SOCIETY OF ACTUARIES

EXAM C CONSTRUCTION AND EVALUATION OF ACTUARIAL MODELS

EXAM C SAMPLE SOLUTIONS

The sample questions and solutions have been modified over time. This page indicates changes made since January 1, 2014.

June 2016
Question 266 was moved to become Question 306 and Question 307 (effective with the October 2016 syllabus) added.

May 2015:
Questions 189 and 244 have been modified to not refer to the Anderson-Darling test

January 14, 2014:
Questions and solutions 300–305 have been added.

Some of the questions in this study note are taken from past examinations. The weight of topics in these sample questions is not representative of the weight of topics on the exam. The syllabus indicates the exam weights by topic.

Question #1
Key: E

The 40th percentile is the 0.4(12) = 4.8th smallest observation. By interpolation it is 0.2(86) + 0.8(90) = 89.2. The 80th percentile is the 0.8(12) = 9.6th smallest observation. By interpolation it is 0.4(200) +0.6(210) = 206.

The equations to solve are

$0.4 = \dfrac{(89.2/\theta)^\gamma}{1+(89.2/\theta)^\gamma}$ and $0.8 = \dfrac{(206/\theta)^\gamma}{1+(206/\theta)^\gamma}$.

Solving each for the parenthetical expression gives $\dfrac{2}{3} = (89.2/\theta)^\gamma$ and $4 = (206/\theta)^\gamma$.

Taking the ratio of the second equation to the first gives $6 = (206/89.2)^\gamma$ which leads to $\gamma = \ln(6)/\ln(206/89.2) = 2.1407$. Then $4^{1/2.1407} = 206/\theta$ for $\theta = 107.8$.

Question #2
Key: E

The standard for full credibility is $\left(\dfrac{1.645}{0.02}\right)^2\left(1+\dfrac{Var(X)}{E(X)^2}\right)$ where X is the claim size variable.

For the Pareto variable, $E(X) = 0.5/5 = 0.1$ and $Var(X) = \dfrac{2(0.5)^2}{5(4)} - (0.1)^2 = 0.015$. Then the standard is $\left(\dfrac{1.645}{0.02}\right)^2\left(1+\dfrac{0.015}{0.1^2}\right) = 16,913$ claims.

Question #3
Key: B

The kernel is a triangle with a base of 4 and a height at the middle of 0.5 (so the area is 1). The length of the base is twice the bandwidth. Any observation within 2 of 2.5 will contribute to the estimate. For the observation at 2, when the triangle is centered at 2, the height of the triangle at 2.5 is .375 (it is one-quarter the way from 2 to the end of the triangle at 4 and so the height is one-quarter the way from 0.5 to 0). Similarly the points at 3 are also 0.5 away and so the height of the associated triangle is also .375. Each triangle height is weighted by the empirical probability at the associated point. So the estimate at 2.5 is (1/5)(3/8) + (3/5)(3/8) + (1/5)(0) = 12/40.

Question #4
Key: A

The distribution function is $F(x)=\int_1^x \alpha t^{-\alpha-1}dt=-t^{-\alpha}\Big|_1^x=1-x^{-\alpha}$. The likelihood function is

$$\begin{aligned}L &= f(3)f(6)f(14)[1-F(25)]^2\\ &=\alpha 3^{-\alpha-1}\alpha 6^{-\alpha-1}\alpha 14^{-\alpha-1}(25^{-\alpha})^2\\ &\propto \alpha^3[3(6)(14)(625)]^{-\alpha}.\end{aligned}$$

Taking logs, differentiating, setting equal to zero, and solving:

$\ln L=3\ln\alpha-\alpha\ln 157{,}500$ plus a constant

$d\ln L/d\alpha=3\alpha^{-1}-\ln 157{,}500=0$

$\hat{\alpha}=3/\ln 157{,}500=0.2507.$

Question #5
Key: C

$\pi(q\,|\,1,1)\propto p(1\,|\,q)p(1\,|\,q)\pi(q)=2q(1-q)2q(1-q)4q^3\propto q^5(1-q)^2$

$\int_0^1 q^5(1-q)^2dq=1/168,\ \pi(q\,|\,1,1)=168q^5(1-q)^2.$

The expected number of claims in a year is $E(X\,|\,q)=2q$ and so the Bayesian estimate is

$E(2q\,|\,1,1)=\int_0^1 2q(168)q^5(1-q)^2dq=4/3.$

The answer can be obtained without integrals by recognizing that the posterior distribution of q is beta with a = 6 and b = 3. The posterior mean is $E(q\,|\,1,1)=a/(a+b)=6/9=2/3$ The posterior mean of $2q$ is then 4/3.

Question #6
Key: D

For the method of moments estimate,

$$386 = \exp(\mu + 0.5\sigma^2),\ 457,480.2 = \exp(2\mu + 2\sigma^2)$$

$$5.9558 = \mu + 0.5\sigma^2, 13.0335 = 2\mu + 2\sigma^2$$

$$\hat{\mu} = 5.3949,\ \hat{\sigma}^2 = 1.1218.$$

Then

$$E(X \wedge 500) = \exp[5.3949 + 0.5(1.1218)]\Phi\left(\frac{\ln 500 - 5.3949 - 1.1218}{\sqrt{1.1218}}\right) + 500\left[1 - \Phi\left(\frac{\ln 500 - 5.3949}{\sqrt{1.1218}}\right)\right]$$

$$= 386\Phi(-0.2853) + 500[1 - \Phi(0.7739)] = 386(0.3877) + 500(0.2195) = 259.$$

Note-these calculations use exact normal probabilities. Rounding and using the normal table that accompanies the exam will produce a different numerical answer but the same letter answer.

Question #7
DELETED

Question #8
Key: C

Let N be the Poisson claim count variable, let X be the claim size variable, and let S be the aggregate loss variable.

$$\mu(\theta) = E(S \mid \theta) = E(N \mid \theta)E(X \mid \theta) = \theta 10\theta = 10\theta^2$$

$$v(\theta) = Var(S \mid \theta) = E(N \mid \theta)E(X^2 \mid \theta) = \theta 200\theta^2 = 200\theta^3$$

$$\mu = E(10\theta^2) = \int_1^\infty 10\theta^2 (5\theta^{-6})d\theta = 50/3$$

$$EPV = E(200\theta^3) = \int_1^\infty 200\theta^3 (5\theta^{-6})d\theta = 500$$

$$VHM = Var(10\theta^2) = \int_1^\infty (10\theta^2)^2 (5\theta^{-6})d\theta - (50/3)^2 = 222.22$$

$$k = 500/222.22 = 2.25.$$

Question #9
DELETED

Question #10
DELETED

Question #11
Key: D

$$\Pr(\theta=1 \mid X=5)=\frac{f(5\mid\theta=1)\Pr(\theta=1)}{f(5\mid\theta=1)\Pr(\theta=1)+f(5\mid\theta=3)\Pr(\theta=3)}$$

$$=\frac{(1/36)(1/2)}{(1/36)(1/2)+(3/64)(1/2)}=16/43$$

$$\Pr(X_2>8\mid X_1=5)=\Pr(X_2>8\mid\theta=1)\Pr(\theta=1\mid X_1=5)+\Pr(X_2>8\mid\theta=3)\Pr(\theta=3\mid X_1=5)$$

$$=(1/9)(16/43)+(3/11)(27/43)=0.2126.$$

For the last line, $\Pr(X>8\mid\theta)=\int_8^\infty \theta(x+\theta)^{-2}\,dx=\theta(8+\theta)^{-1}$ is used.

Question #12
Key: C

The sample mean for *X* is 720 and for *Y* is 670. The mean of all 8 observations is 695.

$$\hat{v}=[(730-720)^2+\cdots+(700-720)^2+(655-670)^2+\cdots+(750-670)^2]/[2(4-1)]=3475$$

$$\hat{a}=[(720-695)^2+(670-695)^2]/(2-1)-3475/4=381.25$$

$$\hat{k}=3475/381.25=9.1148$$

$$\hat{Z}=4/(4+9.1148)=0.305$$

$$P_C=0.305(670)+0.695(695)=687.4.$$

Question #13
Key: B

There are 430 observations. The expected counts are 430(0.2744) = 117.99, 430(0.3512) = 151.02, and 430(0.3744) = 160.99. The test statistic is

$$\frac{(112-117.99)^2}{117.99}+\frac{(180-151.02)^2}{151.02}+\frac{(138-160.99)^2}{160.99}=9.15.$$

Question #14
Key: B

From the information, the asymptotic variance of $\hat{\theta}$ is 1/4*n*. Then

$$Var(2\hat{\theta})=4Var(\hat{\theta})=4(1/4n)=1/n$$

Note that the delta method is not needed for this problem, although using it leads to the same answer.

Question #15
Key: A

$$\pi(p\,|\,1,1,1,1,1,1,1,1) \propto \Pr(1,1,1,1,1,1,1,1\,|\,p)\pi(p) \propto p^8$$

$$\pi(p\,|\,1,1,1,1,1,1,1,1) = \frac{p^8}{\int_0^{0.5} p^8 dp} = \frac{p^8}{0.5^9/9} = 9(0.5^{-9})p^8$$

$$\Pr(X_9 = 1\,|\,1,1,1,1,1,1,1,1) = \int_0^{0.5} \Pr(X_9 = 1\,|\,p)\pi(p\,|\,1,1,1,1,1,1,1,1)dp$$

$$= \int_0^{0.5} p9(0.5^{-9})p^8 dp = 9(0.5^{-9})(0.5^{10})/10 = 0.45.$$

Question #16
Key: A

${}_3\hat{p}_1 = \frac{18}{27}\frac{26}{32}\frac{20}{25} = \frac{13}{30}$. Greenwood's approximation is

$$\left(\frac{13}{30}\right)^2\left(\frac{9}{18(27)} + \frac{6}{26(32)} + \frac{5}{20(25)}\right) = 0.0067.$$

Question #17
Key: D

$\hat{H}(3)$ = 5/30 + 9/27 + 6/32 = 0.6875

$V\hat{a}r(\hat{H}(3)) = 5/30^2 + 9/27^2 + 6/32^2 = 0.02376.$

The 95% log-transformed confidence interval is:

$\hat{H}(3)U$, where $U = \exp\left(\pm\frac{1.96\sqrt{0.02376}}{0.6875}\right) = \exp(\pm 0.43945)$

The confidence interval is:
[0.6875 exp(–0.43945), 0.6875 exp(0.43945)] = [0.443, 1.067].

Question #18
Key: D

The means are 0.5(250) + 0.3(2,500) + 0.2(60,000) = 12,875 and 0.7(250) + 0.2(2,500) + 0.1(60,000) = 6,675 for risks 1 and 2 respectively.
The variances are $0.5(250)^2 + 0.3(2,500)^2 + 0.2(60,000)^2 - 12,875^2 = 556,140,625$ and $0.7(250)^2 + 0.2(2,500)^2 + 0.1(60,000)^2 - 6,675^2 = 316,738,125$ respectively.

The overall mean is (2/3)(12,875) + (1/3)(6,675) = 10,808.33 and so
EPV = (2/3)(556,140,625) + (1/3)(316,738,125) = 476,339,792 and
VHM = $(2/3)(12,875)^2 + (1/3)(6,675)^2 - 10,808.33^2 = 8,542,222$. Then,
k = 476,339,792/8,542,222 = 55.763 and Z = 1/(1 + 55.763) = .017617.
The credibility estimate is .017617(250) + .982383(10,808.33) = 10,622.

Question #19
Key: D

The first two sample moments are 15 and 500, and the first two population moments are $E(X) = 0.5(\theta + \sigma)$ and $E(X^2) = 0.5(2\theta^2 + 2\sigma^2) = \theta^2 + \sigma^2$. These can be obtained either through integration or by recognizing the density function as a two-point mixture of exponential densities. The equations to solve are $30 = \theta + \sigma$ and $500 = \theta^2 + \sigma^2$. From the first equation, $\sigma = 30 - \theta$ and substituting into the second equation gives $500 = \theta^2 + (30-\theta)^2 = 2\theta^2 - 60\theta + 900$. The quadratic equation has two solutions, 10 and 20. Because $\theta > \sigma$ the answer is 20.

Question #20
Key: D

There are four possible samples, (5,5), (5,9), (9,5), and (9,9). For each, the estimator g must be calculated. The values are 0, 4, 4, and 0 respectively. Assuming a population in which the values 5 and 9 each occur with probability 0.5, the population variance is $0.5(5-7)^2 + 0.5(9-7)^3 = 4$. The mean square error is approximated as $0.25[(0-4)^2 + (4-4)^2(4-4)^2 + (0-4)^2] = 8$.

Question #21
Key: B

From the Poisson distribution, $\mu(\lambda) = \lambda$ and $v(\lambda) = \lambda$. Then, $\mu = E(\lambda) = 6/100 = 0.06, EPV = E(\lambda) = 0.06, VHM = Var(\lambda) = 6/100^2 = 0.0006$ where the various moments are evaluated from the gamma distribution. Then, $k = 0.06/0.0006 = 100$ and Z = 450/(450 + 100) = 9/11 where 450 is the total number of insureds contributing experience. The credibility estimate of the expected number of claims for one insured in month 4 is (9/11)(25/450) + (2/11)(0.06) = 0.056364. For 300 insureds the expected number of claims is 300(0.056364) = 16.9.

Question #22
Key: C

The likelihood function is $L(\alpha,\theta)=\prod_{j=1}^{200}\frac{\alpha\theta^{\alpha}}{(x_j+\theta)^{\alpha+1}}$ and its logarithm is

$l(\alpha,\theta)=200\ln(\alpha)+200\alpha\ln(\theta)-(\alpha+1)\sum_{j=1}^{200}\ln(x_j+\theta)$. When evaluated at the hypothesized values of 1.5 and 7.8, the loglikelhood is –821.77. The test statistic is 2(821.77 – 817.92) = 7.7. With two degrees of freedom (0 free parameters in the null hypothesis versus 2 in the alternative), the test statistic falls between the 97.5th percentile (7.38) and the 99th percentile (9.21).

Question #23
Key: E

Assume that $\theta>5$. Then the expected counts for the three intervals are
$15(2/\theta)=30/\theta, 15(3/\theta)=45/\theta,$ and $15(\theta-5)/\theta=15-75/\theta$ respectively. The quantity to minimize is

$\frac{1}{5}[(30\theta^{-1}-5)^2+(45\theta^{-1}-5)^2+(15-75\theta^{-1}-5)^2]$.

Differentiating (and ignoring the coefficient of 1/5) gives the equation
$-2(30\theta^{-1}-5)30\theta^{-2}-2(45\theta^{-1}-5)45\theta^{-2}+2(10-75\theta^{-1})75\theta^{-2}=0.$ Multiplying through by θ^3 and dividing by 2 reduces the equation to
$-(30-5\theta)30-(45-5\theta)45+(10\theta-75)75=-8550+1125\theta=0$ for a solution of
$\hat{\theta}=8550/1125=7.6.$

Question #24
Key: E

$\pi(\theta\,|\,1)\propto\theta(1.5\theta^{0.5})\propto\theta^{1.5}$. The required constant is the reciprocal of $\int_0^1\theta^{1.5}d\theta=0.4$ and so

$\pi(\theta\,|\,1)=2.5\theta^{1.5}$. The requested probability is

$\Pr(\theta>0.6\,|\,1)=\int_{0.6}^{1}2.5\theta^{1.5}d\theta=\theta^{2.5}\Big|_{0.6}^{1}=1-0.6^{2.5}=0.721.$

Question #25
Key: A

k	kn_k / n_{k-1}
0	
1	0.81
2	0.92
3	1.75
4	2.29
5	2.50
6	3.00

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #26
Key: B

The likelihood function is $\frac{e^{-1/(2\theta)}}{2\theta}\frac{e^{-2/(2\theta)}}{2\theta}\frac{e^{-3/(2\theta)}}{2\theta}\frac{e^{-15/(3\theta)}}{3\theta}=\frac{e^{-8/\theta}}{24\theta^4}$. The loglikelihood function is $-\ln(24)-4\ln(\theta)-8/\theta$. Differentiating with respect to θ and setting the result equal to 0 yields $-\frac{4}{\theta}+\frac{8}{\theta^2}=0$ which produces $\hat{\theta}=2$.

Question #27
Key: E

The absolute difference of the credibility estimate from its expected value is to be less than or equal to $k\mu$ (with probability *P*). That is,

$$\left|[ZX_{partial}+(1-Z)M]-[Z\mu+(1-Z)M]\right|\le k\mu$$

$$-k\mu \le ZX_{partial}-Z\mu \le k\mu.$$

Adding μ to all three sides produces answer choice (E).

Question #28
Key: C

In general,

$$E(X^2) - E[(X \wedge 150)^2] = \int_0^{200} x^2 f(x)dx - \int_0^{150} x^2 f(x)dx - 150^2 \int_{150}^{200} f(x)dx$$

$$= \int_{150}^{200} (x^2 - 150^2) f(x)dx.$$

Assuming a uniform distribution, the density function over the interval from 100 to 200 is 6/7400 (the probability of 6/74 assigned to the interval divided by the width of the interval). The answer is

$$\int_{150}^{200} (x^2 - 150^2)\frac{6}{7400}dx = \left(\frac{x^3}{3} - 150^2 x\right)\frac{6}{7400}\Bigg|_{150}^{200} = 337.84.$$

Question #29
Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

$$\Pr(3 \mid I) = \binom{6}{3}(0.1)^3(0.9)^3 = 0.01458$$

$$\Pr(3 \mid II) = \binom{6}{3}(0.2)^3(0.8)^3 = 0.08192$$

$$\Pr(3 \mid III) = \binom{6}{3}(0.4)^3(0.6)^3 = 0.27648$$

The probability of observing three successes is 0.7(.01458) + 0.2(.08192) + 0.1(.27648) = 0.054238. The three posterior probabilities are:

$$\Pr(I \mid 3) = \frac{0.7(0.01458)}{0.054238} = 0.18817$$

$$\Pr(II \mid 3) = \frac{0.2(0.08192)}{0.054238} = 0.30208$$

$$\Pr(III \mid 3) = \frac{0.1(0.27648)}{0.054238} = 0.50975.$$

The posterior probability of a claim is then 0.1(0.18817) + 0.2(0.30208) + 0.4(0.50975) = 0.28313.

Question #30
Key: E

$0.542 = \hat{F}(n) = 1 - \exp[-\hat{H}(n)],\ \hat{H}(n) = 0.78.$ The Nelson-Aalen estimate is the sum of successive *s*/*r* values. From the problem statement, $r = 100$ at all surrender times while the *s*-values follow the pattern 1, 2, 3, …. Then,

$0.78 = \frac{1}{100} + \frac{2}{100} + \cdots + \frac{n}{100} = \frac{n(n+1)}{200}$ and the solution is $n = 12$.

Question # 31
Answer: C

$g = [12(0.45)] = [5.4] = 5$, $h = 5.4 - 5 = 04$.

