TRANSACTIONS OF SOCIETY OF ACTUARIES
1985 VOL. 37

SELECT AND ULTIMATE FINANCIAL ASSUMPTIONS IN PENSION PLAN VALUATIONS: AN ANALYSIS OF THE ISSUES

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I. ABSTRACT

Over the last several years, interest rates have risen and fallen dramatically within very short time periods. Wage increases have also fluctuated considerably. At the same time, actuaries have received increasing numbers of requests for information regarding pension plans. Examples include new accounting standards for reporting pension plan assets and liabilities in financial statements, and multiemployer withdrawal liability calculations. These circumstances have focused increasing attention on financial assumptions used to value pension plans. Greater interest in using select and ultimate financial assumptions for pension plan valuation purposes has resulted.

The objective of this paper is to examine the advantages and disadvantages of using select and ultimate financial assumptions and the technical issues involved. The effect of such assumptions on the incidence of plan cost is studied. The discussion of technical issues and the study of plan cost incidence deal mainly with the interest assumption.

II. ADVANTAGES AND DISADVANTAGES OF SELECT AND ULTIMATE FINANCIAL ASSUMPTIONS

This section will examine the advantages and disadvantages of adopting select and ultimate financial assumptions. Technical issues as well as potential problems and solutions which arise due to such assumptions are considered in a later section.

A. *Advantages*

In recent years, increased attention has been focused on pension actuarial workproducts. The Financial Accounting Standards Board (FASB) has taken a keen interest in plan cost disclosure. The Multiemployer Pension Plan

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Amendments Act of 1980 (MPPAA) can require arbitrators and judges to quantify a plan's vested liability. The Internal Revenue Service (IRS) requires that a plan's minimum contribution and maximum deductible calculations be based on the same set of reasonable actuarial assumptions. It assesses the reasonableness of a matrix of assumptions on the basis of emerging experience. Accordingly, level financial assumptions that are lower than prevailing experience are more likely to be challenged by the IRS regardless of established actuarial practice. Other regulatory agencies such as the Federal Communications Commission and the Defense Contract Audit Agency are also concerned about the use of level interest assumptions that are lower than prevailing experience.

In a volatile economic environment, laymen are sometimes surprised by seemingly anachronistic, long-term actuarial assumptions. Interest rates, salary increases and inflation have typically varied with the business cycle. Until recently, the variation was relatively narrow, and level financial assumptions predominated. In the current environment, the actuary may consider using select and ultimate financial assumptions. His justification would be the phenomenon of the business cycle.

Adopting select and ultimate assumptions can minimize the need to use different sets of assumptions for different purposes. FASB standards apply to plan valuation. The interest rate used for quantifying a pension plan's accrued benefit liability "shall reflect the expected rates of return during the periods for which payment of benefits is deferred and shall be consistent with returns realistically achievable." A select and ultimate approach would satisfy the requirements promulgated by the FASB.

An employer ceasing to have an obligation to make contributions to a multiemployer plan may be liable for an allocable share of the present value of the plan's unfunded vested benefits. Faced with this liability, the employer might challenge the actuarial assumptions on which the plan's determination is based. Recently, opposing parties have advocated disparate points of view on the structure of the interest rate. Such debates might be avoided by using an interest rate assumption that does not appear unreasonable to a withdrawing employer, an arbitrator or a judge. Select and ultimate rates offer some promise here.

Computations to determine minimum funding requirements and maximum deductible limits must be based on the same actuarial assumptions. Such assumptions are held to a general reasonableness standard. Historically, the IRS has assessed the reasonableness of actuarial assumptions on the basis of gains and losses or change in the accrual rate for aggregate methods. The

IRS's assessment criteria have recently been published as a set of actuarial audit guidelines. The IRS instructions contained in the guidelines confirm that the IRS is interested in confronting assumptions that appear unduly optimistic (from a minimum standpoint) or unduly conservative (from a deductibility standpoint). Using select and ultimate assumptions might be helpful in diffusing this potential controversy.

Perhaps the greatest advantage of using select and ultimate financial assumptions is the opportunity to enhance communications with plan sponsors, accountants, participants and others interested in the actuary's results. Increasing public awareness and the volatile economic environment have resulted in closer scrutiny of pension plan valuation. Using select and ultimate assumptions may enhance the actuary's credibility as well as satisfy professional and legal responsibilities.

Select and ultimate interest rates can provide greater consistency in valuing assets and actuarial liabilities. Typical U.S. asset valuation is based on market or some market-related method. There is no direct relationship between the market's short-term outlook for interest rates and the long-term rate at which the actuary is discounting future benefit payments. If select and ultimate financial assumptions are used, the initial rate can be the rate that the market uses to discount assets. Some consistency between the two sides of the balance sheet is then achieved.

B. *Disadvantages*

There are potential disadvantages in using select and ultimate financial assumptions. Traditionally, most pension plan valuations have been based on interest rate and salary scale assumptions that are level over time. These rates are applied prospectively and, to the extent required by the funding method, retrospectively. The recent volatility of market interest rates has damaged the credibility of level interest assumptions and sparked the current interest in select and ultimate assumptions. By separating short-term and long-term financial considerations, the select and ultimate approach may improve credibility and, therefore, communications with plan sponsors, accountants, and participants. The select and ultimate approach also may create an implication of greater accuracy than a level long-term assumption. If so, communications may be hampered if the investment manager views the select assumptions as annual investment targets. Level financial assumptions would be used if there was an expected rapid return to low levels of inflation or a belief that future levels of inflation are unpredictable.

Technically, select and ultimate financial assumptions are more complex than level assumptions. For example, investment return is said to be composed of an inflation rate plus a real rate of return, while salary scales include an inflation rate, longevity increases, and merit and productivity increases. The actuary must decide whether the changing inflation rate would imply that the other components of the salary and investment assumptions also vary over time. Experience studies have suggested that turnover, disability, and death rates are also affected by changing economic conditions.

Changes in Internal Revenue Code section 415 imposed by the Tax Equity and Fiscal Responsibility Act (TEFRA) have caused financial assumptions to reach historical levels. In projecting plan benefits under pay-related plans, the expected benefit at retirement cannot exceed the current section 415 dollar limit. The dollar limit is adjusted annually to reflect changes in the consumer price index. The increased limit can be funded as it becomes effective. TEFRA reduced the existing maximum dollar benefit by more than one-third. Moreover, there is a five-year (1983-1987) moratorium on annual cost-of-living adjustments to this limit, so the ultimate effect on plan funding will be even more pronounced. Long-term financial assumptions which assume an orderly return to past economic experience could improve the fluctuating funding that is otherwise possible. Lower salary increase assumptions mean fewer projected benefits would bump into the lower dollar limits. In Canada, the maximum dollar limit is currently defined in Information Circular 72-13R7, and future increases in the limit cannot be assumed. Nevertheless, inflation and pension industry lobbying may well force adjustments to the limit. Thus, the difficulties with the approach are even more pronounced in Canada.

Select and ultimate assumptions typically would be developed on a valuation year basis. That is, they apply to specific time periods following a particular valuation date. With succeeding valuations, adoption of the same assumptions will lead to a contracting select period. The assumptions will therefore become level rates (the ultimate rate) at the end of the select period unless the select period is regularly lengthened. Select and ultimate assumptions may require more frequent changes than level assumptions. The actuary should consider the potential effects of frequent changes in assumptions on the funding stability of the program.

Extending this analysis to methods requiring retrospective financial assumptions illustrates subtle features of the role of financial assumptions in the valuation. Consider select and ultimate assumptions in three parts—the ultimate rate, the retrospective rates, and the select rates.

The ultimate rate will be a level rate. To minimize gains or losses caused by the difference between current investment experience and the ultimate rate, we would consciously decide that the long-term stability of the assumption should be overridden to the extent that the select rates vary from this level rate. In a new plan, however, the lack of assets would seem to preclude such a decision. It seems more appropriate in a well-funded plan.

It is apparent that a change to select and ultimate assumptions from level rates will alter future costs. The effects of these changes will vary depending upon the distribution of liabilities under an individual plan. Thus, two plans which have comparable cost patterns under identical level rate assumptions may have dissimilar cost patterns under identical select and ultimate assumptions.

The common thread in these observations is that the valuation's financial assumptions can be viewed as defining liabilities, which may be different than viewing them as the prediction of future plan investment performance. This is standard valuation practice, as assets are generally measured in relation to current book or market values. Although an interest assumption which matches future investment experience will minimize investment gains and losses, other measures of liability could be used. For example, the sponsor might wish to value liabilities based on the expected future cost of money. In any event, the actuary should be satisfied that the select and ultimate approach is appropriate for defining liabilities before adopting this technique.

III. TECHNICAL ISSUES INVOLVED IN THE USE OF SELECT AND ULTIMATE FINANCIAL ASSUMPTIONS

There are several technical issues concerning actuarial valuations which do not arise with level financial assumptions but do with variable financial assumptions. Other technical issues that are pertinent to both level and variable financial assumptions have a somewhat different emphasis with the latter.

A. *Nomenclature*

In the literature, the term *select and ultimate interest assumptions* has been interpreted in two different ways. One interpretation is that the select period, analogous to the select period in mortality tables, remains constant. For example, if the rates for the first valuation year 1983 are set as 10 percent for the first ten years and 5 percent thereafter, the rates for 1984 would also be 10 percent for the first ten years, and 5 percent thereafter.

The second interpretation is that the interest rate is variable for a certain calendar period called the select period, after which the interest rate attains its ultimate level. If the rates for the first valuation year 1983 are established as 10 percent for the first ten years, and 5 percent thereafter, the rates for 1984 would be 10 percent for the first nine years, and 5 percent thereafter. In other words, the interest rates are calendar specific.

The first interpretation runs counter to the U.S. reasonable funding regulations, but could be used with Canadian pension plans.

Consider the present value of benefits, PVB, at the beginning of year 1:

$$PVB_1 = \sum_{k=1}^{N} \left(\prod_{S=1}^{k} \frac{1}{1 + i_s} \right) B_k$$

$$= \frac{1}{1 + i_1} B_1 + \frac{1}{(1+i_1)(1+i_2)} B_2 + \ldots \quad .$$

where B_k = benefits projected to be paid at the end of year k.

i_s = .10 for $1 \leq s \leq 10$

.05 for $s > 10$.

