases} 0 & \text{if } Q \le R_1 \\ Q - R_1 & \text{if } R_1 < Q \le R_2, \\ R_2 - R_1 & \text{if } Q > R_2 \end{cases}$$

while the insured firm pays the difference $X-Y$. Under catastrophic cover, both X and Y are expected to be very large.

While the above descriptions are basic policy types, it should be emphasized that business credit insurance policies are very flexible and can be designed to meet the needs of any business. Also, when designing policies, the insurer relies on the various deductibles and coinsurance payments to encourage the insured to develop sound credit management techniques, which include proper credit screening and reasonable credit limits.

3.3 Collections and Settlements

Some business credit insurance policies include a provision for the collection of past-due accounts. Such policies often contain a fee schedule for the collection service. Usually no fee is assessed if the balance due on the account is collected within a certain number of days, say, 10 days. In the past, fees followed the guidelines suggested by the Commercial Law League of America. Pierce [17, p. 89] gives the following example of a fee structure:

- 15 percent on the first $750 or less collected
- 10 percent on the excess of $750
- Minimum charge $40.

However, the Commercial Law League of America no longer publishes fee guidelines.

The ultimate motivation for a business to obtain credit insurance is for indemnification in the event that a buyer defaults (due to slow pay, bankruptcy or insolvency) on a debt. In most cases there is no doubt about the cause of a loss resulting from nonpayment. However, in every case the insurer must verify the reason for the loss before fulfilling its obligations under the terms of the policy.

Cole [6, p. 480–81] lists several steps the creditor should take after learning of a pending petition for bankruptcy by a debtor; these include the following:

1. Stop all further credit to the debtor.
2. Recover as much of the outstandings as possible. Hold or stop payment on all checks and other remittances to the debtor pending bankruptcy. Stop all goods in transit to the debtor. Stop work on all goods ordered by the debtor.
3. Start foreclosure proceedings on all secured claims.
4. File a proof of claim.

Even before learning of a debtor's petition for bankruptcy, insurers demand prompt notice of any adverse changes in the debtor's financial situation. For example, if a creditor has reason to believe that an insured account is unlikely to be paid off or that the payments will be late due to financial difficulties, the insurer requires immediate notification. In addition, insurers usually reserve the right to cancel coverage on further shipments on an insured account if the insurer believes the debtor has experienced financial difficulties since the account was insured. In the event that coverage is canceled, only losses resulting from prior sales on the account are subject to the terms of the contract. The creditor can continue to ship goods to the debtor but at the creditor's own risk.

4. PREMIUM CALCULATION

4.1 A Note of Caution

So far, only the basic features of business credit insurance have been described. No mention has been made of the pricing of such policies. Even though there are obvious similarities between business credit insurance and other forms of indemnity insurance such as automobile, fire, marine, and so on, there are some marked differences. The most obvious difficulties are those associated with accurately estimating default probabilities and the lack of independence among claims.

The difficulty in assessing the underlying probability of a debtor's default within the contract period is captured by Fiedler [9, p. 11]:

> When we are evaluating the probable incidence of collection difficulties, we need information on the risk-related characteristics embodied in all credit outstanding—information that is rarely available from direct measurement. Part of the problem is that the composition of the stock of credit outstanding is constantly changing. New credit is being granted, and old credit is being paid off. The risk-related characteristics in both the inflow and outflow are not typically the same as in the continuing outstanding stock. These flows, consequently, bring about significant changes in the composition of the characteristics in the stocks of credit outstanding. The changes, in turn, continually affect the risk of collection difficulties.

This difficulty is compounded by the fact that default rates vary with overall economic conditions. In economic recessions, some industries are hit harder than others, making default rates vary by industry. In

addition, regardless of the prevailing economic conditions, each business has certain unique characteristics that affect its probability of default.

It is common for several sellers to sell to the same buyer and for each seller to sell to different buyers. This interaction between buyers and sellers leads to a matrix of transactions in which the default of one buyer (debtor) can affect several sellers (creditors) simultaneously. It also presents another source of problems: *dependent claims*; that is, the default of a single debtor may generate multiple claims (from different creditors). This dependency violates an important actuarial assumption: the mutual independence among claim producers. In addition, the default of one debtor may signal an overall weakness in the debtor's industry and an increased probability of default of other debtors in the industry. As a result, the dependence structure inherent in business credit insurance is reinforced. There are other types of insurance that produce dependent claims, for example, earthquake, crop, and windstorm insurance.