$\hat{\pi}_{0.45} = 0.6x_{(5)} + 0.4x_{(6)} = 0.6(360) + 0.4(420) = 384.$

Question # 32
Key: D

N is distributed Poisson(λ)

$\mu = E(\lambda) = \alpha\theta = 1(1.2) = 1.2$

$v = E(\lambda) = 1.2,\ a = Var(\lambda) = \alpha\theta^2 = 1(1.2)^2 = 1.44$

$k = \frac{1.2}{1.44} = \frac{5}{6},\ Z = \frac{2}{2 + 5/6} = \frac{12}{17}$

Thus, the estimate for Year 3 is $\frac{12}{17}(1.5) + \frac{5}{17}(1.2) = 1.41.$

Note that a Bayesian approach produces the same answer.

Question # 33
Key: C

At the time of the second failure,

$$\hat{H}(t) = \frac{1}{n} + \frac{1}{n-1} = \frac{23}{132} \Rightarrow n = 12.$$

At the time of the fourth failure,

$$\hat{H}(t) = \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} = 0.3854.$$

Question # 34
Key: B

The likelihood is:

$$L=\prod_{j=1}^{n}\frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}}\propto\prod_{j=1}^{n}\beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l=\sum_{j=1}^{n}[x_j\ln\beta-(r+x_j)\ln(1+\beta)]$$
$$l'=\sum_{j=1}^{n}\left[\frac{x_j}{\beta}-\frac{r+x_j}{1+\beta}\right]=0$$
$$0=\sum_{j=1}^{n}[x_j(1+\beta)-(r+x_j)\beta]=\sum_{j=1}^{n}x_j-rn\beta=n\bar{x}-rn\beta$$
$$\hat{\beta}=\bar{x}/r.$$

Question # 35
Key: C

The Bühlmann credibility estimate is $Zx+(1-Z)\mu$ where x is the first observation. The Bühlmann estimate is the least squares approximation to the Bayesian estimate. Therefore, Z and μ must be selected to minimize

$$\frac{1}{3}[Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[2Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[3Z+(1-Z)\mu-3]^2.$$

Setting partial derivatives equal to zero will give the values. However, it should be clear that μ is the average of the Bayesian estimates, that is,

$$\mu=\frac{1}{3}(1.5+1.5+3)=2.$$

The derivative with respect to Z is (deleting the coefficients of 1/3):

$$2(-Z+0.5)(-1)+2(0.5)(0)+2(Z-1)(1)=0$$
$$4Z-3=0,\ Z=0.75.$$

The answer is 0.75(1) + 0.25(2) = 1.25.

Question # 36
Key: E

The confidence interval is $(\hat{S}(t_0)^{1/\theta}, \hat{S}(t_0)^{\theta})$.
Taking logarithms of both endpoints gives the two equations

$$\ln 0.695 = -0.36384 = (1/\theta)\ln \hat{S}(t_0)$$
$$\ln 0.843 = -0.17079 = \theta \ln \hat{S}(t_0).$$

Multiplying the two equations gives

$$0.06214 = [\ln \hat{S}(t_0)]^2, \ \ln \hat{S}(t_0) = -0.24928, \ \hat{S}(t_0) = 0.77936.$$

The negative square root is required in order to make the answer fall in the interval (0,1).

Question # 37
Key: B

The likelihood is:

$$L = \frac{\alpha 150^{\alpha}}{(150+225)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+525)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+950)^{\alpha+1}} = \frac{\alpha^3 150^{3\alpha}}{[(375)(675)(1100)]^{\alpha+1}}.$$

The loglikelihood is:

$$l = 3\ln\alpha + 3\alpha\ln 150 - (\alpha+1)\ln[(375)(675)(1100)]$$
$$l' = 3\alpha^{-1} + 3\ln 150 - \ln[(375)(675)(1100)] = 3\alpha^{-1} - 4.4128$$
$$\hat{\alpha} = 3/4.4128 = 0.6798.$$

Question # 38
Key: D

For this problem, r = 4 and n = 7. Then,

$$\hat{v} = \frac{33.60}{4(7-1)} = 1.4,\ \hat{a} = \frac{3.3}{4-1} - \frac{1.4}{7} = 0.9.$$

Then,

$$k = \frac{1.4}{0.9} = \frac{14}{9},\ Z = \frac{7}{7+(14/9)} = \frac{63}{77} = 0.82.$$

Question # 39
Key: B

X is the random sum $Y_1 + Y_2 + \cdots + Y_N$.
N has a negative binomial distribution with r = a = 1.5 and $\beta = \theta = 0.2$.

$$E(N) = r\beta = 0.3, Var(N) = r\beta(1+\beta) = 0.36$$
$$E(Y) = 5{,}000, Var(Y) = 25{,}000{,}000$$
$$E(X) = 0.3(5{,}000) = 1{,}500$$
$$Var(X) = 0.3(25{,}000{,}000) + 0.36(25{,}000{,}000) = 16{,}500{,}000$$

Number of exposures (insureds) required for full credibility
$n_{FULL} = (1.645/0.05)^2(16{,}500{,}000/1{,}500^2) = 7{,}937.67.$

Number of expected claims required for full credibility
$E(N)n_{FULL} = 0.3(7{,}937.67) = 2{,}381.$

Question # 40
Key: E

X	$F_n(x)$	$F_n(x^-)$	$F_0(x)$	$\lvert F_n(x)-F_0(x)\rvert$	$\lvert F_n(x^-)-F_0(x)\rvert$
29	0.2	0	0.252	0.052	0.252
64	0.4	0.2	0.473	0.073	0.273
90	0.6	0.4	0.593	0.007	0.193
135	0.8	0.6	0.741	0.059	0.141
182	1.00	0.8	0.838	0.162	0.038

where:
$\hat{\theta} = \bar{x} - 100$ and $F_0(x) = 1 - e^{-x/100}$.
The maximum value from the last two columns is 0.273.

Question # 41
Key: E

$\mu = E(\lambda) = 1, v = E(\sigma^2) = 1.25, a = Var(\lambda) = 1/12$
$k = v/a = 15, Z = 1/(1+15) = 1/16.$

Thus, the estimate for Year 2 is (1/16)(0) + (15/16)(1) = 0.9375.

Question # 42
DELETED

Question # 43
Key: E

The posterior density, given an observation of 3 is:

$$\pi(\theta|3) = \frac{f(3|\theta)\pi(\theta)}{\int_1^\infty f(3|\theta)\pi(\theta)d\theta} = \frac{\frac{2\theta^2}{(3+\theta)^3}\frac{1}{\theta^2}}{\int_1^\infty 2(3+\theta)^{-3}d\theta} = \frac{2(3+\theta)^{-3}}{-(3+\theta)^{-2}\Big|_1^\infty} = 32(3+\theta)^{-3}, \theta > 1.$$

Then,

$$\Pr(\Theta > 2) = \int_2^\infty 32(3+\theta)^{-3}d\theta = -16(3+\theta)^{-2}\Big|_2^\infty = 16/25 = 0.64.$$

Question # 44
Key: B

$$L = F(1000)^7[F(2000) - F(1000)]^6[1 - F(2000)]^7$$
$$= (1 - e^{-1000/\theta})^7(e^{-1000/\theta} - e^{-2000/\theta})^6(e^{-2000/\theta})^7$$
$$= (1 - p)^7(p - p^2)^6(p^2)^7 = p^{20}(1 - p)^{13}$$

where $p = e^{-1000/\theta}$ The maximum occurs at p = 20/33 and so $\hat{\theta} = -1000\ln(20/33) = 1996.90$.

Question # 45
Key: A

$$E(X \mid \theta) = \theta / 2$$
$$E(X_3 \mid 400, 600) = \int_{600}^{\infty} E(X \mid \theta) f(\theta \mid 400, 600) d\theta = \int_{600}^{\infty} \frac{\theta}{2} 3 \frac{600^3}{\theta^4} d\theta$$
$$= \frac{3(600^3)}{2} \frac{\theta^{-2}}{-2}\Bigg|_{600}^{\infty} = \frac{3(600^3)(600^{-2})}{4} = 450.$$

Question # 46
Key: D

The data may be organized as follows:

T	Y	d	$\hat{S}(t)$
2	10	1	(9/10) = 0.9
3	9	2	0.9(7/9) = 0.7
5	7	1	.07(6/7) = 0.6
6	5	1	0.6(4/5) = 0.48
7	4	1	0.48(3/4) = 0.36
9	2	1	0.36(1/2) = 0.18

Because the product-limit estimate is constant between observations, the value of $\hat{S}(8)$ is found from $\hat{S}(7) = 0.36$.

Question # 47
Key: C

The maximum likelihood estimate for the Poisson distribution is the sample mean:

$$\hat{\lambda} = \bar{x} = \frac{50(0)+122(1)+101(2)+92(3)}{365} = 1.6438.$$

The table for the chi-square test is:

Number of days	Probability	Expected*	Chi-square
0	$e^{-1.6438} = 0.19324$	70.53	5.98
1	$1.6438e^{-1.6438} = 0.31765$	115.94	0.32
2	$\frac{1.6438^2 e^{-1.6438}}{2} = 0.26108$	95.30	0.34
3+	0.22803**	83.23	0.92

*365x(Probability) **obtained by subtracting the other probabilities from 1

The sum of the last column is the test statistic of 7.56. Using 2 degrees of freedom (4 rows less 1 estimated parameter less 1) the model is rejected at the 2.5% significance level but not at the 1% significance level.

Question # 48
Key: D

$$\mu(0) = \frac{0.4(0)+0.1(1)+0.1(2)}{0.6} = 0.5,\ \mu(1) = \frac{0.1(0)+0.2(1)+0.1(2)}{0.4} = 1$$

$$\mu = 0.6(0.5)+0.4(1) = 0.7$$

$$a = 0.6(0.5^2)+0.4(1^2)-0.7^2 = 0.06$$

$$v(0) = \frac{0.4(0)+0.1(1)+0.1(4)}{0.6} - 0.5^2 = 7/12,\ v(1) = \frac{0.1(0)+0.2(2)+0.1(4)}{0.4} - 1^2 = 0.5$$

$$v = 0.6(7/12)+0.4(0.5) = 11/20$$

$$k = v/a = (11/20)/0.06 = 11/1.2 = 55/6,\ Z = \frac{10}{10+55/6} = 60/115 = 12/23$$

Bühlmann credibility premium = $\frac{12}{23}\frac{10}{10} + \frac{11}{23}(0.7) = 0.8565.$.

Question # 49
Key: C

$$\mu = 0.5(0) + 0.3(1) + 0.1(2) + 0.1(3) = 0.8$$
$$\sigma^2 = 0.5(0) + 0.3(1) + 0.1(4) + 0.1(9) - 0.8^2 = 0.96$$
$$E(S_n^2) = \frac{n-1}{n}\sigma^2 = \frac{3}{4}0.96 = 0.72$$
$$bias = 0.72 - 0.96 = -0.24$$

Question # 50
Key: C

The four classes have means 0.1, 0.2, 0.5, and 0.9 respectively and variances 0.09, 0.16, 0.25, and 0.09 respectively.

Then,
$$\mu = 0.25(0.1 + 0.2 + 0.5 + 0.9) = 0.425$$
$$v = 0.25(0.09 + 0.16 + 0.25 + 0.09) = 0.1475$$
$$a = 0.25(0.010.04 + 0.25 + 0.81) - 0.425^2 = 0.096875$$
$$k = 0.1475 / 0.096875 = 1.52258$$
$$Z = \frac{4}{4 + 1.52258} = 0.7243$$

The estimate is 5[0.7243(2/4) + 0.2757(0.425)] = 2.40.

Question # 51
DELETED

Question # 52
Key: A

The distribution used for simulation is given by the observed values.

Question # 53
Key: B

First obtain the distribution of aggregate losses:

Value	Probability
0	1/5
25	(3/5)(1/3) = 1/5
100	(1/5)(2/3)(2/3) = 4/45
150	(3/5)(2/3) = 2/5
250	(1/5)(2)(2/3)(1/3) = 4/45
400	(1/5)(1/3)(1/3) = 1/45

$$\mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105$$

$$\sigma^2 = (1/5)(0^2) + (1/5)(25) + (4/45)(100^2) + (2/5)(150^2) + (4/45)(250^2)$$
$$+(1/45)(400^2) - 105^2 = 8100$$

Question # 54
Key: A

Loss Range	Cum. Prob.
0 – 100	0.320
100 – 200	0.530
200 – 400	0.800
400 – 750	0.960
750 – 1000	0.980
1000 – 1500	1.000

At 400, $F(400) = 0.8 = 1 - e^{-400/\theta}$; solving gives $\theta = 248.53$.

Question # 55
Key: B

$$\Pr(class1\,|\,1) = \frac{(1/2)(1/3)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{3}{4}$$
$$\Pr(class2\,|\,1) = \frac{(1/3)(1/6)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{1}{4}$$
$$\Pr(class3\,|\,1) = \frac{(1/6)(0)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = 0$$

because the prior probabilities for the three classes are 1/2, 1/3, and 1/6 respectively.

The class means are

$$\mu(1) = (1/3)(0)+(1/3)(1)+(1/3)(2) = 1$$
$$\mu(2) = (1/6)(1)+(2/3)(2)+(1/6)(3) = 2$$

The expectation is

$$E(X_2\,|\,1) = (3/4)(1)+(1/4)(2) = 1.25.$$

Question # 56
Key: E

The first, second, third, and sixth payments were observed at their actual value and each contributes $f(x)$ to the likelihood function. The fourth and fifth payments were paid at the policy limit and each contributes $1 - F(x)$ to the likelihood function. This is answer (E).

Question #57
DELETED

Question #58
Key: B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $\mu(\lambda) = \mathrm{v}(\lambda) = \lambda$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$$\mu = v = E(\lambda) = 50(1/500) = 0.1$$
$$a = Var(\lambda) = 50(1/500)^2 = 0.0002$$
$$k = v/a = 500$$
$$Z = 1500/(1500+500) = 0.75$$
$$\overline{x} = (75+210)/(600+900) = 0.19$$

The credibility estimate is 0.75(0.19) + 0.25(0.1) = 0.1675. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter λ times the number of policies)
$\frac{e^{-600\lambda}(600\lambda)^{75}}{75!}\frac{e^{-900\lambda}(900\lambda)^{210}}{210!}\frac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)} \propto \lambda^{105}e^{-2000\lambda}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.

Question #59
Key: E

The *q*-*q* plot takes the ordered values and plots the *j*th point at $j/(n+1)$ on the horizontal axis and at $F(x_j;\theta)$ on the vertical axis. For small values, the model assigns more probability to being below that value than occurred in the sample. This indicates that the model has a heavier left tail than the data. For large values, the model again assigns more probability to being below that value (and so less probability to being above that value). This indicates that the model has a lighter right tail than the data. Of the five answer choices, only E is consistent with these observations. In addition, note that as you go from 0.4 to 0.6 on the horizontal axis (thus looking at the middle 20% of the data), the *q*-*q* plot increases from about 0.3 to 0.4 indicating that the model puts only about 10% of the probability in this range, thus confirming answer E.

Question #60
Key: C

The posterior probability of having one of the coins with a 50% probability of heads is proportional to (0.5)(0.5)(0.5)(0.5)(4/6) = 0.04167. This is obtained by multiplying the probabilities of making the successive observations 1, 1, 0, and 1 with the 50% coin times the prior probability of 4/6 of selecting this coin. The posterior probability for the 25% coin is proportional to (0.25)(0.25)(0.75)(0.25)(1/6) = 0.00195 and the posterior probability for the 75% coin is proportional to (0.75)(0.75)(0.25)(0.75)(1/6) = 0.01758. These three numbers total 0.06120. Dividing by this sum gives the actual posterior probabilities of 0.68088, 0.03186, and 0.28726. The expected value for the fifth toss is then (.68088)(0.5) + (.03186)(0.25) + (.28726)(0.75) = 0.56385.

Question #61
Key: A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,

$$L(\theta) = \frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1-F(100)]^5} = \frac{\theta^{-1}e^{-125/\theta}\theta^{-1}e^{-150/\theta}\theta^{-1}e^{-165/\theta}\theta^{-1}e^{-175/\theta}\theta^{-1}e^{-250/\theta}}{(e^{-100/\theta})^5}$$
$$= \theta^{-5}e^{-365/\theta}.$$

Taking logarithms and then a derivative gives

$$l(\theta) = -5\ln(\theta) - 365/\theta, l'(\theta) = -5/\theta + 365/\theta^2 = 0$$
$$\hat{\theta} = 365/5 = 73.$$

Question #62
Key: D

The number of claims for each insured has a binomial distribution with n = 1 and q unknown. We have

$$\mu(q) = q, v(\theta) = q(1-q)$$
$$\mu = E(q) = 0.1 \text{ (see iv)}$$
$$a = Var(q) = E(q^2) - 0.1^2 = 0.01 \text{ (see v)} \Rightarrow E(q^2) = 0.02$$
$$v = E[q(1-q)] = E(q) - E(q^2) = 0.1 - 0.02 = 0.08$$
$$k = v / a = 8, Z = 10/(10+8) = 5/9$$

Then the expected number of claims in the next one year is (5/9)(0) + (4/9)(0.1) = 2/45 and the expected number of claims in the next five years is 5(2/45) = 2/9 = 0.22.

Question #63
DELETED

Question #64
Key: E

The model distribution is $f(x|\theta) = 1/\theta, 0 < x < \theta$. Then the posterior distribution is proportional to

$$\pi(\theta | 400, 600) \propto \frac{1}{\theta}\frac{1}{\theta}\frac{500}{\theta^2} \propto \theta^{-4}, \theta > 600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible parameter values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} \theta^{-4} d\theta = \frac{1}{3(600)^3}$ and thus the exact posterior density is $\pi(\theta | 400, 600) = 3(600)^3 \theta^{-4}, \theta > 600$. The posterior probability of an observation exceeding 550 is

$$\Pr(X_3 > 550 | 400, 600) = \int_{600}^{\infty} \Pr(X_3 > 550 | \theta)\pi(\theta | 400, 600) d\theta = \int_{600}^{\infty} \frac{\theta - 550}{\theta} 3(600)^3 \theta^{-4} d\theta = 0.3125.$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.

Question #65
Key: C

$$E(N) = r\beta = 0.4$$
$$Var(N) = r\beta(1+\beta) = 0.48$$
$$E(Y) = \theta / (\alpha - 1) = 500$$
$$Var(Y) = \frac{\theta^2 \alpha}{(\alpha-1)^2(\alpha-2)} = 750,000$$

Therefore,
$$E(X) = 0.4(500) = 200$$
$$Var(X) = 0.4(750,000) + 0.48(500)^2 = 420,000.$$

The full credibility standard is $n = \left(\frac{1.645}{0.05}\right)^2 \frac{420,000}{200^2} = 11,365$, $\mathrm{Z} = \sqrt{2,500/11,365} = 0.47.$

Question #66
Key: E

The sample variance is $s^2 = \frac{(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2}{4} = 2.5.$ The estimator of $E[X]$ is the sample mean and the variance of the sample mean is the variance divided by the sample size, estimated here as $2.5/n$. Setting the standard deviation of the estimator equal to 0.05 gives the equation $\sqrt{2.5/n} = 0.05$ which yields $n = 1000$.