The equation for the expected present value of benefits at the end of year 1 is:

Expected $PVB_2 = PVB_1 (1 + i_1) - B_1$.

Under the reasonable funding regulations, the actual PVB_2 must equal expected PVB_2 if experience exactly matches assumptions.

Assuming that the projected benefits B_2, B_3,. . . remain the same at the end of year 1 as at the beginning, actual PVB_2 under the first approach, where interest rates for the 1984 valuation are 10 percent for the first ten years and 5 percent thereafter, can be written as:

$$PVB_2 = \frac{1}{1 + i_1} B_2 + \frac{1}{(1+i_1)(1+i_2)} B_3 + \ldots \quad .$$

Actual PVB_2 under the second approach, where interest rates for the 1984 valuation are 10 percent for the first nine years and five percent thereafter, can be written as

$$PVB_2 = \frac{1}{1 + i_2} B_2 + \frac{1}{(1+i_2)(1+i_3)} B_3 + \ldots \quad .$$

With the second approach, PVB_2 = Expected PVB_2, meeting the requirements of the reasonable funding regulations. This is not the case with the first approach.

The specific approach will have implications for the determination of actuarial gains or losses. In the rest of this paper, references to select and ultimate interest rates will mean calendar specific variable interest rates, unless specified otherwise.

B. *Choosing Select and Ultimate Interest Rates*

Determining specific values for the select and ultimate interest assumption is a more complex process than choosing a level interest assumption. Values must be assigned to the initial rate, the ultimate rate, the select period and the interim rates.

In an economic climate where prevailing rates are much higher than those expected over the long term, the initial rate could be a function of a recognized index. A recent average of the prime as compiled by the Federal Reserve or of Moody's long-term corporate bond yields for high quality issues can serve as the initial rate. Then the initial rate assumption has the authority of the index used. The burden of proof would shift to anyone making the allegation that the assumption does not reflect prevailing rates. An alternate approach to determining the initial interest rate would be to base it upon recent plan experience and the expected short-term rate of return.

The considerations applicable to choosing the ultimate interest rate are the same as in determining a level interest assumption. The ultimate rate can be viewed as the sum of three components: the "riskless" return, the risk premium for potential loss of principal, and the inflation risk premium. The sum of first two sometimes is called the real rate of return. Historically, such a rate has oscillated within the range of 2 percent to 4 percent for quality instruments. Although recent real rates have been well in excess of five percent, it is important to note the cyclical nature of financial markets (illustrated by historical studies relative to a pension plan time frame). In the final analysis, it is the actuary's judgment regarding future levels of inflation and real returns that will determine the level of the ultimate rate.

The Macaulay duration of the fund's bond investments may provide a basis for determining the select period. At current interest rates, this duration for a long-term bond is about one-third of the term of the bond. A bond portfolio which invests mainly in investments thirty to forty-five years long would have a duration of ten to fifteen years. An investment strategy, even

if more aggressive than an immunization strategy, can be assumed to yield returns at least as good as those earned on an immunized portfolio. On this basis, an actuary with a pension fund invested substantially in long-term fixed income securities could choose a select period of ten to fifteen years.

The difference between the initial rate and the ultimate rate will also have a bearing on the length of the select period. In theory, the bigger the difference between the initial rate and the ultimate rate, the longer the period.

The actuary who has decided upon the select period, the initial rate, and the ultimate rate, must now develop interim rates. One consideration is how quickly and smoothly the transition between the initial rate and the ultimate rate is to be made. A slow transition is possible by introducing small changes in the beginning of the select period with such rates applying for relatively longer time intervals. Conversely, the early select rates can apply for relatively short periods with relatively larger differences in successive intervening rates. Of course, the faster the transition between the initial rate and the ultimate rate, the greater the effect the ultimate rate will have on the resultant liability determination. The actuary may decide upon a uniform transition. After determining specific rates for the first few years of the select period, he or she may choose rates for the remaining portion of the select period in an arithmetic or geometric progression, merging into the ultimate rate.

The asset valuation method has a bearing on the choice of the select rates. Often the asset valuation method has substantial smoothing properties. The interest assumption is, of course, the expected actuarial rather than the market rate of return. The smoothing technique to be used for developing the actuarial value of assets can be applied to the expected market rates of return to develop the select interest assumptions.

When select and ultimate interest assumptions are used for the valuation, the salary scale assumption needs to be examined for compatibility with the interest assumptions. It is common to use the same inflation component in the interest and the salary scale assumptions. It may be necessary to revise the salary scale assumption that was used with a level interest assumption.

C. *Valuation Procedures*

Actuarial valuations usually arrive at a present value of benefits by one of two means. The traditional method is to use commutation functions, which are applied to projected benefits to determine the present value. With the advent of high-speed computers, it has become possible to adopt a procedure similar to a cash-flow projection. In this case, the anticipated benefit outgo in each year is determined, and these amounts are then discounted with

interest. This latter approach lends itself more easily to variable interest rates. For instance, let us suppose that the valuation determines a projected benefit payout of $B_1, B_2, \ldots, B_n$ for the next n years. Then, the cash-flow method produces a PVB of

$$\sum_{k=1}^{N} \frac{1}{(1+i)^k} B_k .$$

The only alteration that needs to be made to this formula to accommodate variable interest assumptions is to replace each of the coefficients,

$$\frac{1^k}{1+i}, \quad \text{with} \quad \prod_{s=1}^{k} \frac{1}{(1+i_s)} .$$

Since each of the B_k's has to be isolated and a separate interest discount factor applied, generally there will be no difficulty reflecting the select and ultimate interest discount factor.

With regard to the commutation function approach to actuarial valuations, there appears to be some added complexity when a select and ultimate interest assumption is employed. This is because there is a new commutation function for each attained age. This is not necessarily a serious drawback. It is possible to sort all of the valuation data by attained age and calculate commutation functions based upon attained age. For a valuation with employees ranging from 17 to 65, this means the calculation of forty-nine commutation tables rather than one. However, with the speed of today's computers, the added cost for this type of valuation is generally not significant. In addition, the computer costs are such a small portion of the total that even a fairly large increase in the computer cost is still a minor increase in the cost of the valuation. Thus, it appears feasible to introduce select and ultimate financial assumptions with either of the two valuation procedures.

D. *Amortization of Unfunded Actuarial Liabilities*

There are a number of approaches which could be taken in amortizing unfunded actuarial liabilities, both for minimum funding standard purposes as well as for actual funding purposes. Three such approaches are:

1. Level annual installments determined using the select and ultimate interest rates. The amortization schedule would also use the year-by-year interest rates.

2. Level annual installments determined using the select and ultimate interest rates. However, the amortization schedule would use an average interest rate derived from the relationship of the level installments and the Unfunded Actuarial Liability (UAL).
3. Annual installments which would remain level for the duration of a particular interest rate assumption. When a new select interest rate (or the ultimate rate) becomes effective, a new level amount would be determined using the remaining unamortized UAL, the number of years remaining, and the new interest rate.

In the exhibits which follow, amortization schedules of 10, 15, and 30 year durations are shown for the three approaches noted. Exhibits 1, 2, and 3 have assumed interest rates of 10 percent for ten years and 5 percent thereafter, while exhibits 4, 5, and 6 have assumed an interest rate of 10 percent for the first year, grading down by 1/2 percent per year to 5 percent for years eleven and later.

It should be noted that in the first approach, the UAL increases during the first ten years, since interest requirements are not being met. Approach 2 corrects this problem but introduces another. According to Regulation Section 1.412(c)(3)-1 on reasonable funding methods, the present value of future benefits must equal the present value of future normal costs, plus unamortized portions of amortizable bases, plus assets, less credit balance. Since approach 2 reduces the unamortized bases at a different interest rate than is used for the other pieces, the equation will no longer balance. Approach 3 solves the problems in these first two approaches, but does not use level installments. This is a problem because section 412(b) of the Internal Revenue Code requires that "amounts necessary to amortize in equal annual installments" be used in funding standard account calculations. This suggests that the first approach would be the only allowable method for minimum funding standard purposes. Similarly, under the various provincial regulations in Canada, "special payments consisting of equal annual amounts" would be the acceptable approach.

EXHIBIT 1

USE OF SELECT AND ULTIMATE INTEREST RATES OF 10 PERCENT FOR TEN YEARS AND 5 PERCENT THEREAFTER FOR MINIMUM FUNDING, FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 1 —
Level Annual Payment; Amortization Schedule Using
Select Rates in Effect by Years

Year	Minimum Funding UAL Jan. 1	Payment*
1	$100,000	$ 9,133
2	100,867	
3	101,821	
4	102,870	
5	104,024	
6	105,293	
7	106,689	
8	108,225	
9	109,915	
10	111,773	
11	113,817	
12	110,375	
13	106,761	
14	102,966	
15	98,981	
16	94,798	
17	90,404	
18	85,792	
19	80,948	
20	75,863	
21	70,523	
22	64,916	
23	59,029	
24	52,847	
25	46,356	
26	39,541	
27	32,385	
28	24,871	
29	16,982	
30	8,698	9,133
31	0	0
		273,990

$$*p = \frac{100{,}000}{a_{\overline{10}|\,10\%} + v^{10}_{10\%}\, a_{\overline{20}|\,5\%}}$$

Year	15 Year Funding UAL Jan. 1	Payment†
1	$100,000	$ 12,798
2	97,202	
3	94,124	
4	90,739	
5	87,014	
6	82,918	
7	78,412	
8	73,455	
9	68,002	
10	62,005	
11	55,407	
12	45,379	
13	34,850	
14	23,795	
15	12,187	12,798
16	0	0
		$191,970

$$\dagger p = \frac{100{,}000}{a_{\overline{10}|\,10\%} + v^{10}_{10\%}\, a_{\overline{5}|\,5\%}}$$

