

**AN EXCESS SPREAD APPROACH
TO NONPARTICIPATING INSURANCE PRODUCTS**

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ABSTRACT

In the past two decades many North American life insurance companies have sold primarily nonparticipating and investment-oriented products such as single-premium deferred annuities (SPDAs), universal life, single-premium immediate annuities, single-premium whole life, flexible-premium annuities, guaranteed investment contracts, and pension plan termination annuity contracts. Actuaries have sought to measure the true economic profitability of these products by comparing the market value of the offsetting asset portfolios with the market value of the liabilities. The difficulty of this approach is identifying a credible market value of the liabilities.

The excess spread approach introduced in this paper addresses this problem by measuring expected profitability in terms of a spread between the earnings rate on the assets and the rate required to be earned on the assets to satisfy the liabilities. For many insurance products, an important part of the liability cost is the value of interest rate options embedded in the policies. The excess spread approach incorporates an option-pricing technique that is flexible enough to allow actuaries to study the effect of policyholder (and insurance company) behavior patterns in different interest rate environments. In addition to ongoing profitability analysis, the excess spread approach can be used in product pricing and design and in risk analysis.

The SPDA is chosen to demonstrate the technique because it provides the best example of the flexibility of the technique and the importance of behavior modeling. The SPDA involves an option of the policyholder—the surrender option—and an option of the insurance company—the ability to reset credited rates.

INTRODUCTION

Section I presents a very brief summary of the history of asset/liability matching. Also summarized are the SPDA product, which is used as an example, and a mortgage-backed securities (MBS) valuation technique, which is adapted in the next section into the excess spread technique. Section II describes the purpose of the excess spread technique and the steps involved

in the calculations. Section III is an in-depth example of how the excess spread technique can be used to design and price the SPDA and to evaluate different crediting strategies. The example illuminates the capabilities of the technique. Section IV demonstrates how the excess spread technique can be used to measure the various risks of the SPDA product. Section V discusses briefly how the excess spread technique can be used on an ongoing basis to measure both the overall profitability and the different components of profitability. Section VI is a summary.

I. BACKGROUND

A. Asset/Liability Background

The work done by Macaulay [10] in the late 1930s, and Redington [12] in the 1950s, introduced the actuarial profession to the field of asset/liability matching. Their approach to duration matching is still generally accepted today. Numerous authors have since written on how the calculation of the appropriate duration index depends on the assumed stochastic process for changes in interest rates [1],[2].

The increase in interest rate volatility in the 1970s heightened awareness of the value of interest rate options, both those embedded in callable bonds and MBS, and those in exchange-traded option contracts. As a result, considerable time and effort have been spent in developing option-pricing techniques for fixed-income securities [7],[8]. This research has been greatly aided by the ability to observe, in the fixed-income marketplace, prices on option contracts and the effect of embedded options on the price behavior of callable and puttable securities.

The increase in policy loan activity associated with rising interest rates awakened actuaries to the value of the fixed-rate loan option many policyholders had been granted in traditional life insurance policies. Even though interest rate options clearly exist in many insurance products, the development of insurance liability option-pricing techniques has been slow. This is due at least partly to the lack of a secondary market to provide price data for these options.

In 1985, Clancy [4] suggested that the cost of the option package necessary to protect an insurer against the options granted in its policies could be used to quantify the value of these policy options. The adoption of this approach is an important step for actuaries in studying interest-sensitive products. However, because of the nature of some interest-sensitive insurance

products,¹ the “necessary option package” often cannot be easily identified from the universe of fixed-income options available in the marketplace. Also, many of the option-pricing techniques developed for particular asset classes cannot incorporate the complicated behavior functions necessary for some insurance products. Fortunately, valuing one asset class, the MBS, utilizes a rather complicated behavior function involving both the economic and noneconomic exercise of the mortgage-holder’s option to prepay. An option-pricing mechanism developed for the MBS, and described later in the paper, is general enough to be applied to interest-sensitive insurance products and is incorporated into the excess spread technique.

B. The SPDA

An SPDA is an insurance product that acts like a savings account for the eventual purchase of an annuity. The SPDA purchaser makes a single-premium payment that is credited to the policy account value and accrues interest at rates declared by the insurance company. The initial declared rate is usually guaranteed for a period of between one and ten years, after which the rates are reset periodically (usually annually) by the insurance company. Initial guaranteed rates available in the SPDA market move in general with the overall level of rates on assets available to insurance companies in the fixed-income marketplace. The account value accumulates at the declared rates until either the account value is used to purchase some form of life annuity, the policyholder dies, or the policyholder decides to withdraw the account value. Usually, a scale of fixed surrender charges will be applied to the account value if the money is withdrawn within five to ten years of original deposit. In many cases during the surrender charge period, a portion of the account value can be withdrawn free of surrender charges. If the proceeds of the SPDA are used to purchase a life annuity, there is no current taxation of the interest income earned in the SPDA. The policyholder may also surrender one SPDA and buy another without taxation of any of the proceeds. Depending on the policyholder’s current SPDA rate, applicable surrender charges, and SPDA rates available in the marketplace, it may be advantageous for the policyholder to surrender one SPDA and buy another one.

¹The adjective “interest-sensitive” refers to products with a credited rate reset feature and/or possible disintermediation due to changing interest rates.

C. MBS Valuation

The MBS is a pass-through security of principal and interest payments from a pool of residential mortgages. The pool of mortgages will have very similar coupon rates as well as time remaining to maturity. The majority of MBS comprise 30-year amortization fixed-rate mortgages. Typically, the MBS carries a guarantee of timely payment of principal and interest from an agency of the U.S. Government. Individual mortgage holders ordinarily have the right to prepay their mortgages at any time without penalty, regardless of the level of current interest rates. Therefore, many prepayments result from interest rates falling and mortgage holders refinancing into lower-rate mortgages. Prepayments as well as scheduled interest and principal amortization amounts are received by the holders of the MBS.