Question #67
Key: E

$$\mu(r) = E(X \mid r) = E(N)E(Y) = r\beta\theta / (\alpha - 1) = 100r$$
$$v(r) = Var(X \mid r) = Var(N)E(Y)^2 + E(N)Var(Y)$$
$$= \frac{r\beta(1+\beta)\theta^2}{(\alpha-1)^2} + \frac{r\beta\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = 210,000r$$
$$v = E(210,000r) = 210,000(2) = 420,000$$
$$a = Var(100r) = (100)^2(4) = 40,000$$
$$k = v / a = 10.5,\ Z = 100/(100+10.5) = 0.905$$

Question #68
Key: B

Using all participants, $S^T(4)=\left(1-\frac{35}{300}\right)\left(1-\frac{74}{265}\right)\left(1-\frac{34}{191}\right)\left(1-\frac{32}{157}\right)=0.41667.$

Using only Country B, $S^B(4)=\left(1-\frac{15}{100}\right)\left(1-\frac{20}{85}\right)\left(1-\frac{20}{65}\right)\left(1-\frac{10}{45}\right)=0.35.$

The difference is, $S^T(4)-S^B(4)=0.41667-0.35=0.0667.$

Question #69
Key: B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have

$$E(\bar{X})=E(X)=\theta, Var(\bar{X})=Var(X)/n=\theta^2/n$$

$$cv=SD(\bar{X})/E(\bar{X})=(\theta/\sqrt{n})/\theta=1/\sqrt{n}=1-\sqrt{5}=0.447.$$

If the maximum likelihood estimator is not known, it can be derived:

$$L(\theta)=\theta^{-n}e^{-\Sigma x/\theta}, l(\theta)=-n\ln\theta-n\bar{X}/\theta, l'(\theta)=-n\theta^{-1}+n\bar{X}\theta^{-2}=0\Rightarrow\hat{\theta}=\bar{X}.$$

Question #70
Key: D

Because the total expected claims for business use is 1.8, it must be that 20% of business users are rural and 80% are urban. Thus the unconditional probabilities of being business-rural and business-urban are 0.1 and 0.4 respectively. Similarly the probabilities of being pleasure-rural and pleasure-urban are also 0.1 and 0.4 respectively. Then,

$$\mu=0.1(1.0)+0.4(2.0)+0.1(1.5)+0.4(2.5)=2.05$$

$$v=0.1(0.5)+0.4(1.0)+0.1(0.8)+0.4(1.0)=0.93$$

$$a=0.1(1.0)^2+0.4(2.0)^2+0.1(1.5)^2+0.4(2.5)^2-2.05^2=0.2225$$

$$k=v/a=4.18, Z=1/(1+4.18)=0.193.$$

Question #71
Key: A

No. claims	Hypothesized	Observed	Chi-square
1	250	235	15(15)/250 = 0.90
2	350	335	15(15)/350 = 0.64
3	240	250	10(10)/240 = 0.42
4	110	111	1(1)/110 = 0.01
5	40	47	7(7)/40 = 1.23
6+	10	22	12(12)/10 = 14.40

The last column sums to the test statistic of 17.60 with 5 degrees of freedom (there were no estimated parameters), so from the table reject at the 0.005 significance level.

Question #72
Key: C

In part (ii) you are given that $\mu = 20$. In part (iii) you are given that a = 40. In part (iv) you are given that v = 8,000. Therefore, $k = v/a = 200$. Then,

$$\bar{X} = \frac{800(15) + 600(10) + 400(5)}{1800} = \frac{100}{9}$$

$$Z = \frac{1800}{1800 + 200} = 0.9$$

$$P_C = 0.9(100/9) + 0.1(20) = 12.$$

Question #73
Key: C

j	P_j	n_j	d_j	w_j	e_j
0	10	0	1	2	10 – 3/2 = 8.5
1	7	0	1	2	7 – 3/2 = 5.5
2	4	0	4	0	4 – 4/2 = 2.0

$$\Pr(X > 30{,}000) = p_0 p_1 = e^{-1/8.5} e^{-1/5.5} = 0.741$$

Question #74
Key: C

j	P_j	n_j	d_j	w_j	e_j
0	10	0	1	2	10 – 2/2 = 9.0
1	7	0	1	2	7 – 2/2 = 6.0
2	4	0	4	0	4 – 0/2 = 4.0

$$\Pr(X > 30,000) = p_0 p_1 = \frac{9-1}{9}\frac{6-1}{6} = \frac{20}{27} = 0.741.$$

Question #74
Original version DELETED

Question #75
Key: D

$$E(X) = \int_\delta^\infty \frac{x}{\theta} e^{-(x-\delta)/\theta} d\theta = \int_0^\infty \frac{y+\delta}{\theta} e^{-y/\theta} dy = \theta + \delta$$

$$E(X^2) = \int_\delta^\infty \frac{x^2}{\theta} e^{-(x-\delta)/\theta} d\theta = \int_0^\infty \frac{y^2 + 2y\delta + \delta^2}{\theta} e^{-y/\theta} dy = 2\theta^2 + 2\theta\delta + \delta^2$$

Both derivations use the substitution $y = x - \delta$ and then recognize that the various integrals are requesting moments from an ordinary exponential distribution. The method of moments solves the two equations

$$\theta + \delta = 10$$

$$2\theta^2 + 2\theta\delta + \delta^2 = 130.6$$

$$\hat{\delta} = 4.468.$$

It is faster to do the problem if it is noted that $X = Y + \delta$ where Y has an ordinary exponential distribution. Then $E(X) = E(Y) + \delta = \theta + \delta, Var(X) = Var(Y) = \theta^2.$

Question #76
Key: D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $\pi(\theta \mid N>0) \propto \Pr(N>0 \mid \theta)\pi(\theta) = (1-e^{-\theta})\theta e^{-\theta}$.

The constant of proportionality is obtained from $\int_0^\infty \theta e^{-\theta} - \theta e^{-2\theta} d\theta = \frac{1}{1^2} - \frac{1}{2^2} = 0.75.$
The posterior density is $\pi(\theta \mid N>0) = (1/0.75)(\theta e^{-\theta} - \theta e^{-2\theta}).$

Then,

$$\Pr(N_2>0 \mid N_1>0) = \int_0^\infty \Pr(N_2>0 \mid \theta)\pi(\theta \mid N_1>0)d\theta = \int_0^\infty (1-e^{-\theta})(4/3)(\theta e^{-\theta} - \theta e^{-2\theta})d\theta$$

$$= \frac{4}{3}\int_0^\infty \theta e^{-\theta} - 2\theta e^{-2\theta} + \theta e^{-3\theta} d\theta = \frac{4}{3}\left(\frac{1}{1^2} + \frac{2}{2^2} + \frac{1}{3^2}\right) = 0.8148.$$

Question #77
Key: E

The interval is centered at 2.09 and the plus/minus term is 0.46 which must equal $1.96\hat{\sigma}$ and so $\hat{\sigma} = 0.2347$. For the log-transformed interval we need $\phi = e^{1.96(0.2347)/2.09} = 1.2462$. The lower limit is 2.09/1.2462 = 1.68 and the upper limit is 2.09(1.2462) = 2.60.

Question #78
Key: B

From item (ii), $\mu = 1000$ and $a = 50$. From item (i), $v = 500$. Therefore, $k = v/a = 10$ and $Z = 3/(3+10) = 3/13$. Also, $\bar{X} = (750+1075+2000)/3 = 1275$. Then $P_c = (3/13)(1275) + (10/13)(1000) = 1063.46.$

Question #79
Key: C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10,000}e^{-x/10,000}$$

$$L(100,200) = f(100)f(2000)$$

$$= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$$

Question #80
DELETED

Question # 81
Key: C

Because 0.25 < 0.30, the observation, *Y*, is from the exponential distribution. Then,

$$0.69 = F(y) = 1 - e^{-y/0.5}$$

$$-y / 0.5 = \ln(1 - 0.31) = -1.171$$

$$y = 0.5855.$$

Question #82
Key: B

For the simulated value of *N*, note that $f(1) = F(1) = 0.9$ and because 0.05 < 0.9, $n = 1$. For the single value of *x* and recognizing that it has an exponential distribution,

$$0.30 = 1 - e^{-0.01x}$$

$$-0.01x = \ln(1 - 0.30) = -0.3567$$

$$x - 35.67.$$

The amount of total claims during the year is 35.67.

Question #83
Key: B

$F(0) = 0.8$
$F(t) = 0.8 + 0.00005(t - 1000)$, $1000 \le t \le 5000$
The simulated value of 0.75 is less than 0.8 and so the outcome is 0.
For the second value,

$$0.85 = 0.8 + 0.00005(t - 1000)$$

$$0.05 = 0.00005(t - 1000)$$

$$1000 = t - 1000$$

$$t = 2000$$

Average of those two outcomes is 1000.

Question #84
Key: A

$$B = \begin{cases} c(400-x) & x < 400 \\ 0 & x \geq 400 \end{cases}$$

$$100 = E(B) = c400 - cE(X \wedge 400)$$
$$= c400 - c300\left(1 - \frac{300}{300+400}\right)$$
$$= c\left(400 - 300\frac{4}{7}\right)$$
$$c = \frac{100}{228.6} = 0.44$$

Question #85
Key: C

Let N = number of computers in department
Let X = cost of a maintenance call
Let S = aggregate cost

$$\text{Var}(X) = [\text{Standard Deviation}(X)]^2 = 200^2 = 40{,}000$$
$$E(X^2) = \text{Var}(X) + [E(X)]^2$$
$$= 40{,}000 + 80^2 = 46{,}400$$
$$E(S) = N\lambda E(X) = N(3)(80) = 240N$$
$$\text{Var}(S) = N\lambda \times E(X^2) = N(3)(46{,}400) = 139{,}200N$$

We want

$$0.1 \geq \Pr\left(S > 1.2E(S)\right)$$
$$\geq \Pr\left(\frac{S-E(S)}{\sqrt{139{,}200N}} > \frac{0.2E(S)}{\sqrt{139{,}200N}}\right) \Rightarrow \frac{0.2(240)N}{373.1\sqrt{N}} \geq 1.282 = \Phi(0.9)$$
$$N \geq \left(\frac{1.282(373.1)}{48}\right)^2 = 99.3$$

Question #86
Key: D

The modified severity, X^*, represents the conditional payment amount given that a payment occurs. Given that a payment is required ($X > d$), the payment must be uniformly distributed between 0 and $c(b - d)$.

The modified frequency, N^*, represents the number of losses that result in a payment. The deductible eliminates payments for losses below d, so only $1-F_X(d)=\frac{b-d}{b}$ of losses will require payments. Therefore, the Poisson parameter for the modified frequency distribution is $\lambda\frac{b-d}{b}$. (Reimbursing c% after the deductible affects only the payment amount and not the frequency of payments).

Question #87
Key: E

$$\begin{aligned} f(x) &= 0.01, \quad 0 \le x \le 80 \\ &= 0.01 - 0.00025(x-80) = 0.03 - 0.00025x, \quad 80 < x \le 120 \end{aligned}$$

$$\begin{aligned} E(x) &= \int_0^{80} 0.01x\,dx + \int_{80}^{120}(0.03x - 0.00025x^2)dx \\ &= \frac{0.01x^2}{2}\Big|_0^{80} + \frac{0.03x^2}{2}\Big|_{80}^{120} - \frac{0.00025x^3}{3}\Big|_{80}^{120} \\ &= 32 + 120 - 101.33 = 50.66667 \end{aligned}$$

$$\begin{aligned} E(X-20)_+ &= E(X) - \int_0^{20} x f(x)dx - 20\left[1 - \int_0^{20} f(x)dx\right] \\ &= 50.6667 - \frac{0.01x^2}{2}\Big|_0^{20} - 20\left(1 - 0.01x\Big|_0^{20}\right) \\ &= 50.6667 - 2 - 20(0.8) = 32.6667 \end{aligned}$$

Loss Elimination Ratio = $1-\frac{32.6667}{50.6667}=0.3553$

Question #88
Key: B

First restate the table to be CAC's cost, after the 10% payment by the auto owner:

Towing Cost, x	$p(x)$
72	50%
90	40%
144	10%

Then $E(X) = 0.5(72) + 0.4(90) + 0.1(144) = 86.4$.

$$E(X^2) = 0.5(72^2) + 0.4(90^2) + 0.1(144^2) = 7905.6$$

$$\text{Var}(X) = 7905.6 - 86.4^2 = 440.64$$

Because Poisson,

$$E(N) = Var(N) = 1000$$

$$E(S) = E(X)E(N) = 86.4(1000) = 86,400$$

$$\text{Var}(S) = E(N)\text{Var}(X) + E(X)^2\text{Var}(N) = 1000(440.64) + 86.4^2(1000) = 7,905,600$$

$$\Pr(S > 90,000) + \Pr\left(\frac{S - E(S)}{\sqrt{\text{Var}(S)}} > \frac{90,000 - 86,400}{\sqrt{7,905,600}}\right) = \Pr(Z > 1.28) = 1 - \Phi(1.28) = 0.10$$

Since the frequency is Poisson, you could also have used

$$\text{Var}(S) = \lambda E(X^2) = 1000(7905.6) = 7,905,600.$$

That way, you would not need to have calculated Var(X).

Question #89
Key: C

$$\text{LER} = \frac{E(X \wedge d)}{E(X)} = \frac{\theta(1 - e^{-d/\theta})}{\theta} = 1 - e^{-d/\theta}$$

Last year $\quad 0.70 = 1 - e^{-d/\theta} \Rightarrow -d = \theta \log(0.30)$

Next year: $\quad -d_{new} = \theta \log(1 - \text{LER}_{new})$

Hence $\theta \log(1 - \text{LER}_{\text{new}}) = -d_{\text{new}} = \frac{4}{3}\theta \log(0.30)$

$$\log\left(1 - \text{LER}_{new}\right) = -1.6053$$

$$\left(1 - \text{LER}_{new}\right) = e^{-1.6053} = 0.20$$

$$\text{LER}_{new} = 0.80$$

Question # 90
Key: E

The distribution of claims (a gamma mixture of Poissons) is negative binomial.

$$E(N) = E_\Lambda[E(N \mid \Lambda)] = E_\Lambda(\Lambda) = 3$$
$$Var(N) = E_\Lambda[Var(N \mid \Lambda)] + Var_\Lambda[E(N \mid \Lambda)] = E_\Lambda(\Lambda) + Var_\Lambda(\Lambda) = 6$$
$$r\beta = 3$$
$$r\beta(1+\beta) = 6$$
$$(1+\beta) = 6/3 = 2;\ \beta = 1$$
$$r\beta = 3;\ r = 3$$

$$p_0 = (1+\beta)^{-r} = 0.125$$
$$p_1 = \frac{r\beta}{(1+\beta)^{r+1}} = 0.1875$$
$$\Pr(\text{at most } 1) = p_0 + p_1 = 0.3125.$$

Question # 91
Key: A

$$E(S) = E(N)E(X) = 110(1{,}101) = 121{,}110$$
$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 110(70^2) + 1101^2(750) = 909{,}689{,}750$$
$$StdDev(S) = 30{,}161$$
$$\Pr(S < 100{,}000) = \Pr\left(Z < \frac{100{,}000 - 121{,}110}{30{,}161} = -0.70\right) = 0.242$$

where Z has standard normal distribution.

Question # 92
Key: C

Let N = number of prescriptions then

n	$f_N(n)$	$F_N(n)$	$1-F_N(n)$
0	0.2000	0.2000	0.8000
1	0.1600	0.3600	0.6400
2	0.1280	0.4880	0.5120
3	0.1024	0.5904	0.4096

$$E(N) = 4 = \sum_{j=0}^{\infty}[1-F(j)]$$

$$E[(S-80)_+] = 40E[(N-2)_+] = 40\sum_{j=2}^{\infty}[1-F(j)]$$

$$= 40\left[\sum_{j=0}^{\infty}[1-F(j)] - \sum_{j=0}^{1}[1-F(j)]\right]$$

$$= 40(4-1.44) = 102.40$$

$$E[(S-120)_+] = 40E[(N-3)_+] = 40\left[\sum_{j=0}^{\infty}[1-F(j)] - \sum_{j=0}^{2}[1-F(j)]\right] = 40(4-1.952) = 81.92$$

Because no values of S between 80 and 120 are possible,

$$E[(S-100)_+] = \frac{(120-100)E[(S-80)_+] + (100-80)E[(S-120)_+]}{120-80} = 92.16$$

Alternatively,

$$E[(S-100)_+] = \sum_{j=0}^{\infty}(40j-100)f_N(j) + 100f_N(0) + 60f_N(1) + 20f_N(2)$$

(The correction terms are needed because $(40j-100)$ would be negative for $j = 0, 1, 2$; we need to add back the amount those terms would be negative)

$$= 40\sum_{j=0}^{\infty} jf_N(j) - 100\sum_{j=0}^{\infty} f_N(j) + 100(0.2) + 60(0.16) + 20(0.128)$$

$$= 40E(N) - 100 + 20 + 9.6 + 2.56 = 160 - 67.84 = 92.16$$

Question #93
Key: E

Method 1:

In each round,
N = result of first roll, to see how many dice you will roll
X = result of for one of the N dice you roll
S = sum of X for the N dice

$$E(X) = E(N) = 3.5$$
$$Var(X) = Var(N) = 2.9167$$
$$E(S) = E(N)E(X) = 12.25$$
$$Var(S) = E(N)Var(X) + Var(N)E(X)^2 = 3.5(2.9167) + 2.9167(3.5)^2 = 45.938$$

Let S_{1000} the sum of the winnings after 1000 rounds

$$E(S_{1000}) = 1000(12.25) = 12,250$$
$$SD(S_{1000}) = \sqrt{1000(45.938)} = 214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings for a total of $2,500 + S_{1000}$ Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$\Pr(2,500 + S_{1000} > 14,999.5) = \Pr(S_{1000} > 12,499.5) \approx \Pr\left(Z > \frac{12,499.5 - 12,250}{214.33} = 1.17\right) = 0.12.$$

Question #94
Key: B

$$p_k = \left(a + \frac{b}{k}\right) p_{k-1}$$
$$0.25 = (a+b)0.25 \Rightarrow a = 1 - b$$
$$0.1875 = \left(a + \frac{b}{2}\right)(0.25) \Rightarrow 0.1875 = (1 - 0.5b)(0.25) \Rightarrow b = 0.5,\ a = 0.5$$
$$p_3 = \left(0.5 + \frac{0.5}{3}\right)(0.1875) = 0.125$$

Question #95
Key: E

$\beta = \text{mean} = 4,\ p_k = \beta^k / (1+\beta)^{k+1}$

n	$P(N=n)$
0	0.2
1	0.16
2	0.128
3	0.1024

x	$f^{(1)}(x)$	$f^{(2)}(x)$	$f^{(3)}(x)$
0	0	0	0
1	0.25	0	0
2	0.25	0.0625	0
3	0.25	0.125	0.0156

$f_S(0) = 0.2,\ f_S(1) = 0.16(0.25) = 0.04$
$f_S(2) = 0.16(0.25) + 0.128(0.0625) = 0.049$
$f_S(3) = 0.16(0.25) + 0.128(0.125) + 0.1024(0.0156) = 0.0576$
$F_S(3) = 0.2 + 0.04 + 0.049 + 0.0576 = 0.346$

Question #96
Key: E

Let L = incurred losses; P = earned premium = 800,000
Bonus $0.15\left(0.6 - \frac{L}{P}\right)P = 0.15(0.6P - L) = 0.15(480{,}000 - L)$ if positive.
This can be written $0.15[480{,}000 - (L \wedge 480{,}000)]$. Then,
$E(\text{Bonus}) = 0.15[480{,}000 - E(L \wedge 480{,}000)]$
From Appendix A.2.3.1
E(Bonus)= 0.15{480,000 – [500,000 (1 – (500,000 / (480,000+500,000))]} = 35,265

Question # 97
Key: D

Severity after increase	Severity after increase and deductible
60	0
120	20
180	80
300	200

Expected payment per loss = 0.25(0) +0.25(20) + 0.25(80) + 0.25(200) = 75

Expected payments = Expected number of losses x Expected payment per loss
= 300(75) = 22,500

Question # 98
Key: A

E(S) = E(N)E(X) = 50(200) = 10,000

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 50(400) + 200^2(100) = 4,020,000$

$$\Pr(S < 8,000) \approx \Pr\left(Z < \frac{8,000 - 10,000}{\sqrt{4,020,000}} = -0.998\right) = 0.16$$

Question #99
Key: B

Let S denote aggregate loss before deductible.

