TRANSACTIONS OF SOCIETY OF ACTUARIES
1993 VOL. 45

# THE APPLICATION OF FUZZY SETS TO GROUP HEALTH UNDERWRITING

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ABSTRACT

We use fuzzy sets to model the selection process in group health insurance. Fuzzy sets describe collections of objects whose boundaries are not precisely defined; in particular, we define fuzzy sets that characterize groups that are good underwriting risks. First, we consider single-plan underwriting and then extend the work to multiple-option plans.

## 1. INTRODUCTION

In this paper, we use fuzzy sets to model the selection process in group health insurance. First, we present the underwriting of a group of employees covered by a single plan of health insurance. Next, we extend the work to group selection in a multichoice environment. As more employers offer multiple plans to their employees, an underwriting scheme for such situations becomes important.

Fuzzy sets describe collections of objects whose boundaries are not precisely defined. Indeed, a fuzzy set is a mapping, $f$, from the universe of discourse, $X$, to the unit interval $I=[0, 1]$. The value $f(x)$ represents the degree to which $x$ is a member of the fuzzy set given by $f$. This definition generalizes the identification of a crisp set with its characteristic function.

For example, consider the statement: To underwrite a group, require a minimum percentage participation in the group health plan. Insurers often quantify this by requiring 75 to 85 percent participation. This rule can be represented as a fuzzy set through the function:

$$f(x) = \begin{cases} 0, & 0 \leq x \leq 0.75 \\ 10x - 7.5, & 0.75 \leq x \leq 0.85 \\ 1, & 0.85 \leq x \leq 1.00, \end{cases}$$

in which $f(x)$ is the possibility of accepting the group that has $x$ proportion of participation. This function can be refined to include variations

*Dr. Young was awarded Fellowship Credit for this research paper.

for group size, for participation of employees with dependents, or for both.

See Section 2 for an introduction to fuzzy sets; references for this topic include [1], [5], [6], [11], [12], [14], [15], and [16].

Lemaire [8] provides a model for underwriting individual life insurance using fuzzy sets. We follow Lemaire's lead by creating models for underwriting group health insurance. In Section 3, we examine single-option plans, and in Section 4, multiple-option plans. The Society of Actuaries has published several Study Notes that contain rules for group selection [7, pp. 2, 10–18], [10, pp. 48–52], and [13, pp. 4–9]. One problem is to determine how these underwriting rules interact.

Fuhrer and Shapiro [4] model selection in multiple-option plans, and Mailander [9] lists factors that influence selection in such plans. These include the plan of benefits, access to care, employee costs, and the age, sex, and marital status of the individuals in the group. We define fuzzy sets that measure the possibility of accepting the group based upon such considerations. This measure of possibility can be used to develop underwriting loads for various benefit plans.

Finally, in Section 5, we suggest areas for future research, such as applying fuzzy sets to trend analysis and to credibility theory.

I thank several colleagues: Joseph W. Michel, for direction as the sponsor of my work; David W. Pray, for suggestions about the content of this paper; and Wendell L. Holt, for comments about the underwriting rules. I also thank my Research Paper Committee and Dan Dieterich for recommending valuable changes.

## 2. FUZZY SETS

### *2.1 Definition and Examples*

Zadeh introduced the theory of fuzzy sets in a paper published in 1965 [14]. Research in this area has expanded so that now there are journals devoted to fuzzy set theory. Also, the applications of fuzzy sets cover a broad range, such as artificial intelligence, linguistics, economics, decision-making, and consumer products [5, p. 301], [12, p. 37]. Lemaire's 1990 paper [8] presents some of the first applications of fuzzy set theory in actuarial science.

Fuzzy sets describe concepts that are vague. The fuzziness of a set arises from the lack of well-defined boundaries. This lack is due to the equivocal nature of language and to the subjective interpretation of it.

For example, let $A$ represent the set of good basketball players among the set of teenage basketball players, $X$. Some teenagers clearly do not play basketball well; however, others are decidedly talented players. Between these two extremes lie marginally good players. In set theory, the grade of membership 1 is assigned to elements in the given set, and the grade 0, to those not in the set. In general, the borderline cases have membership values in $A$ between 0 and 1, and the better players have values closer to 1.

People assign membership values subjectively. Indeed, they disagree on what constitutes a good basketball player, thus creating different fuzzy sets to represent $A$. Also, one person may develop distinct fuzzy sets to represent the same concept at varying times. Context, or the underlying universe of discourse, $X$, is another important factor in determining the membership grades of elements in a fuzzy set. If $X$ were the set of all basketball players 13 years of age or older, then the fuzzy set of good basketball players in $X$ would necessarily change. For instance, an otherwise proficient 16-year-old basketball player would not be judged as skilled when compared to a professional player. In this paper, we do not deal with fuzziness that comes from these kinds of subjectivity.

Complexity also can add to the fuzziness of a set. In evaluating a good basketball player, several attributes are measured: ball-handling (dribbling and passing), shooting (field goals and free throws), defensive work (rebounding, guarding, and ability to cause turnovers), physical stamina, and aerobic capacity. In what follows, we consider fuzziness that comes from vague, complex concepts. In this section, we give the definition of a fuzzy set and some examples [14, pp. 339–340], [15, pp. 4–10, 13], [16, pp. 199–201].

### *2.1.1 Definition*

A *fuzzy set*, $A$, in a universe of discourse, $X$, is a function of the form

$$f_A : X \to I = [0, 1].$$

The function $f_A$ is called the *membership function* of $A$, and for any $x$ in $X$, $f_A(x)$ in $[0, 1]$ represents the grade of membership of $x$ in $A$.

This definition generalizes the one for a nonfuzzy, or crisp, set. Such a set is given by a characteristic function:

$$f_A : X \to (0, 1),$$

in which $f_A(x)=1$ if $x$ is in $A$; otherwise, $f_A(x)=0$.

The following two examples illustrate how fuzzy sets can be applied to complex situations that an actuary may encounter.

### *2.1.2 Example*

Among other factors, the following ten influence health-care trend:

- Aging of the insured population
- Higher expectations of medical care and the consequent increase in utilization
- Higher standards of living of the insured population
- Use of more expensive technology
- Cost-shifting from public to private payors
- Rising cost of medical malpractice insurance
- Defensive medical practices due to threat of malpractice
- Leveraging effect of deductibles
- Mandated benefits
- Increased risk due to new diseases, such as AIDS.

These components interact in a complex manner; for example, the costs of defensive medicine can vary directly with the cost of malpractice insurance. As another example, the changes in charges and utilization can vary inversely as physicians attempt to maintain a given standard of living.

Fuzziness exists inherently in these complex relationships. It also arises when the trend for prospective pricing is being determined: If the chosen trend is too high, then good risks may leave the insurance company, and an assessment spiral may develop. If the trend is set too low, then profits may be low or even negative due to inadequate pricing. See Example 2.2.13 below for further discussion of trend.

### *2.1.3 Example*

The evaluation of fixed-income securities is very complex [2, pp. 30–39]. Indeed, risk arises from many sources, but primarily from the following ten items:

- Market risk, or being forced to sell when interest rates are rising
- Reinvesting income when interest rates are falling
- Timing or call risk
- Political risk
- Sector risk
- Purchasing power or inflation risk
- Credit risk

- Currency risk for foreign investments
- Liquidity risk
- Event risk.

To simplify the discussion, look at one part of risk: the risk of call of the security. An investor may think that there is no possibility of call if market interest rates are at least 10 percent when the bond has a coupon rate of 9 percent and that the risk will increase to certainty as market rates fall to 8 percent or less. Note that market interest rates behave randomly, while fuzziness arises from the opinion of the investor about the quality of the investment based on the possibility of call. Let $A$ be the fuzzy set of good bonds from the standpoint of the call risk; one representation of $A$ is

$$f_A(x) = \begin{cases} 1, & 0.10 \le x \\ 100x - 9, & 0.09 \le x \le 0.10 \\ 0, & x \le 0.09, \end{cases}$$

in which $x$ is the market interest rate.

This investor is unwilling to buy a callable 9 percent coupon bond if market rates are less than 9 percent, even though the probability of call is not 1. Other such functions for the remaining risks could be developed and combined through methods introduced below to help this investor decide whether to purchase a given security.

## 2.2 Operations on Fuzzy Sets

To be able to consider several fuzzy conditions simultaneously, we define the ways in which fuzzy sets can be combined or operated upon [14, pp. 340–342], [15, pp. 11–14].

### 2.2.1 Definitions

The *union*, $A \cup B$, of two fuzzy sets, $A$ and $B$, is given by

$$f_{A \cup B}(x) \equiv \max[f_A(x), f_B(x)], \quad x \in X,$$

and the *intersection*, $A \cap B$, is given by

$$f_{A \cap B}(x) \equiv \min[f_A(x), f_B(x)], \quad x \in X.$$

The *complement*, $-A$, of fuzzy set $A$ is given by

$$f_{-A}(x) \equiv 1 - f_A(x), \quad x \in X.$$

Note that these definitions degenerate to those used for crisp sets if $A$ and $B$ are nonfuzzy.