Year	Maximum Funding UAL Jan. 1	Payment‡
1	$100,000	$ 16,275
2	93,725	
3	86,823	
4	79,231	
5	70,880	
6	61,693	
7	51,588	
8	40,472	
9	28,245	
10	14,795	16,275
11	0	0
		$162,750

$$\ddagger p = \frac{100{,}000}{a_{\overline{10}|\,10\%}}$$

EXHIBIT 2

USE OF SELECT AND ULTIMATE
INTEREST RATES OF 10 PERCENT FOR TEN YEARS AND
5 PERCENT THEREAFTER FOR MINIMUM FUNDING
FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 2 —
Level Annual Payment: Amortization Schedule Using
Average Effective Interest Rate

	Minimum Funding*	
	UAL	
Year	Jan. 1	Payment
1	$100,000	$ 9,133
2	99,164	
3	98,259	
4	97,279	
5	96,217	
6	95,068	
7	93,823	
8	92,474	
9	91,014	
10	89,434	
11	87,720	
12	85,868	
13	83,857	
14	81,682	
15	79,326	
16	76,775	
17	74,012	
18	71,020	
19	67,780	
20	64,271	
21	60,471	
22	56,355	
23	51,898	
24	47,071	
25	41,844	
26	36,183	
27	30,052	
28	23,412	
29	16,222	
30	8,435	9,133
31	0	0
		273,990

* i = 8.2972 percent

	15 Year Funding†	
	UAL	
Year	Jan. 1	Payment†
1	$100,000	$ 12,798
2	96,734	
3	93,156	
4	89,238	
5	84,946	
6	80,245	
7	75,096	
8	69,456	
9	63,278	
10	56,512	
11	49,100	
12	40,982	
13	32,091	
14	22,352	
15	11,684	12,798
16	0	0
		$191,970

†i = 9.5319

	Maximum Funding	
	UAL	
Year	Jan. 1	Payment
1	$100,000	$ 16,275
2	93,725	
3	86,823	
4	79,231	
5	70,880	
6	61,693	
7	51,588	
8	40,472	
9	28,245	
10	14,795	16,275
11	0	0
		$162,750

EXHIBIT 3

USE OF SELECT AND ULTIMATE
INTEREST RATES OF 10 PERCENT FOR TEN YEARS AND
5 PERCENT THEREAFTER FOR MINIMUM FUNDING,
FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 3 —
Level Annual Payment for Duration of Interest Rate;
Amortization Schedule Using Select Rates in Effect by Year

	Minimum Funding			15 Year Funding	
Year	UAL Jan. 1	Payment	Year	UAL Jan. 1	Payment
1	$ 100,000	$ 10,608*	1	$ 100,000	$ 13,147†
2	99,392		2	96,853	
3	98,723		3	93,391	
4	97,988		4	89,582	
5	97,179		5	85,393	
6	96,289		6	80,785	
7	95,309		7	75,716	
8	94,233		8	70,140	
9	93,048		9	64,007	
10	91,745	10,608	10	57,260	13,147
11	90,311	7,247**	11	49,839	11,512‡
12	87,580		12	40,820	
13	84,712		13	31,349	
14	81,701		14	21,405	
15	78,539		15	10,964	11,512
16	75,219		16	0	
17	71,733				$ 189,030
18	68,073				
19	64,230				
20	60,195				
21	55,958				
22	51,509				
23	46,838				
24	41,933				
25	36,783				
26	31,375				
27	25,697				
28	17,735				
29	13,475				
30	6,902	7,247			
31	0	0*			
		$ 251,020			

$$*p = \frac{100{,}000}{a_{\overline{30}|10\%}}$$

$$**p = \frac{90{,}311}{a_{\overline{20}|5\%}}$$

$$\dagger p = \frac{100{,}000}{a_{\overline{15}|10\%}}$$

$$\ddagger p = \frac{49{,}839}{a_{\overline{5}|5\%}}$$

Year	Maximum Funding UAL Jan. 1	Payment
1	$ 100,000	$ 16,275
2	93,725	
3	86,823	
4	79,231	
5	70,880	
6	61,693	
7	51,588	
8	40,472	
9	28,245	
10	14,795	16,275
11	0	0
		$ 162,750

EXHIBIT 4

USE OF SELECT RATES OF 10 PERCENT
GRADING DOWN ARITHMETICALLY TO 5 PERCENT
OVER TEN YEARS FOR MINIMUM FUNDING.
FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 1 — Level Annual Payment
Amortization Schedule Using
Select Rates in Effect by Year

Year	Minimum Funding UAL Jan. 1	Payment
1	$ 100,000	$ 8,038
2	101,962	
3	103,611	
4	104,898	
5	105,777	
6	106,201	
7	106,129	
8	105,520	
9	104,341	
10	102,564	
11	100,167	
12	97,138	
13	93,957	
14	90,617	
15	87,110	
16	83,428	
17	79,562	
18	75,502	
19	71,240	
20	66,764	
21	62,065	
22	57,130	
23	51,949	
24	46,509	
25	40,796	
26	34,799	
27	28,501	
28	21,888	
29	14,945	
30	7,654	8,038
31	0	
		$ 241,140

Year	15 Year Funding UAL Jan. 1	Payment
1	$ 100,000	$ 11,651
2	98,349	
3	96,040	
4	93,032	
5	89,289	
6	84,780	
7	79,487	
8	73,400	
9	66,519	
10	58,859	
11	50,445	
12	41,315	
13	31,730	
14	21,665	
15	11,096	11,651
16	0	
		$ 174,765

Year	Maximum Funding UAL Jan. 1	Payment
1	$ 100,000	$ 15,318
2	94,682	
3	88,359	
4	80,994	
5	72,561	
6	63,048	
7	52,459	
8	40,813	
9	28,148	
10	14,519	15,318
11	0	
		$ 153,180

EXHIBIT 5

USE OF SELECT RATES OF 10 PERCENT
GRADING DOWN ARITHMETICALLY TO 5 PERCENT OVER
TEN YEARS FOR MINIMUM FUNDING,
FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 2 — Level Annual Payment
Amortization Schedule Using Average Effective Interest Rate

Minimum Funding*

Year	UAL Jan. 1	Payment
1	$ 100,000	$ 8,038
2	98,936	
3	97,798	
4	96,581	
5	95,279	
6	93,886	
7	92,396	
8	90,803	
9	89,097	
10	87,273	
11	85,322	
12	83,235	
13	81,002	
14	78,613	
15	76,059	
16	73,325	
17	70,401	
18	67,274	
19	63,928	
20	60,348	
21	56,519	
22	52,423	
23	48,042	
24	43,354	
25	38,341	
26	32,977	
27	27,238	
28	21,101	
29	14,535	
30	7,510	8,038
31	0	
		$ 241,140

*i = 6.9741 percent equivalent interest

15 Year Funding†

Year	UAL Jan. 1	Payment
1	$ 100,000	$ 11,651
2	96,304	
3	92,314	
4	88,007	
5	83,357	
6	78,337	
7	72,918	
8	67,068	
9	60,758	
10	53,935	
11	46,574	
12	38,628	
13	30,050	
14	20,790	
15	10,793	11,651
16	0	
		$ 174,765

† i = 7.9552 percent equivalent interest

Maximum Funding‡

Year	UAL Jan. 1	Payment
1	$ 100,000	$ 15,318
2	93,295	
3	86,013	
4	78,103	
5	69,514	
6	60,183	
7	50,049	
8	39,043	
9	27,087	
10	14,103	15,318
11	0	
		$ 153,180

‡i = 8.6312 percent equivalent interest

EXHIBIT 6

USE OF SELECT RATES OF 10 PERCENT
GRADING DOWN ARITHMETICALLY TO 5 PERCENT OVER
TEN YEARS FOR MINIMUM FUNDING.
FIFTEEN YEAR FUNDING AND MAXIMUM FUNDING

Approach 3 — Level Annual Payment for Duration of Interest Rate
Amortization Schedule Using Select Rates in Effect by Year

Year	Minimum Funding UAL Jan. 1	Payment
1	$ 100,000	$ 10,608
2	99,392	10,174
3	98,660	9,753
4	97,786	9,344
5	96,754	8,950
6	95,544	8,571
7	94,139	8,208
8	92,521	7,861
9	90,674	7,530
10	88,584	7,217
11	86,239	6,920
12	83,631	
13	80,893	
14	78,018	
15	74,999	
16	71,829	
17	68,500	
18	65,005	
19	61,335	
20	57,482	
21	53,436	
22	49,188	
23	44,727	
24	40,043	
25	35,125	
26	29,961	
27	24,539	
28	18,846	
29	12,868	
30	6,590	6,920
31	0	
		$ 226,616

Year	15 Year Funding UAL Jan. 1	Payment
1	$ 100,000	$ 13,147
2	96,853	12,791
3	93,263	12,457
4	89,200	12,145
5	84,637	11,856
6	79,552	11,590
7	73,928	11,347
8	67,756	11,128
9	61,032	10,933
10	53,761	10,762
11	45,956	10,615
12	37,639	
13	28,906	
14	19,736	
15	10,108	10,615
16	0	
		$ 171,231

Year	Maximum Funding UAL Jan. 1	Payment
1	$ 100,000	$ 16,274
2	93,726	15,953
3	86,677	15,660
4	78,818	15,399
5	70,119	15,168
6	60,561	14,968
7	50,135	14,801
8	38,843	14,666
9	26,702	14,564
10	13,740	14,995
11	0	0
		$ 152,448

E. *Determination of Option Factors*[1]

Another technical issue which needs to be addressed is the determination of option factors using select and ultimate interest rates. In this discussion, it is assumed that the pension plan makes allowance for optional forms of retirement benefit based on actuarial equivalence to the normal form.

In the case where the select and ultimate rates are 10 percent for ten years and 5 percent thereafter, what rates should be used to determine option factors in the second year? Should 10 percent for nine years and 5 percent thereafter be used, or should the ten year select period be retained? It would seem that if actuarial assumptions are not changed, then it is appropriate to use a shorter select period as time elapses for purposes of determining option factors.

The following table shows the lump-sum value of $1 per month payable from age 65 based on the 1971 GAM table. The ultimate interest rate is 5 percent and the select period is gradually reduced from ten years to zero years.