E(S) = 2(2) = 4, since mean severity is 2.

$f_S(0) = \dfrac{e^{-2}2^0}{0!} = 0.1353$, since must have 0 losses to get aggregate loss = 0.

$f_S(1) = \dfrac{e^{-2}2^1}{1!}\dfrac{1}{3} = 0.0902$, since must have 1 loss whose size is 1 to get aggregate loss = 1.

$$E(S \wedge 2) = 0 f_S(0) + 1 f_S(1) + 2[1 - f_S(0) - f_S(1)]$$
$$= 0(0.1353) + 1(0.0902) + 2(1 - 0.01353 - 0.0902) = 1.6392$$
$$E[(S-2)_+] = E(S) - E(S \wedge 2) = 4 - 1.6392 = 2.3608$$

Question #100
Key: C

Limited expected value =

$$\int_0^{1000}[1 - F(x)]dx = \int_0^{1000} 0.8e^{-0.02x} + 0.2e^{-0.001x}dx = -40e^{-0.02x} - 200e^{-0.001x}\Big|_0^{1000}$$
$$= -0 - 73.576 + 40 + 200 = 166.424$$

Question #101
Key: B

Mean excess loss = $\dfrac{E(X) - E(X \wedge 100)}{1 - F(100)} = \dfrac{331 - 91}{0.8} = 300$

Note that $E(X) = E(X \wedge 1000)$ because F(1000) = 1.

Question #102
Key: E

Expected insurance benefits per factory = $E[(X-1)_+] = 0.2(1) + 0.1(2) = 0.4$

Insurance premium = (1.1)(2 factories (0.4 per factory) = 0.88.

Let *R* = retained major repair costs, then

$f_R(0) = 0.4^2 = 0.16,\ f_R(1) = 2(0.4)(0.6) = 0.48,\ f_S(2) = 0.6^2 = 0.36$

Dividend = 3 – 0.88 – *R* – 0.15(3) = 1.67 – *R*, if positive.

Expected Dividend = 0.16(1.67 – 0) + (0.48)(1.67 – 1) + 0.36(0) = 0.5888

Question #103
DELETED

Question: #104
Key: C

$E(N) = E[E(N \mid \Lambda)] = E(\Lambda) = 2$

$Var(N) = E[Var(N \mid \Lambda)] + Var[E(N \mid \Lambda)] = E(\Lambda) + Var(\Lambda) = 2 + 2 = 4$

The equations to solve are

$\text{mean} = 2 = r\beta$

$\text{variance} = 4 - r\beta(1+\beta)$

This implies

$1 + \beta = 2 \Rightarrow \beta = 1 \Rightarrow r(1) = 2 \Rightarrow r = 2$

Then $1000 p_3 = 1000 \dfrac{r(r+1)(r+2)\beta^3}{3!(1+\beta)^{r+3}} = 1000 \dfrac{2(3)(4)(1^3)}{6(2^5)} = 125$

Question: #105
Key: A

Using the conditional mean and variance formulas and that N has a conditional Poisson distribution:

$$0.2 = E(N) = E[E(N \mid \Lambda)] = E(\Lambda)$$
$$0.4 = Var(N) = E[Var(N \mid \Lambda)] + Var[E(N \mid \Lambda)] = E(\Lambda) + Var(\Lambda) = 0.2 + Var(\Lambda)$$
$$Var(\Lambda) = 0.4 - 0.2 = 0.2$$

Question: #106
Key: B

N = number of salmon in t hours
X = eggs from one salmon
S = total eggs.
$E(N) = 100t$
$Var(N) = 900t$

$$E(S) = E(N)\,\mathrm{E(X)} = (100\,\mathrm{t})(5) = 500\,\mathrm{t}$$
$$\mathrm{Var(S)} = \mathrm{E(N)\,Var(X)} + \mathrm{E(X)}^2 Var(N) = (100t)(5) + (5^2)(900t) = 23{,}000t$$
$$0.95 < \Pr(S > 10{,}000) = \Pr\left(Z > \frac{10{,}000 - 500t}{\sqrt{23{,}000t}} \right) \Rightarrow \frac{10{,}000 - 500t}{\sqrt{23{,}000t}} = -1.645$$
$$10{,}000 - 500t = -1.645(151.66)\sqrt{t} = -249.48\sqrt{t}$$
$$500t - 249.48\sqrt{t} - 10{,}000 = 0$$
$$\sqrt{t} = \frac{249.48 \pm \sqrt{(-249.48)^2 - 4(500)(-10{,}000)}}{2(500)} = 4.73$$
$$t = 22.26$$
Round up to 23

Question: #107
Key: C

X = losses on one life
$E(X) = 0.3(1) + 0.2(2) + 0.1(3) = 1$
S = total losses
$E(S) = 3E(X) = 3(1) = 3$
$$E[(S-1)_+] = E(S) - 1[1 - F_S(0)] = 3 - 1[1 - f_S(0)] = 3 - (1 - 0.4^3) = 2.064$$
Alternatively, the expected retained payment is $0 f_S(0) + 1[1 - f_S(0)] = 0.936$ and the stop-loss premium is 3 – 0.936 = 2.064.

Question: #108
Key: C

$$p(k) = \frac{2}{k} p(k-1) = \left(0 + \frac{2}{k}\right) p(k-1)$$

Thus an (a, b, 0) distribution with a = 0, b = 2.

Thus Poisson with $\lambda = 2$.

$$p(4) = \frac{e^{-2} 2^4}{4!} = 0.09$$

Question: #109
Key: B

By the memoryless property, the distribution of amounts paid in excess of 100 is still exponential with mean 200.

With the deductible, the probability that the amount paid is 0 is $F(100) = 1 - e^{-100/200} = 0.393$.

Thus the average amount paid per loss is (0.393)(0) + (0.607)(200) = 121.4
The expected number of losses is (20)(0.8) = 16.
The expected amount paid is (16)(121.4) = 1942.

Question: #110
Key: E

$$E(N) = 0.8(1) + 0.2(2) = 1.2$$
$$E(N^2) = 0.8(1) + 0.2(4) = 1.6$$
$$Var(N) = 1.6 - 1.2^2 = 0.16$$
$$E(X) = 0.2(0) + 0.7(100) + 0.1(1000) = 170$$
$$E(X^2) = 0.2(0) + 0.7(10,000) + 0.1(1,000,000) = 107,000$$
$$Var(X) = 107,000 - 170^2 = 78,100$$
$$E(S) = E(N)E(X) = 1.2(170) = 204$$
$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 1.2(78,100) + 170^2(0.16) = 98,344$$
$$SD(S) = \sqrt{98,344} = 313.6$$

So Budget = 204 + 314 = 518

Question: #111
Key: E

For a compound Poisson distribution, $Var(S) = E(N)E(X^2)$.

$E(X^2) = (1/2)(1) + (1/3)(4) + (1/6)(9) = 10/3$

$Var(S) = 12(10/3) = 40$

Alternatively, the total is the sum $N_1 + 2N_2 + 3N_3$ where the *N*s are independent Poisson variables with means 6, 4, and 2. The variance is 6 + 4(4) + 9(2) = 40.

Question: #112
Key: A

N = number of physicians	*E*(*N*) = 3	Var (*N*) = 2
X = visits per physician	E(*X*) = 30	Var (*X*) = 30
S = total visits		

$E(S) = E(N)E(X) = 3(30) = 90$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 3(30) + 900(2) = 1890$

$SD(S) = \sqrt{1890} = 43.5$

$$\Pr(S > 119.5) = \Pr\left(Z > \frac{119.5 - 90}{43.5} = 0.68\right) = 1 - \Phi(0.68)$$

Question: #113
Key: E

$E(N) = 0.7(0) + 0.2(2) + 0.1(3) = 0.7$

$Var(N) = 0.7(0) + 0.2(4) + 0.1(9) - 0.7^2 = 1.21$

$E(X) = 0.8(0) + 0.2(10) = 2$

$Var(X) = 0.8(0) + 0.2(100) - 2^2 = 16$

$E(S) = E(N)E(X) = 0.7(2) = 1.4$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 0.7(16) + 4(1.21) = 16.04$

$SD(S) = \sqrt{16.04} = 4$

$\Pr(S > 1.4 + 2(4) = 9.4) = 1 - \Pr(S = 0) = 1 - 0.7 - 0.2(0.8)^2 - 0.1(0.8)^3 = 0.12$

The last line follows because there are no possible values for *S* between 0 and 10. A value of 0 can be obtained three ways: no claims, two claims both for 0, three claims all for 0.

Question: #114
Key: A

$$P(0) = \frac{1}{5}\int_0^5 e^{-\lambda} d\lambda = \frac{1}{5}(-e^{-\lambda})\Big|_0^5 = \frac{1}{5}(1-e^{-5}) = 0.1987$$

$$P(1) = \frac{1}{5}\int_0^5 \lambda e^{-\lambda} d\lambda = \frac{1}{5}(-\lambda e^{-\lambda} - \mathrm{e}^{-\lambda})\Big|_0^5 = \frac{1}{5}(1-6e^{-5}) = 0.1919$$

$$P(N \geq 2) = 1 - 0.1987 - 0.1919 = 0.6094$$

Question: #115
Key: D

Let X be the occurrence amount and Y = max(X – 100, 0) be the amount paid.

$E(X) = 1000, Var(X) = 1000^2$, $\Pr(X > 100) = \mathrm{e}^{-100/1000} = 0.904837$

The distribution of Y given that $X > 100$, is also exponential with mean 1,000 (memoryless property). So Y is 0 with probability 1 – 0.904837 = 0.095163 and is exponential with mean 1000 with probability 0.904837.

$$E(Y) = 0.095163(0) + 0.904837(1000) = 904.837$$

$$E(Y^2) = 0.095163(0) + 0.904837(2)(1000^2) = 1{,}809{,}675$$

$$Var(Y) = 1{,}809{,}675 - 904.837^2 = 990{,}944$$

Question: #116
Key: C

Expected claims under current distribution = 500/(2 – 1) = 500
K= parameter of new distribution
X = claims

$$E(X) = K/(2-1) = K$$

$$E(\text{claims} + \text{bonus}) = E\{X + 0.5[500 - (X \wedge 500)]\} = K + 0.5\left[500 - K\left(1 - \frac{K}{500+K}\right)\right] = 500$$

$$(K + 250 - 0.5K)(500 + K) + 0.5K^2 = 500(500 + K)$$

$$0.5K^2 + 500K + 125{,}000 + 0.5K^2 = 250{,}000 + 500K$$

$$K^2 = 125{,}000$$

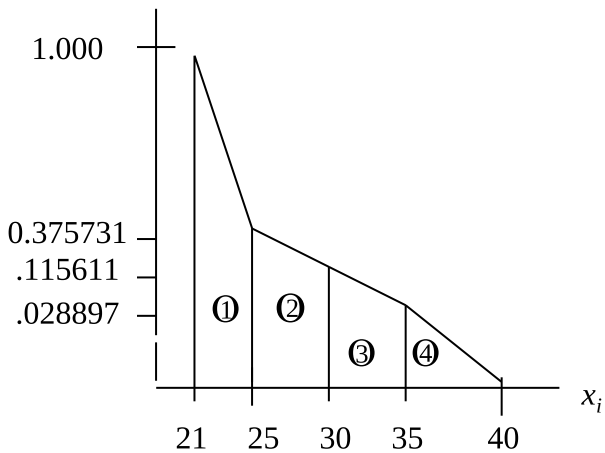
$$K = \sqrt{125{,}000} = 354$$

Question: #117
Key: A

$$UDD \Rightarrow l_{21} = 0.8(53,488) + 0.2(17,384) = 46,267.2$$

$$MRL(21) = \int_0^\infty \frac{S(21+t)}{S(21)} dt = \sum \text{areas} = 2.751 + 1.228 + 0.361 + 0.072 = 4.412$$

$S(X_i) \mid S(21)$



Question: #118
Key: D

$$E(N) = 25$$

$$Var(N) = 25$$

$$E(X) = (5+95)/2 = 50$$

$$Var(X) = (95-5)^2 / 12 = 675$$

$$E(S) = E(N)E(X) = 25(50) = 1250$$

$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 25(675) + 50^2(25) = 79,375$$

$$SD(S) = \sqrt{79,375} = 281.74$$

$$P(S > 2000) = P\left(Z > \frac{2000-1250}{281.74} = 2.66 \right) = 1 - \Phi(2.66)$$

Question: #119
DELETED

$$E(X) = 2000(1!)/(1!) = 2000$$

$$E(X \wedge 3000) = \frac{2000}{1}\left(1 - \frac{2000}{3000+2000}\right) = 1200$$

So the fraction of the losses expected to be covered by the reinsurance
is (2000 – 1200)/2000 = 0.4.
The expected ceded losses are 4,000,000 the ceded premium is 4,400,000.

Question: #120
Key: E

For 2001:

$$E(X) = 2000/(2-1) = 2000$$

$$E(X \wedge 3000) = \frac{2000}{1}\left(1 - \frac{2000}{3000+2000}\right) = 1200$$

So the fraction of the losses expected to be covered by the reinsurance
is (2000 – 1200)/2000 = 0.4. The expected ceded losses are 4,000,000 and the ceded premium is 4,400,000.

For 2002:
Inflation changes the scale parameter, here to 2000(1.05) = 2100. The revised calculations are

$$E(X) = 2100/(2-1) = 2100$$

$$E(X \wedge 3000) = \frac{2100}{1}\left(1 - \frac{2100}{3000+2100}\right) = 1235$$

The revised premium is 1.1(10,500,000)(2100 – 1235)/2100 = 4,757,500.
The ratio is 4,757,500/4,400,000 = 1.08.

Question #121
DELETED

Question #122
Key: C

$E(N) = mq = 1.8 \Rightarrow q = 1/8/3 = 0.6$

x	$f_N(x)$	$F_N(x)$
0	0.064	0.064
1	0.288	0.352
2	0.432	0.784
3	0.216	1.000

First: 0.352 < 0.7 < 0.784 so N = 2. Use 0.1 and 0.3 for amounts
Second: 0.064 < 0.1 < 0.352 so N = 1. Use 0.9 for amount
Third: 0.352 < 0.5 < 0.784 so N = 2. Use 0.5 and 0.7 for amounts

For the discrete uniform, a random number between 0 and 0.2 simulates a 1, 0.2 to 0.4 a 2 and so on. Because 0.5 is between 0.4 and 0.6 the simulated amount is 3. Because 0.7 is between 0.6 and 0.8 the simulated amount is 4. The total is 7.

Question #123
Key: C

$$E\left(X \wedge x\right) = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right] = \frac{2000}{1}\left[1 - \frac{2000}{x+2000}\right] = \frac{2000x}{x+2000}$$

x	$E(X \wedge x)$
∞	2000
250	222
2250	1059
5100	1437

$$0.75[E(X \wedge 2250) - E(X \wedge 250)] + 0.95\left[E(X) - E(X \wedge 5100)\right]$$
$$0.75(1059 - 222) + 0.95(2000 - 1437) = 1162.6$$

The 5100 breakpoint was determined by when the insured’s share reaches 3600:

3600 = 250 + 0.25 (2250 – 250) + (5100 – 2250)

Question #124
DELETED

Question #125
Key: A

N_I, N_{II} denote the random variables for # of claims for Types I and II in one year
X_I, X_{II} denote the claim amount random variables for Types I and II
S_I, S_{II} denote the total claim amount random variables for Types I and II
$S = S_I + S_{II}$
$E(N_I) = Var(N_I) = 12,\ E(N_{II}) = Var(N_{II}) = 4$
$E(X_I) = (0+1)/2 = 1/2, Var(X_I) = (1-0)^2/12 = 1/12$
$E(X_{II}) = (0+5)/2 = 5/2, Var(X_{II}) = (5-0)^2/12 = 25/12$
$E(S) = E(N_I)E(X_I) + E(N_{II})E(X_{II}) = 12(1/2) + 4(5/2) = 16$
$Var(S) = E(N_I)Var(X_I) + E(X_I)^2 Var(N_I) + E(N_{II})Var(X_{II}) + E(X_{II})^2 Var(N_{II})$
$\quad = 12(1/12) + (1/2)^2(12) + 4(25/12) + (5/2)^2(4) = 37.33$

$$\Pr(S > 18) = \Pr\left(Z > \frac{18-16}{\sqrt{37.33}} = 0.327\right) = 1 - \Phi(0.327) = 0.37$$

Question #126
Key: C

Let X be the loss random variable. Then, $(X-5)_+$ is the claim random variable.

$$E(X) = \frac{10}{2.5-1} = 6.667$$

$$E(X \wedge 5) = \left(\frac{10}{2.5-1}\right)\left[1 - \left(\frac{10}{5+10}\right)^{2.5-1}\right] = 3.038$$

$$E[(X-5)_+] = E(X) - E(X \wedge 5) = 6.667 - 3.038 = 3.629$$

Expected aggregate claims = $E(N)E[(X-5)_+] = 5(3.629) = 18.15$.