### *2.2.2 Example*

Let $X=\{1, 2, 3, 4, 5\}$, and define fuzzy sets $A$ and $B$ on $X$ by

$$f_A = \{(1, 0.2), (2, 0.5), (3, 1.0), (4, 0.2), (5, 0.0)\}$$

and

$$f_B = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.8), (5, 1.0)\}.$$

One can interpret the fuzzy set $A$ as the collection of small integers very close to 3, and $B$ as the set of integers in $X$ close to 5. Some combinations of $A$ and $B$ are

$$f_{A \cup B} = \{(1, 0.2), (2, 0.5), (3, 1.0), (4, 0.8), (5, 1.0)\}$$

$$f_{A \cap B} = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.2), (5, 0.0)\}$$

and

$$f_{-A} = \{(1, 0.8), (2, 0.5), (3, 0.0), (4, 0.8), (5, 1.0)\}.$$

The operation of union acts as an "or" operator; intersection, as "and"; and complement, as "not." Thus, for example, $f_{A \cap B}$ represents the fuzzy set of small integers that are very close to 3 and close to 5.

In what follows, fuzzy sets represent characteristics of a group that must be simultaneously "good" for the group to be classified as a good risk. We want the characteristics to interact when combined, especially when intersected; however, the above definition of intersection does not allow interaction. We present other definitions of intersection below, which take into account some or all of the following properties [8, pp. 41–42]:

**Property 1** (cumulative effects):

$$f_{A \cap B}(x) < \min[f_A(x), f_B(x)], \quad \text{if } f_A(x), f_B(x) < 1.$$

This property states that if two characteristics are both "less than preferred," then the two taken together are worse than each separately.

**Property 2** (interactions between criteria): The effect of a change of $f_A(x)$ upon $f_{A \cap B}(x)$ also depends on $f_B(x)$. In other words, the effects of $A$ and $B$ are not independent.

**Property 3** (compensation between criteria): The effect of a decrease of $f_A(x)$ upon $f_{A\cap B}(x)$ can be eliminated by an increase of $f_B(x)$.

**Property 4:** If $A$ and $B$ are crisp sets, then

$$f_{A\cap B}(x) = \begin{cases} 1, & \text{if } x \in A \quad \text{and} \quad x \in B \\ 0, & \text{else.} \end{cases}$$

Note that the minimum operator satisfies only Property 4. Lemaire [8, p. 42] follows the description of these properties with alternative definitions of intersection:

### *2.2.3 Definition*

The *algebraic product*, $AB$, is given by

$$f_{AB}(x) \equiv f_A(x) \times f_B(x), \quad x \in X.$$

The algebraic product satisfies all four properties.

### *2.2.4 Definition*

The *bounded difference*, $A \ominus B$, is given by

$$f_{A\ominus B} \equiv \max[0, f_A(x) + f_B(x) - 1], \quad x \in X.$$

The bounded difference satisfies all but Property 2. Indeed, a change effected upon $A \ominus B$ by $f_A(x)$ is independent of the value of $f_B(x)$ as long as $f_A(x)+f_B(x)-1$ is greater than 0.

### *2.2.5 Definition*

The *Hamacher operator*, $H$, which depends on $p$, is given by

$$H(A, B; p)(x) \equiv \frac{f_A(x) f_B(x)}{p + (1 - p)[f_A(x) + f_B(x) - f_A(x) f_B(x)]}, \quad 0 \leq p \leq 1.$$

The Hamacher operator satisfies all four properties. In general, $f_{AB} \leq f_H^p \leq f_H^q \leq f_{A\cap B}$, if $0 \leq q \leq p \leq 1$. Note that the interaction between $A$ and $B$ depends upon the parameter $p$; the degree of interaction decreases as $p$ decreases. When $p=1$, the Hamacher operator reduces to the algebraic product—the intersection that affords maximal interaction.

### *2.2.6 Definition*

The *Yager operator*, $Y$, which depends on $p$, is given by

$$Y(A, B; p)(x) \equiv 1 - \min(1, \{[1 - f_A(x)]^p + [1 - f_B(x)]^p\}^{1/p}), \quad p \geq 1.$$

The Yager operator satisfies all four properties as long as $1<p<\infty$. It reduces to the minimum operator as $p$ goes to infinity and to $A \ominus B$ when $p=1$.

### *2.2.7 Example*

Let $A$, $B$, and $X$ be as in Example 2.2.2:

$$f_{AB} = \{(1, 0.04), (2, 0.2), (3, 0.6), (4, 0.16), (5, 0.0)\}$$

$$f_{A \ominus B} = \{(1, 0.0), (2, 0.0), (3, 0.6), (4, 0.0), (5, 0.0)\}$$

$$H(A, B; 0.5) = \{(1, 0.059), (2, 0.235), (3, 0.6), (4, 0.174), (5, 0.0)\}$$

and

$$Y(A, B; 2) = \{(1, 0.0), (2, 0.219), (3, 0.6), (4, 0.175), (5, 0.0)\}.$$

Fuzzy sets can be combined in other ways to allow for interaction among criteria [13, p. 345], [14, pp. 14–15]:

### *2.2.8 Definition*

Given fuzzy sets $A_1, A_2, \ldots, A_n$, the *convex combination*, $B$, is defined by

$$f_B(x) \equiv w_1(x) f_{A_1}(x) + \ldots + w_n(x) f_{A_n}(x)$$

in which

$$\sum_{i=1}^{n} w_i(x) = 1$$

and $0 \leq w_i(x)$, for all $x \in X$.

A special case of the above occurs when $w_i(x) = w_i$, a constant, $i=1, \ldots, n$. In this case, $B$ is called a *convex linear combination* of the $A_i$. If $w_i = 1/n$, $i=1, \ldots, n$, then $B$ is the *arithmetic mean* of the $A_i$.

### 2.2.9 Definition

Given fuzzy sets $A_1$, $A_2$, ..., $A_n$, their *geometric mean*, $B$, is defined by

$$f_B(x) \equiv [f_{A_1}(x) \ldots f_{A_n}(x)]^{1/n}, \quad x \in X.$$

If a given criteria $A_j$ is more important than the others, then one can account for this difference when intersecting fuzzy sets or forming convex combinations of them. In the second case, $A_j$ can be weighted with a larger value of $w_j$. In the first, one can concentrate $A_j$ before intersecting, where the operation of concentration is defined below.

### 2.2.10 Definition

The *concentration*, *CON(A)*, of a fuzzy set $A$ is given by

$$f_{CON(A;a)}(x) \equiv [f_A(x)]^a, \quad a > 1.$$

Concentration reduces the grade of membership of all elements $x$, with $f_A(x) < 1$, so that the closer $f_A(x)$ is to 0, the more its grade of membership is reduced. In most applications of this operation on fuzzy sets, $a$ is set equal to 2.

The inverse operation of concentration is called dilation, and it reduces the importance of a given criteria by increasing the grades of membership.

### 2.2.11 Definition

The *dilation*, *DIL(A)*, of a fuzzy set $A$ is given by

$$f_{DIL(A;a)}(x) \equiv [f_A(x)]^a, \quad 0 < a < 1.$$

For most applications, $a$ is set equal to $1/2$.

### 2.2.12 Example

Again, let $A$, $B$, and $X$ be as in Example 2.2.2, and let $C$ be

$$f_c = \{(1, 0.6), (2, 0.8), (3, 0.2), (4, 0.1), (5, 0.5)\}.$$

Form the convex combination:

$$D = C \times A + (1 - C) \times B;$$

$$f_D = \{(1, 0.2), (2, 0.48), (3, 0.68), (4, 0.74), (5, 0.5)\}.$$

Calculate the geometric mean of $A$ and $B$, or equivalently, the dilation of the algebraic product of $A$ and $B$:

$$f_E = \{(1, 0.2), (2, 0.447), (3, 0.775), (4, 0.4), (5, 0.0)\}.$$

The concentration of $A$ is the fuzzy set of very small integers in $X$ that are also very close to 3:

$$f_{CON(A;2)} = \{(1, 0.04), (2, 0.25), (3, 1.0), (4, 0.04), (5, 0.0)\}.$$

Dilation has the opposite effect of concentration. The dilation of $B$ is the fuzzy set of integers in $X$ that are somewhat close to 5:

$$f_{DIL(B;0.5)} = \{(1, 0.447), (2, 0.633), (3, 0.775), (4, 0.894), (5, 1.0)\}.$$

### *2.2.13 Example*

We base the model of this example upon one used to determine washing time for a particular Japanese washing machine [12, p. 37]. We continue the discussion of Example 2.1.2 by presenting two rules for estimating trend:

(1) If the increase in the medical consumer price index (CPI) is high and the increase in the reimbursement of providers under Medicare is low, then the trend will be high.

(2) If the increase in the medical CPI is moderate and the increase in the reimbursement of providers under Medicare is moderate, then the trend will be moderate.

