A MATHEMATICAL ANALYSIS
OF *FINANCIAL ACCOUNTING STANDARD* No. *88*

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ABSTRACT

This paper builds on the analysis of Berin and Lofgren of *Financial Accounting Standard No. 87* [3]. The relationships between the quantities defined in *FAS 87* are further explored, and the analysis is extended to cover *FAS 88*, "Employers' Accounting for Settlements and Curtailments of Defined-Benefit Pension Plans and for Termination Benefits." The paper will be useful for actuaries working with U.S. pension plans and will be an aid in understanding the impact of *FAS 87* and *FAS 88*.

I. INTRODUCTION

Statements of Financial Accounting Standards No. 87 and *No. 88* (*FAS 87* and *88*) were issued in December 1985 by the Financial Accounting Standards Board (see, for example, [4]). The titles of *FAS 87* and *88* are "Employers' Accounting for Pensions" and "Employers' Accounting for Settlements and Curtailments of Defined-Benefit Pension Plans and for Termination Benefits," respectively. The statements have had a substantial impact on the accounting for pension costs in employers' financial statements and are of significance to actuaries [1], [2].

FAS 87 and *88* are expressed in prose style with numerical examples but no formulas. Understanding and precision are enhanced by a mathematical representation of their logical content. Berin and Lofgren [3] have developed a very useful mathematical representation of *FAS 87*. In this paper, a few additions are made to their work, and then the formulas are extended to cover *FAS 88*. This paper extends my description given in [5].

II. *FAS 87*

The notation, given in Appendix A, and the formulas, given in Appendix B, are based on the analysis of Berin and Lofgren [3]. In this section are described the areas in which the formulas in Appendix B constitute extensions of the work of those authors.

In the calculations of pension expense, Equation (87.1) in Appendix B uses items available at the beginning of the relevant year. Equation (87.2) gives the same sum, but the components are those required to be disclosed at the end of the year under paragraph 54b of *FAS 87*.

Equation (87.4) expresses the actual return on fair value as a sum of expected return and the interest gain. Equation (87.5) gives the interest gain as the required "hybrid," involving elements of the return on fair value and the return on market-related value.

At transition to the use of *FAS 87* procedures, an adjustment is made if market-related value and fair value are not equal. Equation (87.11) expresses the gain subject to amortization with the required adjustment. Equation (87.17) gives the required minimum amortization of the gain; amortization increases the pension expense if the sign of the gain is negative (a loss).

III. *FAS 88*

FAS 88 is applicable to settlements, curtailments and termination benefits. These are considered in turn. Mention is then made of asset reversions prior to adoption of *FAS 87* and *88* and to plan termination.

A. Settlement

Under *FAS 88*, a settlement is a transaction that
(a) Is irrevocable
(b) Relieves the employer or plan of primary responsibility for a pension benefit obligation
(c) Eliminates significant risks related to the obligation and the assets used to effect the settlement [4].

One example of a settlement is the making of lump-sum cash payments to plan participants in exchange for their benefit rights. Another example is the purchasing of nonparticipating annuity contracts to cover vested benefits. The settlement occurs when the exchange has been accomplished. Other examples include the purchase of nonparticipating annuity contracts to cover vested benefits and the transfer of assets to a successor employer's plan.

The settlement ratio is the proportion of the *PBO* that is settled:

$$SR = \frac{PBO_S^B - PBO_S^A}{PBO_S^B} \tag{88.1}$$

where superscripts B and A refer to the *PBO* before and after the settlement, respectively.

The maximum gain recognized on a settlement is defined as

$$MAXGAIN = UG_S^B - \min [UT_S^B, 0]. \tag{88.2}$$

In the above, UG_S^B is the cumulative unrecognized gain at the time, S, of settlement. It includes the effect of any assumption change necessary to make the PBO for settled benefits equal the cost of settlement. It can be seen that the maximum gain is increased by any positive unrecognized transition asset (shown here as a reduction by a negative unrecognized transition obligation). The maximum gain (but not the maximum loss) is reduced by the cost of participation rights when participating annuities are purchased.

The occurrence of a settlement requires treatment of any deferred gains, UG (possibly negative), and any unrecognized transition liability, UT (only if negative). The accounting recognition is given by

$$RECSET = SR \times MAXGAIN. \tag{88.3}$$

The deferred gain, UG, may have arisen from many sources, and on a settlement some of it must be recognized immediately. After settlement, any asset reversion to the employer or transfer of excess assets to a successor employer's plan is treated as a negative contribution.