The prepayment right of mortgage holders is identical in nature to a call option on an amortizing bond. However, option valuation models developed for callable corporate bonds and option contracts on bonds are not well suited for use directly on MBS for two reasons:

- (i) Cash flows from MBS are generally recognized to be path-dependent. At a given time, the expected cash flow from a particular MBS depends on the level of interest rates during the life of the underlying mortgages.
- (ii) The prepayment rights held by the mortgage holders are exercised with only partial efficiency. Homeowners often prepay their mortgages for reasons other than refinancing with a cheaper mortgage. Also, some homeowners will not prepay when their mortgage carries a much higher than current rate and it would seem advantageous to refinance.² In general, however, prepayments on MBS with higher rates than current coupon MBS do tend to increase as interest rates go down.³

These mortgage-holder behavior characteristics of the MBS make it necessary to use a Monte Carlo cash flow simulation technique to most accurately value the security [9]. MBS valuation can be organized into four steps:

1. A set of possible future Treasury interest rate paths that do not permit riskless arbitrage is calculated. (For brevity "Treasury" is used hereafter in this paper to refer to Treasury bill, Treasury note, or Treasury bond as appropriate.) The mean of future rates is implied by the initial Treasury term structure, and changes are assumed to be lognormal. Along each interest rate path, at each point in time, short-term (usually 90-day) rates are calculated as well as at least one other longer maturity

²The underlying mortgages ordinarily contain a "due-on-sale" clause that forces repayment when the property is sold. Any mortgages that default will effectively be prepaid by the guarantor. Also, if the property value has fallen, it may not be possible for the homeowner to refinance due to loan-to-value limitations.

³The term "current coupon MBS" is used to describe MBS that would be securitized from currently originated mortgages.

(say five years). Given the assumed volatility and correlation of 90-day and five-year rates, normal deviates can be generated that allow calculation of the required yields at each point in time along each interest rate path. The set of paths calculated represent a finite sample from the underlying distribution. To ensure that the arbitrage conditions are met (within tolerable limits) by this finite sample, some adjustments have to be made.

2. Along each interest rate path, the expected cash flows from the MBS are calculated. Considerations in projecting cash flows include: the coupon rate of the MBS relative to current coupon MBS in that interest rate environment, the proportion of the original principal of the pool that remains, seasonality, geographic location, and other factors.
3. The option-adjusted spread (OAS) is calculated to be the spread that, when added to the short-term Treasury rates along each path, will discount the MBS cash flows to its market price.
4. Once the OAS has been calculated, the option-adjusted duration of the MBS can be determined. This is done by shocking the initial Treasury yield curve both up and down. Typically, the shocks are parallel shocks to the forces of interest, but non-parallel shocks could be used if desired. For each shock, a set of interest rate paths is calculated as in Step 1. For each shock, the expected market price is determined by adding the OAS to the short-term Treasury rates and discounting the cash flows to get the new market values. Duration can then be calculated as negative 100 times the relative market value change for a change in interest rates of 1 percent. If desired, convexity can also be calculated from the set of market values.

The purpose of the OAS calculation is to provide a measure of the relative value between MBS with different coupon rates, times to maturity, and market prices. The OAS reflects the expected value of the mortgage holder's call option more accurately than a spread calculated by using a static prepayment assumption. The option-adjusted duration is calculated to measure the expected price effect of small changes in Treasury rates.

In some respects the SPDA is similar to the MBS. Both instruments can be "terminated" at any point by the exchange of a fixed percentage of the "account" value, although in both cases this option is not always executed efficiently. In fact, the typical Canadian mortgage is in some ways more similar to the SPDA because it involves resetting the rate, usually every five years [3].

II. THE EXCESS SPREAD TECHNIQUE

If there were an identifiable market value of insurance liabilities, an option-adjusted spread on the liabilities could be calculated by a method very similar to the MBS method. The OAS on the liabilities could then be

compared to the OAS on the supporting assets. Also, a market value of liabilities, when subtracted from the market value of assets, would give the market value of surplus. The market value of surplus could be calculated periodically to measure the true economic profitability of the insurance product.

However, in the vast majority of cases there is no unambiguous market value of insurance liabilities, so another approach must be adopted. The excess spread technique uses a known quantity, the market value of assets, to calculate the required spread on assets (RSA). The RSA is the spread over Treasuries that must be earned on the assets in order to satisfy the liabilities. The RSA is calculated as follows:

- a. Calculate the market value of the asset portfolio as of a certain date. At the point of product pricing and design, the market value of assets is simply the premium assumed to be received on the product, less up-front expenses.
- b. Calculate the Treasury forward rates as of the same date.
- c. For interest-sensitive liabilities, develop a set of Treasury interest rate paths (as described above in Step 1).
- d. Calculate liability and expense cash flows. For non-interest-sensitive liabilities, this will be simply a vector of cash flows corresponding to different points in time. For interest-sensitive liabilities this will be a matrix of cash flows, a cash flow at each future period of time along each interest rate path.
- e. Determine the spread that, when added to the corresponding Treasury rates (vector or matrix), will discount the liability and expense cash flows (vector or matrix) to the market value of assets. This spread is the RSA.

The profit target, or any expenses that are expressed in terms of spread, can be added directly to the RSA. If the profit target has a different form, it must be incorporated into the liability cash flows. In this paper, the profit target is expressed as a spread and is known as the excess spread target. In addition, provision for the credit risk of the assets to be purchased must be added to the RSA. The total of these is the total spread target to be used in the selection of the asset portfolio.

Over time, the relative performance of the liabilities and the asset portfolio can cause the excess spread of the product to vary from the original target. The RSA can be recalculated later by using the market value of assets and the projected liabilities at that point. The RSA, adjusted for any effect of expenses, can then be subtracted from the spread above Treasuries being earned on the assets to determine the excess spread.

III. PRODUCT PRICING, DESIGN AND STRATEGY

A. SPDA Pricing

The first step in pricing the sample SPDA product is to calculate a set of arbitrage-free Treasury interest rate paths. For this application each path consists of a short-term (three-month) Treasury rate, as well as a one-year coupon Treasury rate at each time interval (quarterly) over the 24-year period being studied. The set of interest rate paths is used to calculate the RSA, duration, and mean term of the SPDA product shown in Table 1.

TABLE 1
SPDA PRODUCT PRICING

Step Number	Description	RSA (basis points)	Marginal Effect on RSA (basis points)	Duration (years)	Mean Term (years)
1	Five-year Treasury	0		4.0	4.0
2	"Bare bones" SPDA	0	0	4.0	8.9
3	"Bare bones" SPDA (different base withdrawal assumption)	0	0	4.2	10.6
4	Deduct \$1,800 of up-front commissions and \$300 of up-front expenses from market value of assets	73	+73	4.2	10.2
5	Add annual renewal expenses of \$100 to liability cash flows	101	+28	4.2	10.0
6	Credit 50 bp below three-year Treasuries	51	-50	4.2	10.0
7	Collect surrender charges upon policy lapse	39	-12	4.2	10.1
8	Include interest-sensitive surrenders	81	+42	3.0	7.4

Table 1 shows the stepwise pricing of the SPDA product, each line representing one "step." Ordinarily, some of the steps in Table 1 could be combined. However, more steps are shown here to help the reader gain an understanding of the technique and develop intuition for other product features or other applications. For each step, four figures are shown:

1. RSA: The RSA at the time of product pricing can be thought of as the borrowing cost of the liabilities expressed as a spread over Treasuries. The RSA is expressed in terms of basis points per year (1 basis point = 0.01 percent).