Question #127
Key: B

A Pareto $(\alpha = 2, \theta = 5)$ distribution with 20% inflation becomes Pareto with $\alpha = 2, \theta = 5(1.2) = 6$. In 2004

$$E(X) = \frac{6}{2-1} = 6$$

$$E(X \wedge 10) = \frac{6}{2-1}\left[1 - \left(\frac{6}{10+6}\right)^{2-1}\right] = 3.75$$

$$E[(X-10)_+]_+ = E(X) - E(X \wedge 10) = 6 - 3.75 - 2.25$$

$$\text{LER} = 1 - \frac{E[(X-10)_+]}{E(X)} = 1 - \frac{2.25}{6} = 0.625$$

Question #128
DELETED

Question #129
DELETED

Question #130
Key: E

Begin with $E(W) = E(2^N) = E[E(2^N \mid \Lambda)]$. The inner expectation is the probability generating function of Poisson distribution evaluated at 2, $P_N(2) = e^{\Lambda(2-1)} = e^{\Lambda}$. Then,

$$E(e^{\Lambda}) = \int_0^4 e^{\lambda}(0.25)d\lambda = 0.25e^{\lambda}\Big|_0^4 = 0.25(e^4 - 1) = 13.4.$$

If the pgf is not recognized, the inner expectation can be derived as
$E(2^N \mid \Lambda) = \sum_{n=0}^{\infty} 2^n \frac{e^{-\Lambda}\Lambda^n}{n!} = \sum_{n=0}^{\infty} \frac{e^{-\Lambda}(2\Lambda)^n}{n!} = e^{\Lambda}\sum_{n=0}^{\infty} \frac{e^{-2\Lambda}(2\Lambda)^n}{n!} = e^{\Lambda}$ noting that the sum is over the probabilities for a Poisson distribution with mean 2Λ and so must be 1.

Question #131
Key: E

$E(S)=\lambda E(X)=2/3(1/4+2/4+3/2)=2/3(9/4)=3/2$

$Var(S)=\lambda E(X^2)=2/3(1/4+4/4+9/2)=23/6$

So cumulative premium to time 2 is $2(3/2+1.8\sqrt{23/6})=10$, where the expression in parentheses is the annual premium.

Times between claims are determined by $-(1/\lambda)\log(1-u)$ and are 0.43, 0.77, 1.37, 2.41. So 2 claims before time 2 (second claim is at 1.20; third is at 2.57)

Sizes are 2, 3, 1, 3, where only the first two matter.

The gain to the insurer is 10 – (2 + 3) = 5.

Question #132
Key: C

To get number of claims, set up cdf for Poisson:

x	$f(x)$	$F(x)$
0	0.135	0.135
1	0.271	0.406
2	0.271	0.677
3	0.180	0.857

0.80 simulates 3 claims.

$F(\mathrm{x})=1-[500/(x+500)]^2=u,\ \text{so } x=500[(1-u)^{-1/2}-1]$

0.6 simulates 290.57
0.25 simulates 77.35
0.7 simulates 412.87

So total losses equals 780.79

Insurer pays 0.80(750) + (780.79 – 750) = 631.

Question #133
Key: C

$$E(X \mid q) = 3q, Var(X \mid q) = 3q(1-q)$$
$$\mu = E(3q) = \int_0^1 3q2qdq = 2q^3\Big|_0^1 = 2$$
$$v = E[3q(1-q)] = \int_0^1 3q(1-q)2qdq = 2q^3 - 1.5q^4\Big|_0^1 = 0.5$$
$$a = Var(3q) = E(9q^2) - \mu^2 = \int_0^1 9q^2 2qdq - 2^2 = 4.5q^4\Big|_0^1 - 4 = 4.5 - 4 = 0.5$$
$$k = v / a = 0.5 / 0.5 = 1, Z = \frac{1}{1+1} = 0.5$$
The estimate is 0.5(0) + 0.5(2) = 1.

Question #134
Key: D

0.35(13 + 1) = 4.9
$$\hat{\pi}_{0.35} = 0.1(216) + 0.9(250) = 246.6$$

Question #135
Key: E

At the first two *y*-values
$$y_1 = 0.9, r_1 = 7, s_1 = 1$$
$$y_2 = 1.5, r_2 = 6, s_2 = 1$$
$$S_{10}(1.6) = \frac{7-1}{7}\frac{6-1}{6} = 0.714$$

Question #136
Key: B

$$\Pr(\text{class } 1 \mid \text{claim} = 250) = \frac{\Pr(\text{claim} = 250 \mid \text{class } 1)\Pr(\text{class } 1)}{\Pr(\text{claim} = 250 \mid \text{class } 1)\Pr(\text{class } 1) + \Pr(\text{claim} = 250 \mid \text{class } 2)\Pr(\text{class } 2)}$$
$$= \frac{0.5(2/3)}{0.5(2/3) + 0.7(1/3)} = \frac{10}{17}$$
$$E(\text{claim} \mid \text{class } 1) = 0.5(250) + 0.3(2,500) + 0.2(60,000) = 12,875$$
$$E(\text{claim} \mid \text{class } 2) = 0.7(250) + 0.2(2,500) + 0.1(60,000) = 6,675$$
$$E(\text{claim} \mid 250) = (10/17)(12,875) + (7/17)(6,675) = 10,322$$

Question #137
Key: D

$$L(p) = f(0.74)f(0.81)f(0.95) = (p+1)0.74^p(p+1)0.81^p(p+1)0.95^p$$
$$= (p+1)^3(0.56943)^p$$
$$l(p) = \ln L(p) = 3\ln(p+1) + p\ln(0.56943)$$
$$l'(p) = \frac{3}{p+1} - 0.563119 = 0$$
$$p+1 = \frac{3}{0.563119} = 5.32747,\ p = 4.32747.$$

Question #138
Key: E

The sample mean is 1 and therefore mq = 1.
For the smoothed empirical 33rd percentile, (1/3)(5 + 1) = 2 and the second smallest sample item is 0. For the 33rd percentile of the binomial distribution to be 0, the probability at zero must exceed 0.33. So, $(1-q)^m > 0.33$ and then $(1-m^{-1})^m > 0.33$. Trial and error gives m = 6 as the smallest value that produces this result.

Question #139
Key: C

Let X be the number of claims.
$$E(X \mid I) = 0.9(0) + 0.1(2) = 0.2$$
$$E(X \mid II) = 0.8(0) + 0.1(1) + 0.1(2) = 0.3$$
$$E(X \mid III) = 0.7(0) + 0.2(1) + 0.1(2) = 0.4$$
$$Var(X \mid I) = 0.9(0) + 0.1(4) - 0.2^2 = 0.36$$
$$Var(X \mid II) = 0.8(0) + 0.1(1) + 0.1(4) - 0.3^2 = 0.41$$
$$Var(X \mid III) = 0.7(0) + 0.2(1) + 0.1(4) - 0.4^2 = 0.44.$$
$$\mu = (1/2)(0.2 + 0.3 + 0.4) = 0.3$$
$$v = (1/3)(0.36 + 0.41 + 0.44) = 0.403333$$
$$a = (1/3)(0.2^2 + 0.3^2 + 0.4^2) - 0.3^2 = 0.006667$$
$$k = 0.403333 / 0.006667 = 60.5$$
$$Z = \frac{50}{50 + 60.5} = 0.45249.$$
For one insured the estimate is 0.45249(17/50) + 0.54751(0.3) = 0.31810.
For 35 insureds the estimate is 35(0.31810) = 11.13.

Question #140
Key: A

For the given intervals, based on the model probabilities, the expected counts are 4.8, 3.3, 8.4, 7.8, 2.7, 1.5, and 1.5. To get the totals above 5, group the first two intervals and the last three. The table is

Interval	Observed	Expected	Chi-square
0-500	3	8.1	3.21
500-2498	8	8.4	0.02
2498-4876	9	7.8	0.18
4876-infinity	10	5.7	3.24
Total	30	30	6.65

Question #141
Key: E

Let $\hat{H} = \hat{H}(t)$ and $\hat{v} = V\hat{a}r(\hat{H}(t))$. The confidence interval is $\hat{H}U$ where $U = \exp(\pm z_{\alpha/2}\sqrt{\hat{v}}/\hat{H})$. Multiplying the two bounds gives $0.7(0.357) = \hat{H}^2$ for $\hat{H} = 0.49990$. Then,
$\hat{S} = \exp(-0.49990) = 0.60659.$

Question #142
Key: C

$$0.575 = \Pr(N=0) = \int_0^k \Pr(N=0\,|\,\theta)\pi(\theta)d\theta$$

$$= \int_0^k e^{-\theta}\frac{e^{-\theta}}{1-e^{-k}}d\theta = -\frac{e^{-2\theta}}{2(1-e^{-k})}\Bigg|_0^k = -\frac{e^{-2k}}{2(1-e^{-k})} + \frac{1}{2(1-e^{-k})}$$

$$= \frac{1-e^{-2k}}{2(1-e^{-k})} = \frac{1+e^{-k}}{2}$$

$$e^{-k} = 2(0.575) - 1 = 0.15$$

$$k = 1.90.$$

Question #143
Key: C

The sample -1 moment is $\frac{1}{6}\left(\frac{1}{15}+\frac{1}{45}+\frac{1}{140}+\frac{1}{250}+\frac{1}{560}+\frac{1}{1340}\right)=0.017094.$ The sample -2 moment is $\frac{1}{6}\left(\frac{1}{15^2}+\frac{1}{45^2}+\frac{1}{140^2}+\frac{1}{250^2}+\frac{1}{560^2}+\frac{1}{1340^2}\right)=0.00083484.$

Then the equations are

$$0.017094=\frac{1}{\theta(\tau-1)},$$

$$0.00083484=\frac{2}{\theta^2(\tau-1)(\tau-2)}.$$

Divide the square of the first equation by the second equation to obtain

$0.35001=\frac{\tau-2}{2(\tau-1)}$ which is solved for $\tau=4.33356.$ From the first equation,

$$\theta=\frac{1}{3.33356(0.017094)}=17.55.$$

Question #144
Key: A

For each simulation, estimate the LER and then calculate the squared difference from the estimate, 0.125.

Simulation	First claim	Second claim	Third claim	LER	Squared difference
1	600	600	1500	0.111111	0.000193
2	1500	300	1500	0.090909	0.001162
3	1500	300	600	0.125000	0.000000
4	600	600	300	0.200000	0.005625
5	600	300	1500	0.125000	0.000000
6	600	600	1500	0.111111	0.000193
7	1500	1500	1500	0.066667	0.003403
8	1500	300	1500	0.090909	0.001162
9	300	600	300	0.250000	0.015625
10	600	600	600	0.166667	0.001736

The last column has an average of 0.002910, which is the bootstrap estimate.

Question #145
Key: B

The subscripts denote the three companies.

$$x_{I1} = \frac{50,000}{100} = 500, \quad x_{I2} = \frac{50,000}{200} = 250, \quad x_{II1} = \frac{150,000}{500} = 300$$

$$x_{II2} = \frac{150,000}{300} = 500, \quad x_{III1} = \frac{150,000}{50} = 3,000, \quad x_{III2} = \frac{150,000}{150} = 1,000$$

$$\bar{x}_I = \frac{100,000}{300} = 333.33, \quad \bar{x}_{II} = \frac{300,000}{800} = 375, \quad \bar{x}_{III} = \frac{300,000}{200} = 1,500, \quad \bar{x} = \frac{700,000}{1,300} = 538.46$$

$$\hat{v} = \frac{\begin{array}{c}100(500-333.33)^2 + 200(250-333.33)^2 + 500(300-375)^2 + 300(500-375)^2 \\ + 50(3,000-1,500)^2 + 150(1,000-1,500)^2\end{array}}{(2-1)+(2-1)+(2-1)} = 53,888,888.89$$

$$\hat{a} = \frac{\begin{array}{c}300(333.33-538.46)^2 + 800(375-538.46)^2 + 200(1,500-538.46)^2 \\ -53,888,888.89(3-1)\end{array}}{1,300 - \dfrac{300^2+800^2+200^2}{1,300}} = 157,035.60$$

$$k = \frac{53,888,888.89}{157,035.60} = 343.1635, \ Z = \frac{200}{200+343.1635} = 0.3682$$

Question #146
Key: D

Let α_j be the parameter for region *j*. The likelihood function is $L = \left(\prod_{i=1}^{n} \frac{\alpha_1}{x_i^{\alpha_1+1}}\right)\left(\prod_{i=1}^{m} \frac{\alpha_2}{y_i^{\alpha_2+1}}\right)$.

The expected values satisfy $\frac{\alpha_2}{\alpha_2-1} = 1.5\frac{\alpha_1}{\alpha_1-1}$ and so $\alpha_2 = \frac{3\alpha_1}{2+\alpha_1}$. Substituting this in the likelihood function and taking logs produces

$$l(\alpha_1) = \ln L(\alpha_1) = n\ln\alpha_1 - (\alpha_1+1)\sum_{i=1}^{n}\ln x_i + m\ln\left(\frac{3\alpha_1}{2+\alpha_1}\right) - \frac{2+4\alpha_1}{2+\alpha_1}\sum_{i=1}^{m}\ln y_i$$

$$l'(\alpha_1) = \frac{n}{\alpha_1} - \sum_{i=1}^{n}\ln x_i + \frac{2m}{\alpha_1(2+\alpha_1)} - \frac{6}{(2+\alpha_1)^2}\sum_{i=1}^{m}\ln y_i = 0.$$

Question #147
Key: D

Let $K_y(x)$ be the contribution at *x* of the data point at *y*. It is

$$K_y(x)=\begin{cases}0, & x<y-1.4\\ \dfrac{x-y+1.4}{2.8}, & y-1.4\le x\le y+1.4\\ 1, & x>y+1.4.\end{cases}$$

For the particular points,
$K_2(4)=1,\ K_{3.3}(4)=\dfrac{4-3.3+1.4}{2.8}=0.75,\ K_4(4)=0.5,\ K_{4.7}(4)=0.25.$ The kernel estimate is the weighted average $\dfrac{1}{8}(1)+\dfrac{2}{8}(0.75)+\dfrac{2}{8}(0.5)+\dfrac{3}{8}(0.25)=0.53125.$

Question #148
Key: B

The mean is *mq* and the variance is *mq*(1 – *q*). The mean is 34,574 and so the full credibility standard requires the confidence interval to be ±345.74 which must be 1.96 standard deviations. Thus,

$$345.74=1.96\sqrt{mq(1-q)}=1.96\sqrt{34,574}\sqrt{1-q}$$
$$1-q=0.9,\quad q=0.1.$$

Question #149
DELETED

Question #150
Key: E

The sample average is (14 + 33 + 72 + 94 + 120 + 135 + 150 + 150)/8 = 96. The model average is $E(X\wedge 150)=\int_0^{150}x\frac{1}{\theta}dx+\int_{150}^{\theta}150\frac{1}{\theta}dx=\frac{150^2}{2\theta}+150\frac{\theta-150}{\theta}=150-\frac{11,250}{\theta}$. The equation to solve is $150-\dfrac{11,250}{\theta}=96,\quad \dfrac{11,250}{\theta}=54,\quad \theta=\dfrac{11,250}{54}=208.3.$

Question #151
Key: C

$$E(N|1) = 5,\ E(N|2) = 8(0.55) = 4.4,\ \mu = 0.5(5) + 0.5(4.4) = 4.7$$
$$Var(N|1) = 5, Var(N|2) = 8(0.55)(0.45) = 1.98,\ v = 0.5(5) + 0.5(1.98) = 3.49$$
$$a = 0.5(5)^2 + 0.5(4.4)^2 - 4.7^2 = 0.09,\ k = 3.49/0.09 = 38.7778$$
$$Z = \frac{3}{3 + 38.7778} = 0.0718,\ 4.6019 = 0.0718\frac{7+r}{3} + 0.9282(4.7)$$
$$Var(N|1) = 5, Var(N|2) = 8(0.55)(0.45) = 1.98,\ v = 0.5(5) + 0.5(1.98) = 3.49$$
$$a = 0.5(5)^2 + 0.5(4.4)^2 - 4.7^2 = 0.09,\ k = 3.49/0.09 = 38.7778$$
$$Z = \frac{3}{3 + 38.7778} = 0.0718,\ 4.6019 = 0.0718\frac{7+r}{3} + 0.9282(4.7)$$
The solution is $r = 3$.

Question #152
Key: A

These observations are truncated at 500. The contribution to the likelihood function is $\frac{f(x)}{1-F(500)} = \frac{\theta^{-1}e^{-x/\theta}}{e^{-500/\theta}}$. Then the likelihood function is

$$L(\theta) = \frac{\theta^{-1}e^{-600/\theta}\theta^{-1}e^{-700/\theta}\theta^{-1}e^{-900/\theta}}{\left(e^{-500/\theta}\right)^3} = \theta^{-3}e^{-700/\theta}$$
$$l(\theta) = \ln L(\theta) = -3\ln\theta - 700\theta^{-1}$$
$$l'(\theta) = -3\theta^{-1} + 700\theta^{-2} = 0$$
$$\theta = 700/3 = 233.33.$$

Question #153
DELETED

Question #154
Key: E

For a compound Poisson distribution, *S*, the mean is $E(S\,|\,\lambda,\mu,\sigma)=\lambda E(X)=\lambda e^{\mu+0.5\sigma^2}$ and the variance is $Var(S\,|\,\lambda,\mu,\sigma)=\lambda E(X^2)=\lambda e^{2\mu+2\sigma^2}$. Then,

$$E(S)=E[E(S\,|\,\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda e^{\mu+0.5\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma$$
$$=\int_0^1\int_0^1 e^{\mu+0.5\sigma^2}\sigma\, d\mu\, d\sigma=\int_0^1 (e-1)e^{0.5\sigma^2}\sigma\, d\sigma$$
$$=(e-1)(e^{0.5}-1)=1.114686$$
$$v=E[Var(S\,|\,\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda e^{2\mu+2\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma$$
$$=\int_0^1\int_0^1 e^{2\mu+2\sigma^2}\sigma\, d\mu\, d\sigma=\int_0^1 0.5(e^2-1)e^{2\sigma^2}\sigma\, d\sigma$$
$$=0.5(e^2-1)0.25(e^2-1)=0.125(e^2-1)^2=5.1025$$
$$a=Var[E(S\,|\,\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda^2 e^{2\mu+\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma-E(S)^2$$
$$=\int_0^1\int_0^1 \frac{2}{3}e^{2\mu+\sigma^2}\sigma\, d\mu\, d\sigma-E(S)^2=\int_0^1 \frac{1}{3}(e^2-1)e^{\sigma^2}\sigma\, d\sigma-E(S)^2$$
$$=\frac{1}{3}(e^2-1)\frac{1}{2}(e-1)-E(S)^2=(e^2-1)(e-1)/6-E(S)^2=0.587175$$
$$k=\frac{5.1025}{0.587175}=8.69.$$

Question #155
Key: D

The equations to solve are $0.4=e^{-(\theta/1.82)^\tau}, 0.8=e^{-(\theta/12.66)^\tau}$. Taking logs yields $0.91629=(\theta/1.82)^\tau, 0.22314=(\theta/12.66)^\tau$. Taking the ratio of the first equation to the second equation gives $4.10635=(12.66/1.82)^\tau=6.95604^\tau$. Taking logs again, $1.41253=1.93961\tau$ and then $\tau=0.72825$. Returning to the first (logged) equation, $0.91629=(\theta/1.82)^\tau, 0.88688=\theta/1.82, \theta=1.614.$

Question #156
Key: C

There are n/2 observations of N = 0 (given N = 0 or 1) and n/2 observations of N = 1 (given N = 0 or 1). The likelihood function is

$L=\left(\frac{e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2}\left(\frac{\lambda e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2}=\frac{\lambda^{n/2}e^{-n\lambda}}{(e^{-\lambda}+\lambda e^{-\lambda})^{n}}=\frac{\lambda^{n/2}}{(1+\lambda)^{n}}$. Taking logs, differentiating and solving provides the answer.