Define the following fuzzy set functions:

$$CPI\text{-}High(x) = \begin{cases} 0, & 0 \le x \le 0.05 \\ 20x - 1, & 0.05 \le x \le 0.10 \\ 1, & 0.10 \le x, \end{cases}$$

$$CPI\text{-}Mod(x) = \begin{cases} 20x, & 0 \le x \le 0.05 \\ -20x + 2, & 0.05 \le x \le 0.10 \\ 0, & 0.10 \le x, \end{cases}$$

in which $x$ is the annual increase in the medical CPI.

$$Medicare\text{-}Low(y) = \begin{cases} -25y + 1, & 0 \le y \le 0.04 \\ 0, & 0.04 \le y, \end{cases}$$

$$Medicare\text{-}Mod(y) = \begin{cases} 40y, & 0 \leq y \leq 0.025 \\ -40y + 2, & 0.025 \leq y \leq 0.05 \\ 0, & 0.05 \leq y, \end{cases}$$

in which $y$ is the annual increase in the reimbursement of providers under Medicare.

$$Trend\text{-}High(z) = \begin{cases} 0, & z \leq 0.10 \\ 10z - 1, & 0.10 \leq z \leq 0.20 \\ 1, & 0.20 \leq z \leq 0.25 \\ -20z + 6, & 0.25 \leq z \leq 0.30 \\ 0, & 0.30 \leq z, \end{cases}$$

$$Trend\text{-}Mod(z) = \begin{cases} 0, & z \leq 0.05 \\ 20z - 1, & 0.05 \leq z \leq 0.10 \\ -10z + 2, & 0.10 \leq z \leq 0.20 \\ 0, & 0.20 \leq z, \end{cases}$$

in which $z$ is the annual trend. See the graphical representations of the above fuzzy sets in Figure 1.

Suppose that $x$ is 0.075 and $y$ is 0.025; then $CPI\text{-}High(0.075)=0.5$ and $Medicare\text{-}Low(0.025)=0.375$. Intersect these two values through the minimum operator to get 0.375. Truncate the graph of the function *Trend-High* at 0.375 from above. Similarly, $CPI\text{-}Mod(0.075)=0.5$ and $Medicare\text{-}Mod(0.025)=1.0$. Intersect these two to obtain 0.5; again, truncate the graph of the function *Trend-Mod* at 0.5 from above. Form the union of the two planar regions (see Figure 2), and set the trend equal to the abscissa of the center of gravity of the union. The trend thus determined in this example is 0.168=16.8%.

### *2.3 Fuzzy Set Theory Versus Probability Theory*

Fuzzy set theory and probability theory are related in that they both deal with uncertainty. Fuzzy sets represent the uncertainty that comes from vagueness and the extent to which an event occurs. On the other hand, probability theory looks at the uncertainty that arises from randomness and regards simply whether an event will occur.

FIGURE 1

GRAPHS FOR EXAMPLE 2.2.13

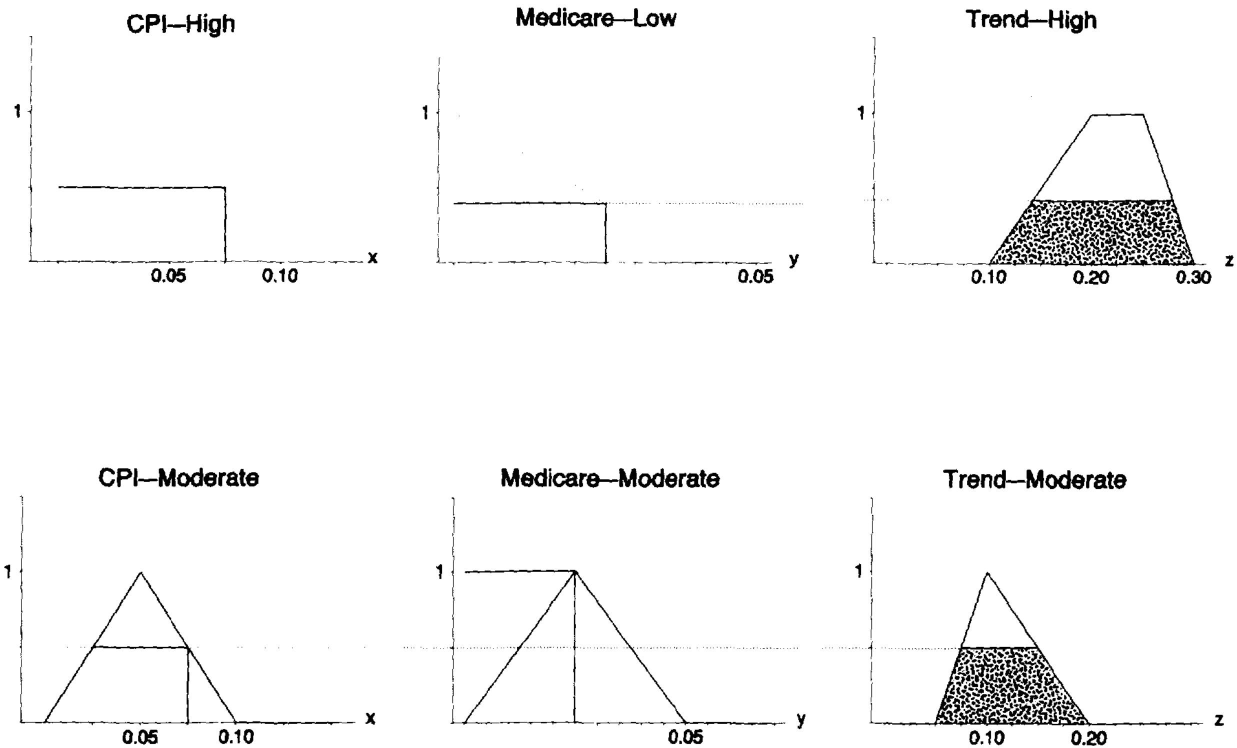
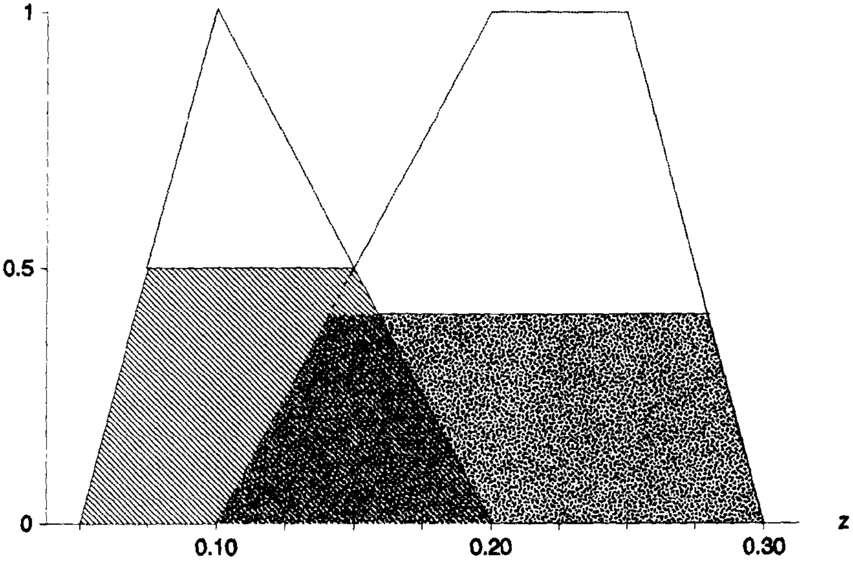


FIGURE 2

GRAPH OF RESULTING FUZZY SET FOR EXAMPLE 2.2.13




After the given event occurs (or not), randomness no longer exists; however, fuzziness does not decrease after additional information has been acquired. For example, suppose that 1 in 5 men is at least 6 feet tall; therefore, if a man is selected at random, the probability that the man is at least 6 feet tall is 0.20. If the height of the man is measured at 5 ft. 11 in., then we are no longer uncertain about his actual height. The vagueness related to whether the man is tall, however, remains; for example, his membership value in the fuzzy set of tall men can be set at 0.80.

In general, fuzziness arises when a concept, $A$, and its opposite, $-A$, overlap, thus violating the law of noncontradiction:

### *2.3.1 Example*

Let $A$ be as in Example 2.2.2.

$$f_{A\cap -A} = \{(1, 0.2), (2, 0.5), (3, 0.0), (4, 0.2), (5, 0.0)\}.$$

In probability theory, an event and its opposite cannot both occur; yet in fuzzy set theory, each may occur to some degree. For example, a man can be both tall and not tall to some extent.

In the sections that follow, we address the problem of characterizing what constitutes a group that is a good risk for group health insurance. From the viewpoint of probability theory, a group that is a good risk is one that is expected to be profitable. From this perspective, uncertainty is connected with claim and expense fluctuation relative to the price charged.

From the standpoint of fuzzy set theory, a group is a good risk if it is stable and unlikely to select against the insurer. (Note that these qualities are themselves fuzzy in nature.) This judgment is unrelated to the premium charged. In fact, a group may still be considered a good risk even if it is unprofitable in a given year—or for several years. Events beyond the control of a group can lead to unprofitability, such as a severe epidemic attacking members of the group, an automobile accident involving one or more members, higher-than-expected expenses associated with administration, or inadequate pricing due to competitive pressure or to lack of expertise within the pricing unit of the group health insurer.