2. **Marginal Effect on RSA:** This is the marginal effect on RSA due to the feature introduced in that step.
3. **Duration:** The interest rate duration is calculated as described in Step 4 of the MBS valuation procedure. In this case new present values of the liability are calculated while the RSA is held constant.
4. **Mean Term of Liabilities:** The mean term of liabilities is the Macaulay duration (present-value-weighted time to maturity) averaged over the set of interest rate paths. The RSA is added to the short-term Treasury rates for this calculation. The duration and the mean term will be equal only for products for which there are no interest-sensitive surrenders and the credited rate on the product does not change. The mean term of liabilities measures the persistency of the liabilities. For example, the mean term can be used as the appropriate time period for amortizing acquisition expenses.

The RSA and duration are calculated to provide earnings and duration targets used in choosing an investment strategy and the eventual selection of particular assets. A convexity target may also be calculated. The mean term is calculated to demonstrate that it can be used for expense amortization but not as the target duration for the asset portfolio.

A detailed explanation of each step in Table 1 follows.

1. The insurance company hypothetically issues a current-coupon five-year Treasury bond at its current market price of \$100. Projecting the cash flows from the liability along the interest rate paths is trivial because they are simply the coupons and principal repayment of the five-year Treasury and are independent of the path. In this case, if a spread of 0 basis points is added to the short-term Treasury rates along each path, the cash flows discount to the market price of the assets—hence the RSA is zero. The assets are the single premium received for the policy, which is assumed to be \$30,000. The duration and mean term of this simplified liability are the same, equal to that of the five-year current-coupon Treasury bond. This trivial case demonstrates why following the arbitrage conditions in building the interest rate paths is essential. The model indicates that investing the proceeds from the sale of this product in five-year Treasuries would eliminate interest rate risk (duration match) and provide sufficient earnings power ($RSA = 0$) to back the product.⁴ (Note that there are no expenses and no profit target at this point.) This is the expected result. Sets of paths that do not meet the same criteria can give very different results.
2. This is a “bare-bones” SPDA; there are no expenses, no profit target, and no surrender charges. On this SPDA product the current five-year Treasury rate is guaranteed for the first five years. After five years, the credited rate is reset annually to the then-prevailing one-year Treasury rate. The model assumes an annual “base” surrender rate equal to the credited interest rate. At the end of 24 years any business that remains is assumed to surrender.

⁴In fact, modeling any of the Treasury bonds that were used in setting the initial Treasury term structure would yield an RSA of zero and the correct interest rate duration.

The surrender rate assumption and credited rate assumption have been chosen such that the SPDA product being offered at this point is really nothing more than a five-year Treasury that, when it matures, becomes a series of rolling one-year Treasuries. Initially an asset portfolio of five-year Treasuries will give an earnings and duration match for this product. Thus the model would be expected to give an RSA that is zero and a duration that is the same as the five-year Treasury bond, which it does. The mean term of liabilities is longer because the business is “on the books” for a much longer period due to the reset procedure. This example clearly demonstrates that for a product with a rate reset like the SPDA, the duration (sometimes known as the price sensitivity duration) is the proper measure to use for asset management purposes, not the mean term of liabilities.

3. This is the same “bare-bones” SPDA product, but with a base surrender assumption of 6 percent per year for the first ten years and 10 percent per year thereafter. In this example, this change results in no change in the RSA, a small change in the duration, and an increase in the mean term of liabilities due to an overall reduction in surrender rates. Often, such a change in the base surrenders also has a small effect on the RSA.
4. The first of a number of pricing steps, this step includes the effect of deducting \$1,800 of commission expense and \$300 of issue expense from the \$30,000 of single premium. A positive spread must now be added to short-term Treasury rates to discount the liability cash flows to the new, lower market value of assets (\$27,900). The increase in RSA of 73 basis points means that an additional 73 basis points above Treasuries has to be earned to recoup the up-front expenses. A shortcut method for estimating this number is very useful. First, the up-front expenses are expressed as an average percentage of the single premium, $7.25\% = \$2,100 \div (\$30,000 - 0.5 \times \$2,100)$, and then in effect they are amortized over the average mean term of the liabilities $[(10.6 + 10.2) \div 2 = 10.4 \text{ years}]$, to get 70 basis points. The RSA is expressed on a bond-equivalent or semiannual basis, so the 70 basis point estimate should be adjusted from a force of interest to a bond-equivalent rate, giving 73 basis points.
5. This step shows the effect of adding an annual renewal expense of \$100 per policy.
6. The assumption that five-year Treasury rates would be credited initially and one-year Treasury rates credited on all the reset dates was just a starting point for demonstrating the model. In fact, the insurer may decide to credit 50 basis points less than Treasuries both initially and on all reset dates. Step 6 shows that the RSA calculation produces the expected effect when this change is made: a cost reduction of 50 basis points. To the extent that there are significant liability cash flows not dependent on the credited rate, such as renewal expenses, this relationship may be less exact.
7. One feature that makes the product “cheaper” is that, in the early years, surrender charges are collected when policies are surrendered. This SPDA policy has a surrender charge scale of 6 percent, 5 percent, 4 percent, 3 percent, 2 percent, and 1

percent for the first six years, and then 0 percent for the remainder of the life of the policy. The policyholder has a return of principal guarantee. Step 7 shows the effect of collecting these surrender charges.

8. In Step 8, interest-sensitive surrenders are introduced. It is assumed that whenever the credited rate falls too far below prevailing new money rates for SPDAs (assumed to be 50 basis points below one-year Treasuries), there will be interest-sensitive surrenders in addition to the base surrenders. The policyholder is presumed to make his or her surrender decision based on the ability to recover surrender charges over a three-year period. For example, in the first year, when the surrender charge is 6 percent, it would take a spread between the policy's credited rate and new money rates of at least 2 percent ($6\% \div 3 = 2\%$) before interest-sensitive surrenders would begin to occur. Interest-sensitive surrenders are assumed to be four times the square of the rate gap that exists beyond the surrender charge amortization threshold (expressed as a percent). So, if a rate gap of 1 percent exists, annualized interest-sensitive surrenders of 4 percent are assumed to occur. A rate gap of 2 percent would cause annualized interest-sensitive surrenders of 16 percent. Interest-sensitive surrenders are capped at an annualized rate of 50 percent.