Because the claims produced by creditors are expected to be dependent, one should be wary of using "individual" pricing formulas unless they are used very cautiously and conservatively. Conservative estimates of default probabilities must be used along with a sufficiently high risk-loading. It may be better to place creditors and debtors in homogeneous groups than to price the entire group. Homogeneous grouping is often not possible because creditors usually approach insurers or are approached by insurers on an individual basis.

4.2 The Model

Consider a diverse group of creditors who sell their products to other businesses on credit with the same terms of sale, say, net 30 days. Specifically, assume there are M creditors C_i, $i=1, \ldots, M$, and N debtors D_j, $j=1, \ldots, N$. In consultation with the insurer, each creditor sets a credit limit for each debtor. In particular, there is an aggregate annual credit limit of L_{ij} established between creditor, C_i, and debtor, D_j. To explain what this means, suppose that at any some time during the year, D_j's outstanding balance with C_i is B_{ij}. Suppose also that D_j now orders goods and services from C_i totaling T_{ij}, which is in excess of $L_{ij} - B_{ij}$. It follows that D_j must pay the excess $[T_{ij}-(L_{ij}-B_{ij})]$ in cash before or on delivery of the goods. Thus L_{ij} is the maximum loss from D_j that C_i can experience in a year. The following is the creditor-debtor matrix, L, of credit limits

$$L = \begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \cdots & L_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ L_{M1} & L_{M2} & \cdots & L_{MN} \end{pmatrix}.$$

For simplicity, it is assumed that no credit arrangement is hidden from the insurer and that each L_{ij} is reviewed and adjusted at the end of each policy year. In addition, if D_j appears to be experiencing financial difficulties during the year, each creditor may decide to reduce its existing credit limit to D_j or to eliminate D_j's credit entirely. If D_j's payment is overdue for more than 90 days, further shipments to D_j cease, and the account is given to the insurer for payment and collections. The insurer pays the claim according to the policy provisions; see Section 2.3 above.

If the insurer knows that debtor D_j has other forms of financial indebtedness but with creditors who are not covered by this insurance and the insurer does not have access to these other arrangements, the insurer should take this information into account and increase the default probability of D_j and assume that, in the event of D_j's default, the amount of monies recovered by the insurance company will necessarily be reduced. Other calculations should not be affected.

Insurance is assumed to be offered on a 1-year forward cover and spread cover basis. All accounts are offered for insurance. Each account is scrutinized to determine whether the debtor satisfies the minimum capital and credit rating (on the Dun & Bradstreet ratings) specified in the policy. If the debtor's rating subsequently falls below the minimum rating specified in the policy, insurance on all further shipments to the debtor is immediately canceled. Note that the minimum rating may vary according to the debtor; for example, for D_1 it may be (CC, good), while for D_2 it may be (BB, fair).

The account between C_i and D_j ($i=1, 2, \ldots, M$ and $j=1, 2, \ldots, N$) is assumed to be insured on a first-loss deductible basis (as described in Section 2.2 above) with a deductible of d_{ij} and a coinsurance percentage of α_{ij}. Without loss of generality, the *minimum retention* cover will be assumed. The ideas and techniques used can easily be extended to the

other types of cover. The normal loss on credit to D_j from C_i is assumed to be proportional to the credit limit L_{ij}; that is,

$$\text{normal loss} = \eta_{ij} L_{ij} \tag{1}$$

where the η_{ij}'s are a sequence of known constants with $0<\eta_{ij}<1$. Let X_{ij} be the actual loss experienced by C_i due to the default of D_j; note $X_{ij} \leq L_{ij}$. The total qualified loss, Q_{ij}, is given by

$$Q_{ij} = \text{Max}(0, X_{ij} - \eta_{ij} L_{ij}). \tag{2}$$

The amount paid by the insurer, Y_{ij}, is given by

$$Y_{ij} = \begin{cases} 0 & \text{if } Q_{ij} \leq d_{ij} \\ Q_{ij} - d_{ij} & \text{if } d_{ij} < Q_{ij} \leq d_{ij}/\alpha_{ij} \\ (1 - \alpha_{ij})Q_{ij} & \text{if } Q_{ij} > d_{ij}/\alpha_{ij}. \end{cases} \tag{3}$$