For further information on the relationship between probability theory and fuzzy set theory, read Kosko [6].

## 3. FUZZY SET MODEL FOR UNDERWRITING SINGLE-OPTION PLANS

In modeling the group selection process for health insurance via fuzzy sets, we first consider the case of an employer that offers a single plan of insurance to its employees. In this portion of the paper, we outline desirable characteristics of such a group [7, pp. 2, 10–18].

A. Obtaining insurance is incidental to the purpose of the group. An employer-employee group usually satisfies this criterion.

B. Employment status determines eligibility for health coverage. For example, the employer or insurer may require that an employee be actively at work at least 30 hours per week to receive benefits, and only the spouse and dependent children of such an employee are eligible for dependent coverage. Underwriters use an actively-at-work rule because someone who works needs to be healthy.

C. A minimum number of people are in the group. For example, some companies require that the group contain at least five employees.

D. Benefits are determined automatically. For example, this criterion is met in the case of a single-option health plan in which the class of employment determines the benefits.

We assume that the above rules are most appropriately described by nonfuzzy sets, although $C$ lends itself to fuzzy set representation. As for the remainder of the criteria, we present possible fuzzy set characterizations in the following discussion. One difficulty in fuzzy set theory is to create a function that accurately describes the given condition or characteristic. In this paper, we essentially derive the functions by working backwards. We consider four categories of groups: preferred risk; normal, or acceptable, risk; substandard risk; and unacceptable, or declinable, risk. The fuzzy set function has a membership value of 1.0 for a preferred risk. Normal risks have values between 0.5 and 1.0; substandard risks, between 0.25 and 0.5. Finally, unacceptable risks have membership values from 0.0 to 0.25.

The developed functions are illustrative only and are not intended to represent any particular company's underwriting guidelines.

E. Young lives flow constantly into the group and old lives out. Such a requirement helps to ensure stable morbidity. This rule can be verified by determining whether the age/sex factor has been relatively stable and whether the group size has fluctuated greatly during the past few years.

Let $a/x$ equal the annual percentage change in the age/sex factor for the past two years and $g/s$ the annual percentage change in the group size during the same period:

$$e_1(a/x) = \begin{cases} 1, & a/x \leq 0.05 \\ -5a/x + 1.25, & 0.05 \leq a/x \leq 0.25 \\ 0, & \text{else.} \end{cases}$$

$$e_2(g/s) = \begin{cases} 1, & -0.05 \leq g/s \\ 5g/s + 1.25, & -0.25 \leq g/s \leq -0.05 \\ 0, & \text{else.} \end{cases}$$

Examine the information contained in the above functions: An increase of up to 5 percent or any decrease in the age/sex factor describes a preferred risk. A group is normal if the percentage increase lies between 5 and 15 percent ($0.5 \leq e_1 \leq 1.0$), while an unacceptable group has an age/sex factor increase of 20 percent or more ($0.0 \leq e_1 \leq 0.25$). For a change in group size, we take the opposite viewpoint; that is, a decrease of down to 5 percent or any increase is preferred, while a 5 to 15 percent decrease is acceptable. These two functions are linear, but for a particular insurance company's underwriting guidelines, the functions are not necessarily expected to be of that form.

One way to combine these two functions, which give equal weight to $e_1$ and $e_2$, is

$$e(a/x, g/s) = \sqrt{e_1(a/x) \times e_2(g/s)}.$$

F. There is a minimum participation in the plan. An insurer usually requires that all eligible employees enroll in a noncontributory, or employer-pay-all, plan. Since there is no reason for an employee not to join such a plan, we focus more on contributory ones. A typical rule in this case requires 100 percent enrollment for groups of five or fewer grading to 75 or 85 percent for groups of 10 or more.

In addition, insurers often want a minimum percentage of employees who have dependents to cover them. For example, suppose there is a group of 20 eligible employees; then 15 employees must elect coverage if the insurer mandates 75 percent participation. If 16 of the 20 are enrolled and 12 of them are eligible to cover their dependents, then at least 9 must do so.

If at least 90 percent of the employees participate, then the group is preferred. A normal group has 80 to 90 percent participation; a substandard group, 75 to 80 percent. This distribution corresponds to the requirement that a group has 85 percent participation with possible bending of the rule down to 75 percent. Let $p$ be the proportion of employees that select group coverage:

$$f(p) = \begin{cases} 1, & 0.90 \leq p \\ 5p - 3.5, & 0.70 \leq p \leq 0.90 \\ 0, & \text{else.} \end{cases}$$

For small groups, higher percentages of participation than those given in the above equation may be desired. Also, the same or a similar function may account for participation of employees who are eligible for dependent coverage; then some intersection of the two could integrate the criteria.

G. The employer pays some or all of the cost of insurance; contributing at least one-fourth of the premium is often a requirement. A preferred risk is one for which the employer pays all of the employee's cost and 75 percent or more of the dependent's cost. For an acceptable risk, the percentages go down to 75 and 50 percent, respectively; for a substandard risk, down to 50 and 25 percent, respectively. Let $r_1$ equal the proportion of the employee's premium paid by the employer and $r_2$ the proportion of the dependent's premium paid by the employer:

$$g_1(r_1) = \begin{cases} 0, & r_1 \leq 0.25 \\ r_1 - 0.25, & 0.25 \leq r_1 \leq 0.75 \\ 2r_1 - 1, & 0.75 \leq r_1 \leq 1.0. \end{cases}$$

$$g_2(r_2) = \begin{cases} r_2, & r_2 \leq 0.50 \\ 2r_2 - 0.50, & 0.50 \leq r_2 \leq 0.75 \\ 1, & 0.75 \leq r_2. \end{cases}$$

As in criterion $E$, we link the two functions by

$$g(r_1, r_2) = \sqrt{g_1(r_1) \times g_2(r_2)}.$$

The function $g$ allows trade-offs between $r_1$ and $r_2$. For example, if an employer pays all of the employee's cost but only 40 percent of the dependent's cost, then

$$g(1.0, 0.4) = \sqrt{1 \times 0.4} \cong 0.6325 \in [0.5, 1.0],$$

from which we infer that the group is an acceptable risk. Remember that if the employee's contribution decreases, then the participation is likely to increase.

H. A strong, central department of the employer helps the insurer administer billing, enrollment, and certification of eligibility. Fuzziness arises by the use of the term "strong"; such a word cannot be quantified easily. An underwriter will use judgment to assign a value between 0 and 1 to represent the administrative ability of the group. Some factors to consider include the size and qualifications of the personnel staff, the function it normally performs, and any knowledge about previous experience of the insurer with the employer's staff. Call the developed fuzzy set function $h$.

I. The industry of the group is acceptable. A company may either use a list of industries it considers uninsurable or apply a load to the medical manual rates. Note that the size of the load may be restricted for small groups due to state regulation or law.

If the underwriting department has a list of unacceptable industries, then any group in one of those industries has a membership value of 0 in the fuzzy set of groups in industries that can be underwritten. Similarly, a group in a questionable industry has a membership value between 0 and 1. Call the corresponding fuzzy set function $i$.

J. The policyholder has a good credit rating that helps to ensure that the premium will be paid in a timely manner. The Dun & Bradstreet credit reports or financial statements of the prospect can be examined to determine whether the prospect is a good credit risk. Based upon the willingness of the company to accept this risk, develop a fuzzy set function for this criterion and call it $j$.

K. Ongoing claims are not large as a proportion of total expected claims. For the initial underwriting of a small group, no employees are health risks as determined by a health questionnaire or by an attending physician's statement or physical examination. In renewal underwriting, information is available from the claim file.

The number and size of ongoing claims that an insurer can tolerate depend upon the group size and the nature of the claims. An acute condition may be resolved in a relatively short time, whereas a chronic condition may continue much longer. The function we define presupposes the following: The expected payments for ongoing claims are less than $\frac{1}{2}$ percent of the total expected paid claims for a preferred group. For a normal group, the percentage lies between $\frac{1}{2}$ and $1\frac{1}{2}$ percent. If the percentage of ongoing claims is above 2 percent, then the group is unacceptable. Let $c$ equal the percentage of ongoing claims as a proportion of the total expected claims:

$$k(c) = \begin{cases} 1, & c \le \frac{1}{2} \\ -\frac{1}{2}c + \frac{5}{4}, & \frac{1}{2} \le c \le 2\frac{1}{2} \\ 0, & 2\frac{1}{2} \le c. \end{cases}$$

(Note that 100 times the decimal representation of the proportion of ongoing claims is used to evaluate $k$; for example, if ongoing claims $= ^3/_4$ percent, then use $c=0.75$ to determine $k$.)

L. The claim experience of the group has been good. One measure of such a characteristic is the loss ratio. Experience for small groups fluctuates more than that for large groups; therefore, a high loss ratio for a large group is more indicative of poor experience in the future than a high loss ratio for a small group.