In some SPDA products, a portion of the account value can be withdrawn each year free of surrender charges (this feature is not included in this example). This free-surrender portion of the business may therefore exhibit different surrender behavior in the first six years than the remainder of the account value on which surrender charges would be levied and should be modeled accordingly.

Introducing interest-sensitive surrenders into the cash flows is how disintermediation risk can be incorporated into product pricing and asset management targets. The increase of 42 basis points in the RSA shows that there is definitely a "cost" to disintermediation. Taking account of interest-sensitive surrenders also results in a lower duration. Also, increased surrenders lead to a lower mean term of liabilities.

There are really two components to the marginal cost of 42 basis points. One component is the option cost, which is the effect of higher surrenders in higher-interest-rate environments. The second component is the expense amortization effect, which is the effect of amortizing the up-front expenses over a shorter time due to interest-sensitive surrenders in higher-interest-rate environments.

B. SPDA Design and Insurance Company Strategy

The excess spread technique is very useful in addressing product design and insurance company strategy questions. The effect on RSA of different combinations of commission, surrender charges, and credited rate levels can be tested. It may be possible to calculate a number of essentially "RSA neutral" combinations of product features. For example:

Product Description			RSA (basis points)	Duration (years)
Commission	Surrender Charge Scale	Initial and Renewal Spread below Treas. (basis points)		
6%	6%,5%,4%,...	50	81*	3.0*
7.25%	7%,7%,6%,5%,4%,...	50	81	3.2
5.5%	7%,7%,6%,5%,4%,...	25	80	3.4

*This is the product priced in Table 1.

The calculations above assume that sales volume does not change for the different product descriptions. The effect on RSA of different sales volumes resulting from more or less aggressive credited rates also could be tested. Higher volumes of sales would decrease initial and periodic expenses to some extent, but these savings would presumably be at least partly offset by the more aggressive credited rates necessary to generate that volume.

The rate resetting strategy is also a very important aspect of insurance company strategy. Rather than resetting the credited rate completely to one-year Treasuries minus 50 basis points (new money rate) on every reset date as shown in Table 1, a strategy could be adopted of changing the rate by a fixed percentage of the difference between the new money rate and the previous credited rate. By using the excess spread approach, more dynamic strategies can also be tested; one is shown in Table 2.

TABLE 2
DIFFERENT CREDITED RATE RESET STRATEGIES

Resetting Strategy	RSA (basis points)	Duration (years)	Mean Term (years)
Static Resetting Strategies			
Reset completely to the new money rate (pricing assumption shown in Table 1)	81	3.0	7.4
Reset to: two-thirds of the new money rate plus one-third of the previous credited rate	84	3.2	7.3
Reset to: one-third of the new money rate plus two-thirds of the previous credited rate	98	3.5	7.0
Dynamic Resetting Strategy			
Reset to: the new money rate if rates go down, but if rates go up, reset to the mean of the previous credited rate and the new money rate	71	3.1	7.0

The first three lines show that for this sample product and surrender behavior assumption, following new money rates less closely gives a longer interest rate duration but a shorter mean term. Following new money rates less closely makes the policy more like a longer-term fixed-rate financial instrument and produces a longer interest rate duration. However, this causes the business to run off the books more quickly (shorter mean term) due to higher interest-sensitive surrenders. In this situation, completely following new money rates produces a lower RSA than resetting rates one-third or two-thirds of the way towards new money rates.

At the bottom of Table 2, a dynamic resetting strategy that lowers the RSA is shown. The lower RSA of the dynamic strategy is evidence of the value of the insurance company's option—to reset rates differently based on the path of interest rates. With this dynamic strategy the insurer is more selective about changes in the credited rate. In effect, the insurer is taking advantage of inefficient policyholder behavior. Depending on the surrender rate function assumed, in rising interest rate environments there may be a point at which the savings of crediting less to those policyholders who stay more than offsets the cost incurred by those who surrender.

However, the strategy with the lowest RSA may not necessarily be the best strategy for two reasons:

- (i) Different asset OASs may be achievable at different sales volumes and at different durations. For example, higher asset spreads may be achievable for lower sales volume through the ability to put a larger proportion of the asset portfolio into attractive, but scarce, high-yielding private placements. Also, different asset spreads may be available at different durations simply as a market phenomenon of the public and/or private debt market.
- (ii) The profit goal may be to maximize total excess spread and not excess spread per dollar of business. In that case, the total excess spread of different strategies should be compared by using the product of the excess spread and the sales volume. Surplus considerations will probably also affect the level of sales volume that can be considered.

IV. MEASURING RISK

The risks that affect the SPDA or other products can be determined by measuring the exposure of the excess spread to various risk factors. The change in excess spread caused by the change in a risk factor provides a relative measure of risk that can be compared against the expected profit and also against the other risks that are present. The SPDA product and strategy used in Table 1 are the ones used in the following examples of risk analysis.

A. Interest Rate Risk

To measure interest rate risk, there must be an asset to pair with the liability. Assume the insurance company is able to buy acceptable credit-quality noncallable bonds that have a duration of 3.0 years and a spread of 150 basis points over Treasuries. The bonds are investment grade and the insurance company's credit analysts believe that 5 basis points is the appropriate deduction from the yield for credit risk. Investment expense is assumed to be 14 basis points per year. Also assume that the insurance company has a profit target of at least 50 basis points per year for this product. This asset (ABC bond) gives the following expected profitability picture when used to support the SPDA product:

Spread on Assets	150 bp
– Credit Risk	– 5
– Investment Expense	– 14
– Required Spread on Assets	<u>– 81</u>
Excess Spread	50 bp

It is a coincidence if working back from available spreads gives an excess spread of exactly the insurer's target, as above. Doing the calculation shown in Table 3 and comparing the "available" excess spread to the profit target is a very useful feasibility check of the product.

Interest rate risk can be measured by the effect on excess spread of parallel shocks in interest rates. The first step would be to select the shock levels to use and to calculate the new market value of the asset position at each shock level. For a noncallable bond such as the ABC bond, the market value calculation is relatively easy. The second step is to generate a set of interest rate paths for each interest rate shock, in order to calculate the RSA. This produces the analysis shown in Table 3.

TABLE 3
INTEREST RATE RISK

Change in interest rates	– 3%	– 1%	0%	+ 1%	+ 3%
Market value of assets	\$30,534	\$28,751	\$27,900	\$27,081	\$25,516
Spread on assets	150 bp	150 bp	150 bp	150 bp	150 bp
– Credit risk	– 5	– 5	– 5	– 5	– 5
– Investment expense	– 14	– 14	– 14	– 14	– 14
– RSA	<u>– 97</u>	<u>– 84</u>	<u>– 81</u>	<u>– 83</u>	<u>– 146</u>
Excess spread	34 bp	47 bp	50 bp	48 bp	15 bp

Duration is a measure of price sensitivity to small changes in interest rates. Table 3 is a good example of how a simple duration-matching strategy can give good results within a certain range of interest rate changes, but less acceptable results outside the range.