Select Period	Lump-Sum ($1 per month)
10	91.61
9	92.75
8	94.12
7	95.76
6	97.72
5	100.04
4	102.79
3	106.01
2	109.79
1	114.20
0	119.23

The lump-sum value increases steadily, raising the question of equity in plan participants' optional benefits. This question is brought into sharper focus when considering two employees whose retirements at age 65 are one month apart with one employee retiring before and the other retiring after the valuation date.

[1]Several questions arise regarding the appropriate application of existing regulations to option factors calculated using select and ultimate interest rates. To clarify such issues, the Pension Committee submitted a letter to the Internal Revenue Service requesting guidance. The letter and the IRS response are published in the appendix.

For options other than lump-sum, the difference between select and ultimate rates and level rates may not be material enough to warrant the extra complexity. For example, compare the following factors at age 65 using the 1971 GAM Table.

	Factor For Converting From Life Only	
	Select and Ultimate	Ultimate Only
Life only	1.0000	1.0000
Life guaranteed five years	.9777	.9760
Life guaranteed ten years	.9207	.9121
Joint and 100 percent survivor (spouse three years younger)	.7566	.7216
Joint and 50 percent survivor (spouse three years younger)	.8614	.8383

Other important considerations are provisions of the Internal Revenue Code. Code Section 411(d)(6) requires that the accrued benefit of a participant shall not be decreased by an amendment of the plan. Revenue Rulings 79-90 and 81-12 give specific advice on the effect of changes in actuarial assumptions on accrued benefits.

Revenue Ruling 79-90 states that a defined benefit plan must specify the actuarial assumptions used to compute the amounts of optional benefits which are "actuarially equivalent" in order for the benefits to be definitely determinable. If the actuarial assumptions stated in the plan for determination of optional benefits are later modified by a plan amendment, the accrued benefits resulting from the use of the new option factors must satisfy the anti-cutback and grandfathering rules. Revenue Ruling 81-12 describes two methods of stating the assumptions for determination of optional benefits: adoption of a fixed standard which requires a plan amendment to change the option factors, and adoption of a variable standard which permits the underlying assumptions for the option factors to float with some outside source, such as the prime rate.

Thus, select and ultimate assumptions could be stated in one of the following forms:

Fixed Standards

1. 10 percent for the next ten years, and 5 percent thereafter.
2. 10 percent for years 1983 to 1992 and 5 percent thereafter.

Variable Standards

3. Prime rate in effect on January 1 of the calendar year of retirement for the next ten years and 50 percent of such prime rate thereafter.
4. Prime rate in effect on January 1 of the calendar year of retirement until 1992 and 50 percent of such prime rate thereafter.

If a fixed standard is used, a set of option factors is produced which will not change without a plan amendment. This approach provides a set of option factors which will remain constant (or at least predictable) from year to year, eliminating the need for frequent recalculation of the option factors and permitting consistent estimates of future benefits for plan participants.

Fixed standards, however, will need to be reviewed frequently in order to follow changing economic conditions. When a change is made, significant differences may occur and the application of the Revenue Ruling 81-12 rules may result in the grandfathering of many accrued benefits. If frequent changes are not anticipated, it would seem that a simple interest assumption would be more appropriate than a fixed select and ultimate standard.

A variable select and ultimate standard can be designed to follow a reliable market index. Once a suitable index is found, the option factors will automatically reflect current market conditions. This is a particularly important consideration when the plan provides for a lump-sum option, and the plan sponsor wants to reflect current interest rates in the calculation. A variable standard has the added advantage that when the automatic changes occur, there is no need to apply the Revenue Ruling 81-12 rules. Even though the variable method can result in option factors that decrease, the cutback rules or grandfathering are not triggered unless a plan amendment that changes the outside source results in a decrease in currently applicable option factors.

The main disadvantages of variable standards are the frequent recalculation of option factors and the unpredictability of benefits. With the availability of sophisticated computer programs, the frequent recalculations are not of as much concern as they were previously. However, if the necessary programs are not readily available, they would still be a major consideration. The unpredictability of future benefits is a more difficult problem to overcome. Communication with plan participants will be needed to explain that the value of their optional benefits (particularly lump sums) will depend on the value of the chosen index at the time of retirement. This problem can be lessened somewhat by providing that the index will not change more frequently than once a year. Thus, once the values are set for a given year, accurate projections can be made for that year.

F. *Effect of the Use of Select and Ultimate Assumptions on Selection of Funding Methods*

Assumptions about prior periods have no bearing on current results under the majority of actuarial cost methods. Under the accrued benefit cost method, for example, the calculations are based on accrued benefits which are independent of the actuarial assumptions. This independence is also a characteristic of the aggregate method, the projected unit credit service prorata method and the attained age normal method.

This independence is not characteristic of all cost methods. Under the entry age normal cost method, the partition of plan costs between prior and future periods is a function of assumptions about prior experience. The normal cost is determined as a level percentage of pay or a level dollar amount over both past and future periods that will accumulate to the needed sum at retirement. This requires an interest rate assumption (and a salary increase assumption for pay-related funding) applicable to prior periods as well as future periods.

What rates and assumptions are appropriate for past periods? Alternatives include (a) ultimate rates only; (b) the select rates as to experience anticipated from prior valuations; or (c) some other independent set of assumptions. While (a) or (c) might be easy to apply, they would introduce a systematic actuarial gain or loss from one valuation to the next. For plans subject to the minimum funding standards of ERISA, this could result in a technical violation of reasonable funding regulations. On the other hand, the effect of this systematic gain or loss is to shift costs between accrued actuarial liabilities and future normal costs rather than to increase or decrease total expected plan costs. It might have only an immaterial effect on the current level of plan contribution requirements or on the incidence of plan costs from year to year. Use of the ultimate rate for past periods would come closer to the commonly used approach. If the assumption for salary increase were 10 percent for the next five years and 5 percent thereafter, the 5 percent ultimate rate could be applied to prior periods. The assumption would then be explained as being a long-term, all-period rate of 5 percent, with an adjustment for the next five years of an additional 5 percent to reflect short-term expectations. It is possible that as select and ultimate assumptions gain greater recognition, other practical approaches will be deemed acceptable by the regulatory authorities.

G. *Numerical Illustrations*

ASSUMPTIONS

The following tables illustrate the effect of experience on contribution

patterns under different benefit formulas and two different cost methods when either level or select and ultimate interest assumptions are chosen to value the plan. Four different benefit formulas were studied under four different combinations of valuation assumptions and experience rates. Contribution rates as a percent of payroll were produced under both the entry age normal and aggregate cost methods. The benefit formulas studied were as follows:

Formula 1 1 percent of final five year average pay times years of service (Tables 1 and 2).
Formula 2 1 percent of career average pay times years of service (Tables 3 and 4).
Formula 3 $10 per month per year of service (Tables 5 and 6).
Formula 4 1 percent of final five year average pay minus 1 percent of primary Social Security times years of service (Tables 7 and 8).

To simplify calculations, the plan population was chosen to have no retired lives. This is not expected to materially affect the conclusions. The average active age was 38 and the average amount of past service was twelve years. The average salary was $24,873. The initial level of assets was assumed to equal 25 percent of the present value of future benefits. Each table displays contribution requirements for the next ten years. Two different assumption sets were used in the analysis:

Set 1 10 percent interest the first year, decreasing 1 percent per year to an ultimate rate of 6 percent.
Set 2 8 percent interest in every year.

The assumed rate of salary increase was 1 percent less than the assumed interest rate in every instance. Under the entry age normal method, the initial unfunded accrued liability was funded over a period of twenty-five years. The initial payroll for active employees was $24,872,769. Section 415 limits were ignored for purposes of these illustrations.

OBSERVATIONS

Certain observations that may be made from the tables are independent of the methodology used for the valuation assumptions but are characteristic of the cost method, benefit formula, or level of assumptions chosen.

Contribution rates under the entry age normal cost method include amortization of the unfunded accrued liability. When assumptions exactly match experience, costs are level under the aggregate cost method while they decrease monotonically under the entry age normal method. Accrual rates are higher under Set 1 assumptions than Set 2, since Set 1 represents a lower

level of interest rates. The tables show that costs will be lower and decrease faster for benefit formulas which are less responsive to inflation (such as formulas 2 and 3).

The tables may be used to examine whether the effect of variation of experience from assumptions is different when select and ultimate interest assumptions rather than level interest assumptions are used. Such an examination is made in the following tabulation:

EFFECT OF EXPERIENCE UPON ACCRUAL RATES
- EAN COST METHOD

	FINAL AVERAGE PLAN		CAREER AVERAGE PLAN		FLAT-DOLLAR PLAN		SOCIAL SECURITY OFFSET PLAN	
	Valuation Assumption							
	Set 1	Set 2	Set 1	Set 2	Set 1	Set 2	Set 1	Set 2
Initial Rate (1)	1113	9.24	5.79	4.42	2.58	1.94	6.91	5.58
Rate After Ten Years—Different Experience (2)	9.62	8.46	4.97	3.98	1.76	1.86	5.85	5.19
Rate After Ten Years When Experience Matches Assumptions (3)	10.11	8.09	5.21	3.80	2.19	1.55	6.26	4.87
(4) = (2)/(3)	0.95	1.04	0.95	1.05	0.80	1.20	0.93	1.07
Effect of Experience On Rate After Ten Years |1.0-(4)| (5)	5%	4%	5%	5%	20%	20%	7%	7%

It is inappropriate to draw any generally applicable conclusions from Tables 1 through 8 or the last tabulation regarding which benefit formula is subject to the greatest variability. For example, in these illustrations the flat-dollar formula shows the greatest volatility over the years when experience does not match assumptions. The reason is that only the change in interest rates (actual versus experienced) affects the results. For the other formulas, the results reflect the assumed 1 percent spread between experience salary rates and experience interest rates which dampens the variability of the interest rates and makes the formulas appear less volatile.