Let $LR$ equal the previous year's loss ratio and $s$ the group size. Here, the loss ratio refers to incurred claims divided by expected claims for the given time, that is, the actual to expected ratio. The inclusion of retention in the denominator would distort the loss ratio because retention, as a percentage of gross premium, varies with the size of the premium:

$$L_1(LR) = \begin{cases} 1, & LR \le 0.95 \\ -5\,LR + 5.75, & 0.95 \le LR \le 1.15 \\ 0, & 1.15 \le LR. \end{cases}$$

In this function, a group is preferred if its loss ratio is 95 percent or less. An acceptable group has a loss ratio between 95 and 105 percent. If a given company's guidelines are looser than the above indicates, then the underwriter will broaden the intervals.

Because the loss ratio is not as indicative of future claims for small groups as it is for large ones, we account for size through

$$L_2(s) = \begin{cases} \sqrt{\dfrac{s}{500}}, & 0 \le s \le 500 \\ 1, & 500 \le s. \end{cases}$$

Another function for $L_2$ could be derived from a credibility table. We combine the two functions:

$$L(LR, s) = L_1(LR) \times L_2(s) + [1.0 - L_2(s)].$$

Note that $L(LR, s)$ is a weighted average of $L_1(LR)$ and 1.0 with $L_2(s)$ as the weight.

M. The group does not change insurers often. A high turnover rate may mean that group has poor experience and frequently shops around for better rates.

Let $n$ equal the number of insurers the group has had in the past five years.

$$m(n) = \begin{cases} 1, & n = 1 \\ 0.5, & n = 2 \\ 0, & n \ge 3. \end{cases}$$

The remainder of this section deals with the problem of combining the above rules so that we can decide whether to underwrite the group. In other words, we create a single fuzzy set to describe the set of good risks. We assume that criteria A, B, C, and D are satisfied from the outset and do not consider them further.

First, contemplate whether a 0 in any one of the categories leads to outright rejection of the group. If not, then include the operation of convex combination together with intersection; otherwise, use only intersection. Suppose that a grade of 0 in any of the following rules does not automatically disqualify a group from being insured:

H. The employer has a strong, central administrative department.

I. The industry of the group is acceptable.

K. The ongoing claims are not large.

The manual rates can be loaded to account for the effects of criteria I and K, and an extra margin can be added for expenses to compensate for any deficiency concerning criterion H. Create the linear combination:

$$P = \frac{1}{6}h + \frac{1}{3}i + \frac{1}{2}k.$$

By the choice of weights, we view the amount of ongoing claims to be somewhat more important than the industry of the group but judge strong administration less important than either. We can intersect $P$ with the remaining criteria in many ways as in Section 2.2. The fuzzy set used in deciding whether to accept the group can have the form:

$$Q = P * e * f * g * j * L * m,$$

in which $*$ is the selected intersection. We present two options below:

### *3.1 Option One*

$$Q_1 = P \cap e \cap f \cap g \cap j \cap L \cap m,$$

in which $\cap$ denotes the minimum operator. Note that a group preferred in each category is preferred in total. Since the minimum operator does not allow the variables to interact, the relations among them are missed.

The following matrix represents the pair-wise interactions between the criteria:

| Interaction between Criteria | $P$ | $e$ | $f$ | $g$ | $j$ | $L$ | $m$ |
|---|---|---|---|---|---|---|---|
| $P$ | | | | | Some | | Little |
| Flow of lives ($e$) | | | Some | | | | |
| Participation ($f$) | | Some | Important | Max | | | |
| Employer contribution ($g$) | | | Max | | | | |
| Good credit ($j$) | Some | | | | | | Little |
| Loss ratio ($L$) | | | | | | | |
| Turnover ($m$) | Little | | | | Little | | |

We symbolize the label of "some" by the Hamacher operator ($p$=0.5); "little" by the Hamacher operator ($p$=0); "max" by the algebraic product; and no interaction by the minimum operator. Finally, "important," means that the criterion warrants special emphasis.

One possible fuzzy set function representing the above table is as follows:

### 3.2 Option Two

$$Q_2 = \{H[H(P, j; 0.5), m; 0]\}^{1/3} \cap [f \times g]^{1/2} \cap [H(f, e; 0.5)]^{1/2} \cap [L^a],$$

in which $a$ changes the importance of the fuzzy set representing the loss ratio. The less important it is, the less $a$ will be.

To develop this function, first group the variables according to whether they interact. In this case, partition the variables as follows: $\{P, j, m\}$, $\{e, f\}$, $\{f, g\}$, and $\{L\}$. Note that this is not a true partition because $f$ appears in two distinct subsets.

Since $P$ and $j$ interact somewhat and each interacts little with $m$, first combine $P$ and $j$ via the Hamacher operator ($p$=0.5), then join the result with $m$ by using the Hamacher operator ($p$=0). Take the cube root of the outcome to negate the effect of the product $P \times j \times m$, making the result comparable to other terms that involve fewer than three factors. If, instead, the interaction between $P$ and $m$ had been "some" and not "little," $m$ and $j$ could first be combined via the Hamacher operator ($p$=0) and then that result could be joined with $P$ by using the Hamacher operator ($p$=0.5).

As an aside, note that the Hamacher operator is not associative if the parameter changes; that is, $H[H(A,B;p),C;q]$ is not necessarily equal to $H[A,H(B,C;q);p]$. This inequality can be seen by taking $f_B \equiv 1$; in this case, the first term is $H(A,C;q)$, while the second term is $H(A,C;p)$. For a general fuzzy set $B$, if $p$=0.5 and $q$=0, then an interpretation of the first term is that $A$ and $B$ interact somewhat and each interacts little with $C$. Similarly, the second term may mean that $B$ and $C$ interact little and each interacts somewhat with $A$. From this perspective, the two terms are not expected to be equal.

Because $f$ and $g$ interact maximally, intersect them through the algebraic product. Form the square root of this product to make the term commensurate with the others.

The functions $f$ and $e$ interact to some extent; therefore, combine them by using the Hamacher operator ($p$=0.5); again, take the square root of this result. The importance of $f$ is reflected in its appearance in the above two terms. Alternatively, the concentration $CON(f;2)$ could be used in either or both of the two terms in place of $f$, or an extra term of the form $CON(f;2)$ could be included, in which the importance of $f$ is made explicit. Ambiguity arises in this example because $e$ and $g$ are each related to $f$ but not to each other.

Since $L$ does not interact with any other variable, it is a term in the final intersection by itself. However, $L$ can be concentrated or dilated to reflect the significance of the loss ratio. The degree of dilation or concentration is influenced by any considerations of credibility not taken into account by the variable of group size, for example, a high turnover rate within the plan.

Finally, intersect the four terms, $H[H(P,j;0.5),m;0]^{1/3}$, $(fg)^{(1/2)}$, $H(f,e;0.5)^{(1/2)}$, and $L^a$, through the minimum operator. Apply this operator because the four terms do not interact, except by means of $f$.

In general, the cutoff point for choosing or not choosing a group depends upon the function selected for $Q$. An alternative to using a particular number between 0 and 1 is to implement a fuzzy decision scheme. For example, if $Q$ lies between 0.75 and 1.0, then the group is definitely acceptable. If $Q$ is in the range from 0.50 to 0.75, then the group is most likely acceptable, and if $Q$ is less than 0.25, then the group is definitely unacceptable. Otherwise, if $Q$ lies between 0.25 and 0.50, rely upon the discretion of the underwriter. (This outline roughly follows the concepts of preferred, normal, substandard, and unacceptable risks introduced at the beginning of this section.)

Within this fuzzy decision strategy, an insurer can use the value of $Q$ to indicate how to load the manual rates, change the plan of benefits, or require an alternative funding scheme. This process of making decisions could be modeled by using the method in Example 2.2.13.

### *3.3 Example*

Consider a group with the characteristics:

| | |
|---|---|
| Group size ($s$) | = 250 |
| Age/sex factor change ($a/x$) | = 10% |
| Size change ($g/s$) | = −15% |
| Participation ($p$) | = 85% |
| Employer contribution ($r_1/r_2$) | = 100%/40% |
| Strong administration ($h$) | = 0.9 |
| Industry ($i$) | = 1.0 |
| Credit rating ($j$) | = 0.95 |
| Ongoing claims ($c$) | = 0.75% |
| Loss ratio ($LR/a$) | = 1.05/0.75 |
| Number of carriers ($n$) | = 1. |

After applying the fuzzy functions presented above, we obtain:

$$Q_2 = \min[0.8765^{1/3}, 0.4743^{1/2}, 0.4827^{1/2}, 0.6464^{3/4}]$$
$$= 0.6887.$$

Since $Q_2$=0.6887 lies between 0.50 and 0.75, the group is most likely acceptable. The employer may be interested in knowing how to improve the group's acceptability. Determine this information by examining the function $\sqrt{(fg)}$ because its value yields the minimum $Q_2$. The contribution made by the employer to the single rate is 100 percent, so $g_1$ cannot be improved. To measure the effect of an increase in the participation rate or in the employer's contribution to the dependent's cost, consider the following first partial derivatives:

$$\frac{\partial}{\partial p}\sqrt{fg} = \frac{5}{2}\sqrt{\frac{g}{f}} = 2.2958.$$

$$\frac{\partial}{\partial r_2}\sqrt{fg} = \frac{1}{4}\sqrt{\frac{f}{g}} \times \sqrt{\frac{g_1}{g_2}} = 0.4305.$$

The greater improvement in $\sqrt{(fg)}$ comes from an increase in participation rate because 2.2958>0.4305. On the other hand, this change may be effected most easily if the employer contributes more. Suppose the variable $r_2$=0.50 and, as a result, $p$=0.90, then

$$Q_2 = \min[0.8765^{1/3}, 0.7071^{1/2}, 0.6124^{1/2}, 0.6464^{3/4}]$$
$$= 0.7209.$$

The above process may be continued by considering how the importance of $L$ can be decreased; for example, a strong preexisting-condition exclusion could be implemented or the plan design could be changed to lower utilization.