B. Curtailment

A curtailment is "an event that significantly reduces the expected years of future service of present employees or eliminates for a significant number of employees the accrual of defined benefits for some or all of their future service" [4]. An example is the closing of a facility with the resulting termination of employees' service. Another example is a plan amendment that results in a reduction of future benefit accruals. Usually a curtailment will result in a reduction of the *PBO*. A curtailment loss is recognized when a commitment is made to take the action, while a curtailment gain is not recognized until the related employees terminate or the plan amendment becomes effective.

The curtailment ratio for the net transition obligation depends on the before and after curtailment expected future service values, EFS_{UT}. The expected future service values are calculated for those who were participants in the plan at transition to *FAS 87*:

$$CR_{UT} = \frac{EFS^B_{UT} - EFS^A_{UT}}{EFS^B_{UT}}. \tag{88.4}$$

The curtailment ratio applicable to the k-th amendment depends on the expected future service of those who were participants at the time of the amendment. The expected future service is measured immediately before and immediately after the curtailment. Then the curtailment ratio is given by

$$CR_{PSC(k)} = \frac{EFS^B_{PSC(k)} - EFS^A_{PSC(k)}}{EFS^B_{PSC(k)}}. \tag{88.5}$$

In the case of curtailments, account must be taken not only of curtailment gains and any unrecognized transition liability but also of unrecognized prior service cost resulting from plan amendments:

$$TOTRECCUR = RECCUR_{GAIN} + RECCUR_{UT} + RECCUR_{PSC}. \tag{88.6}$$

The recognition of any positive curtailment gain $(PBO^B_C > PBO^A_C)$ is, by *FAS 88* paragraph 13, reduced in consideration of any unrecognized net loss $(UG^B_C < 0)$. The latter is adjusted with respect to any remaining unrecognized net asset existing at the date of transition to *FAS 87*:

$$RECCUR_{GAIN} = \max\{0; PBO^B_C - PBO^A_C + \min[0; UG^B_C - \min(0; UT^B_C)]\}$$

$$\text{for } PBO^B_C > PBO^A_C. \tag{88.7a}$$

If the curtailment gain is negative, then the recognition is reduced by any unrecognized net gain at the time C of curtailment $(UG^B_C > 0)$ and is given by

$$RECCUR_{GAIN} = -\max\{0; PBO^A_C - PBO^B_C - \max[0; UG^B_C$$

$$- \min(0; UT^B_C)]\} \text{ for } PBO^B_C < PBO^A_C. \tag{88.7b}$$

Note that (88.7a) and (88.7b) form a pair of equations for $RECCUR_{GAIN}$; which is used depends on the sign of $PBO^B_C - PBO^A_C$. Examining (88.7a), one sees that any unamortized loss satisfying $UG^B_C - \min(0; UT^B_C) < 0$ is netted against the gain $PBO^B_C - PBO^A_C$. Any unamortized gain would not be recognized at curtailment but would continue to be amortized after the curtailment.

The prior service cost recognized due to curtailment is given by

$$RECCUR_{UT} = -CR_{UT} \times \max\ (0;\ UT_C^B). \qquad (88.8)$$

It is necessary to treat plan amendments separately in view of the different ratios applicable to each amendment:

$$RECCUR_{PSC} = -\sum_k CR_{PSC(k)} \times UPSC_{C(k)}^B. \qquad (88.9)$$

C. Termination Benefits

Special termination benefits may be offered for a short time. The liability and loss are recognized when employees accept the offer and the amount can be reasonably estimated. *Contractual termination benefits* are required by the terms of a plan only if a specified event, such as a plant closing, occurs. The liability and loss are recognized when it is probable that employees will be entitled to benefits and the amount can be reasonably estimated [4].

D. Asset Reversion at Transition

A situation can arise in which, prior to transition to *FAS 87*, some pension obligations were settled and plan assets reverted to the employer. Then the asset reversion was being amortized, that is, reducing pension expense, at the time of transition, with outstanding unamortized amount UAR_{00}. Then at transition to *FAS 87* a gain, *ARGAIN*, should be recognized (paragraph 20 of *FAS 88*):

$$ARGAIN = \min\ [UAR_{00};\ \max\ (-UT_{00},\ 0)]. \qquad (88.10)$$

E. General

In the event that a settlement, curtailment or termination benefit situation occurs during an accounting period, the pension cost for the period equals the sum of the prorated net periodic pension costs under *FAS 87* for the portions of the accounting period before and after the event plus the loss (minus the gain) associated with the event.