Changes in interest rates are generally thought of as changes in Treasury rates. The excess spread technique also allows the study of the effect of changes in bond spreads only, not Treasury rates. In general, if the change in spreads is an overall marketplace phenomenon, the effect will be the same as the identical change in Treasury rates. A change in spreads for any other reason, such as a change in credit classification, can have a very important effect on the excess spread. The assumed level of future interest rate volatility in the set of interest rate paths is a very important assumption. The sensitivity of results to changes in the volatility assumption should be tested. A number of other risks can also be studied by using the excess spread technique [5], [11].

B. Policyholder Surrender Risk

The sensitivity of the RSA to different surrender assumptions should be tested. In pricing the SPDA product, an assumption about policyholder surrender behavior was made by using both a base and interest-sensitive component. Changes both in the base component and in the interest-sensitive component should be tested. The sensitivity of the results to changes in the interest-sensitive component could be tested by simply taking a multiple (say 0.5 and 2.0) of the assumption used in Step 8 of Table 1.

However, it is often interesting to test some different types of surrender functions, such as those shown in Figure 1 and Table 4.

Sometimes the results of testing different surrender behaviors can be surprising. Note that interest-sensitive surrenders represent policyholder exercise of a very long-dated option. Therefore, quick exercise of the surrender option as soon as it is slightly advantageous to the policyholder (such as assumption C) does not always lead to the most expensive (highest RSA) result. The three assumptions shown all have mean terms that are very close, thereby removing the expense amortization effect from the comparison.

There may be as many different possible surrender rate functions as there are actuaries. One possible approach not shown is to start the analysis with two groups of policyholders, a "hot-money" group and a "cold-money" group, and apply different surrender functions to each.

FIGURE 1

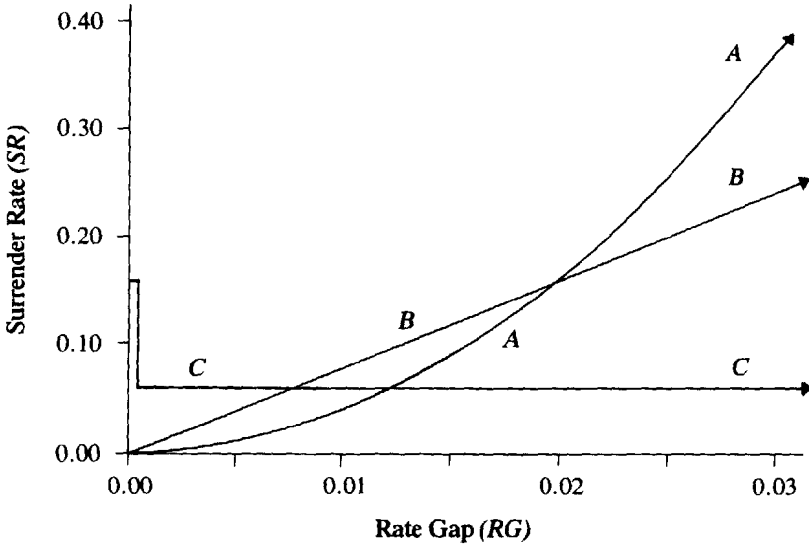


TABLE 4

SURRENDER FUNCTIONS

ICR = Insurance company's credited rate

CCR = Competitor's credited rate (current one-year Treasury minus 50 basis points)

SC = Prevailing surrender charge

RG = Rate Gap = $CCR - [ICR + (SC/3)]$

Assumption	Surrender Rate*	RSA (basis points)	Duration (years)
A (Table 1 line 9)	$400 \times (RG^2)$	81	3.0
B	$8 \times RG$	79	3.1
C	0.06 when $RG > 0$ + 0.10 first time $RG > 0$	69	3.6

*Calculated when $RG > 0$ and always capped at 50%.

C. Expense Risk

Steps 4 and 5 in Table 1 indicate the effect of changing the up-front and ongoing expense assumptions. It may be instructive to calculate the effect on RSA of different-than-assumed per-policy expenses. Per-policy expense misestimation may result from different-than-assumed sales volume for the particular rate crediting strategy or different-than-assumed average policy size.

V. MEASURING EXPERIENCE

The excess spread technique can be used to measure the experience on a block of business at any time after issue. The technique can be used not only to measure the overall experience, but also to determine the respective contributions to the overall performance from a number of sources.

A. Asset Performance

The first comparison is between the spread above Treasury levels at which it was assumed one could invest and the actual spread at which investments were made. The actual initial spread on assets establishes an excess spread for comparison against actual results.

The path that interest rates have taken since the policy was priced or profitability was last studied will be known. To measure the contribution of asset performance to profitability, the actuary does not need to know anything about actual liability behavior—it is presumed that the liabilities behave as assumed, given the interest rate environment that occurred. The market value of the assets is measured at the end of the time that they would have been held if the liability had behaved as assumed. This calculation involves “surrendering” the appropriate percentage of the block of policies based on the assumed surrender rate function and the actual path of rates and then deducting expected profits, expenses, and so on. A new set of interest rate paths based on current interest rate levels has to be generated in order to project the liabilities and recalculate the RSA. The change in excess spread that results from this calculation can be ascribed to asset performance over the year. Among the factors contributing to this change are the movement of rates and the relationship between asset and liability durations. Other possible contributing factors include: any spread change on the assets, the realization of the credit risk charge (unless the assets defaulted or were downgraded), and the effect of a change in the market’s implied volatility outlook if the assets involve options.

B. Liability Performance

The liability performance component of the overall profitability result is measured by the change in RSA due to the use of actual liabilities in the calculation rather than assumed liabilities. In the case of the SPDA the major component of this performance will likely be the degree of surrenders experienced. Depending on the level of interest rates and the surrender charges collected, actual surrenders will have a positive or negative effect on the RSA. Unfortunately, a number of observations of surrender experience in different interest rate and surrender charge environments are required to draw any conclusions about the appropriateness of the originally assumed surrender rate function. For other nonparticipating insurance products, the major component of liability performance is often mortality.