Another application of the first partial derivatives inherent in the above discussion is sensitivity analysis. In addition to calculating $Q$ for various scenarios to determine its appropriateness, the partial derivatives can be evaluated to determine whether $Q$'s sensitivity to the variables follows the underwriting guidelines of the company.

Such an evaluation is important if several factors interact intricately. For instance, suppose the fuzzy sets A, B, C, and D interact as follows:

| Variable | A | B | C | D |
|---|---|---|---|---|
| A | | Some | Little | Some |
| B | Some | | Some | Little |
| C | Little | Some | | Some |
| D | Some | Little | Some | |

Ambiguity exists because B and D interact to some degree with each of A and C, but B and D interact little with each other, as do A and C. These relationships can be represented by $H[H(\mathrm{A},\mathrm{C};0),H(B,D;0);0.5]$. The vagueness of this example emphasizes the importance of verifying that the chosen functions characterize the qualities correctly and that the combination of those functions accurately reflects the given underwriting process.

## 4. FUZZY SET MODEL FOR UNDERWRITING MULTIPLE-OPTION PLANS

The factors that receive the most attention in the underwriting of multiple-option plans are those that affect participation, such as the level of access to care, the employee contribution, the plan of benefits, and the age, sex, and dependent coverage of the employee. These factors are also at work in single-option selection because an employee can choose whether to accept coverage. The existence of a working spouse's plan and that plan's benefits, employee contribution, and access to care influence an employee's decision. Underwriting against shadow plans is nearly impossible; therefore, external factors that affect participation in single-option plans are not usually considered. They are more visible, however, in multiple-option cases because the underwriter is normally aware of the competition.

The level of access to care encompasses the size of the provider panel relative to the provider population in the area, the geographic distribution of the panel relative to the location of the employees, the operating hours of the panel, the particular gatekeeper mechanism used, and the ease of obtaining referrals. In general, a staff model or small, closed-panel health maintenance organization (HMO) with tight controls has restricted access. Moderate control is found in a large, open-panel HMO, for example, and loose control in an HMO that is based upon an independent practice association. Little or no control describes a preferred provider organization or an indemnity plan.

We measure the richness of the plan of benefits relative to the competing plans and consider the scope of coverage and any copayments, deductibles, and coinsurance. The contribution to the premium required from the employee may or may not be tied to the benefits and access to care. For example, the employer may contribute a flat dollar amount for each employee—say, the cost of single coverage for the lowest-priced option. In this case, the excess paid by the employee varies with the benefit design and access to care, assuming that the tighter the control, the lower the premium. On the other hand, the employer may seek to direct employees to a particular option by requiring that employees contribute less for that option.

In what follows, we use the above variables to develop an age factor that denotes possible participation in the plan. To simplify the presentation of the age factor table, we combine the categories of restricted access and moderate control into one of limited access; the categories of loose and no control, into one of free access. Also, we confine benefit design to either rich or poor; employee contribution to the premium, to high or low. We consider only three employee age groups: younger (under 40), middle-aged (40 to 55), and older (over 55).

We associate a number between 0 and 1 with each age group that varies with the level of access to care, the plan design, the employee contribution, and whether the employee has single or family coverage. The number represents the possibility of the employee's participating in the described plan. We obtain single and family age factors by forming the weighted averages of these individual factors, where the weights are the proportions of employees in the corresponding single and family age brackets. Sex can be taken into account if the composition of the block of business warrants doing so. For example, in some areas, a married female employee is more likely to be covered through her husband's plan than vice versa.

| | | | Single | | | Family | | |
|---|---|---|---|---|---|---|---|---|
| Access | Benefits | Cost | Young | Middle | Older | Young | Middle | Older |
| Free | Rich | High | 0.2 | 0.5 | 0.8 | 0.3 | 0.6 | 0.9 |
| | | Low | 0.9 | 0.9 | 0.9 | 1.0 | 1.0 | 1.0 |
| | Poor | High | 0.1 | 0.4 | 0.7 | 0.1 | 0.5 | 0.8 |
| | | Low | 0.5 | 0.6 | 0.7 | 0.6 | 0.7 | 0.8 |
| Limited | Rich | High | 0.2 | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 |
| | | Low | 0.8 | 0.5 | 0.2 | 0.7 | 0.4 | 0.2 |
| | Poor | High | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| | | Low | 0.7 | 0.4 | 0.1 | 0.6 | 0.3 | 0.1 |

We emphasize that these possibility distributions are theoretical only and are not based upon empirical data. The following is a list of assumptions implicit in the table:

(1) Older people prefer less control to more control regardless of the cost, because they have established physician relations and want no restrictions of the providers they use. Within a given level of access, benefits are more important than the amount of employee contribution.

(2) Younger people are more interested in low-cost options. Within a particular cost bracket, less control and richer benefits are preferred, with the latter taking precedent.

(3) An employee with family coverage is more interested in easy access to care than one with single coverage.

(4) Middle-aged people balance the preferences of younger and older employees.

The health status of an individual also plays a role in selection: Less healthy employees and dependents favor greater access to care, and more healthy ones look for lower required contributions. In fact, the tacit assumption in the age factors is that health status is related to age. For small groups, an underwriter may know the health status of individuals, but for large groups, gaining such information would be administratively costly.

The relative differences among the factors in the various categories can be adjusted based on the actual variations among the plans. For example, if the average difference in the employee contributions between two given plans is $50 and between two other plans is $25, then the relativities between the factors in the low and high employee contribution categories for the first case should be greater than those for the second. Indeed, the level of access to care, the plan of benefits, and the employee contribution can be represented by fuzzy set functions and the participation factors varied according to the membership values of those fuzzy sets.

### *4.1 Example*

Employees may choose from two plans: plan 1 with free access, rich benefits, and high employee contribution; and plan 2 with limited access, rich benefits, and low employee contribution. The census for the group is

| | Single | Family |
|---|---|---|
| Younger | 10 | 20 |
| Middle-aged | 20 | 35 |
| Older | 15 | 15 |

| | Single | | | Family | | |
|---|---|---|---|---|---|---|
| | Employees | Factor | Product | Employees | Factor | Product |
| Younger | 10 | 0.2 | 2.0 | 20 | 0.3 | 6.0 |
| Middle-aged | 20 | 0.5 | 10.0 | 35 | 0.6 | 21.0 |
| Older | 15 | 0.8 | 12.0 | 15 | 0.9 | 13.5 |
| Total | 45 | | 24.0 | 70 | | 40.5 |

Calculate the single/family age factors for plan 1:

$$\text{Single factor} = 24/45 = 0.533; \quad \text{family factor} = 40.5/70 = 0.579.$$

Similarly, the factors for plan 2 are:

$$\text{Single factor} = 21/45 = 0.467; \text{family factor} = 31/70 = 0.443.$$

Plan 1 is the favorite; if neither were clearly preferred, then we would combine the single and family factors to obtain one age factor for each plan. One possible combination is the weighted average, with the weight equal to the relative premium size, say, 2.7 = (family rate)/(single rate). If we follow this scheme, the age factor for plan 1 is

$$(0.533 + 2.7 * 0.579)/(1 + 2.7) = 0.567,$$

and that for plan 2 is

$$(0.467 + 2.7 * 0.443)/(1 + 2.7) = 0.450.$$

Since there is a 26 percent difference between the factors, roughly 5/9 of the participants can be expected to chose plan 1 and 4/9, plan 2; however, the desirability of a certain level of participation can differ from plan to plan. A given participation may be good for plan 2 because it expects to attract the younger and supposedly healthier lives, whereas the same participation may be less than adequate for plan 1 because older and presumably sicker lives will be drawn to it. (Percentage participation in the different options is with respect to those employees selecting some type of coverage, and the corresponding age factors are also based upon that subset.)

To account for variable levels of participation for different types of acceptable plans, we define a fuzzy set for each plan to reflect the desired participation. For instance, create the following fuzzy set for plan 1 in the above example:

$$f_{11}(p_1) = \begin{cases} 1, & 0.6 \le p_1 \\ 5p_1 - 2, & 0.4 \le p_1 \le 0.6 \\ 0, & p_1 \le 0.4 \end{cases}$$

in which $p_1$ is the expected percentage participation in plan 1. Similarly, for plan 2, define

$$f_{12}(p_2) = \begin{cases} 1, & 0.5 \le p_2 \\ 4p_2 - 1, & 0.25 \le p_2 \le 0.5 \\ 0, & p_2 \le 0.25, \end{cases}$$

in which $p_2$ is the expected percentage participation in plan 2. Calculate the variables $p_1$ and $p_2$ based upon the participation age factors of the plan. For example, assume that $p_1 = 5/9$ and $p_2 = 4/9$ for the given group.