To complete the description of the model, let q_j denote the probability that D_j defaults on payment for goods/services sold during the insurance contract period. It is assumed that if D_j defaults on any of its creditors, all creditors immediately cancel all further credit and shipments to D_j. This assumption is not unreasonable given the obvious risks associated with further sales to D_j. Of course, creditors can continue to ship goods to D_j but at their own risk; these shipments will not be insured.

Even though the insurance period is 12 months, defaults can occur up to 120 days beyond the end of the year, assuming D_j receives an invoice on 30 days net terms and the creditor sends the account to the insurer for collections and payment (on a slow pay basis) after it is 90 days overdue.

4.3 The Gross Premium

In business credit insurance, gross premiums are typically derived with reference to a table of premium rates. These rates vary by the type of industry involved and by the size of the anticipated sales (and coinsurance percentage) during the 12-month period of the contract. These rates can be further refined for each particular creditor. For example, for a business in the leather and leather products industry with expected sales of $3 million, the initial gross may be determined from a table of rates as follows: 0.5 percent of the first $1 million in sales, 0.410 percent of the next $1.5 million, and 0.325 percent of the final $0.5 million. The initial gross premium is thus

$$(0.5 \times 1 + 0.410 \times 1.5 + 0.325 \times 0.5) \times 10{,}000 = \$12{,}775$$

The final premium is determined after considering the competing rates, reinsurance costs and the creditor's own loss experience.*

In practice, the gross premiums used in business credit insurance are generally not grounded in any particular actuarial theory. Because it is such a small and highly specialized line of business, it is thought that there is no need to use actuarial methods. However, as the business expands, there may be a greater need for a more actuarial approach to pricing.

Casualty actuaries use two basic methods to derive gross premiums: the pure premium method and the loss ratio method; see McClenahan [14]. Due to the short-term nature of the contracts, these pricing methodologies usually ignore interest; that is, claims and expenses are not discounted.

The following notation will be used for the insured account representing the credit from C_i to D_j, $i=1, 2, \ldots, M$, and $j=1, 2, \ldots, N$

G_{ij} = gross premium
μ_{ij} = $E[Y_{ij}|D_j$ defaults]
q_j = probability D_j defaults on payments to any of its creditors
f_{ij} = fixed expenses incurred on insuring
v_{ij} = variable expenses factor on G_{ij}
r_{ij} = risk (contingencies) factor on G_{ij}
p_{ij} = profit factor on G_{ij}.

4.3.1 The Pure Premium Method

The pure premium is $\mu_{ij}q_j$. From McClenahan's Equation (5),

$$G_{ij} = \mu_{ij}q_j + f_{ij} + v_{ij}G_{ij} + r_{ij}G_{ij} + p_{ij}G_{ij}$$

$$= \frac{\mu_{ij}q_j + f_{ij}}{1 - v_{ij} - r_{ij} - p_{ij}}. \tag{4}$$

This gives the total premium paid by creditor C_i as

$$G_i = \sum_{j=1}^{N} G_{ij}. \tag{5}$$

*I am grateful for Angela Taylor, FCAS, for her assistance with this example.

4.3.2 The Loss Ratio Method

This method requires the previous year's gross premium, $G_{ij}^{(0)}$, from which it develops the current gross premium, G_{ij}. Following McClenahan's Equation (6),

$$G_{ij} = a_{ij}G_{ij}^{(0)} \tag{6}$$

where a_{ij} is an adjustment factor. In particular, a_{ij} is the ratio of the "experience loss ratio (w_{ij})" to the "targeted loss ratio (t_{ij})."