$l=\ln L=(n/2)\ln\lambda-n\ln(1+\lambda)$

$l'=\frac{n}{2\lambda}-\frac{n}{1+\lambda}=0$

$n(1+\lambda)-n2\lambda=0$

$1-\lambda=0,\quad \lambda=1.$

Question #157
Key: D

The posterior density function is proportional to the product of the likelihood function and prior density. That is, $\pi(q\,|\,1,0)\propto f(1\,|\,q)f(0\,|\,q)\pi(q)\propto q(1-q)q^{3}=q^{4}-q^{5}$. To get the exact posterior density, integrate this function over its range:

$\int_{0.6}^{0.8}q^{4}-q^{5}dq=\frac{q^{5}}{5}-\frac{q^{6}}{6}\Big|_{0.6}^{0.8}=0.014069$ and so $\pi(q\,|\,1,0)=\frac{q^{4}-q^{5}}{0.014069}$. Then,

$\Pr(0.7<q<0.8\,|\,1,0)=\int_{0.7}^{0.8}\frac{q^{4}-q^{5}}{0.014069}dq=0.5572.$

Question #158
Key: B

The cumulative hazard function for the exponential distribution is $H(x) = x/\theta$. The maximum likelihood estimate of θ is the sample mean, which equals (1227/15) = 81.8. Therefore $\hat{H}_2(75) = 75/81.8 = 0.917$. To calculate $\hat{H}_1(75)$ use the following table.

j	1	2	3	4	5	6
y_j	11	22	36	51	69	92
s_j	1	3	1	1	3	2
r_j	15	14	11	10	9	6

Therefore,

$$\hat{H}_1(75) = \frac{1}{15} + \frac{3}{14} + \frac{1}{11} + \frac{1}{10} + \frac{3}{9} = 0.805.$$

Thus, $\hat{H}_2(75) - \hat{H}_1(75) = 0.917 - 0.805 = 0.112$.

Question #159
Key: A

The sample mean is $\dfrac{0(2000) + 1(600) + 2(300) + 3(80) + 4(20)}{3000} = 0.5066667 = \hat{\mu} = \hat{v}$ and the sample variance is

$$\frac{2000(0-\hat{\mu})^2 + 600(1-\hat{\mu})^2 + 300(2-\hat{\mu})^2 + 80(3-\hat{\mu})^2 + 20(4-\hat{\mu})^2}{2999} = 0.6901856.$$ Then,

$$\hat{a} = 0.6901856 - 0.5066667 = 0.1835189,\ k = \frac{0.5066667}{0.1835189} = 2.760842$$ and

$$Z = \frac{1}{1 + 2.760842} = 0.2659.$$

Question #160
Key: E

The cdf is $F(x) = \int_0^x 4(1+t)^{-5}\,dt = -(1+t)^{-4}\Big|_0^x = 1 - \dfrac{1}{(1+x)^4}$.

Observation (x)	$F(x)$	compare to:	Maximum difference
0.1	0.317	0, 0.2	0.317
0.2	0.518	0.2, 0.4	0.318
0.5	0.802	0.4, 0.6	0.402
0.7	0.880	0.6, 0.8	0.280
1.3	0.964	0.8, 1.0	0.164

K-S statistic is 0.402.

Question #161
Key: D

This follows from the formula $MSE(\hat{\theta}) = Var(\hat{\theta}) + [bias(\hat{\theta})]^2$. If the bias is zero, then the mean-squared error is equal to the variance.

Question #162
Key: B

$E[X-d \mid X>d]$ is the expected payment per payment with an ordinary deductible of d
It can be evaluated (for Pareto) as

$$\frac{E(X)-E(X\wedge d)}{1-F(d)} = \frac{\dfrac{\theta}{\alpha-1} - \dfrac{\theta}{\alpha-1}\left[1-\left(\dfrac{\theta}{d+\theta}\right)^{\alpha-1}\right]}{1-\left[1-\left(\dfrac{\theta}{d+\theta}\right)^{\alpha}\right]} = \frac{\dfrac{\theta}{\alpha-1}\left(\dfrac{\theta}{d+\theta}\right)^{\alpha-1}}{\left(\dfrac{\theta}{d+\theta}\right)^{\alpha}} = \frac{d+\theta}{\alpha-1} = d+\theta$$

$$E[X-100 \mid X>100] = (5/3)E[X-50 \mid \mathrm{X}>50]$$

$$100+\theta = (5/3)(50+\theta)$$

$$300+3\theta = 250+5\theta, 50 = 2\theta, \theta = 25$$

$$E[X-150 \mid X>150] = 150+\theta = 150+25 = 175$$

Question #163
Key: D

Let S = score

$$E(S) = E[E(S \mid \theta)] = E(\theta) = 75$$

$$Var(S) = E[Var(S \mid \theta)] + Var[E(S \mid \theta)] = E(8^2) + Var(\theta) = 64 + 6^2 = 100$$

S is normally distributed (a normal mixture of normal distributions with constant variance is normal).

$$\Pr(\mathrm{S}<90 \mid \mathrm{S}>65) = \frac{F(90)-F(65)}{1-F(65)} = \frac{\Phi[(90-75)/10)] - \Phi[(65-75)/10]}{1-\Phi[(65-75)/10]}$$

$$\frac{\Phi(1.5)-\Phi(-1.0)}{1-\Phi(-1.0)} = \frac{0.9332-(1-0.8413)}{1-(1-0.8413)} = 0.9206$$

Question #164
Key: B

Losses in excess of the deductible occur at a Poisson rate of $\lambda^* = [1 - F(30)]\lambda = 0.75(20) = 15$
The expected payment squared per payment is

$$E[(X-30)^2 \mid X > 30) = E(X^2 - 60X + 900 \mid X > 30)$$

$$E[X^2 - 60(X-30) - 900 \mid X > 30]$$

$$E[X^2 \mid X > 30] - 60\frac{E(X) - E(X \wedge 30)}{1 - F(30)} - 900$$

$$= 9000 - 60\frac{70-25}{0.75} - 900 = 4500$$

The variance of *S* is the expected number of payments times the second moment, 15(4500) = 67,500.

Question #165
Key: A

$$E[(S-3)_+] = E(S) - 3 + 3f_S(0) + 2f_S(1) + 1f_S(2)$$

$$E(S) = 2[0.6 + 2(0.4)] = 2.8$$

$$f_S(0) = e^{-2},\ f_S(1) = e^{-2}(2)(0.6) = 1.2e^{-2}$$

$$f_S(2) = e^{-2}(2)(0.4) + \frac{e^{-2}2^2}{2!}(0.6)^2 = 1.52e^{-2}$$

$$E[(S-3)_+] = 2.8 - 3 + 3e^{-2} + 2(1.2e^{-2}) + 1(1.52e^{-2}) = -0.2 + 6.92e^{-2} = 0.7365$$

Question #166
Key: C

Write (i) as $\dfrac{p_k}{p_{k-1}} = c + \dfrac{c}{k}$

This is an (*a*, *b*, 0) distribution with *a* = *b* = *c*.
Which?

1. If Poisson, *a* = 0, so *b* = 0 and thus $p_0 = 0.5$ and $p_1 = p_2 = \cdots = 0$. The probabilities do not sum to 1 and so not Poisson.
2. If Geometric, *b* = 0, so *a* = 0. By same reasoning as #1, not Geometric.
3. If binomial, *a* and *b* have opposite signs. But here *a* = *b*, so not binomial.
4. Thus negative binomial.

$$1 = \frac{a}{b} = \frac{\beta/(1+\beta)}{(r-1)\beta/(1-\beta)} = \frac{1}{r-1} \text{ so } r = 2.$$

$$p_0 = 0.5 = (1+\beta)^{-r} = (1+\beta)^{-2} \Rightarrow \beta = \sqrt{2} - 1 = 0.414$$
$$c = a = \beta / (1+\beta) = 0.29$$

Question #167
Key: B

The number of repairs for each boat type has a binomial distribution.
For power boats:

$$E(S) = 100(0.3)(300) = 9,000, Var(S) = 100(0.3)(10,000) + 100(0.3)(0.7)(300^2) = 2,190,000$$

For sail boats:

$$E(S) = 300(0.1)(1,000) = 30,000, Var(S) = 300(0.1)(400,000) + 300(0.1)(0.9)(1,000^2) = 39,000,000$$

For luxury yachts:

$$E(S) = 50(0.6)(5,000) = 150,000, Var(S) = 50(0.6)(2,000,000) + 50(0.6)(0.4)(5,000^2) = 360,000,000$$

The sums are 189,000 expected and a variance of 401,190,000 for a standard deviation of 20,030. The mean plus standard deviation is 209,030.

Question #168
Key: B

$$S_X(150) = 1 - 0.2 = 0.8$$
$$f_{Y^P}(y) = \frac{f_X(y+150)}{S_X(150)},\ f_{Y^P}(50) = \frac{0.2}{0.8} = 0.25,\ f_{Y^P}(150) = \frac{0.6}{0.8} = 0.75$$
$$E(Y^P) = 0.25(50) + 0.75(150) = 125$$
$$E\left[(Y^P)^2\right] = 0.25(50^2) + 0.75(150^2) = 17,500$$
$$Var(Y^P) = 17,500 - 125^2 = 1,875$$

Question #169
Key: A

$$F(200) = 0.8\left[1 - \left(\frac{100}{200+100}\right)^2\right] + 0.2\left[1 - \left(\frac{3000}{3000+200}\right)^4\right] = 0.7566$$

Question #170
Key: B

Let C denote child; ANS denote Adult Non-Smoker; AS denote Adult Smoker.

$$P(3|C)P(C)=\frac{3^3e^{-3}}{3!}(0.3)=0.067$$
$$P(3|ANS)P(ANS)=\frac{1^3e^{-1}}{3!}(0.6)=0.037$$
$$P(3|AS)P(AS)=\frac{4^3e^{-4}}{3!}(0.1)=0.020$$
$$P(AS|N=3)=\frac{0.020}{0.067+0.037+0.020}=0.16$$

Question #171
Key: C

$$E(S)=E(N)E(X)=3(10)=30$$
$$Var(S)=E(N)Var(X)+E(X)^2Var(N)=3\frac{400}{12}+100(3.6)=100+360=460$$
For 95th percentile, $E(S)+1.645\sqrt{Var(S)}=30+1.645\sqrt{460}=65.28$

Question #172
Key: D

The CDF is $F(x)=1-\dfrac{1}{(1+x)^4}$

Observation (x)	$F(x)$	compare to:	Maximum difference
0.2	0.518	0, 0.2	0.518
0.7	0.880	0.2, 0.4	0.680
0.9	0.923	0.4, 0.6	0.523
1.1	0.949	0.6, 0.8	0.349
1.3	0.964	0.8, 1.0	0.164

The K-S statistic is the maximum from the last column, 0.680.
Critical values are: 0.546, 0.608, 0.662, and 0.729 for the given levels of significance. The test statistic is between 0.662 (2.5%) and 0.729 (1.0%) and therefore the test is rejected at 0.025 and not at 0.01.

Question #173
Key: E

For claim severity,
$\mu_S = 1(0.4) + 10(0.4) + 100(0.2) = 24.4,$
$\sigma_S^2 = 1^2(0.4) + 10^2(0.4) + 100^2(0.2) - 24.4^2 = 1,445.04.$
For claim frequency,
$\mu_F = r\beta = 3r, \quad \sigma_F^2 = r\beta(1+\beta) = 12r.$
For aggregate losses,
$\mu = \mu_S\mu_F = 24.4(3r) = 73.2r,$
$\sigma^2 = \mu_S^2\sigma_F^2 + \sigma_S^2\mu_F = 24.4^2(12r) + 1,445.04(3r) = 11,479.44r.$
For the given probability and tolerance, $\lambda_0 = (1.96/0.1)^2 = 384.16.$
The number of observations needed is
$\lambda_0\sigma^2/\mu^2 = 384.16(11,479.44r)/(73.2r)^2 = 823.02/r.$
The average observation produces $3r$ claims and so the required number of claims is
$(823.02/r)(3r) = 2,469.$

Question #174
Key: A

$\hat{H}(t_2) = \frac{1}{n} + \frac{1}{n-1} = \frac{2n-1}{n(n-1)} = \frac{39}{380} \Rightarrow 39n^2 - 799n + 380 = 0 \Rightarrow n = 20, n = 0.487$.
Discard the non-integer solution to have $n = 20$. The Kaplan-Meier Product-Limit Estimate is:
$\hat{S}(t_9) = \frac{19}{20}\frac{18}{19}\cdots\frac{11}{12} = \frac{11}{20} = 0.55.$

Question #175
Key: E

There are 27 possible bootstrap samples, which produce four different results. The results, their probabilities, and the values of g are:

Bootstrap Sample	Prob	g
1, 1, 1	8/27	0
1, 1, 4	12/27	2
1, 4, 4	6/27	-2
4, 4, 4	1/27	0

The third central moment of the original sample is 2. Then,

MSE = $\frac{8}{27}(0-2)^2 + \frac{12}{27}(2-2)^2 + \frac{6}{27}(-2-2)^2 + \frac{1}{27}(0-2)^2 = \frac{44}{9}$.

Question #176
Key: A

Pick one of the points, say the fifth one. The vertical coordinate is $F(30)$ from the model and should be slightly less than 0.6. Inserting 30 into the five answers produces 0.573, 0.096, 0.293, 0.950, and something less than 0.5. Only the model in answer A is close.

Question #177
Key: E

The distribution of Θ is Pareto with parameters 1 and 2.6. Then,

$$v = EPV = E(\Theta) = \frac{1}{2.6-1} = 0.625, \quad a = VHM = Var(\Theta) = \frac{2}{1.6(0.6)} - 0.625^2 = 1.6927,$$

$$k = v/a = 0.625/1.6927 = 0.3692, \quad Z = \frac{5}{5+0.3692} = 0.9312.$$

Question #178
DELETED

Question #179
Key: D

For an exponential distribution, the maximum likelihood estimate of θ is the sample mean, 6. Let $Y = X_1 + X_2$ where each X has an exponential distribution with mean 6. The sample mean is $Y/2$ and Y has a gamma distribution with parameters 2 and 6. Then

$$\Pr(Y/2 > 10) = \Pr(Y > 20) = \int_{20}^{\infty} \frac{xe^{-x/6}}{36} dx$$

$$= -\frac{xe^{-x/6}}{6} - e^{-x/6} \Big|_{20}^{\infty} = \frac{20e^{-20/6}}{6} + e^{-20/6} = 0.1546.$$

Question #180
Key: A

Let $W = X_1 + X_2$ where each X has an exponential distribution with mean θ. The sample mean is $Y = W/2$ and W has a gamma distribution with parameters 2 and θ. Then

$$g(\theta) = F_Y(10) = \Pr(Y \le 10) = \Pr(W \le 20) = \int_0^{20} \frac{we^{-w/\theta}}{\theta^2} dw$$

$$= -\frac{we^{-w/\theta}}{\theta} - e^{-w/\theta} \Bigg|_0^{20} = 1 - \frac{20e^{-20/\theta}}{\theta} - e^{-20/\theta} = 1 - \mathrm{e}^{-20/\theta}(1 + 20\theta^{-1}).$$

$$g'(\theta) = -\frac{20}{\theta^2} e^{-20/\theta}(1 + 20\theta^{-1}) + e^{-20/\theta} \frac{20}{\theta^2} = -\frac{400e^{-20/\theta}}{\theta^3}.$$

At the maximum likelihood estimate of 6, $g'(6) = -0.066063$.
The maximum likelihood estimator is the sample mean. Its variance is the variance of one observation divided by the sample size. For the exponential distribution the variance is the square of the mean, so the estimated variance of the sample mean is 36/2 = 18. The answer is $(-0.066063)^2(18) = 0.079.$

Question #181
Key B

$$\mu(\lambda,\theta) = E(S \mid \lambda,\theta) = \lambda\theta,$$
$$v(\lambda,\theta) = Var(S \mid \lambda,\theta) = \lambda 2\theta^2,$$
$$v = EPV = E(\lambda 2\theta^2) = 1(2)(1+1) = 4,$$
$$a = VHM = Var(\lambda\theta) = E(\lambda^2)E(\theta^2) - [E(\lambda)E(\theta)]^2 = 2(2) - 1 = 3,$$
$$k = v / a = 4/3.$$

Question #182
Key: B

The distribution is binomial with m = 100 and q = 0.03. The first three probabilities are:

$$p_0 = 0.97^{100} = 0.04755,\ p_1 = 100(0.97)^{99}(0.03) = 0.14707,$$

$$p_2 = \frac{100(99)}{2}(0.97)^{98}(0.03)^2 = 0.22515.$$

Values between 0 and 0.04755 simulate a 0, between 0.04755 and 0.19462 simulate a 1, and between 0.19462 and 0.41977 simulate a 2. The three simulated values are 2, 0, and 1. The mean is 1.

Question #183
DELETED

Question #184
Key: D

The posterior distribution can be found from

$$\pi(\lambda\,|\,10) \propto \frac{e^{-\lambda}\lambda^{10}}{10!}\left(\frac{0.4}{6}e^{-\lambda/6}+\frac{0.6}{12}e^{-\lambda/12}\right) \propto \lambda^{10}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right).$$

The required constant is

$$\int_0^\infty \lambda^{10}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right)d\lambda = 0.8(10!)(6/7)^{11}+0.6(10!)(12/13)^{11} = 0.395536(10!).$$

The posterior mean is

$$E(\lambda\,|\,10) = \frac{1}{0.395536(10!)}\int_0^\infty \lambda^{11}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right)d\lambda$$

$$= \frac{0.8(11!)(6/7)^{12}+0.6(11!)(12/13)^{12}}{0.395536(10!)} = 9.88.$$

Question #185
Key: A

$\hat{H}(4.5) = \frac{2}{12}+\frac{1}{10}+\frac{2}{9}+\frac{2}{7} = 0.77460.$ The variance estimate is $\frac{2}{12^2}+\frac{1}{10^2}+\frac{2}{9^2}+\frac{2}{7^2} = 0.089397.$

The confidence interval is $0.77460 \pm 1.96\sqrt{0.089397} = 0.77460 \pm 0.58603.$ The interval is from 0.1886 to 1.3606.