F. Plan Termination

Plan terminations can be curtailments and/or settlements. A shutdown without plan termination may be an example of a curtailment without a

settlement. A plan termination with settlement of the obligation may be both a settlement and a curtailment.

If a defined-benefit plan is terminated and replaced by a defined-contribution plan, there is a curtailment with $EFS^A_{UT}=0$ and $EFS^A_{PSC(k)}=0$, as well as a settlement with $SR=1$. The net effect of these two events is that a gain is recognized equal to the assets reverted, minus the PPC existing immediately prior to the transaction.

If a defined-benefit plan is terminated and replaced with another defined-benefit plan, then there is a settlement but no curtailment. Then Equation (88.3) is used.

IV. SUMMARY

It is hoped that the formulas presented here will provide a more precise meaning to the pension accounting standards. In addition, it will be less time-consuming to review the set of formulas than to re-read the original statements.

ACKNOWLEDGMENTS

REFERENCES

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3. BERIN, B.N., AND LOFGREN, E.P. "Financial Accounting Standards No. 87: Recursion Formulas and Other Related Matters," *TSA* XXXIX (1987): 9–18.
4. PEAT MARWICK MITCHELL & CO. *Interpretation of Pension Statements*. Issued as Study Note 461-32-87, Society of Actuaries, 1987.
5. SHARP, K.P. *Actex Manual for P-461*. Edited by D. London. Winsted, Conn.: Actex Publications, 1991.

APPENDIX A

NOTATION

ABO	= Accumulated benefit obligation calculated using the projected unit credit method
AEFS	= Averaged expected future service
AL	= Additional liability
AMENDT	= Increase in *PBO* from plan amendments
AMORT	= Amortization
AMORTGAIN	= Amortization of the transition gain
AMORTTRANS	= Amortization of the UGA
APC	= Accrued pension cost (if *PPC<0*)
ARGAIN	= Asset reversion gain on transition
$\Delta ASSUMPTNS$	= Decrease in *PBO* from assumption changes
B^{DR}	= Benefit payments made with interest to the end of the year at rate i_{DR}
B^{ELTR}	= Benefit payments made with interest to the end of the year at rate i_{ELTR}
C	= Contributions with interest to the end of the year at rate i_{ELTR}
CR	= Curtailment ratio
CTE	= Charge to equity (pretax)
CUMAMORT	= Accumulated amortization
EB	= Expected (at start of year) benefit payments with interest to the end of the year at rate i_{ELTR}
EFS	= Expected future service (in person-years)
EI^{PA}	= Expected interest (based on fair value)
EI^{MRV}	= Expected interest (based on market-related value)
EPA	= Expected plan assets (at fair value)
EPBO	= Expected projected benefit obligation
G	= Gain (= −loss)
HIG	= Hybrid interest gain (hybrid between *PA* and *MRV*)
i_{DR}	= Discount rate
i_{ELTR}	= Expected long-term rate
IA	= Intangible asset
IG^{PA}	= Interest gain (based on fair value)
IG^{MRV}	= Interest gain (based on market-related value)
LG	= Liability gain

MAXGAIN	= Maximum gain recognized on settlement
ML	= Minimum (balance sheet) liability
MRV	= Market-related value of plan assets
NC	= Current service cost or normal cost, calculated at year-end, that is, annually in arrears, using the projected unit credit method
PA	= Market (fair) value of plan assets
PBO	= Projected benefit obligation calculated using the projected unit credit method
PE	= Pension expense
PPC	= Prepaid pension cost
PPC_{00}	= Prepaid pension cost on books at transition to *FAS 87*
$RECCUR_{GAIN}$	= Recognition of curtailment gain
$RECCUR_{LOSS}$	= Recognition of curtailment loss
RECSET	= Recognition of settlement gain
SR	= Settlement ratio
TOTRECCUR	= Total recognition of curtailment gain
UAR	= Unamortized asset reversion
UG	= Cumulative unrecognized gain from experience and assumption changes ($UG_{00} = 0$)
UGA	= Cumulative unrecognized gain subject to amortization
UPSC	= Unrecognized prior service cost, increased only by amendments ($UPSC_{00} = 0$)
UT	= Unrecognized transition obligation
00	: date of transition to *FAS 87*
0	: start of a general year
1	: end of a general year.