C. Other Contributors to Performance

A number of other factors can contribute to the overall performance of the block of business. The marginal contribution of each is determined by measuring the change in RSA caused by incorporating that aspect of experience into the RSA calculation. For example, the use of actual versus assumed expenses shows the marginal effect on profitability of expense misestimation. Of course, expense misestimation can derive from a number of sources such as misestimation of volume, average policy size, actual costs, or allocation of expenses among lines of business. Expense (and for that matter surrender) experience may also prompt a reevaluation of the assumptions used for future periods. Changing prospective assumptions during the lifetime of a block of business also has an immediate effect on the RSA and hence on expected future profitability.

In the case of the SPDA and similar products, an aspect of insurance company behavior that must be monitored is the resetting of credited rates. Resetting to a higher or lower level of rates than previously assumed will interact with the surrender experience and could cause incremental profits or losses. Any change in rate resetting strategy should be reflected immediately in the RSA calculation.

VI. SUMMARY

This paper proposes a pricing and valuation technique that measures the total cost of an insurance product, including any interest-sensitive features. The technique can be used for product design, product pricing, strategy assessment, and risk analysis. The excess spread technique can also be used

throughout the life of the product to provide an ongoing report card on its economic health. The same methods discussed in this paper for the SPDA can also be used for other nonparticipating products [5], [6].

With the emergence of interest-sensitive products, the study of policyholder and insurance company behavior in different interest rate environments will become an important part of actuarial science. The excess spread approach provides not just a pricing and valuation methodology, but also a tool to measure the economic effect of different behavior patterns.

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DISCUSSION OF PRECEDING PAPER

PHILIPPE ARTZNER*:

Our discussion concentrates on the part related to mortgage-backed securities (MBS), specifically the prepayment risk, and suggests that a difference between model prices and market prices can be explained by the risk of variation of the proportion of prepayment *around* the value forecast by the prepayment function, *in the cases* in which risk cannot be diversified.

This idea is explained on the spreadsheet, which gives an example of a pool of *bonds* callable at par, where the call option can be either unduly exercised (some issuers want to be freed of their debt) or only partially exercised (some issuers neglect the opportunity of cheap refinancing). The case of MBS would differ only by the amortization component.

1	2	3	4	5	6	7	8	9	10	11
1	Date	Date	Rational	Physical	Possible	Conditional	Risk neutral	Possible	Conditional	Conditional
2	$t = 0$	$t = 1$	cash-flows	proportion	proportion	risk neutral	average	cash-flows	cash-flow	average
3	short rate	short rate	to the	of	of	probabilities:	probation	for the	to the	cash-flow
4	$r(t)$	$r(t)$	investor	prepayments:	prepayments:		of	investor:	to the	investor
5							prepayments:		according	investor
6									to the	due
7									physical	to the
8									prepayment	risk neutral
9									function:	variation
10					10%	0.10		1.106173		in prepay:
11										
12		12.00%	0.106173	5%	5%	0.60	4.00%	1.106173	1.094444	1.084321
13										
14						0%	0.30	1.106173		
15										
16										
17										
18										
19										
20	10.00%									
21										
22										
23										
24	Fixed rate δ	for	default-free	bond with	maturity at	date 2				
25	price 1	at date 0,	and call	option	at date 1:		90%	0.60	1.106173	
26										
27										
28	$\delta =$	10.6173%	8.00%	1.106173	60%	60%	0.35	64.50%	1.106173	1.115866
29										1.114776
30							30%	0.25	1.106173	
31										
32	Value P at	date 0	of the	future	cash-flows					
33										
34	according	to the	prepayment	function:						
35		$P =$	1.004687							
36										
37	according	to the	variations	around	the	prepayment	function:			
38		$P =$	1.004135							

The short-term interest rate process is assumed to be 10 percent between date 0 and date 1, and either 12 percent or 8 percent between date 1 and date 2, with *risk-adjusted* probabilities $\frac{1}{2}$ and $\frac{1}{2}$ [2]—[4]; the market will

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then require on a bond issued at date 0, maturing at date 2 and callable by the issuer at date 1, a coupon rate δ given by the relation:

$$1 = \frac{1}{2}[\delta + (1 + \delta)/(1 + 0.12)] + 1 + \delta/1.10,$$

that is, $\delta = 10.6173\%$, which takes into account the call at date 1 that would occur if the interest rate moves to the low value.

Assume now that an econometric study forecasts a *prepayment function* indicating (see Column 5 in the spreadsheet) that the proportion of calls within the pool at date 1 shall be: 5 percent in the high-interest case and 60 percent in the low-interest case. Mr. Griffin's paper, in Section I.C.2, suggests that the variety of cash flows at date 1 (as well as the possible remaining cash flows at date 2) be taken into account as follows (see also Column 10):

$$1.094444 = 0.95[\delta + (1 + \delta)/(1 + 0.12)] + 0.05[1 + \delta]$$

$$1.115866 = 0.40[\delta + (1 + \delta)/(1 + 0.08)] + 0.60[1 + \delta],$$

which finally gives a model value at date 0 equal to $1.004687 = \frac{1}{2}(1.094444 + 1.115866)/1.10$.

Notice that the valuation formulas above are *linear* in the cash flows, which is consistent with the linear pricing principle in finance and with *most* of the chapters in [1]. However, utility theory, as in the first chapter of [1] as well as in its financial counterpart of risk-adjusted probabilities [4], either (1) averages *utilities* of (discounted) cash flows with respect to the actual, statistically amenable, probabilities or (2) averages (discounted) cash flows with respect to a risk-neutral probability, reflected in market prices.

The latter probability will "put more weight on defavorable events" [2] and, as early as 1880, was not unknown in the use of "risk-adjusted" life tables, as mentioned, for example, in [5]. In the case of MBS or callable bonds, it does not allow consideration of the statistician's forecasted average proportion of prepayments (for a given interest rate path) as the proper expected cash flows; here as elsewhere, it is crucial to distinguish between *description* and *evaluation*.

Assume, for instance, that equally probable percentages 10, 5 and 0 are forecast for prepayment in the high-interest case of the example presented, with similar percentages of 90, 60 and 30 in the low-interest case (see Column 6). Adjusting the probabilities of variations around the forecast average proportions (5 percent and 60 percent, Column 5) for this supposedly

nondiversifiable risk (see Columns 7 and 8) gives lower discounted cash flows for date 1, found in Column 11, than those in Column 10:

$$1.094321 = 0.96[\delta + (1 + \delta)/(1 + 0.12)] + 0.04[1 + \delta]$$

$$1.114776 = 0.355[\delta + (1 + \delta)/(1 + 0.08)] + 0.645[1 + \delta].$$

This gives a market value at date 0 equal to $1.004135 = \frac{1}{2}(1.094321 + 1.114776)/1.10$.

Future papers might elaborate upon possible diversification of the risk of variable proportions of prepayment.