In addition to or in modification of the underwriting rules presented in Section 3 for single-option plans, we discuss the following guidelines in the case of multiple-option plans [9, pp. 4–9], [10, pp. 48–52]. Again, we contemplate only an employer-employee group.

D1. Benefits within each option are determined automatically. Since this rule is nonfuzzy in nature, we assume that D1 is satisfied from the beginning.

E1. The group has an average age less than a stated number of years and female participation less than a given percentage. A large proportion of females and older participants leads to high claim costs and may induce selection against the plan that has greater access to care or richer benefits. To combat higher expected claims, the manual rates can be adjusted by an appropriate age/sex factor. In addition, the age/sex factor may have to be close to the average of one's block of business or to the nationwide average for the industry of the group.

Such a rule is more important for a plan against which selection is more likely. For example, a plan with freer access to care may attract those with higher expected claim costs, as mentioned above. Different plans therefore will require distinct fuzzy sets to represent the above criterion.

Let *asf* be the age/sex factor of the group. Define a fuzzy set representation for plan 1 by:

$$e_{11}(asf) = \begin{cases} 1, & asf \leq 1.1 \\ -5\,asf + 6.5, & 1.1 \leq asf \leq 1.3 \\ 0, & 1.3 \leq asf. \end{cases}$$

Similarly, define one for plan 2 by:

$$e_{12}(asf) = \begin{cases} 1, & asf \leq 1.3 \\ -2.5\,asf + 4.25, & 1.3 \leq asf \leq 1.7 \\ 0, & 1.7 \leq asf. \end{cases}$$

F1. There is a minimum participation when all benefit options are being considered, as well as a minimum enrollment in the given option. We discuss this topic at the beginning of this section and mention the rule here for the sake of completeness.

N. The employee contributions do not differ greatly among the plans. A large difference may lead to selection against a higher-cost plan. Let *ec* be the difference between the employee contributions to plan 1 and plan 2. For the higher-cost plan, plan 1, define the fuzzy set:

$$n_1(ec) = \begin{cases} 1, & ec \leq 25 \\ -0.04\,ec + 2, & 25 \leq ec \leq 50 \\ 0, & 50 \leq ec. \end{cases}$$

For the lower-cost plan, plan 2, define the fuzzy set identically equal to 1:

$$n_2 \equiv 1.$$

Note that we may use the same function for both plans by allowing $ec$ to assume negative values.

O. The benefits of one plan are not overly rich in relation to the other(s), particularly with respect to selected benefits, such as prescription drugs and organ transplants. This requirement helps to reduce selection against the plan and premium differentials.

One measure of the relative richness is the ratio of the manual claims of one plan divided by the other, where provider discounts are not considered. Let $rr$ be this ratio, and define the fuzzy set:

$$o(rr) = \begin{cases} 1, & rr \leq 1.2 \\ -2.5\, rr + 4, & 1.2 \leq rr \leq 1.6 \\ 0, & 1.6 \leq rr. \end{cases}$$

As in Section 3, we use a matrix to relate the above rules:

| Interaction between Criteria | $e_{1i}$ | $f_{1i}$ | $n_i$ | $o$ | $Q_2$ |
|---|---|---|---|---|---|
| Age/sex ($e_{1i}$) | | Some | | | Some |
| Participation ($f_{1i}$) | Some | Important | Max | Max | Some |
| Contribution ($n_i$) | | Max | | | Some |
| Benefits ($o$) | | Max | | | Some |
| $Q_2$ | Some | Some | Some | Some | |

To define a fuzzy set function representing the above table, first group the variables associated with an option's characteristics according to whether they interact: $\{e_{1i}, f_{1i}\}$ and $\{f_{1i}, n_i, o\}$. Since $e_{1i}$ and $f_{1i}$ interact somewhat, merge them by means of the Hamacher operator ($p$=0.5). Take the square root to make the result comparable to the next term.

Because $f_{1i}$, $n_i$, and $o$ interact maximally, intersect them through the algebraic product. Form the cube root of the product, so that the two terms are commensurate. The importance of $f_{1i}$ is implicit in its appearance in both of the above terms. As in Section 3, this importance becomes explicit by using the concentration of $f_{1i}$, either as a substitute for $f_{1i}$ in the two terms or as an extra term.

Combine the two terms through the minimum operator; the resulting function represents an option's qualities. Join this function with $Q_2$ by the Yager operator ($p=2$); the Hamacher operator ($p=0.5$) also may be used. The resulting combination is

$$R_i = Y(Q_2, [f_{1i} \times n_i \times o]^{1/3} \cap [H(f_{1i}, e_{1i}; 0.5)]^{1/2}; 2),$$

in which $i=1,2$. Note that $R_i$ generalizes the results of Section 3 by embedding $Q_2$, the fuzzy set related to group characteristics, within it. The other four functions, $e_{1i}$, $f_{1i}$, $n_i$, and $o$, depend upon the plan's characteristics as well as those of the group.

### *4.2 Example*

Assume the group is as given in Example 3.3, and use the plans and single/family age distributions from Example 4.1. Let $asf=1.2$, $ec=40$, and $rr=1$.

$$\begin{aligned} R_1 &= Y(0.6887, \min[0.6776, 0.6417]; 2) \\ &= 1 - \min\{1, [(1 - 0.6887)^2 + (1 - .6417)^2]^{1/2}\} \\ &= 1 - \min\{1, 0.4746\} \\ &= 0.5254. \\ R_2 &= Y(0.6887, \min[0.9196, 0.8819]; 2) \\ &= 1 - \min\{1, [(1 - 0.6887)^2 + (1 - .8819)^2]^{1/2}\} \\ &= 1 - \min\{1, 0.3329\} \\ &= 0.6671. \end{aligned}$$

According to the above results, the sponsor of plan 2 is happier about offering that plan against plan 1 than vice versa.

## 5. CONCLUSIONS AND AREAS FOR FURTHER RESEARCH

The fuzzy set approach to underwriting presented here generalizes the method of debits and credits: For each group characteristic, a given number of points is added or subtracted, and the resulting number is used to categorize the group and to determine a load or credit on the manual rates for that group. Fuzzy set theory allows for more interaction between the variables because operations other than addition can be used. The flexibility allowed in creating the fuzzy set functions and the variety of ways in which to combine them also add to the attractiveness of this theory.

Another advantage of fuzzy set models is that they can be used to make decisions. Fuzzy sets representing external forces, such as the economy or market conditions, could be combined with those describing the characteristics of the group. Then, marketing and pricing decisions could be based on the resulting fuzzy sets, as in Example 2.2.13.

This work can immediately be extended by using the resulting fuzzy set functions as the basis of underwriting loads. Keep in mind that if the employee contribution increases with the load, then the selection being guarded against may be more likely to occur. Underwriters may be more effective in the long run if they change the plan design or require employers to contribute more than they would have if they loaded the rates to anticipate selection against the plan.

The mathematics of fuzzy set theory used in this paper are simple enough for someone to readily implement them. A PC spreadsheet could be created to calculate membership values for all the fuzzy functions needed. Testing the appropriateness of the particular functions would then be straightforward. Other forms of intersection may be more suitable than those given here. Also, more multivariate functions may prove useful in modeling asymmetrical situations. For example, an increase in age/sex factor may be less harmful if the group is growing than if it is decreasing.

Other areas in actuarial science that lend themselves to fuzzy sets are trend analysis and credibility theory. Fuzzy sets could represent the relations among trend and economic indicators, such as the medical CPI. In credibility theory, the credibility factor is usually based on the exposure months, premium, or claims during the experience period. Other variables can also influence credibility, such as turnover within the group, plan design, or chronic versus acute ongoing claims [3]; fuzzy sets could symbolize how credibility is affected by these factors.

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## DISCUSSION OF PRECEDING PAPER

**DAVID W. ERBACH* AND ERIC SEAH:**

There is little question that the techniques of fuzzy logic provide the best opportunities currently available to model the underwriting process, and not only for group health business. While underwriting is, in principle, an affair of statistics, the inexactness of the underlying data, and even of the underlying phenomena, invite a treatment that does not depend on the precise specification of the values of defining parameters. Fuzzy logic has the cardinal virtue of not imposing a burden of precision where circumstances do not justify it.

Furthermore, as Dr. Young mentions, the mathematics are not specially complicated, even if they may be unfamiliar in appearance. This is a happy consequence of the fact that it seems adequate to define the membership functions in piecewise linear components.

Inevitably, she had to restrict her scope somewhat in a survey article, but other formally defined notions of fuzzy analysis, such as "plausibility" and "possibility," also have natural and useful interpretations in the context of underwriting.

Fuzzy logic is a very promising tool waiting to be exploited.

### *Some Caveats*

The methods do have some weaknesses. One is the analysis of the likelihood of catastrophic (from an underwriting perspective) single events. Fuzzy methods are not so easy to apply to these, which is no doubt part of the reason that Dr. Young has concentrated her attention on group health underwriting. They can even be applied to individual life business, but some of the emphases need to be different.