Question #186
Key: D

$$bias = E(\hat{\theta})-\theta = \frac{k}{k+1}\theta-\theta = -\frac{\theta}{k+1}, \quad Var(\hat{\theta}) = Var\left(\frac{k\theta}{k+1}\right) = \frac{k^2\theta^2}{25(k+1)^2},$$

$$MSE = Var(\hat{\theta})+bias^2 = \frac{k^2\theta^2}{25(k+1)^2}+\frac{\theta^2}{(k+1)^2},$$

$$MSE = 2bias^2 = \frac{2\theta^2}{(k+1)^2}.$$

Setting the last two equal and canceling the common terms gives $\frac{k^2}{25}+1=2$ for $k = 5$.

Question #187
Key: D

For the geometric distribution $\mu(\beta)=\beta$ and $v(\beta)=\beta(1+\beta)$. The prior density is Pareto with parameters α and 1. Then,

$$\mu = E(\beta) = \frac{1}{\alpha-1},$$

$$v = EPV = E[\beta(1+\beta)] = \frac{1}{\alpha-1} + \frac{2}{(\alpha-1)(\alpha-2)} = \frac{\alpha}{(\alpha-1)(\alpha-2)},$$

$$a = VHM = Var(\beta) = \frac{2}{(\alpha-1)(\alpha-2)} - \frac{1}{(\alpha-1)^2} = \frac{\alpha}{(\alpha-1)^2(\alpha-2)},$$

$$k = v/a = \alpha-1, \quad Z = \frac{1}{1+k} = \frac{1}{\alpha}.$$

The estimate is

$$\frac{1}{\alpha}x + \left(1-\frac{1}{\alpha}\right)\frac{1}{\alpha-1} = \frac{x+1}{\alpha}.$$

Question #188
DELETED

Question #189
Key: E

A is false. Using sample data gives a better than expected fit and therefore a test statistic that favors the null hypothesis, thus increasing the Type II error probability. The K-S test works only on individual data and so B is false. D is false because the critical value depends on the degrees of freedom which in turn depends on the number of cells, not the sample size.

Question #190
Key: B

$E(\theta) = 0.05(0.8) + 0.3(0.2) = 0.1,$
$E(\theta^2) = 0.05^2(0.8) + 0.3^2(0.2) = 0.02,$
$\mu(\theta) = 0(2\theta) + 1(\theta) + 2(1-3\theta) = 2-5\theta,$
$v(\theta) = 0^2(2\theta) + 1^2(\theta) + 2^2(1-3\theta) - (2-5\theta)^2 = 9\theta - 25\theta^2,$
$\mu = E(2-5\theta) = 2-5(0.1) = 1.5,$
$v = EPV = E(9\theta - 25\theta^2) = 9(0.1) - 25(0.02) = 0.4,$
$a = VHM = Var(2-5\theta) = 25Var(\theta) = 25(0.02 - 0.1^2) = 0.25,$
$k = v/a = 0.4/0.25 = 1.6,\ Z = \frac{1}{1+1.6} = \frac{5}{13},$
$P = \frac{5}{13}2 + \frac{8}{13}1.5 = 1.6923.$

Question #191
Key: B

$f(\lambda \mid 5,3) \propto \frac{e^{-\lambda}\lambda}{5!}\frac{e^{-\lambda}\lambda^3}{3!}\frac{2^5\lambda^5 e^{-2\lambda}}{24\lambda} \propto \lambda^{12}e^{-4\lambda}$. This is a gamma distribution with parameters 13 and 0.25. The expected value is 13(0.25) = 3.25.

Alternatively, if the Poisson-gamma relationships are known, begin with the prior parameters $\alpha = 5$ and $\beta = 2$ where $\beta = 1/\theta$ if the parameterization from *Loss Models* is considered. Then the posterior parameters are $\alpha' = 5+5+3 = 13$ where the second 5 and the 3 are the observations and $\beta' = 2+2 = 4$ where the second 2 is the number of observations. The posterior mean is then 13/4 = 3.25.

Question #192
Key: D

Because the kernel extends one unit in each direction there is no overlap. The result will be three replications of the kernel. The middle one will be twice has high due to having two observations of 3 while there is only one observation at 1 and 5. Only graphs A and D fit this description. The kernel function is smooth, which rules out graph A.

Question #193
Key: C

The two moment equations are

$$508 = \frac{\theta}{\alpha - 1}, \quad 701,401.6 = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}.$$

Dividing the square of the first equation into the second equation gives

$\frac{701,401.6}{508^2} = 2.7179366 = \frac{2(\alpha-1)}{\alpha-1}$. The solution is $\alpha = 4.785761$. From the first equation, $\theta = 1,923.167$. The requested LEV is

$$E(X \wedge 500) = \frac{1,923.167}{3.785761}\left[1 - \left(\frac{1,923.167}{1,923.167+500}\right)^{3.785761}\right] = 296.21.$$

Question #194
Key: B

$$\hat{v} = \frac{50(200-227.27)^2 + 60(250-227.27)^2 + 100(160-178.95)^2 + 90(200-178.95)^2}{1+1}$$

$$= 71,985.647,$$

$$\hat{k} = 71,985.647 / 651.03 = 110.57,$$

$$\hat{Z} = \frac{110}{110+110.57} = 0.499.$$

Question #195
Key: B

$$F_{100}(1,000) = 0.16,\ F_{100}(3,000) = 0.38,\ F_{100}(5,000) = 0.63,\ F_{100}(10,000) = 0.81,$$
$$F_{100}(2,000) = 0.5(0.16) + 0.5(0.38) = 0.27,$$
$$F_{100}(6,000) = 0.8(0.63) + 0.2(0.81) = 0.666.$$
$$\Pr(2,000 < X < 6,000) = 0.666 - 0.27 = 0.396.$$

Question #196
Key: C

$$L=\left[\frac{f(750)}{1-F(200)}\right]^3 f(200)^3 f(300)^4[1-F(10,000)]^6\left[\frac{f(400)}{1-F(300)}\right]^4$$

$$=\left[\frac{\alpha 10,200^\alpha}{10,750^{\alpha+1}}\right]^3\left[\frac{\alpha 10,000^\alpha}{10,200^{\alpha+1}}\right]^3\left[\frac{\alpha 10,000^\alpha}{10,300^{\alpha+1}}\right]^4\left[\frac{10,000^\alpha}{20,000^\alpha}\right]^6\left[\frac{\alpha 10,300^\alpha}{10,400^{\alpha+1}}\right]^4$$

$$=\alpha^{14}10,200^{-3}10,000^{13\alpha}10,300^{-4}10,750^{-3\alpha-3}20,000^{-6\alpha}10,400^{-4\alpha-4}$$

$$\propto \alpha^{14}10,000^{13\alpha}10,750^{-3\alpha}20,000^{-6\alpha}10,400^{-4\alpha},$$

$$\ln L = 14\ln\alpha+13\alpha\ln(10,000)-3\alpha\ln(10,750)-6\alpha\ln(20,000)-4\alpha\ln(10,400)$$

$$=14\ln\alpha-4.5327\alpha.$$

The derivative is $14/\alpha-4.5327$ and setting it equal to zero gives $\hat{\alpha}=3.089$.

Question #197
Key: C

$$\hat{v}=\bar{x}=\frac{30+30+12+4}{100}=0.76.$$

$$\hat{a}=\frac{50(0-0.76)^2+30(1-0.76)^2+15(2-0.76)^2+4(3-0.76)^2+1(4-0.76)^2}{99}-0.76$$

$$=0.090909,$$

$$\hat{k}=\frac{0.76}{0.090909}=8.36,\quad \hat{Z}=\frac{1}{1+8.36}=0.10684,$$

$$P=0.10684(1)+0.89316(0.76)=0.78564.$$

The above analysis was based on the distribution of total claims for two years. Thus 0.78564 is the expected number of claims for the next two years. For the next one year the expected number is 0.78564/2 = 0.39282.

Question #198
DELETED

Question #199
Key: E

The density function is $f(x)=\dfrac{0.2x^{-0.8}}{\theta^{0.2}}e^{-(x/\theta)^{0.2}}$. The likelihood function is

$$
\begin{aligned}
L(\theta) &= f(130)f(240)f(300)f(540)[1-F(1000)]^2 \\
&= \frac{0.2(130)^{-0.8}}{\theta^{0.2}}e^{-(130/\theta)^{0.2}}\frac{0.2(240)^{-0.8}}{\theta^{0.2}}e^{-(240/\theta)^{0.2}}\frac{0.2(300)^{-0.8}}{\theta^{0.2}}e^{-(300/\theta)^{0.2}}\frac{0.2(540)^{-0.8}}{\theta^{0.2}}e^{-(540/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}} \\
&\propto \theta^{-0.8}e^{-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})},
\end{aligned}
$$

$$
\begin{aligned}
l(\theta) &= -0.8\ln(\theta)-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2}) \\
&= -0.8\ln(\theta)-20.2505\theta^{-0.2},
\end{aligned}
$$

$$l'(\theta)=-0.8\theta^{-1}+0.2(20.2505)\theta^{-1.2}=0,$$

$$\theta^{-0.2}=0.197526, \quad \hat{\theta}=3,325.67.$$

Question #200
Key: A

Buhlmann estimates are on a straight line, which eliminates E. Bayes estimates are never outside the range of the prior distribution. Because graphs B and D include values outside the range 1 to 4, they cannot be correct. The Buhlmann estimates are the linear least squares approximation to the Bayes estimates. In graph C the Bayes estimates are consistently higher and so the Buhlmann estimates are not the best approximation. This leaves A as the only feasible choice.

Question #201
Key: C

The expected counts are 300(0.035) = 10.5, 300(0.095) = 28.5, 300(0.5) = 150, 300(0.2) = 60, and 300(0.17) = 51 for the five groups. The test statistic is

$$\frac{(5-10.5)^2}{10.5}+\frac{(42-28.5)^2}{28.5}+\frac{(137-150)^2}{150}+\frac{(66-60)^2}{60}+\frac{(50-51)^2}{51}=11.02.$$

There are 5 – 1 = 4 degrees of freedom. From the table, the critical value for a 5% test is 9.488 and for a 2.5% test is 11.143. The hypothesis is rejected at 5%, but not at 2.5%.

Question #202
Key: A

To simulate a lognormal variate, first convert the uniform random number to a standard normal. This can be done be using the table of normal distribution values. For the six values given, the corresponding standard normal values are 0.3, -0.1, 1.6, -1.4, 0.8, and -0.2. Next, multiply each number by the standard deviation of 0.75 and add the mean of 5.6. This produces random observations from the normal 5.6, 0.75^2 distribution. These values are 5.825, 5,525, 6.8, 4.55, 6.2, and 5.45. To create lognormal observations, exponentiate these values. The results are 339, 251, 898, 95, 493, and 233. After imposing the policy limit of 400, the mean is (339 + 251 + 400 + 95 + 400 + 233)/6 = 286.

Question #203
Key: C

For the geometric distribution, $\Pr(X_1 = 2 \mid \beta) = \dfrac{\beta^2}{(1+\beta)^3}$ and the expected value is β.

$$\Pr(\beta = 2 \mid X_1 = 2) = \frac{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2)}{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2) + \Pr(X_1 = 2 \mid \beta = 5)\Pr(\beta = 5)}$$

$$= \frac{\frac{4}{27}\frac{1}{3}}{\frac{4}{27}\frac{1}{3} + \frac{25}{216}\frac{2}{3}} = 0.39024.$$

The expected value is then 0.39024(2) + 0.60976(5) = 3.83.

Question #204
Key: D

Let *X* be the random variable for when the statistic is forgotten. Then $F_X(x \mid y) = 1 - e^{-xy}$
For the unconditional distribution of *X*, integrate with respect to y

$$F_X(x) = \int_0^\infty (1 - e^{-xy}) \frac{1}{\Gamma(2) y}\left(\frac{y}{2}\right)^2 e^{-y/2} dy = 1 - \frac{1}{4}\int_0^\infty y e^{-y(x+1/2)} dy = 1 - \frac{1}{4(x+1/2)^2}$$

$$F(1/2) = 1 - \frac{1}{4(1/2 + 1/2)^2} = 0.75$$

There are various ways to evaluate the integral (1) Integration by parts. (2) Recognize that $\int_0^\infty y(x+1/2) e^{-y(x+1/2)} dy$ is the expected value of an exponential random variable with mean $(x+1/2)^{-1}$. (3) Recognize that $\Gamma(2)(x+1/2)^2 y e^{-y(x+1/2)}$ is the density function for a gamma random variable with $\alpha = 2$ and $\theta = (x+1/2)^{-1}$, so it would integrate to 1.

Question #205
Key: D

State#	Number	Probability of needing Therapy	Mean Number of visits E(X)	E(N)	Var(N)	Var(X)	E(S)	Var(S)
1	400	0.2	2	80	64	6	160	736
2	300	0.5	15	150	75	240	2,250	52,875
3	200	0.3	9	60	42	90	540	8,802
							2,950	62,413

$$\text{Std Dev}(S) = \sqrt{62413} = 250$$

$$\Pr(S > 3000) = \Pr\left(\frac{S-2950}{250} > \frac{50}{250}\right) = 1 - \Phi(0.2) = 0.42$$

The Var(X) column came from the formulas for mean and variance of a geometric distribution. Using the continuity correction, solving for Pr(S > 3000.5) is theoretically better but does not affect the rounded answer.

Question #206
Key: B

Frequency is geometric with $\beta = 2$, so $p_0 = 1/3,\ p_1 = 2/9,\ p_2 = 4/27$.

Convolutions of $f_X(x)$ needed are

x	$f(x)$	$f^{*2}(x)$
5	0.2	0
10	0.3	0.04

$$f_S(0) = 1/3 = 0.333,\ f_S(5) = (2/9)(0.2) = 0.044,\ f_S(10) = (2/9)(0.3) + (4/27)(0.04) = 0.073$$

$$E(X) = 0.2(5) + 0.3(10) + 0.5(20) = 14$$

$$E(S) = 2E(X) = 28$$

$$E[(S-15)_+] = E(S) - 5[1-F(0)] - 5[1-F(5)] - 5[1-F(10)]$$

$$= 28 - 5(1-0.333) - 5(1-0.333-0.044) - 5(1-0.333-0.044-0.073) = 18.81$$

Alternatively,

$$E[(S-15)_+] = E(S) - 15 + 15f_S(0) + 10f_S(5) + 5f_S(10)$$

$$= 28 - 15 + 15(0.333) + 10(0.044) + 5(0.073) = 18.81$$

Question #207
Key: E

$$S_X(4) = 1 - \int_0^4 f_X(s)dx = 1 - \int_0^4 0.02x dx = 1 - 0.01x^2 \Big|_0^4 = 0.84$$

$$f_{Y^P}(y) = \frac{f_X(y+4)}{S_X(4)} = \frac{0.02(y+4)}{0.84} = 0.0238(y+4)^2$$

$$E(Y^P) = \int_0^6 y[0.0238(y+4)]dy = 0.0238\left(\frac{y^3}{3} + \frac{4y^2}{2}\right)\Big|_0^6 = 3.4272$$

Question #208
Key: E

The probability of zero claims is the pgf of the negative binomial distribution applied to the probability that a Poisson variable equals 0.

For the Poisson, $f(0) = e^{-\lambda}$

So $0.067 = \left[1 - \beta(e^{-\lambda} - 1)\right]^{-r} = \left[1 - 3(e^{-\lambda} - 1)\right]^{-2}$

Solving gives $\lambda = 3$

Question #209
Key: D

For any deductible *d* and the given severity distribution

$$E[(X-d)_+] = E(X) - E(X \wedge d) = 3000 - 3000\left(1 - \frac{3000}{3000+d}\right) = 3000\left(\frac{3000}{3000+d}\right) = \frac{9,000,000}{3000+d}$$

So $P_{2005} = 1.2\dfrac{9,000,000}{3000+600} = 3000$

Let *r* denote the reinsurer's deductible relative to insured losses. Thus, the reinsurer's deductible is 600 + *r* relative to losses. Thus

$$R_{2005} = 1.1\left(\frac{9,000,000}{3000+600+r}\right) = 0.55P_{2005} = 0.55(3000) = 1650 \Rightarrow r = 2400$$

In 2006, after 20% inflation, losses will have a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 1.2(3000) = 3600$. The general formula for expected claims with a deductible of *d* is

$$E[(X-d)_+] = 3600\left(\frac{3600}{3600+d}\right) = \frac{12,960,000}{3600+d}$$

$$P_{2006} = 1.2\frac{12,960,000}{3000+600} = 3703,\ R_{2006} = 1.1\frac{12,960,000}{3000+600+2400} = 2160,\ \frac{R_{2006}}{R_{2005}} = \frac{2160}{3703} = 0.583$$

Question #210
Key: C

Consider Disease 1 and Other Diseases as independent Poisson processes with respective lambdas 016(1/16) = 0.01 and 0.16(15/16) = 0.15. Let *S* denote aggregate losses from Disease 1 and *T* denote aggregate losses from other diseases. Let *W* = *S* + *T*.

$$E(S) = 100(0.01)(5) = 5, \quad Var(S) = 100(0.01)(50^2 + 5^2) = 2525$$
$$E(T) = 100(0.15)(10) = 150, \quad Var(T) = 100(0.15)(20^2 + 10^2) = 7500$$

If no one gets the vaccine:
$$E(W) = 5 + 150 = 155, \quad Var(W) = 2525 + 7500 = 10,025$$
$$\Phi(0.7) = 1 - 0.24, \quad A = 155 + 0.7\sqrt{10,025} = 225.08$$

If all get the vaccine, the vaccine cost = 100(0.15) = 15. Then,
$$B = 15 + 150 + 0.7\sqrt{7500} = 225.62, \quad A / B = 0.998$$

Question #211
Key: A

For the current model $f(x) = (1/4)e^{-x/4}$.

Let $g(x)$ be the new density function, which has

(i) $g(x) = c, \quad 0 \le x \le 3$

(ii) $g(x) = ke^{-x/4}, \quad x > 3$

(iii) $c = ke^{-3/4}$, since the function is continuous at $x = 3$

Since g is density function, it must integrate to 1.
$$1 = \int_0^3 c\,dx + \int_3^\infty ke^{-x/4}dx = 3c + 4ke^{-3/4} = 3c + 4c \Rightarrow c = 1/7$$
$$F(3) = \int_0^3 c\,dx = 3c = 3/7 = 0.43$$

Question #212
Key: C

Since loss amounts are uniform on (0, 10), 40% of losses are below the deductible (4), and 60% are above. Thus, claims occur at a Poisson rate $\lambda^* = 0.6(10) = 6$.

Since loss amounts were uniform on (0, 10), claims are uniform on (0, 6).