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While thanking Professor D. Heath for a useful discussion, the author bears all responsibility for the opinion expressed here.

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HAL W. PEDERSEN* AND ELIAS S. W. SHIU:

Mr. Griffin has written an intriguing paper; however, we are unable to follow it completely. Perhaps the following remarks and questions will bring to light some details of which we are unsure. We restrict most of our comments to the two pages on the valuation of mortgage-backed securities (Section I.C of the paper).

Our main question is whether there is a theoretical foundation for the concept of the option-adjusted spread. Because statements such as "do not permit riskless arbitrages" appear in the paper, we assume that the principle of no arbitrage plays a central role in Mr. Griffin's valuation model.

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To fix ideas, let us describe a general finite-state discrete-time security market model. We assume that trades occur only at the times $t = 0, 1, 2, \dots$. Let $\delta(t)$ denote the one-period risk-free force of interest at time t ; that is, if one invests \$1 at time t , one will receive $\$e^{\delta(t)}$ at time $t + 1$. We also assume that there are a finite number of primitive securities. Let $S_j(t)$ denote the value of the j -th primitive security at time t , and let $D_j(t)$ denote the dividend or interest payment for the j -th security at time t . (We assume that $S_j(t)$ is the value of the security after $D_j(t)$ is paid.) Note that, as seen from time s , $s < t$, $\delta(t)$, $S_j(t)$ and $D_j(t)$ are random variables. It can be shown ([2]–[5], [8], [10], [12], [17]) that the assumption of no arbitrage is *equivalent* to the existence of a probability measure under which the conditional expectation

$$E_t[S_j(t + 1) + D_j(t + 1)]$$

equals $e^{\delta(t)}S_j(t)$, $t = 0, 1, 2, \dots$, and $j = 1, 2, 3, \dots$; that is,

$$S_j(t) = E_t\{e^{-\delta(t)}[S_j(t + 1) + D_j(t + 1)]\}. \tag{1}$$

It follows from (1) that, for each j and n ,

$$S_j(0) = E\left[\sum_{t=0}^n e^{-\sum_{k=0}^t \delta(k)} D_j(t + 1) + e^{-\sum_{k=0}^n \delta(k)} S_j(n + 1)\right]. \tag{2}$$

Now, consider a (stochastic) cash-flow stream, denoted by $\{D(t); t = 1, 2, 3, \dots\}$. If this cash-flow stream can be replicated by the primitive securities, then its value at time 0 is given by the formula

$$E\left[\sum_{t=0}^{\infty} e^{-\sum_{k=0}^t \delta(k)} D(t + 1)\right]; \tag{3}$$

see [2, section 3.5] for technical details. To get a better understanding of (3), we rewrite it as

$$\sum_{\text{all } \omega} \text{Prob}(\omega) \left[\sum_{t=0}^{\infty} e^{-\sum_{k=0}^t \delta(k, \omega)} D(t + 1, \omega) \right]. \tag{4}$$

Here, each event ω can be identified as an interest-rate path or scenario path; $\{\delta(0, \omega), \delta(1, \omega), \delta(2, \omega), \dots\}$ and $\{D(1, \omega), D(2, \omega), D(3, \omega), \dots\}$ are the one-period forces of interest and cash flows along the path. We note that (3) and (4), in different notation, can be found in Tilley's address to the 23rd International Congress of Actuaries in Helsinki [16].

If we have correctly understood Mr. Griffin's concept of the option-adjusted spread, it is the number s such that

$$E \left[\sum_{t=0}^{\infty} e^{-\sum_{k=0}^t [b(k)+s]} D(t+1) \right] \quad (5)$$

equals the market price of the cash-flow stream $\{D(t)\}$. Let us call the value given by Formula (3) or (4) the *model price*. Observe that, if the model price is less than the market price, s is a negative number. Conversely, if the model price is greater than the market price, s is a positive number. If the valuation model is correct, then riskless arbitrages exist by longing MBS with positive spreads and shorting Treasuries, by shorting MBS with negative spreads and longing Treasuries, or by longing MBS with high spreads and shorting MBS with low spreads! We now have a contradiction—the valuation model is based on the principle of no arbitrage, but the existence of a nonzero spread implies a riskless-arbitrage opportunity. The contradiction seems to arise as follows. A model is constructed to value the stochastic cash-flow stream $\{D(t)\}$. The price that comes out from the model turns out to be different from the market price of $\{D(t)\}$. Of course, one cannot say that the market is wrong. Therefore, one has to fudge the model so that the model will produce the market price. Our question is: What is the theoretical foundation for such “fudging”?

Perhaps another way to explain an option-adjusted spread is to say that it is the number s that equates the model price of the cash-flow stream $\{e^{-st}D(t)\}$ with the market price of the cash-flow stream $\{D(t)\}$. Is there a theoretical justification for a nonzero spread? One can always propose a model, parallel to the one in the paper, by picking an arbitrary function $f(s, t)$ and computing a number s that equates the model price of the weighted cash-flow stream $\{f(s, t)D(t)\}$ with the market price of the cash-flow stream $\{D(t)\}$. Is there a theoretical justification for choosing $f(s, t) = e^{-st}$? The principle of no arbitrage dictates that $f(s, t) \equiv 1$.

It is perhaps plausible that a nonzero spread exists because there are uncertainties in projecting cash flows. If uncertainties are the cause for a nonzero spread, then would the spread become zero if one extends the valuation model by incorporating random noise components in the prepayment function? If the spread could be made zero by extending the model, the thesis of this paper might become vacuous. By the way, it seems difficult to explain what a negative spread would mean; we do not understand enough of the paper to see that negative spreads could not occur in such a model.

In practice, it may be difficult to apply Formula (4) to value a path-dependent cash-flow stream. To value an MBS pool, we would probably need 360 time periods, each time period corresponding to one month. If the one-period interest rate process is generated by a binomial model, there are 2^{360} paths. Thus one needs to estimate (4) by means of simulation; that is, one picks a subset Ω' of all paths and calculates

$$\frac{1}{\sum_{\omega' \in \Omega'} \text{Prob}(\omega')} \sum_{\omega' \in \Omega'} \text{Prob}(\omega') \left[\sum_{t=0}^t e^{-\sum_{k=0}^t \delta(k, \omega')} D(t+1, \omega') \right]. \quad (6)$$

Since $2^{10}/2^{360} = 2^{-350}$, a thousand paths are merely a very very small subset of all paths. As the option-adjusted spread s is a relatively small number, is it possible to calculate it with a sufficient degree of accuracy by sampling only several thousand paths? Although we understand that there are variance-reduction techniques such as antithetic variates, we are somewhat skeptical that the spread can be calculated accurately. We certainly would appreciate Mr. Griffin shedding some light on this technical point.