Another is that fuzzy analysis rather depends on the underlying phenomena being "ordered," in the mathematical sense. When it comes to things like participation rates, more is dependably better. But when it comes to assessing industry risk, the ordering depends on knowing the answer in advance; industries cannot easily be set on a scale that makes it easy to locate a new one. The same problem makes it difficult to bring occupations into the model (much less names of occupations, if one is

*Mr. Erbach, not a member of the Society, is head of the Department of Business Computing at the University of Winnipeg, Winnipeg, Manitoba.

thinking in terms of customers filling out electronic application forms), with implications for applicability to, say, individual life business.

A third is that the interplay of factors analyzed can be quite complicated. With only a dozen factors in the analysis, the situation is much worse than simply the number of pairs would imply. For instance, if the membership functions are defined as simply as possible, as three piecewise linear parts (one constant 0, one linear from 0 to 1, and one constant 1), the interplay of two already has numerous boundaries whose location requires some attention. Mix in the possibility that factors may tend to reinforce or may tend to attenuate each other in terms of aggregate risk indication. Combine with this some software that makes the definition and addition of new risk indicators easy, and one would have a stew that would require very close attention from the cooks—and the management.

### *Other Advantages*

To set against this, the techniques have several advantages in addition to those Dr. Young mentions.

One of the most important is that one can record precisely why an underwriting decision was made. This has many implications. If you want to test your proposed underwriting of a new product, you can run the analysis against a past set of cases to see precisely what decisions you would have made. If you have the corresponding case history, you can determine how you would have fared financially against the actual claims experience. Even a matter as vague as deciding to increase or decrease underwriting stringency a bit can become an accurately defined operation.

Another is that one can show, if it ever came to a legal question, precisely how a decision was taken. An investigation of potential legal implications is well beyond the scope of this note. But it is not news that companies in many industries have become vulnerable to post hoc arguments that certain results implied certain decision-making along the way. At least from a mathematical point of view, fuzzy underwriting makes it conceivable to prove how decisions were taken, and what the results might have been had other decision policies been in place, even long after the fact.

Finally, from a software point of view, it seems clear that designing for "economies of scope" has become one of the most important considerations in the preparation of computer systems with a long half-life.

A skillfully designed fuzzy underwriter, which could take electronic input equally from the field for current underwriting purposes, and from historical data for pricing and product development needs, would be a marvel of virtuosity.

### *An Historical Note*

In 1987, with the aid of two colleagues, Douglas Holmes and Robert J. Purdy, one of us (Erbach) led a "skunk works" team that developed *Zeno*, a prototype life automated underwriter using a mixture of fuzzy and other techniques. Done on behalf of a Canadian firm, this work seems to have been the first practical work done in Canada and among the earliest in the industry. It was carried as far as developing an underwriter (coded in C++) that ran on portable computers. *Zeno* was intended to do final underwriting of the majority of individual life cases on the spot. With the aid of electronic links to the home office, *Zeno* aimed to make turnaround on cases complicated enough to require human intervention a matter of minutes, while agent and prospect had a leisurely cup of coffee. *Zeno* was carried far enough to show that it could work, at which point the company turned it over to the regular systems people, who promptly abandoned it. Why? The reasons undoubtedly include both my lack of persuasiveness and management's inability to understand the implications of unfamiliar techniques. The location of the data point with respect to these fuzzy membership functions is doubtless best left undefined.

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#### CHARLES S. FUHRER:

Dr. Young has written an excellent paper. Fuzzy sets is an intriguing area of research that has many applications. Unfortunately, fuzzy sets cannot be usefully applied to insurance underwriting.

The goal of every underwriter is to help achieve the financial goals of the insurance company. The measure of the achievement of these goals is the financial results of the company. The success or failure of the underwriter is measured by the premium versus the claims and other expenses of the company. These are not vague or fuzzy. They are, after they have occurred, exact numbers. No underwriter should ever care that the block of business written has some sort of high membership in the set of good risks. The uncertainty that exists before the claims have been reported is best modeled stochastically. In Section 2.3 the author distinguishes between fuzzy set theory and probability theory and states: "After a given event occurs (or not), randomness no longer exists; however, fuzziness does not decrease after additional information has been acquired." I submit that in underwriting there is not only some decrease in uncertainty after the claims are reported, but also almost no remaining uncertainty.

The author correctly ascertained that there is a problem with current methods of group underwriting. A perusal of Lehman (Young's ref. [7], the Society of Actuaries main Study Note on group underwriting) yields a set of subjective opinions about group characteristics without any systematic method of combining these evaluations. The author's fuzzy set scheme, which systematically combines these characteristics, is a step in the right direction. Nevertheless, the author's method is actually confined by the fuzzy set concept. There are ways in which the author's scheme can be improved. These improvements will necessarily leave behind the fuzzy set terminology in order to use a more general technique.

### *The Basic Limitation of Fuzzy Sets*

The main problem with fuzzy sets is that a set $A$ is defined by $f_A: X \rightarrow [0, 1]$; that is, there is a mapping to the unit interval I. My methodology uses functions (no longer identified as set functions) $f_A: X \rightarrow \Re$, that is, mappings to the whole real line. This greater flexibility prevents having functions that have corners or are non-smooth at 0 or 1. Most real phenomena are best modeled with smooth functions. For example, let us look at the participation function used by the author in Rule F of Section 3. The group is deemed more preferred if the participation rises from 70 percent to 90 percent. The same level of preference (that is, 1) is given for 90 percent through 100 percent. If a 90 percent participation is preferable to an 89 percent participation, why would not a 91 percent be preferred to 90 percent? There is no answer. Of course, the fuzzy set theorist could rescale all the functions to the open interval (0, 1). This is analogous to how truly subjective ratings, such as for fine wines, will always avoid using a 100 percent. Unfortunately, this would not allow the use of the simple linear functions that are preferred by all. The confinement to [0, 1] also leads to very complicated combining operators such as the Yager and Hamacher. A simpler operator, $S$, would be $S(A, B)=aA+bB+cAB$ for three constants $a$, $b$, and $c$. Note that the relative amount of interaction is controlled by the size of $c$ versus $a$ and $b$.

## *Using Data*

The severest problem with current techniques is that the various opinions on group suitability are completely subjective. Many insurance companies load or decline groups based on characteristics of the groups. Often, there is no known objective information that these characteristics have a negative influence on financial results. Some of the characteristics used by the author would fit this description. There is an easy way to use actual group insurance data to determine the loads for various group characteristics. Merely set up a linear or quadratic function of the characteristics such as $y=\sum_{i=1}^{n} a_i x_i+\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$, where the $x$'s are characteristics such as number of people in the group, participation percentage, employer contribution, etc. and the $a$'s are unknown constant coefficients. The $x$'s can be discrete such as 1 for actively-at-work rule and 0 for no rule. Now the $a$'s can be determined by least squares regression on some group data. There are many techniques for determining the coefficients besides least squares. One can also use Bayesian techniques to combine a subjective belief about certain coefficients with the data.

The author's fuzzy set functions would be much harder to use. Furthermore, the author's fuzzy set paradigm gives the illusion of a scientific approach to underwriting when actually it is not even an attempt to substitute demonstrations for impressions. The use of fuzzy sets for credibility factors as suggested by the author would also be a mistake. Most good methods of setting credibility [author's ref. 3] currently use functions of group size (and sometimes other characteristics) that are determined using least squares based on group insurance data. See [1] for some other methods of fitting credibility levels to size of case.

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## (AUTHOR'S REVIEW OF DISCUSSION)

**VIRGINIA R. YOUNG:**

I thank Mr. Erbach and Dr. Seah for complementing my writing with their comments. I am glad that they reinforce my intent, namely, to describe how fuzzy sets may serve as the basis of a fuzzy expert underwriter.

It is unfortunate, however, that Mr. Fuhrer misunderstands my purpose. In this article, I am not describing how a company might develop its underwriting rules. Rather, I am showing how an underwriter might model the insurer's existing rules using fuzzy sets. Mr. Erbach and Dr. Seah list advantages of this technique beyond those I discuss.

At the end of Section 2.3, I address the topic in the second paragraph of Mr. Fuhrer's discussion. I propose that an unprofitable group may still be one that a company wants to write. Please refer to my paper for further comments.

I do not agree with Mr. Fuhrer that the main problem with fuzzy sets is that the range of the set is limited to the unit interval [0, 1]. This restriction does not necessarily lead to nondifferentiable functions or to complicated combining operators. In actuarial practice, one encounters the property of boundedness in such areas as limits on rate increases, limits on premium capacity, etc., so limiting the range of a fuzzy set is not a handicap. Also, cumulative distribution functions are required to increase from 0 to 1, and their power is not unduly hampered.

Mr. Fuhrer takes issue with one of my illustrative fuzzy sets. As I clearly state in the sentence preceding Rule E, Section 3, the fuzzy sets are examples only.

Again, the main intent of my paper is to describe how one may use fuzzy sets to model existing underwriting rules. I did not give procedures for adjusting the functions to reflect given data. That area is one that I am currently researching.