Row (5) of the tabulation shows that under the entry age normal method the effect of variation of experience from assumed rates on the accrual rate after ten years is independent of whether select and ultimate or level valuation assumptions were used. When experience differs from the assumptions, the effect on the pattern of accrual rates is a function of the difference between the valuation interest rates and experience interest rates rather than the methodology (i.e., select and ultimate versus level interest rates). Calculations under the aggregate cost method bear out the same conclusion.

TABLE 1

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG
1983.....	8.0%	8.0%	8.28	6.87	10.0%	8.28	6.87
1984.....	8.0	8.0	8.18	6.87	9.0	8.18	6.87
1985.....	8.0	8.0	8.08	6.87	8.0	8.07	6.87
1986.....	8.0	8.0	7.97	6.87	7.0	7.95	6.86
1987.....	8.0	8.0	7.84	6.87	6.0	7.83	6.86
1988.....	8.0	8.0	7.72	6.87	6.0	7.74	6.88
1989.....	8.0	8.0	7.60	6.87	6.0	7.65	6.90
1990.....	8.0	8.0	7.48	6.87	6.0	7.58	6.95
1991.....	8.0	8.0	7.35	6.87	6.0	7.53	7.01
1992.....	8.0	8.0	7.24	6.87	6.0	7.51	7.08

Unfunded Accrued Liability = $12,212,102

Benefit formula–1 percent of final five-year average pay times years of service.

Valuation assumptions–Constant interest rate and salary scale.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 2

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.
1983.....	10.0%	10.0%	9.57	8.28	8.0%	9.57	8.28
1984.....	9.0	9.0	9.38	8.28	8.0	9.38	8.29
1985.....	8.0	8.0	9.23	8.28	8.0	9.24	8.29
1986.....	7.0	7.0	9.11	8.28	8.0	9.12	8.30
1987.....	6.0	6.0	9.01	8.28	8.0	9.02	8.30
1988.....	6.0	6.0	8.95	8.28	8.0	8.94	8.28
1989.....	6.0	6.0	8.89	8.28	8.0	8.84	8.25
1990.....	6.0	6.0	8.82	8.28	8.0	8.71	8.20
1991.....	6.0	6.0	8.75	8.28	8.0	8.56	8.13
1992.....	6.0	6.0	8.69	8.28	8.0	8.41	8.04

Unfunded Accrued Liability = $14,619,309

Benefit formula–1 percent of final five-year average pay times years of service.

Valuation assumptions–Select and ultimate interest rate and salary scale uniformly decreasing to the ultimate rate.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 3

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG
1983	8.0%	8.0%	3.99	3.23	10.0%	3.99	3.23
1984	8.0	8.0	3.93	3.23	9.0	3.93	3.23
1985	8.0	8.0	3.88	3.23	8.0	3.88	3.23
1986	8.0	8.0	3.82	3.23	7.0	3.82	3.22
1987	8.0	8.0	3.76	3.23	6.0	3.75	3.22
1988	8.0	8.0	3.70	3.23	6.0	3.70	3.23
1989	8.0	8.0	3.63	3.23	6.0	3.65	3.24
1990	8.0	8.0	3.56	3.23	6.0	3.60	3.26
1991	8.0	8.0	3.49	3.23	6.0	3.57	3.29
1992	8.0	8.0	3.42	3.23	6.0	3.55	3.33

Unfunded Accrued Liability = $5,442,889

Benefit formula–1 percent of career average pay times years of service.

Valuation assumptions–Constant interest rate and salary scale.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 4

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG
1983	10.0%	10.0%	5.05	4.30	8.0%	5.05	4.30
1984	9.0	9.0	4.95	4.30	8.0	4.95	4.30
1985	8.0	8.0	4.87	4.30	8.0	4.88	4.31
1986	7.0	7.0	4.80	4.30	8.0	4.81	4.31
1987	6.0	6.0	4.75	4.30	8.0	4.76	4.31
1988	6.0	6.0	4.71	4.30	8.0	4.71	4.30
1989	6.0	6.0	4.67	4.30	8.0	4.65	4.29
1990	6.0	6.0	4.63	4.30	8.0	4.58	4.26
1991	6.0	6.0	4.58	4.30	8.0	4.49	4.23
1992	6.0	6.0	4.54	4.30	8.0	4.40	4.19

Unfunded Accrued Liability = $6,962,396

Benefit formula–1 percent of career average pay times years of service.

Valuation assumptions–Select and ultimate interest rate and salary scale uniformly decreasing to the ultimate rate.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 5

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.
1983	8.0%	8.0%	1.72	1.28	10.0%	1.72	1.28
1984	8.0	8.0	1.68	1.28	9.0	1.64	1.25
1985	8.0	8.0	1.65	1.28	8.0	1.58	1.23
1986	8.0	8.0	1.61	1.28	7.0	1.54	1.23
1987	8.0	8.0	1.57	1.28	6.0	1.53	1.25
1988	8.0	8.0	1.52	1.28	6.0	1.54	1.29
1989	8.0	8.0	1.48	1.28	6.0	1.55	1.34
1990	8.0	8.0	1.43	1.28	6.0	1.57	1.39
1991	8.0	8.0	1.39	1.28	6.0	1.58	1.44
1992	8.0	8.0	1.35	1.28	6.0	1.60	1.50

Unfunded Accrued Liability = $2,886,412

Benefit formula–$10 per month times years of service.

Valuation assumptions–Constant interest rate and salary scale.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 6

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG.
1983	10.0%	10.0%	2.19	1.75	8.0%	2.19	1.75
1984	9.0	9.0	2.13	1.75	8.0	2.19	1.80
1985	8.0	8.0	2.08	1.75	8.0	2.16	1.82
1986	7.0	7.0	2.04	1.75	8.0	2.12	1.82
1987	6.0	6.0	2.00	1.75	8.0	2.04	1.79
1988	6.0	6.0	1.97	1.75	8.0	1.95	1.73
1989	6.0	6.0	1.94	1.75	8.0	1.85	1.67
1990	6.0	6.0	1.90	1.75	8.0	1.74	1.60
1991	6.0	6.0	1.87	1.75	8.0	1.64	1.53
1992	6.0	6.0	1.84	1.75	8.0	1.53	1.46

Unfunded Accrued Liability = $3,662,902

Benefit formula–$10 per month times years of service.

Valuation assumptions–Select and ultimate interest rate and salary scale uniformly decreasing to the ultimate rate.

NOTE.—Experience salary rate assumed to be 1 percent lower than the experience interest rate.

TABLE 7

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG
1983	8.0%	8.0%	4.99	4.12	10.0%	4.99	4.12
1984	8.0	8.0	4.92	4.12	9.0	4.92	4.12
1985	8.0	8.0	4.86	4.12	8.0	4.86	4.12
1986	8.0	8.0	4.80	4.12	7.0	4.79	4.12
1987	8.0	8.0	4.72	4.12	6.0	4.72	4.12
1988	8.0	8.0	4.65	4.12	6.0	4.67	4.14
1989	8.0	8.0	4.57	4.12	6.0	4.63	4.16
1990	8.0	8.0	4.49	4.12	6.0	4.60	4.20
1991	8.0	8.0	4.42	4.12	6.0	4.58	4.25
1992	8.0	8.0	4.34	4.12	6.0	4.58	4.31

Unfunded Accrued Liability = $7,584,907

Benefit formula–1 percent of final five-year average pay less 1 percent of Social Security times years of service.

Valuation assumptions–Constant interest rate and salary scale.

NOTE.—In computing the Social Security offset, the salary scale, Social Security wage base, and CPI were assumed to be lower than the interest rate by 1 percent, 2 percent, and 3 percent respectively.

TABLE 8

ILLUSTRATION OF EFFECT OF ASSUMPTIONS AND EXPERIENCE ON CONTRIBUTION PATTERN

YEAR	VALUATION INTEREST RATE	EXPERIENCE MATCHES ASSUMPTIONS			EXPERIENCE DIFFERS FROM ASSUMPTIONS		
		Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG	Experience Interest Rate	Contribution Rate -EAN	Contribution Rate -AGG
1983	10.0%	10.0%	5.92	5.10	8.0%	5.92	5.10
1984	9.0	9.0	5.80	5.10	8.0	5.80	5.10
1985	8.0	8.0	5.70	5.10	8.0	5.71	5.11
1986	7.0	7.0	5.62	5.10	8.0	5.63	5.11
1987	6.0	6.0	5.56	5.10	8.0	5.56	5.10
1988	6.0	6.0	5.53	5.10	8.0	5.50	5.08
1989	6.0	6.0	5.49	5.10	8.0	5.43	5.05
1990	6.0	6.0	5.44	5.10	8.0	5.33	5.01
1991	6.0	6.0	5.40	5.10	8.0	5.22	4.95
1992	6.0	6.0	5.36	5.10	8.0	5.11	4.88

Unfunded Accrued Liability = $9,305,282

Benefit formula–1 percent of final five-year average pay less 1 percent of Social Security times years of service.

Valuation assumptions–Select and ultimate interest rate and salary scale uniformly decreasing to the ultimate rate.

NOTE.—In computing the Social Security offset, the salary scale, Social Security wage base, and CPI were assumed to be lower than the interest rate by 1 percent, 2 percent, and 3 percent respectively.

APPENDIX

Letter to the IRS

Dear Mr. Cohen:

The Society of Actuaries Committee on Pensions is currently preparing for publication a paper outlining our research on philosophical and technical considerations involved in using select and ultimate financial assumptions. When select interest assumptions are applied on a calendar-specific basis (i.e., 10 percent in 1984, 9 percent in 1985, and 8 percent thereafter), the actuarial equivalence factors created by these assumptions will vary over time. We are unable to determine the appropriate application of existing regulations to these factors and request your guidance on the questions described below.

As Revenue Ruling 81-12 indicates, all of the factors affecting the computation of a participant's accrued benefit, including actuarial factors used for determining optional benefits, are taken into account when considering the requirements of Code section 411(d)(6) that accrued benefits cannot be decreased. The first question then, is:

1. How is the term accrued benefit defined? Is it the definition at regulation section 1.411(a)-7(a)(1)(ii)?

The regulatory definition cited in 1 states, "The accrued benefit includes any optional settlement at normal retirement." It is not uncommon for defined benefit plans to offer participants a variety of actuarially equivalent optional settlement forms at retirement; ERISA requires the joint and survivor option. Other common options include lump sums, period certain and life thereafter, early commencement provisions, and so on. Two questions arise in this connection.