Let $Y_{ij}^{(0)}$ denote the actual previous year's losses on the account. It follows from McClenahan's Equations (7) and (8) that

$$w_{ij} = Y_{ij}^{(0)}/G_{ij}^{(0)} \tag{7}$$

$$t_{ij} = \frac{1 - v_{ij} - r_{ij} - p_{ij}}{1 + (f_{ij}/\mu_{ij}q_j)} \tag{8}$$

and

$$a_{ij} = w_{ij}/t_{ij}. \tag{9}$$

The total premium paid by creditor C_i is G_i given in Equation (5) but with G_{ij} given in Equation (6).

4.4 Premium Adequacy Check

After the insurer has individually priced each account to derive G_{ij}, how can the insurer check the adequacy of the *total gross premium?* If the premiums appear to be out of line, how can they be adjusted? I am suggesting that the insurer construct homogeneous (or nearly so) portfolios of insured accounts. Each portfolio comprises accounts grouped according to certain common characteristics such as the same group of debtors, the same line of business or industry, similar policies, and so on. The more homogeneous a portfolio, the lesser the variability in losses. Also, because of the increased homogeneity, the aggregate losses will be approximately normally distributed if there is a large number of insured accounts in the portfolio. This can be justified by appealing to the central limit theorem.

4.4.1 Safety Considerations

Consider a homogeneous portfolio consisting of m creditors and n debtors. Suppose the insurer sets a safety constraint that requires that

there must be a probability of at most ϵ^* that the total portfolio premium is *not* sufficient to cover the total claims, expenses and profits generated by the portfolio. For this portfolio, let

Y = total insurer losses
G = total gross premium
f = total fixed expenses
v = total variable expense factor
p = total profit factor
r = total risk factor
ϵ^* = desired minimum safety probability
ϵ = actual safety probability.

Examples of ϵ^* are 10 percent, 5 percent, 1 percent, and so on.

The actual safety probability must be calculated:

$$Pr[Y + f + (v + r)G \leq G] = 1 - \epsilon, \quad 0 < \epsilon < 1 \tag{10}$$

where

$$Y = \sum_{i=1}^{m} \sum_{j=1}^{n} Y_{ij}$$

$$G = \sum_{i=1}^{m} \sum_{j=1}^{n} G_{ij}$$

$$f = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}$$

$$v = \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij} G_{ij}/G$$

$$r = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} G_{ij}/G$$

$$p = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} G_{ij}/G$$

Equation (10) can be rewritten as

$$Pr\left[\frac{Y - \mu}{\sigma} \leq \frac{(1 - v - r)G - f - \mu}{\sigma}\right] = 1 - \epsilon$$

where $\mu=E[Y]$ and $\sigma^2=Var[Y]$. The actual probability ϵ can now be calculated. Let $\xi(z)$ denote the $100z\%$ percentile point of the standard normal distribution and Φ denote its *cdf*, that is,

$$\Phi[\xi(z)] = z.$$

For a sufficiently large portfolio, the normal approximation can be used. This gives the actual safety probability as

$$\epsilon = 1 - \Phi\left[\frac{(1 - v - r)G - f - \mu}{\sigma}\right]. \tag{11}$$

If $\epsilon \leq \epsilon^*$, then the insurer views the total portfolio premium as adequate. If, on the other hand, $\epsilon > \epsilon^*$, the insurer must increase the premium charged on the accounts making up the portfolio. Of course the insurer always has the option to accept the premium as is and take the increased level of risk. If the insurer decides to increase the premium so that $\epsilon = \epsilon^*$, the total premium must be set to G^* given by

$$G^* = \frac{\mu + f + \xi(1 - \epsilon^*)\sigma}{1 - v - r}. \tag{12}$$

The aggregate premium, G^*, can now be spread across all creditors in the portfolio. There are several ways of doing this. For example, one can use a simple proportional allocation to adjust G_{ij} and derive the new premium G_{ij}^*:

$$G_{ij}^* = \frac{G^*}{G} G_{ij}. \tag{13}$$

Another way is to ensure creditor i's premium is proportional to its pure premium plus fixed expenses. Thus if μ_i and f_i are C_i's pure premium and fixed expenses, respectively, where

$$\mu_i = \sum_{j=1}^{n} \mu_{ij} q_j$$

$$f_i = \sum_{j=1}^{n} f_{ij},$$

it follows that creditor i's gross premium will be

$$G_i^* = \left(\frac{\mu_i + f_i}{\mu + f}\right) G^*. \tag{14}$$

Of course, the insurer can develop its own approach to spreading the premium charge.