APPENDIX B

SUMMARY OF FORMULAS FOR *FAS 87*

1. Components of Pension Expense

Pension Expense

A. *Computation at Beginning of Year*

$$_0PE_1 = {_0}NC_1 + i_{DR}PBO_0 - {_0}EI_1^{MRV}$$

$$+ {_0}AMORTUPSC_1 - {_0}AMORTGAIN_1$$

$$+ {_0}AMORTTRANS_1 \qquad (87.1)$$

B. *Disclosure of End of Year*

$$_0PE_1 = {_0}NC_1 + i_{DR}\,PBO_0 - [{_0}EI_1^{PA} + {_0}IG_1^{PA}]$$

$$+ [{_0}EI_1^{PA} + {_0}IG_1^{PA} - {_0}EI_1^{MRV}] + {_0}AMORTUPSC_1$$

$$- {_0}AMORTGAIN_1 + {_0}AMORTTRANS_1 \qquad (87.2)$$

Expected Return on MRV and PA

$$_0EI_1^{MRV} = i_{ELTR}MRV_0 \qquad (87.3a)$$

$$_0EI_1^{PA} = i_{ELTR}PA_0 \qquad (87.3b)$$

In these formulas, as in others, end-of-year benefit payments and contributions are assumed.

Actual Return on Fair Value

$$_0EI_1^{PA} + {_0}IG_1^{PA} = PA_1 - PA_0 - ({_0}C_1 - {_0}B_1^{ELTR}) \qquad (87.4)$$

Hybrid Interest Gain [in Pension Expense Formula (87.2)]

$$_0HIG_1 = {_0}EI_1^{PA} + {_0}IG_1^{PA} - {_0}EI_1^{MRV}$$

$$= PA_1 - PA_0 - ({_0}C_1 - {_0}B_1^{ELTR}) - i_{ELTR}MRV_0 \qquad (87.5)$$

Liability Gain

$$EPBO_1 = PBO_0 + {}_0NC_1 + i_{DR}PBO_0 - {}_0B_1^{DR}$$

$$+ AMENDT_1 - \Delta ASSUMPTNS_1 \qquad (87.6)$$

$${}_0LG_1 = EPBO_1 - PBO_1 \qquad (87.7)$$

Asset Gain (Fair Value)

$$EPA_1 = PA_0 + {}_0EI_1^{PA} + {}_0C_1 - {}_0B_1^{ELTR} \qquad (87.8)$$

$${}_0IG_1^{PA} = PA_1 - EPA_1 = PA_1 - PA_0 - {}_0EI_1^{PA} - {}_0C_1 + {}_0B_1^{ELTR} \quad (87.9)$$

Cumulative Unrecognized Gain

$$UG_1 = UG_0 + [\Delta ASSUMPTNS_1 + {}_0LG_1 + {}_0IG_1^{PA}]$$

$$- {}_0AMORTGAIN_1 \qquad (87.10)$$

where $UG_0 = UG_{00} = 0$ at the date of FAS 87 adoption.

Gain Subject to Amortization

$$UGA_1 = UG_1 - (PA_1 - MRV_1)$$

$$+ [MRV_{00} - PA_{00} - CUMAMORT(MRV_{00} - PA_{00})] \quad (87.11)$$

Formula (87.11) results because MRV is used in calculating the pension expense, Formula (87.1), while the amortization of UPSC [Formula (87.16)] and UT [Formula (87.14)] is of amounts based on PA.

Net Transition Obligation

$$UT_{00} = PBO_{00} - PA_{00} + PPC_{00} \qquad (87.12)$$

Note: 00 refers to the date of FAS 87 adoption.

Amortization of Transition Obligation

$$UT_1 = UT_0 - {}_0AMORTTRANS_1 \qquad (87.13)$$

$${}_0AMORTTRANS_1 = UT_{00}/AEFS_{00} \qquad (87.14)$$

Prior Service Cost

$$UPSC_1 = UPSC_0 - {}_0AMORTUPSC_1 + AMENDT_1 \quad (87.15)$$

$${}_0AMORTUPSC_1 = UPSC_0/AEFS_0 \quad (87.16)$$

with $UPSC_{00}=0$.