2. Do the requirements of 411(d)(6) and 81-12 apply to all optional settlement forms and how is optional settlement form defined?
3. Can a plan sponsor eliminate an optional settlement form with respect to benefits accrued to date or for future accruals only? Put another way, is the eligibility to elect an optional benefit form itself part of the accrued benefit, and, therefore, subject to 411(d)(6)?

Many plans offer benefits that are not subject to the vesting and accrual requirements; a variety of subsidized early retirement benefits including early out provisions and so-called ancillary benefits, for example. The definition cited in 1 provides, "accrued benefits do not include (either) ancillary ben-

efits . . . (or) a subsidized early retirement benefit." Plans that offer such benefits do not have to extend them to participants who terminate employment, with a vested interest, prior to the plan's normal retirement age. We assume that the rationale for this conclusion is that these benefits are not part of a participant's accrued benefit and, therefore, do not have to vest. Applying this rationale one step further,

4. If a plan provides benefits that are not part of a participant's accrued benefit, can such benefits be eliminated by plan amendment, for participants not in pay status, without violating 411(d)(6), recognizing that this provision only applies to accrued benefits?

Finally, we do not understand Example (1) in 81-12 in light of the regulatory definition of accrued benefit cited in 1. The example cites an increase from 4 percent to 5 percent in the interest rate used to calculate early retirement reduction factors; this would have the effect of decreasing early retirement benefits. The example suggests that the amendment-effected change violates 411(d)(6), without stating that either the 4 percent or 5 percent based early retirement factors are inappropriate. The accrued benefit does not include early retirement subsidies. Changing the basis for determining actuarial equivalence changes the amount of subsidy provided, and no part of the subsidy is part of the accrued benefits. This leads us to conclude that the 4 percent to 5 percent change in 81-12 would not violate 411(d)(6) because the change only affects the amount of early retirement subsidy involved.

5. How is the regulatory phrase "subsidized early retirement benefit" defined and how is this reconciled with the findings in 81-12?

The Committee believes that resolution of these questions will not only be of considerable use to our study, but will provide important guidance to the actuarial profession in the application of existing regulations.

A copy of the most recent draft of our research paper is enclosed; we hope to submit the paper for publication in the first half of 1984. Thank you for your assistance in this matter.

Sincerely,
Jeffrey J. Furnish
Chairman, Committee on Pensions

IRS Response

Dear Mr. Furnish:

This is in response to your letter of April 2, 1984, addressed to Ira Cohen, concerning actuarial equivalence factors and the use of select and ultimate assumptions in determining such factors. The following comments address the questions and statements in your letter, although not in the same order you presented them.

Section 411(d)(6) of the Code states that, with certain exceptions, plan amendments that decrease a participant's accrued benefit result in a violation of the minimum vesting requirements of section 411. As stated in section 1.411(d)-3(b) of the Income Tax Regulations and in Revenue Ruling 81-12, for purposes of determining whether a participant's accrued benefit has been decreased by a plan amendment, all provisions of the plan shall be taken into consideration. It is further stated that factors indirectly affecting accrued benefits include actuarial factors for determining optional and early retirement benefits. Thus, if a plan amendment changes the actuarial factors used for determining optional and early retirement benefits, provision must be included in the plan so that no decrease in accrued benefits occurs.

As has been noted, the requirements of section 411(d)(6) and Revenue Ruling 81-12 apply to all optional forms of benefit payment that are allowed under the terms of the plan. Consequently, an optional settlement form cannot be eliminated by the plan amendment except for future accruals.

In general, the term accrued benefit is defined in section 1.411(a)-7(a) of the regulations. As stated in that section, the term accrued benefit refers only to pension or retirement benefits and, consequently, accrued benefits do not include ancillary benefits not directly related to retirement benefits. Examples of such ancillary benefits are medical expenses, incidental death benefits, etc. Such benefits and only such benefits may be reduced or removed from a plan without violating section 411(d)(6).

In section 1.411(a)-7(a) of the regulations it is stated that a subsidized early retirement benefit is not taken into account (except to the extent of determining the normal retirement benefit). This is consistent with section 401(a)14 of the Code and means that a plan is not required to provide vesting in an early retirement subsidy until the participant satisfies the eligibility criteria for receiving such subsidized early retirement benefits. Thus, if a vested participant separates from service after satisfying the service requirements for early retirement but prior to satisfying the age requirement for early retirement, it is not required that the participant be vested in the early

retirement subsidy. However, under section 411(d)(b) of the Code and section 1.411(d)-3(b) of the regulations, an early retirement subsidy is considered part of the accrued benefit and cannot be reduced or removed by a plan amendment with respect to benefits that have already accrued. Thus, where you have a plan that freezes accruals, a participant would still be entitled to the subsidized benefit once he met the age and service requirements at any future time.

In a telephone conversation subsequent to your letter you presented an example where select and ultimate interest assumptions of 5 percent for *n* years and 4 percent thereafter are used in determining actuarial equivalence factors. The use of select and ultimate interest assumptions for determining actuarial equivalence factors is permissible under the Code and regulations. In the example presented, the select period is originally *n* years, and decreases by one year for each year in the future to the end of the *n* year select period. This method of determining actuarial equivalence factors in the second and subsequent years would not be considered a change in such factors. Thus for example, if a plan, established in year 1, adopts an interest rate of 5 percent for the first ten years and 4 percent thereafter for purposes of determining actuarial equivalence there would be no change in the actuarial basis and therefore no change subject to the anticutback provisions if in year 2 the interest rate was 5 percent for the next nine years and 4 percent thereafter. However, if in year 2, the plan used 5 percent for the first ten years and 4 percent thereafter, the actuarial basis would be changed from that contemplated in year 1 and such change would be subject to the anticutback requirements of section 411(d)(6) of the Code. The use of any other assumptions in the second and later years would result in a change in actuarial equivalence factors.

This is not a ruling letter and should not be treated as such. It has been written in response to your letter asking questions of a general nature.

Sincerely,
Winfield C. Burley
Chief, Pension Actuarial Branch

TRANSACTIONS OF SOCIETY OF ACTUARIES
1985 VOL. 37

DISCUSSION OF PRECEDING PAPER

NICK FRANCESCHINE:

Three approaches are exhibited in this paper for amortizing unfunded actuarial liabilities, and it is suggested that approach 1 may be the only method allowable for minimum funding standard purposes in the United States. I believe that approach 1 is not only allowed but mandated in this situation. The calendar-specific interest rates which are used to discount actuarial liabilities also must be used to discount future amortization payments; otherwise, the funding standard account is not being maintained in compliance with Regulation Section 1.412(b)-1(b)(2). Approach 2 employs a different interest rate in each year of the amortization than that used for discounting liabilities over the same interval. The overriding objection to approach 3, that of nonlevel payments, has already been noted. The perceived problem with approach 1 is that the outstanding balance of the amortization base may increase with time over some duration. This is not really a problem because the outstanding balance serves only two purposes: to monitor the fundamental equation of the funding standard account and to track the eventual amortization of the base. Neither purpose is impeded by the behavior noted previously. A similar phenomenon occurs with respect to maximum deductible bases under Code Section 404(a) in years when the full-funding limitation prohibits a deductible contribution from being made. If no contribution is made in such a year, the outstanding balances of the bases increase, which merely extends the remaining amortization periods of those bases.

I congratulate the authors on a job well done. A scholarly analysis of this subject is long overdue.

PAUL H. JACKSON:

The Society of Actuaries Committee on Pension Principles and Related Research is to be congratulated for publishing helpful information regarding the use of select and ultimate financial assumptions.

The committee states that level financial assumptions would be used if there was a belief that future levels of inflation are unpredictable. This does not seem logical. We are clearly unable to predict inflation over long periods of time. This suggests that as our ability to predict declines with duration, we should rely less on our predictions and more on long-term averages.

The paper notes the clear advantage in the computational ease offered by

level financial assumptions. Computational ease would also be furthered if we were to assume average long-term mortality rates of, say, a constant 5 percent per year. Thirty years ago, with far less computational power, actuaries developed generation mortality tables. An assumption that mortality rates tend to decline in the long term adds conservatism to the valuation process. The cumbersome methodology necessary at the time did not preclude the use of such tables, probably because interest rates were lower then and mortality rates higher. It is hoped that investment yields are of sufficient importance today that arguments regarding computational complexity will not divert actuaries from the natural process of refining their professional tools.

In a paper titled "Variable Actuarial Assumptions and the Solvency of Private Pension Plans," printed in Vol. 3 of the *Transactions* of the 22nd International Congress of Actuaries (Sydney, 1984), I discussed general advantages and disadvantages of variable economic assumptions. I concluded that variable economic assumptions grading off from current levels to more conservative ultimate levels are always preferable for valuations. Appropriate economic assumptions completely avoid the unrealistic results that develop when pension plans with level economic assumptions are faced with immunization, dedicated bond portfolios, or annuity purchases. They minimize criticism from the IRS, accountants, and stockholders. These assumptions more closely match the values produced by the Pension Benefit Guaranty Corporation. They develop more appropriate costs for retiree increases and vesting improvements. Finally, they minimize differences between ongoing plan and termination valuations. Having used variable economic assumptions, I can attest to their complexity. I can also attest to the satisfaction of doing something better.

FRANK KOVACS:

The committee paper provides a discussion of technical issues involved with the use of select and ultimate financial assumptions in pension plan valuations, supplementing the discussion previously published in volume 9 of the *Record*. Neither noted an additional constraint on the assumptions which would make the approach more practical for the entry age normal cost method for pay-related funding.

The paper notes that "under the entry age normal cost method, the partition of plan costs between prior and future periods is a function of assumptions about prior experience." However, the partition becomes a function of the spread between the salary increase assumption and the interest rate assumption, provided the spread is fixed. More precisely, the spread i' is such that $(1 + i')^k = \prod_{t=1}^{k} (1 + i_t)/(1 + s_t)$ at each period k. Thus, the

assumption could be stated without reference to past periods, and only a single set of commutation functions needs to be developed to partition the present value of future benefits. However, several sets of commutation functions would still be needed to develop the present value of future benefits.