4.4.2 The Variance of Aggregate Losses

The key to checking the gross premiums' adequacy is to determine the variance of Y. Because the Y_{ij}'s are not mutually independent, it is better to compute σ^2 by grouping the losses according to debtors. Let

$$Y_{\cdot j} = \sum_{i=1}^{m} Y_{ij}, \quad j = 1, 2, \ldots, n,$$

be the total losses the insurer pays due to the default of D_j, and let

$$\mu_{\cdot i} = E[Y_{\cdot j}]$$

$$\sigma_{\cdot j}^2 = Var[Y_{\cdot j}]$$

and

$$\rho_{\cdot jk} = Corr[Y_{\cdot j}, Y_{\cdot k}]$$

be the correlation coefficient between $Y_{\cdot j}$ and $Y_{\cdot k}$. The correlation $\rho_{\cdot jk}$ is expected to be non-negative because the debtors are in the same industry and are subjected to similar economic conditions. It follows that

$$\sigma^2 = \sum_{j=1}^{n} \sigma_{\cdot j}^2 + 2 \sum_{j=2}^{n} \sum_{k=1}^{j-1} \rho_{\cdot jk} \sigma_{\cdot j} \sigma_{\cdot k} \tag{15}$$

Next, $\sigma_{\cdot j}^2$ needs to be evaluated. Clearly

$$\sigma_{\cdot j}^2 = Var\left[\sum_{i=1}^{m} Y_{ij}\right]$$

$$= \sum_{i=1}^{m} \sigma_{ij}^2 + 2 \sum_{i=2}^{m} \sum_{l=1}^{i-1} Cov[Y_{ij}, Y_{lj}] \tag{16}$$

where

$$Cov[Y_{ij}, Y_{lj}] = E[Y_{ij} Y_{lj}] - E[Y_{ij}] E[Y_{lj}].$$

But for fixed j, the $Y_{1j}, Y_{2j}, \ldots, Y_{mj}$'s are not independent, because if D_j *does not* default, then each of these random variables is zero. However, given D_j *does* default, the $Y_{1j}, Y_{2j}, \ldots, Y_{mj}$'s are assumed to be mutually independent. This gives

$$E[Y_{ij}Y_{lj}|D_j \text{ defaults}] = \mu_{ij}\mu_{lj}$$

$$E[Y_{ij}Y_{lj}|D_j \text{ does not default}] = 0.$$

Thus

$$\begin{aligned} Cov[Y_{ij}, Y_{lj}] &= \mu_{ij}\mu_{lj}q_j - \mu_{ij}p_j\mu_{lj}q_j \\ &= \mu_{ij}\mu_{lj}q_j(1 - q_j). \end{aligned} \tag{17}$$

Substituting Equation (17) into Equation (16) yields

$$\sigma^2_{\cdot j} = \sum_{i=1}^{m} \sigma^2_{ij} + 2\sum_{i=2}^{m}\sum_{l=1}^{i-1} \mu_{ij}\mu_{lj}q_j(1 - q_j). \tag{18}$$

This equation can now be substituted into Equation (15) to calculate the variance of aggregate claims.

5. COMMENTS

Because of the introductory nature of this paper, several actuarial tasks in the pricing process were not dealt with; these include the following:

- Estimating default probabilities
- Estimating the correlations in Section 4.4.2
- Experience-rating policies to ensure "good" and "bad" risks are recognized and treated differently
- Determining the required expense, profit and risk-loading factors to be used when calculating gross premiums
- Setting reserves to ensure claims are paid
- Establishing reserves for unreported losses (IBNR reserves) and for reported but unsettled losses.

Research is needed in each of these areas.

I hope that this paper will arouse actuarial interest in business credit insurance. There is a dire need for actuarial expertise in this field.