Mr. Griffin seems to say that the short-term interest rate process in his model is lognormal. A problem with a lognormal stochastic process is that the variance grows with time; as the variance becomes large, the probability for very high or very low interest rates becomes substantial. "There is reason to think interest rates are mean reverting, since abnormally high rates will lead to a shift in monetary policy to reduce rates while unusually low rates will lead to a less restrained policy which will lead rates to increase" [1, p. 21].

We are puzzled by the sentence "Given the assumed volatility and correlation of 90-day and five-year rates, ...". In the model above, once the probability measure and the one-period interest rates are prescribed, the other interest rates automatically follow, because the conditional expectation

$$E_t \left\{ \exp \left[- \sum_{k=t}^{n-1} \delta(k) \right] \right\}$$

gives the price of a zero-coupon bond at time t that pays 1 at time n , $n > t$. How are the volatility and correlation prescribed exogenously?

Our next comment concerns duration-matching strategies. A main purpose of the present paper is to propose a methodology for managing interest-sensitive assets and liabilities. To this end, it presents a generalization of Macaulay's duration, which was defined for non-interest-sensitive cash-flow

streams. However, many researchers ([11], [9], [6]) have questioned the effectiveness of duration-matching strategies for the classical case of non-interest-sensitive cash flows, which is, of course, much simpler than the general case of interest-sensitive cash flows. Indeed, Gultekin and Rogalski [7] claim that "business people are wasting time and effort implementing duration-based analysis." It is perhaps also of interest to note the following result in [15]. Consider a company that issues single-premium immediate annuity policies and invests all the premiums that it receives for the annuities in a noncallable and default-free bond. It is proved in [15] that, by matching the asset duration with the liability duration, the company will guarantee itself that it will lose money under any parallel shift of the yield curve.

Our final comment is motivated by the statement that the "excess spread technique uses a known quality, the market value of assets." A substantial portion of many insurance companies' investments consists of private-placement bonds and mortgages, for which it is difficult to obtain market values. Indeed, even corporate bond portfolios are difficult to value accurately ([13], [14]). The spread is a small number; a small change in the asset market value can translate into a big change in the spread value.

We thank Mr. Griffin for a thought-provoking paper.

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(AUTHOR'S REVIEW OF DISCUSSION)

MARK W. GRIFFIN:

I thank Dr. Artzner as well as Mr. Pedersen and Dr. Shiu for their discussions.

I believe that Dr. Artzner's comments and the "main question" of Mr. Pedersen and Dr. Shiu concern the same topic, which I address first. The common question appears to be: If the probability distribution of cash flows (due to credit risk or prepayment uncertainty) is accounted for properly, couldn't one use Treasury rates only (OAS of zero) for discounting and get the proper prices for everything? Despite the conceptual appeal of such a world, if one believes that the cash-flow uncertainty of corporate bonds and MBS is not totally diversifiable, then there should be some compensation for that cash-flow risk. OAS models are widely used in the fixed-income

market to compare the expected compensation for investing in risky securities such as MBS, callable and noncallable public corporate debt, private placements, and even junk bonds. OAS-based models are used as indicators of relative value among securities and typically do not address cash-flow variability around either the expected prepayment or credit risk function.

Mr. Pedersen and Dr. Shiu have gone to great length to express their belief that the valuation model is incorrect due to the existence of riskless arbitrages using MBS. Yet, the discussants themselves point out the uncertainties of MBS cash flows (to which MBS investors will attest). The existence of cash-flow uncertainties in MBS means that there is, by definition, no opportunity to effect a riskless arbitrage using such securities! In fact, the investment strategy of many insurance companies is essentially to use risky securities such as MBS, corporate bonds, private placements, or even junk bonds to outperform a specific maturity Treasury or some other benchmark. Such a strategy is tantamount to a risky arbitrage and should not be mistaken for a risk-free arbitrage.

Mr. Pedersen and Dr. Shiu are skeptical of the accuracy of the OAS calculation because of the reasonably small number of paths used. I suggest they consider the task of estimating the mean of a normal distribution. Although there may be an uncountable infinity of possible outcomes, the mean can be very closely estimated with a reasonably small finite sample. The calculation of the OAS can be thought of as the same type of exercise.

Their points regarding the volatility and correlation of yields for different maturity bonds are generally (but not precisely) valid for a one-factor model. However, I have used a multifactor model in the analysis presented in the paper.

The generalization of Macaulay's duration is referred to in the paper as "mean term." Indeed, references abound on the problems of duration-matching strategies using Macaulay's duration. The paper clearly states, "The mean term is calculated to demonstrate that it can be used for expense amortization but not as the target duration for the asset portfolio." My experience is that the large difference between mean term and duration for a product like the SPDA is not clearly understood by all actuaries and is therefore a very important part of the article. Both the mean term and duration are deliberately tracked through Table 1 to reinforce the difference. I hope this point is clear to other readers.

I am particularly interested in the discussants' reference [15], "On Redington's Theory of Immunization" by Shiu. Shiu shows in his paper that a duration-matching portfolio of noncallable bonds does not have as much

convexity as a particular insurance product and states that this situation guarantees that the company will lose money if one looks at immediate parallel yield curve changes. I find this perspective of profitability to be narrow and dangerous. If one's sole basis for choosing among assets were to look at immediate parallel yield curve changes, with no account of the yield of the different assets, one would conclude that callable bonds or MBS should never be purchased whenever noncallable bonds are available! In reality, the fixed-income marketplace determines the yields at which investors are willing to assume the price performance characteristics of a wide variety of assets. Table 3 in the paper demonstrates how the excess spread approach combines both the price change and yield components of asset performance into a risk management tool.

Mr. Pedersen and Dr. Shiu's final comment provides an opportunity to reinforce an important point. Whenever one is seeking a market value of a particular security at a point in time and there is no publicly available trade information, one must determine a spread relative to Treasuries to use in calculating an assumed market value. Although there is usually good information available to aid in the spread determination, some degree of uncertainty may remain. The excess spread approach uses both the assets' market value and spread over Treasuries in the determination of profitability, so that an error in asset spread estimation will only cause an error in the excess spread measure *to the extent the durations of assets and liabilities are not the same*. For example, suppose a spread over Treasuries of 1.00 percent is used to calculate a "market value" of a bond to be \$100. Suppose a more appropriate spread to use would have been 1.10 percent, giving a "market value" of \$99. For a liability with the same duration as the bond, changing the market value of assets by \$1 will change the required spread on assets by 0.10 percent. Therefore, the error does not affect the excess spread.