By taking this approach one step further and applying it to a nonintegrated benefit formula (e.g., 50 percent of the final year's pay), the present value of future benefits becomes a function of the spread and the lump-sum factor at the assumed retirement age. Thus, in this limited case, only one set of commutation functions would be needed. An adjustment factor would be applied to the present value of future benefits to account for the duration to retirement. The adjustment factor could also adjust for a 5-year final average pay formula. However, this approach would break down in developing the present value of future benefits once the dollar limit of section 415 was exceeded. The approach would still be valid to partition the present value of future benefits.

GERALD RICHMOND:

Double-digit inflation from 1979 to 1981 has stimulated the interest of actuaries in select and ultimate assumptions. Inflation has since subsided to the 4 percent level. Long-term estimates of the inflation for Social Security projections are also presently close to 4 percent. Real interest rates have increased substantially above the historical level of 2–4 percent. Presently, actuaries have much less reason to expect long-term interest rates to decrease substantially from present levels. There is, thus, much less reason now compared to a few years ago to adopt select and ultimate interest assumptions. However, high rates of inflation may recur in the future. The paper thoroughly outlines the *theoretical* considerations in deciding whether or not to adopt select and ultimate assumptions. Unfortunately, a scientific, probabilistic basis for estimating future rates of inflation and future year-by-year interest yields has not yet been made available to the actuarial profession. Until it is made available, I question how confidently actuaries can present select and ultimate assumptions to the public and explain deviations of actual from expected or estimated experience. Perhaps the lack of credibility in volatile economic conditions of level long-term interest assumptions results from our failing to communicate clearly to the public the nature of pension plan costs and related valuation data.

Terminology is important in communicating among ourselves and with the public. In section III, the committee adds a precise definition of select and ultimate interest assumptions. Unfortunately, throughout the paper, it has failed to use terminology recommended by the American Academy of Actuaries Committee on Pension Terminology. The terms *cost, costs, cost*

pattern, expected plan costs, current level of plan contribution requirements, contribution rates, accrual rates, incidence of plan costs, and *liabilities* appear throughout the paper with no clear distinctions among them.

Knowledgeable actuaries can usually read into these terms whether actual ultimate costs or theoretical expected costs are referenced. Actuaries must clearly distinguish pension plan costs that flow from actuarial assumptions from actual or ultimate costs. The statement "a change to select and ultimate assumptions from level rates will alter future *costs* [my emphasis]" clearly references theoretical, not ultimate, costs.

The nonpension actuary and the public are likely to interpret undefined cost as actual or ultimate cost. Ultimate cost is defined by most actuaries as actual benefits plus expenses minus actual investment results over the life of a pension plan. The cost usually referenced in the paper is the theoretical cost flowing from a set of assumptions. If assumptions are reasonably correct over the long run, theoretical costs will converge to ultimate costs. Pension plans are expected to continue indefinitely. Recent experience with plan terminations shows how expected and actual costs may differ.

I would like to see such cost always referenced as theoretical cost, expected cost, or just pension plan contribution (normal cost plus amortization payment).

If we do this and clearly explain to the public that actuaries only make best estimates of long-run plan experience, the public will not misinterpret out-of-context statements, such as "a change to select and ultimate assumptions. . .will alter [future] costs." This is not to say that a meteorologist's judgment regarding future levels of rainfall determines the amount of the rain that actually falls! It is more clear to say that a change in assumptions will alter *expected* future costs.

Also, liabilities should be clearly referenced as either "actuarial accrued liabilities" or "actuarial present values," whichever is intended. Actuaries do not define accounting liabilities; we estimate actuarial liabilities.

I suggest that until we can better predict year-to-year changes in interest rates and future rates of inflation, we may appear more credible to use level long-term assumptions and explain clearly to the public the nature of actuarial estimates. Pension plans are long-run propositions requiring long-run assumptions. Immediate plan experience can differ substantially from long-term expectations. In the current state of our scientific knowledge concerning the basis for predicting future interest rates and inflation, who will appear more credible to the public—an actuary who explains year-to-year fluctuations from long-term expectations, or an actuary who tries to explain year-to-year fluctuations from year-to-year expectations?

The committee is to be complimented for laying out the theoretical considerations in deciding whether or not to adopt select and ultimate assump-

tions. It is hoped that further research to establish a sound scientific (actuarial) basis for making actual decisions will be forthcoming.

RICHARD G. SCHREITMUELLER:

This excellent paper covers an important subject. In a changing economy, assumptions as to rates of inflation, interest, and salary increases should reflect both short-range and long-range expectations. Under some conditions, actuaries can best bridge this gap by varying the year-by-year assumptions.

Pension actuaries long ago began using select and ultimate interest assumptions for single-premium group annuities, insured pension funding contracts, guaranteed investment contracts, and lump-sum cash-outs. Although this required surmounting technical and regulatory obstacles, the result was to provide greater equity to the public. A full set of select and ultimate economic assumptions for valuing pension plans raises new questions, but one may trust that satisfactory answers are forthcoming. As is true for most kinds of pension plan assumptions, large plans seem to require the most sophisticated treatment and can best afford to pay for it.

When interest rates are rising, interest assumptions that do not realistically reflect current yields may well overstate the liabilities for retired lives. This situation has encouraged uninsured plans to transfer pensioner liabilities to insurance companies, creating instant, although temporary, actuarial gains.

Trust fund administrators found that they too could play this game by using dedicated bond portfolios to simulate single-premium group annuity contracts. Actuaries can now preempt such arrangements by using realistic select and ultimate interest rates to value the plan. Conversely, when interest rates are falling, insurance companies wishing to sell single-premium annuities may welcome the use of select and ultimate assumptions that do not inhibit the purchase of annuities by generating temporary actuarial losses.

In summary, select and ultimate economic assumptions are not for everyone, but they can add a dimension that gives a more complete and useful actuarial picture.

JAMES A. STINCHCOMB:

Select and ultimate interest rates can be a useful tool in helping actuaries deal with the year-to-year volatility of available investment yields.

I share the committee's concern that select and ultimate assumptions may carry a misleading implication of greater-than-warranted accuracy. However, I am also concerned that the usual process of setting such assumptions is inaccurate. Most select and ultimate interest assumptions I have seen seem to understate substantially the actuary's best estimates of future experience.

To illustrate, let us apply some of the committee's suggestions on choosing

select and ultimate interest rates. Use Moody's current long-term corporate bond yield as our initial rate; assume it is 10 percent. Use the pension trust's duration as a basis for determining the select period: the trust is like a 30-year bond, so assume a duration and, therefore, a select period of ten years. We believe that over the ten-year period, Moody's long-term yields will drop gradually to an ultimate level of 5 percent, so we pick interim rates grading down arithmetically from 10 to 5 percent over ten years—half a percent per year.

Are we ready to do our valuation? I do not think so.

I share the committee's view that there may be no justification for using select and ultimate rates for a new plan with no assets. Say the plan in this example is well funded. For a well-funded plan, the actual yield on plan assets is a suitable discount rate for defining the plan's liability. What do I expect my actual yields to be? Assuming the trust does behave like a 30-year bond with a 10 percent coupon, the actual first-year return on market will be about 15.25 percent, not 10 percent. We will get the Moody's yield, plus asset appreciation, as the long-term yield drops. The second-year return will be 14.75 percent, not 9.5 percent. The tenth-year return will be 11 percent, not 5.5 percent. Actual returns thereafter will be 5 percent, in line with our ultimate rate.

Of course, if we are using a smoothed asset valuation rate, it will take a few years to build up this spread between actual returns and the beginning-of-each-year Moody's yield. But the investment gains will then continue a few years after the ten-year select period as the smoothing on the later years' investment gains works its way out of the adjusted asset calculation.

The problem: if expected yields are high and they are expected to fall, the returns we expect are higher still. The point is that projected Moody's long-term corporate bond yields are not projected total returns. Our expected returns depend not only on the projected yields but also on the appreciation or depreciation they imply.

Another way to look at this problem: if I used 10, 9.5, . . ., 5.5, and 5 percent thereafter as my select and ultimate interest rate assumption, what would have to happen to make me right? One possibility is that the Moody's rate would stay level the first year, then increase year-after-year. Another possibility is ever-worsening recession, with some of my bonds defaulting and equity holdings depreciating. A third possibility is that the trust's investment managers would keep almost all of the trust in money market instruments for the entire 10-year period.

So in choosing select and ultimate interest rates, it seems that the actuary needs to follow the committee's approach and then add something. If he's not assuming a serious ten-year recession or very disappointing investment management, he should either substantially increase his select rates or sub-

stantially lengthen the select period. In this context, using 10 percent for ten years and 5 percent thereafter makes more sense than 10 percent grading to 5 percent, if the underlying assumption about the year-to-year progression of Moody's long-term yield is 10 percent graded to 5 percent. But even the flat 10 percent rate for ten years seems conservative. In fact, if our bond holdings are immunized, i.e., if their cash flow has the same duration as the next 30 years of projected pension payouts (with a "balloon" at the end of 30 years representing the present value of payouts projected to occur more than 30 years out), most of us would probably use our initial select rate for 30 years, regardless of the duration itself.

Except where "dedicated" assets are involved, our current practices regarding interest assumptions are often very conservative whether our assumed rates are level or select and ultimate. There may be very good reasons to be conservative about investment return assumptions. The danger in using select and ultimate rates of the usual sort is that we forget—or our clients never realize—that despite appearances to the contrary, we are by and large maintaining a very conservative approach. Without clear explanation, this danger can undercut the improvement in credibility we hope to achieve.

STEWART WILDER:

The authors must be congratulated for a most readable, concise discussion of the issues related to the use of select and ultimate financial assumptions. The beginning of the article focused on how many of the concerns of the various regulatory bodies such as FASB and the IRS may be met by the use of such assumptions. I will add my thoughts on considerations relevant to such assumptions for a particular pension plan.