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DISCUSSION OF PRECEDING PAPER

MICHEL ROCHETTE:

As an actuary involved solely in business credit insurance since 1992, I have read Dr. Ramsay's article with great interest. It may finally be an official confirmation that this field of operations could represent great opportunities for actuaries. With more and more international trade, there is no doubt that it is a promising area for actuaries.

In the following paragraphs, I outline what is being implemented at my company—an export credit agency—for the pricing and the valuation of short-term business credit insurance. We are also involved in medium-term (more than a year) business, but I limit my comments to the short-term operations.

If we ignore what we label "country risks"—expropriation, transfer of funds, cancellation of export/import permits, war, and insurrection—for North America, Western Europe, and Southeast Asia, we apply similar techniques to price our policies as mentioned in the paper.

Our pricing approach essentially applies what is mentioned under "Premium Adequacy Check" in Section 4.4. It is just easier to apply this technique directly to the pricing of a policy than to use it afterwards to check the adequacy of it.

In terms of matrix L of risks, we price horizontally for a given exporter. However, we do the valuation by considering the whole matrix, and the concentration of debtor risks would be calculated from the sum of each column for a given product. In the paper, it is essentially discussed in Section 4.4.2. In day-to-day operations, it would be difficult to apply this approach to each policy, so we estimate it globally and impose a charge on each policy for this risk as well as for capital investment in the business.

We use this approach because each creditor—exporter in our case—represents a portfolio of many buyers. This is done for marketing and selling purposes. The drawback of this approach is that each underwriter analyzes only his own portfolio of buyers without necessarily considering what the others do.

We essentially apply the collective-risk model approach in *Actuarial Mathematics* (Ramsay's ref. [4]), commonly called risk theory. Contrary to what Dr. Ramsay mentions in the paper, we do not calculate an

individual premium for each buyer as mentioned under "The Pure Premium Method" in Section 4.3.1. We also do not apply the "Loss Ratio Method" as mentioned in Section 4.3.2, because we do not have enough historical data to justify specific loss ratios for the different underwriting categories.

Our model needs two basic assumptions:

- One assumption for the number of claims, the frequency. We are using the Poisson distribution. For the probability of default, we have so far relied on Dun and Bradstreet's default statistics, which are published by industrial sector. According to our own experience studies, we consider them appropriate for the pricing of our short-term policies in the U.S. market; consequently, the product of the average default probability times the number of debtors for a given creditor gives us the average expected number of claims that we use as the value for the parameter in the Poisson distribution.
- One assumption for the value of a claim, the severity. We essentially use what Dr. Ramsay calls the maximum loss—in our case, the credit approval. Depending on the number of creditors for a given exporter, we either use the actual distribution of credit approvals or we try to fit a curve to represent the distribution of them. In most cases, the lognormal is appropriate.

Then we do a simulation of aggregate losses, taking into account the type of cover of the policy, which is, most of the time, an *aggregate first-loss* cover, in addition to using other assumptions for the value of rejected claims and recoveries.

We determine the net premium—risk premium—from the distribution of aggregate losses from the customer portfolio such that the probability that the total losses will be lower than a certain level is at least 95 percent. This is the equivalent to what Dr. Ramsay calls the *desired minimum safety probability*, although, in our case, it is applied only to the net claims. Contrary to what is mentioned in Section 4.4, we do not use the central limit theorem because, most of the time, the lognormal is the appropriate distribution for our portfolios.

Since as a corporation we control administrative expenses, both fixed and variable, we add these assumptions to the previous estimate to determine the desired total gross premium. Then this preliminary gross premium is compared to what the client is willing to pay and what the market can bear.

Finally, I concur that there is a great deal that actuaries could do in this field with the help of other specialists like economists. For example,

an elaboration of a simple model to relate default probabilities to the economic conditions would be extremely appropriate for the pricing and the valuation—both for the policy and the claims liabilities—of this line of business. It is an open field. There are no rules. It is up to us to take the lead and establish them.

(AUTHOR'S REVIEW OF DISCUSSION)

COLIN M. RAMSAY:

I thank Mr. Rochette for his comments on my paper. As an actuary actively involved in this very interesting field, Mr. Rochette has given us a glimpse of some practical aspects of business credit insurance. I hope that our efforts will result in more actuaries exploring this nontraditional area.