Minimum Amortization of Gains

$${}_0AMORTGAIN_1 = \text{sign }(UGA_0) \times \text{max } [0; |UGA_0|$$

$$- 0.10 \text{ max } (PBO_0, MRV_0)]/AEFS_0 \quad (87.17)$$

Projected Future Service

Count only service that leads to an employer-funded benefit (from the plan) for amortization purposes. To handle this, we give $E_{x+t}^{(d)}$ the value of 1 if an employer-funded benefit is awarded on decrement d at age $x+t$, and value 0 otherwise. The value 0 is used if the benefit is wholly employee-funded and optionally also if a termination benefit is not significant.

The projected future service summed over the set of employees A is

$$EFS = \sum_A \sum_{s=0}^{r-x-1} {}_sp_x^{(T)}(s + 1) \sum_d q_{x+s}^{(d)} E_{x+s}^{(d)}$$

$$= \sum_A \sum_{t=0}^{r-x-1} \left\{ \sum_{s=t}^{r-x-1} \left[{}_sp_x^{(T)} \sum_d q_{x+s}^{(d)} E_{x+s}^{(d)} \right] \right\} \quad (87.18)$$

This formula can be seen to be correct for the special case of a single employee, for example,

$$q_{x+s}^{(d)} \begin{cases} = 1 & \text{if } x + s = r - 1, \quad d = ret \\ = 0 & \text{otherwise} \end{cases}$$

$${}_sp_x^{(T)} = 1 \qquad x < r - s$$

where it gives the answer $(r-x)$. To get $AEFS$, one needs to average the above over all employees:

$$AEFS = \frac{EFS}{\sum\limits_{A} \sum\limits_{s=0}^{r-x-1} {}_sp_x^{(T)} \sum\limits_{d} q_{x+s}^{(d)} E_{x+s}^{(d)}} \qquad (87.19)$$

If $q_{x+s}^{(d)}$ corresponds to a vested termination, then $E_{x+s}^{(d)}$ represents the probability of living to retirement age, so it is neither zero nor one. For a defined-contribution plan, $E_{x+s}^{(d)}$ could represent the probability of living long enough to collect more than employee contributions.

According to paragraph 25 of FAS 87, the amortization is over the remaining life expectancy of participants (rather than employees) "if all or almost all of a plan's participants are inactive." Paragraph 26 allows "consistent use of an alternative amortization approach that more rapidly reduces the unrecognized cost of retroactive amendments." FAS 87 paragraphs 27, 32, 33 and 77 concern other possible variations from the formula.

2. Disclosure and Balance Sheet Items

Prepaid Pension Cost

$$PPC_0 = -PBO_0 + PA_0 + UPSC_0 - UG_0 + UT_0 \qquad (87.20)$$

$$PPC_1 = PPC_0 + {}_0C_1 - {}_0PE_1 \qquad (87.21)$$

The prepaid pension cost asset (or, if negative, the unfunded accrued pension cost) tracks the accumulated difference of pension funding minus pension expensing. Formula (87.20) is true for all times 0, including times 00 and 1.

Minimum Liability

$$ML_1 = \max[0, ABO_1 - PA_1] \qquad (87.22)$$

The employee recognizes in the balance sheet a liability equal to the unfunded accumulated benefit obligation. No net asset is recognized if plan assets exceed the ABO.

Additional Liability

$$AL_1 = \begin{cases} \max[0, ML_1 + PPC_1] & \text{if } ML_1 > 0 \\ 0 & \text{if } ML_1 = 0 \end{cases} \tag{87.23}$$

The "additional liability" is additional to the accrued pension cost liability (treated here as $-PPC_1$). Note that the additional liability is not additional to the minimum liability. Instead it is the amount that is required in addition to any accrued pension cost $(-PPC_1)$ to give a total liability equal to the minimum liability.

Intangible Asset and Charge to Equity

$$IA_1 = \min[AL_1, \max(0, UT_1) + UPSC_1] \tag{87.24}$$

$$CTE_1 = \max[0, AL_1 - UT_1 - UPSC_1] \tag{87.25}$$

The intangible asset offsets (at least partially) the additional liability; the logic is that an asset can reasonably be held because of improved employee morale and so on, up to the unamortized portion of plan amendments and so on. Formula (87.25) is the charge to equity before tax recognition; it includes such items as worse-than-expected asset returns.