Clearly in the case of the investment return, a select and ultimate assumption is not suitable for all plans. With a new plan, there is no investment experience to analyze, making any assumption other than a flat rate hard to justify. Indeed, as the article suggests, one has to assemble much more evidence to support a select and ultimate financial assumption over a flat one. I would have a basic criterion: Is another actuary with access to the same information about the plan likely to endorse as reasonable the assumptions taken as a whole?

With respect to the investment return assumption, I envision such an actuary asking two questions. First, what support can be offered for the contention that, in general, investment returns will follow a particular pattern over the next several years? Second, why is it believed that the assets of this particular plan will achieve a return close to that of the proposed assumption? The second question is just as important as the first since there are numerous factors that influence how a particular portfolio, and its in-

vestment manager, will respond to a given swing in the economy. For example, one might expect much greater volatility if the plan assets are part of a self-directed trust than if they are part of the general account of an insurance company. Also the investment manager may have structured the portfolio to respond in ways contrary to the expected future path of the economy.

As the article points out, other assumptions, especially the financial ones, are also likely to be revised to more explicitly reflect the course of the economic environment. For example, if the investment return assumption is select, usually the assumptions for the projection of the Social Security Wage Base (Maximum Pensionable Earnings in Canada) and CPI will have to be select as well. What should be done with the salary increase assumption is much less clear. The direction a particular industry can be expected to take may be quite different from that of the economy as a whole. In many cases the select rates of salary increase may be developed somewhat independently of the investment return assumption.

As for the rates of decrement, my experience has been that actuaries tend not to alter these rates by calendar year despite the dynamic nature of the financial assumptions. While this may be warranted in many cases, actuaries do have an obligation to consider the sensitivity of decrements such as retirement, disability, and withdrawal to the economic cycle as manifested in the experience of the plan. It should not matter that the accountants and others exhibit much less interest in this area (presumably because they feel decrement rates are the domain of the actuary). The goal in setting assumptions should remain to achieve a consistent degree of recognition of expected experience.

This is not to minimize the difficulty in selecting decrement rates. Frequently, there is little information to go on. Decrements such as retirement and layoff may be highly attuned to events such as the cancellation of major contracts or the imposition of import/export quotas. In many cases, the value of the actuarial gains or losses due to these discontinuities is magnified by the presence of early retirement subsidies or severance benefits.

If the uncertainty surrounding one or more of these cyclical decrements is such that introduction of a calendar year dependent element is not feasible, some types of sensitivity analysis is called for if select and ultimate financial assumptions are being considered, especially for larger plans. One way to proceed would be for the actuary to develop defensible alternatives to the current decrement rate assumptions. Valuations could be done on each of these bases and compared with the impact of switching to select and ultimate financial assumptions. If the potential variation from the current decrement rates appears to be much more important than the use of select and ultimate

financial assumptions, the actuary is in a poor position to recommend use of such financial assumptions given the uncertainty in the decrement rates.

The actuary's obligation as I see it in such a case is to impress upon the sponsor the wide variation possible in the ultimate costs of the plan and not to foster an illusory sense of credibility by adopting more sophisticated financial assumptions. In less extreme cases, this uncertainty would at least argue for keeping the select rates of the financial assumptions as simple as possible—perhaps just maintaining constant initial rates for five to ten years, followed by the ultimate rates.

Another area I will comment on is the valuation approach to be used with select and ultimate financial assumptions. I suspect the majority of existing valuation programs depend on the use of commutation functions rather than the explicit development of benefit payout streams. In those cases, the suggested modification of the program to recompute the commutation functions upon a change in attained age appears easiest to implement, so long as it is possible to have the data sorted by attained age. Decrement rates that vary by calendar year would also appear to be easy to implement with this type of sorting. Though the resulting computer costs, other things being equal, should not increase substantially, the initial development costs may be considerable depending on the size and organization of the program.

Regarding the recalculation of the commutation functions, I suggest use of a formula that will duplicate the standard commutation functions in the case where all interest rates are equal. Thus, if, for a given attained age x, the rate i_y represents the interest rate assumed to apply between ages y and $y+1$, the postretirement commutation function D_z could be defined as

$$D_z = \left(\prod_{y=0}^{z-1} \frac{1}{1 + i_y} \right) l_z.$$

Then the commutation functions N_z, M_z, and so on could be developed via the usual relationships. If all of the i_y values were equal, then the computation of D_z reduces to the standard formula

$$D_z = v^z l_z,$$

where $v = 1/(1+i)$ and $i = i_y$ for all ages.

(AUTHORS' REVIEW OF DISCUSSION)
SOCIETY OF ACTUARIES COMMITTEE ON PENSION PRINCIPLES AND RELATED RESEARCH:

We are gratified that this paper elicited a considerable response from the actuarial profession. The comments raise a variety of interesting

philosophical and technical points concerning the use of select and ultimate financial assumptions.

Mr. Franceschine states that of the three approaches discussed for amortization of unfunded actuarial liabilities, approach 1 would be the mandated approach in the United States. A strict interpretation of Regulation Section 1.412(C)(3)-1 on reasonable funding methods and of section 412(b) would indeed lead to this conclusion. However, the IRS code and regulations were published prior to the wide-spread use of select and ultimate interest assumptions. Often the prevailing regulatory structure is modified when faced with new practices. The appendix of the paper showing correspondence between the committee and the IRS is a case in point. In response to the committee's letter, the IRS clarified the application of current legal requirements on actuarial equivalence factors when select and ultimate interest rates are used. The IRS clarification was later released as an amendment to the Internal Revenue Code.

We thank Mr. Jackson for the reference he provides on previous work done on the subject. The increased sophistication introduced by the use of select and ultimate interest rates can be compared to that gained when level mortality rates were first replaced by generation mortality tables. Although generation mortality tables are now the norm, we suspect that the size of the plan will determine whether the technical complexity associated with the use of select and ultimate interest rates is warranted.

Mr. Kovacs is quite correct to limit use of his spread-based calculations to situations involving nonintegrated final pay formulas not involving section 415 limits. Consulting actuaries might still develop a generalized valuation program if they choose to use select and ultimate financial assumptions for a particular client (as the simplified mathematical structure will be inadequate when the client asks for cost analysis of alternative benefit designs) or when pay increases or changes in the section 415 limits result in projected benefits striking the maximum benefit limitations. In cases where a complex salary scale assumption based on an employee's years of service is used, the constant spread between the interest and salary scale assumptions may not hold, and the analysis will not apply. In such a case, several sets of commutation functions will be necessary.

Mr. Richmond believes that the disadvantages of using select and ultimate interest assumptions outweigh the advantages. The paper has not advocated the use of either select and ultimate or level interest assumptions but, instead, has discussed the philosophical and technical considerations in the use of select and ultimate assumptions. Both level interest assumptions and select and ultimate interest assumptions have plenty of support.

As Mr. Schreitmueller points out, consideration of select and ultimate financial assumptions may lead to a more realistic rate structure in the par-

ticular actuarial application involved. To the extent that such analysis provides a greater understanding of the conservatism or prior assumptions, the actuary may be able to choose a more realistic level rate (which will help reduce the attractiveness of some of the transactions Mr. Schreitmueller describes).

Mr. Stinchcomb's application of Moody's long-term bond rate in developing a select and ultimate interest rate differs from the committee's understanding of the nature of select and ultimate rates. In a given situation, select rates are calendar specific and reflect expected yield for a particular calendar year. If the suggested Moody's rate pattern is applied prospectively to the assets, in the first year, all assets are assumed to be invested in 10 percent 30-year bonds, so the select rate for the first year would be 10 percent. In the second year, the 9.5 percent assumption for 30-year bonds would apply only to investments for that year, probably comprised of the excess of contributions over benefits payments and bond dividends to be reinvested. The calendar-specific select rate for the second year would therefore be a blend of the 10 and 9.5 percent rates based on the assumed asset mix. This approach will produce significantly higher select and ultimate rates than suggested by Mr. Stinchcomb, resolving the concern of understatement of assumptions.

The alternative development suggested, producing select rates ranging from 15.25 to 11 percent and an ultimate rate of 5 percent, is theoretically correct if the entire pension trust portfolio were to be liquidated and reinvested in 30-year bonds at the end of each year. However, this would not be an appropriate assumption for most investments in long-term bonds, particularly under the interest rate forecast assumed by Mr. Stinchcomb. Also, the discontinuity created by the alternative method between the final select rate and the ultimate rate would be highly undesirable in most applications.

While the peer review suggested by Mr. Wilder is useful input in assessing the appropriateness of select and ultimate financial assumptions, the committee would hope that the actuary would also do objective analysis to determine the appropriateness of the select and ultimate approach. This objective analysis should include traditional gain and loss analysis. The committee agrees that sensitivity analysis and development of underlying support data for more complex financial and decrement assumptions would be a useful part of this objective analysis.

The IRS recently issued a publication entitled *Actuarial Guidelines Handbook*. The handbook contains several worksheets that are used in assisting the IRS in enforcing the minimum funding standards and the maximum limitations on deductible contributions which apply to certain pension plans. The handbook will be used in conjunction with an audit of the actuarial assumptions used for the pertinent calculations. With reference to the interest

assumptions, worksheet III compares the expected return on plan assets to the actual yield.

When a select and ultimate interest assumption is used, computing the expected return on plan assets follows the same general rule that applies with assumed level rates. With an assumed level rate, the expected return for the year is the assumed rate multiplied by the asset value at the beginning of the year with adjustments to reflect contributions made and benefits paid during the year. With a select and ultimate assumption, the yield on assets is specifically assumed to vary during the select period. Accordingly, where the first rate applies during the full first year, the expected return for the first year is the first rate multiplied by the corresponding asset value with adjustments to reflect the cash flow. If there is no change in the interest assumption, the expected return for the second year is determined the same way using the interest rate in effect for the second year.

The IRS actuarial worksheet underscores one of the advantages of using select and ultimate interest rates. Interest rates in the early select years that reflect prevailing rates will, almost by definition, be reasonable: actuarial yields will more closely approximate those which are expected.