Let *N* = number of claims; *X* = claim amount; *S* = aggregate claims.

$E(N) = Var(N) = \lambda^* = 6$

$E(X) = (6-0)/2 = 3$

$Var(X) = (6-0)^2/12 = 3$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 6(3) + 3^2(6) = 72$

Question #213
Key: E

N	p_n	np_n	$n^2 p_n$
0	0.1	0	0
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$E(N) = 1.6$	$E(N^2) = 3.4$

$Var(N) = 3.4 - 1.6^2 = 0.84$

$E(X) = Var(X) = \lambda = 3$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 1.6(3) + 3^2(0.84) = 12.36$

Question #214
Key: D

$\hat{S}(300) = 3/10$ (there are three observations greater than 300)

$\hat{H}(300) = -\ln[\hat{S}(300)] = -\ln(0.3) = 1.204$.

Question #215
Key: A

$$E(X \mid \lambda) = Var(X \mid \lambda) = \lambda$$

$$\mu = v = E(\lambda) = \alpha\theta;\ a = Var(\lambda) = \alpha\theta^2;\ k = v/a = 1/\theta$$

$$Z = \frac{n}{n + 1/\theta} = \frac{n\theta}{n\theta + 1}$$

$$0.15 = \frac{\theta}{\theta+1}(1) + \frac{1}{\theta+1}\mu = \frac{\theta+\mu}{\theta+1}$$

$$0.20 = \frac{2\theta}{2\theta+1}(2) + \frac{1}{2\theta+1}\mu = \frac{4\theta+\mu}{2\theta+1}$$

From the first equation,

$0.15\theta + 0.15 = \theta + \mu$ and so $\mu = 0.15 - 0.85\theta$

Then the second equation becomes

$$0.4\theta + 0.2 = 4\theta + 0.15 - 0.85\theta$$

$$0.05 = 2.75\theta;\ \theta = 0.01818$$

Question #216
Key: E

$$0.75 = \frac{1}{1 + (100/\theta)^\gamma};\ 0.25 = \frac{1}{1 + (500/\theta)^\gamma}$$

$$(100/\theta)^\gamma = 1/3;\ (500/\theta)^\gamma = 3$$

Taking the ratio of these two equalities produces $5^\gamma = 9$. From the second equality,

$$9 = [(500/\theta)^2]^\gamma = 5^\gamma;\ (500/\theta)^2 = 5;\ \theta = 223.61$$

Question #217
Key: E

Begin with

y	350	500	1000	1200	1500
s	2	2	1	1	1
r	10	8	5	2	1

Then $\hat{S}_1(1250) = \frac{8}{10}\frac{6}{8}\frac{4}{5}\frac{1}{2} = 0.24$

The likelihood function is

$$L(\theta) = \left[\theta^{-1}e^{-350/\theta}\right]^2 \left[\theta^{-1}e^{-500/\theta}\right]^2 e^{-500/\theta}\theta^{-1}e^{-1000/\theta}\left[e^{-1000/\theta}\right]^2 \theta^{-1}e^{-1200/\theta}\theta^{-1}e^{-1500/\theta} = \theta^{-7}e^{-7900/\theta}$$

$$l(\theta) = -7\ln\theta - \frac{7900}{\theta};\ l'(\theta) = -\frac{7}{\theta} + \frac{7900}{\theta^2} = 0;\ \hat{\theta} = 7900/7$$

$$\hat{S}_2(1250) = e^{-1250(7)/7900} = 0.33$$

The absolute difference is 0.09.

Question #218
Key: E

$$f(x) = -S'(x) = \frac{4x\theta^4}{(\theta^2 + x^2)^3}$$

$$L(\theta) = f(2)f(4)S(4) = \frac{4(2)\theta^4}{(\theta^2 + 2^2)^3}\frac{4(4)\theta^4}{(\theta^2 + 4^2)^3}\frac{\theta^4}{(\theta^2 + 4^2)^2} = \frac{128\theta^{12}}{(\theta^2 + 4)^3(\theta^2 + 16)^5}$$

$$l(\theta) = \ln 128 + 12\ln\theta - 3\ln(\theta^2 + 4) - 5\ln(\theta^2 + 16)$$

$$l'(\theta) = \frac{12}{\theta} - \frac{6\theta}{\theta^2 + 4} - \frac{10\theta}{\theta^2 + 16} = 0;\ 12(\theta^4 + 20\theta^2 + 64) - 6(\theta^4 + 16\theta^2) - 10(\theta^4 + 4\theta^2) = 0$$

$$0 = -4\theta^4 + 104\theta^2 + 768 = \theta^4 - 26\theta^2 - 192$$

$$\theta^2 = \frac{26 \pm \sqrt{26^2 + 4(192)}}{2} = 32;\ \theta = 5.657$$

Question #219
Key: A

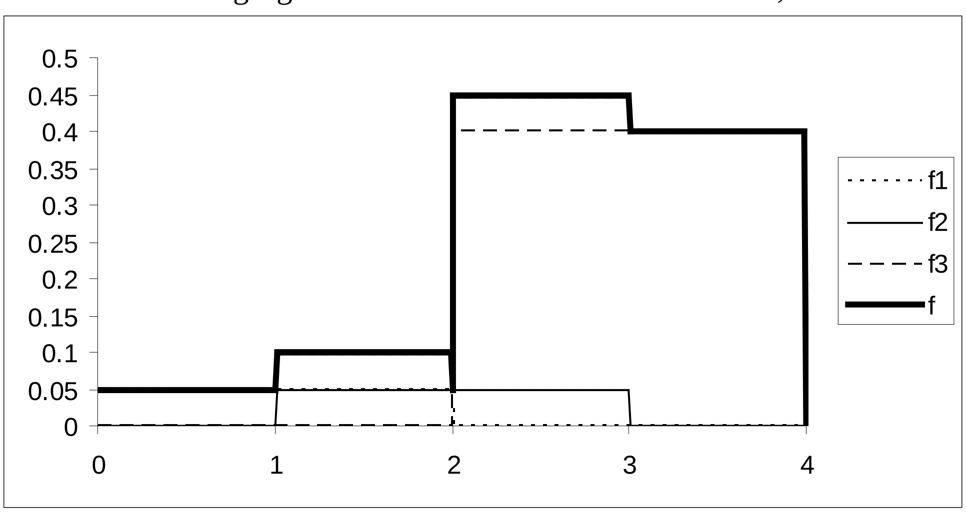
$$E(X\mid\theta)=\int_0^\theta x\frac{2x}{\theta^2}dx=\frac{2\theta}{3};Var(X\mid\theta)=\int_0^\theta x^2\frac{2x}{\theta^2}dx-\frac{4\theta^2}{9}=\frac{\theta^2}{2}-\frac{4\theta^2}{9}=\frac{\theta^2}{18}$$
$$\mu=(2/3)E(\theta)=(2/3)\int_0^1 4\theta^4 d\theta=8/15$$
$$EPV=v=(1/18)E(\theta^2)=(1/18)\int_0^1 4\theta^5 d\theta=1/27$$
$$VHM=a=(2/3)^2 Var(\theta)=(4/9)\left[4/6-(4/5)^2\right]=8/675$$
$$k=\frac{1/27}{8/675}=25/8;\ Z=\frac{1}{1+25/8}=8/33$$
Estimate is $(8/33)(0.1)+(25/33)(8/15)=0.428.$

Question #220
Key: D

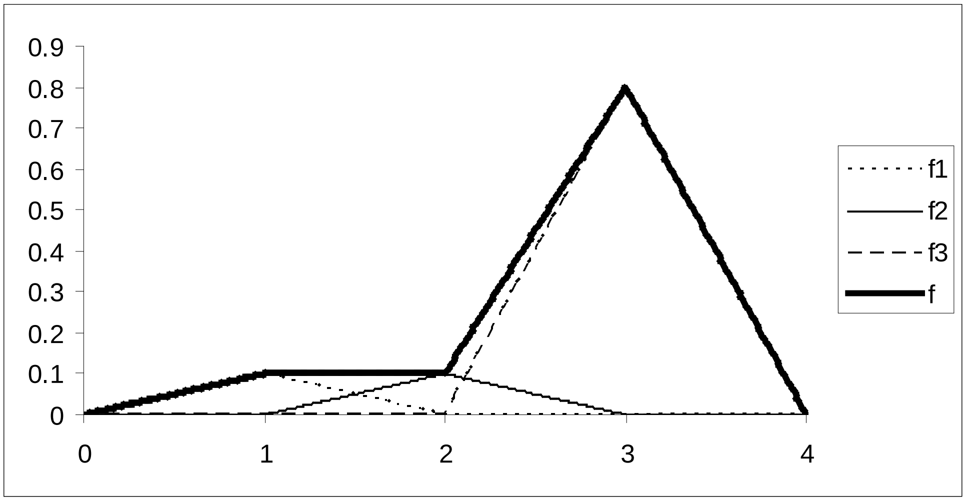
From the Poisson(4) distribution the probabilities at 0, 1, and 2 are 0.0183, 0.0733, and 0.1463. The cumulative probabilities are 0.0183, 0.0916, and 0.2381. Because 0.0916 < 0.13 < 0.2381 the simulated number of claims is 2. Claim amounts are simulated from solving $u=1-e^{-x/1000}$ for $x=-1000\ln(1-u)$. The two simulated amounts are 51.29 and 2995.73 for a total of 3047.02

Question #221
Key: B

It may be easiest to show this by graphing the density functions. For the first function the three components are each constant. One is of height 1/20 from 0 to 2 (representing the empirical probability of 1/10 at 1, one is height 1/20 from 1 to 3 and one is height 8/20 from 2 to 4. The following figure shows each of them and their sum, the kernel density estimator.



The triangular one is similar. For the triangle from 0 to 2, the area must be 1/10. With a base of 2, the height is 1/10. the same holds for the second triangle. The third has height 8/10. When added they look as follows;



The question asks about cumulative probabilities. From 0 to 1 the first is linear and the second is quadratic, but by $x = 1$ both have accumulated 0.05 of probability. Because the cumulative distribution functions are the same at 1 and the density functions are identical from 1 to 2, the distribution functions must be identical from 1 to 2.

Question #222
Key: E

For the Poisson distribution, the mean, λ, is estimated as 230/1000 = 0.23.

# of Days	Poisson Probability	Expected # of Workers	Observed # of Workers	Chi-square
0	0.794533	794.53	818	0.69
1	0.182743	182.74	153	4.84
2	0.021015	21.02	25	0.75
3 or more	0.001709	1.71	4	3.07
Total			1000	9.35

The chi-square distribution has 2 degrees of freedom because there are four categories and the Poisson parameter is estimated (d.f. = 4 – 1 – 1 = 2). The critical values for a chi-square test with two degrees of freedom are shown in the following table.

Significance Level	Critical Value
10%	4.61
5%	5.99
2.5%	7.38
1%	9.21

9.35 is greater than 9.21 so the null hypothesis is rejected at the 1% significance level.

Question #223
Key: D

$EPV = \hat{v} = \dfrac{25(480-472.73)^2 + 30(466.67-472.73)^2}{2-1} = 2423.03$ where 480 = 12,000/25, 466.67 = 14,000/30, and 472.73 = 26,000/55.

$k = 2423.03/254 = 9.54; \; Z = \dfrac{55}{55+9.54} = 0.852$

Question #224
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Question #225
Key: C

The quantity of interest is $P = \Pr(X \le 5000) = \Phi\left(\dfrac{\ln 5000 - \mu}{\sigma}\right)$. The point estimate is

$\Phi\left(\dfrac{\ln 5000 - 6.84}{1.49}\right) = \Phi(1.125) = 0.87$.

For the delta method:

$\dfrac{\partial P}{\partial \mu} = \dfrac{-\phi(1.125)}{1.49} = -0.1422; \; \dfrac{\partial P}{\partial \sigma} = \dfrac{-1.125\phi(1.125)}{1.49} = -0.1600$ where $\phi(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}$.

Then the variance of $\hat{P}$ is estimated as $(-0.1422)^2 0.0444 + (-0.16)^2 0.0222 = 0.001466$ and the lower limit is $P_L = 0.87 - 1.96\sqrt{0.001466} = 0.79496$.

Question #226
Key: A

$$\Pr(\theta = 0.1 \mid X_1 = 1) = \frac{\Pr(X_1 = 1 \mid \theta = 0.1)\Pr(\theta = 0.1)}{\Pr(X_1 = 1 \mid \theta = 0.1)\Pr(\theta = 0.1) + \Pr(X_1 = 1 \mid \theta = 0.3)\Pr(\theta = 0.3)}$$

$$= \frac{0.1(0.8)}{0.1(0.8) + 0.3(0.2)} = \frac{4}{7}$$

Then,

$E(X_2 \mid \theta = 0.1) = 0(0.2) + 1(0.1) + 2(0.7) = 1.5$

$E(X_2 \mid \theta = 0.3) = 0(0.6) + 1(0.3) + 2(0.1) = 0.5$

$E(X_2 \mid X_1 = 1) = (1.5)(4/7) + (0.5)(3/7) = 1.071$

Question #227
Key: D

The requirement is that

$$0.01\hat{F}(1500) \geq 1.96\sqrt{\frac{\hat{F}(1500)\hat{S}(1500)}{N}}$$

$$0.0001\frac{P^2}{N^2} \geq 3.8416\frac{P(N-P)}{N^3}$$

$$\frac{NP}{N-P} \geq 38,416.$$

For the five answer choices, the left hand side is 34,364, 15,000, 27,125, 39,243, and 37,688. Only answer D meets the condition.

Question #228
Key: D

$$\frac{s_4}{r_4} = \hat{H}(y_4) - \hat{H}(y_3) = 0.5691 - 0.4128 = 0.1563.$$

$$\frac{s_4}{r_4^2} = \hat{V}[\hat{H}(y_4)] - \hat{V}[\hat{H}(y_3)] = 0.014448 - 0.009565 = 0.004883.$$

Therefore, $s_4 = \dfrac{(s_4/r_4)^2}{s_4/r_4^2} = \dfrac{0.1563^2}{0.004833} = 5.$

Question #229
Key: A

$$\ln f(x) = \ln\theta - 2\ln(\theta + x)$$

$$\frac{\partial \ln f(x)}{\partial\theta} = \frac{1}{\theta} - \frac{2}{\theta + x}$$

$$\frac{\partial^2 \ln f(x)}{\partial\theta^2} = -\frac{1}{\theta^2} + \frac{2}{(\theta + x)^2}$$

$$E\left[\frac{\partial^2 \ln f(x)}{\partial\theta^2}\right] = -\frac{1}{\theta^2} + \int_0^\infty \frac{2\theta}{(\theta+x)^4}dx = -\frac{1}{\theta^2} + \left[-\frac{2\theta}{3(\theta+x)^3}\right]\Bigg|_0^\infty = -\frac{1}{\theta^2} + \frac{2}{3\theta^2} = -\frac{1}{3\theta^2}$$

$$I(\theta) = \frac{n}{3\theta^2}; \quad Var = \frac{3\theta^2}{n}$$

Question #230
Key: B

$$\mu = E[E(X \mid \lambda)] = E(\lambda) = 1(0.9) + 10(0.09) + 20(0.01) = 2$$
$$EPV = v = E[Var(X \mid \lambda)] = E(\lambda) = 2$$
$$VHM = a = Var[E(X \mid \lambda)] = Var(\lambda) = 1(0.9) + 100(0.09) + 400(0.01) - 2^2 = 9.9$$
$$Z = \frac{1}{1 + 2/9.9} = 0.83193; \quad 11.983 = 0.83193x + 0.16807(2); \quad x = 14$$

Question #231
Key: A

The given interval for H can be written as $0.775 \pm 1.96\sqrt{0.063}$ and therefore the estimated variance of $\hat{H}$ is 0.063. To apply the delta method,
$$S = e^{-H}; \quad \frac{dS}{dH} = -e^{-H}; \quad V\hat{a}r(\hat{S}) \approx (-e^{-\hat{H}})^2 Var(\hat{H}) = (-e^{-0.775})^2(0.063) = 0.134.$$
The point estimate of S is $e^{-0.775} = 0.4607$ and the confidence interval is $0.4607 \pm 1.96\sqrt{0.0134} = 0.2269$ or (0.23, 0.69).

Question #232
Key: B

The first step is to trend the year 1 data by 1.21 and the year 2 data by 1.1. The observations are now 24.2, 48.4, 60.5, 33, 44, 99, and 132.
The first two sample moments are 63.014 and 5262.64. The equations to solve are
$$63.014 = e^{\mu + 0.5\sigma^2}; \quad 4.14336 = \mu + 0.5\sigma^2$$
$$5262.64 = e^{2\mu + 2\sigma^2}; \quad 8.56839 = 2\mu + 2\sigma^2.$$
Taking four times the first equation and subtracting the second gives 2μ and therefore
$$\mu = \frac{4(4.14336) - 8.56839}{2} = 4.00.$$

Question #233
Key: A

$$\hat{\mu} = \bar{x} = 12/60 = 0.2, \; EVPV = \hat{v} = \bar{x} = 0.2$$
$$VHM = \hat{a} = \frac{10(0.4-0.2)^2 + 20(0.25-0.2)^2 + 30(0.1-0.2)^2 - (3-1)(0.2)}{60 - \frac{10^2 + 20^2 + 30^2}{60}} = 0.009545$$
$$\hat{k} = 20.9524; \quad Z = \frac{10}{10 + 20.9524} = 0.323$$

Question #234
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Question #235
Key: C

$l(\tau,\theta)=\sum_{j=1}^{5}\ln f(x_j)=\sum_{j=1}^{5}\ln\tau+(\tau-1)\ln x_j-\tau\ln\theta-(x_j/\theta)^{\tau}$. Under the null hypothesis it is $l(2,\theta)=\sum_{j=1}^{5}\ln 2+\ln x_j-2\ln\theta-(x_j/\theta)^2$. Inserting the maximizing value of 816.7 for θ gives –35.28. The likelihood ratio test statistic is 2(– 33.05 + 35.28) = 4.46. There is one degree of freedom. At a 5% significance level the critical value is 3.84 and at a 2.5% significance level it is 5.02.

Question #236
Key: C

It is given that $n = 4$, $v = 8$, and $Z = 0.4$. Then, $0.4=\dfrac{4}{4+\dfrac{8}{a}}$ which solves for $a = 4/3$. For the covariance,

$$\begin{aligned}Cov(X_i,X_j)&=E(X_iX_j)-E(X_i)E(X_j)\\&=E[E(X_iX_j\mid\theta)]-E[E(X_i\mid\theta)]E[E(X_j\mid\theta)]\\&=E[\mu(\theta)^2]-E[\mu(\theta)]^2=Var[\mu(\theta)]=a=4/3.\end{aligned}$$

Question #237
Key: A

U	z	x	lognormal	with deductible
0.6217	0.31	5.8325	341.21	241.21
0.9941	2.52	7.49	1790.05	1690.05
0.8686	1.12	6.44	626.41	526.41
0.0485	–1.66	4.355	77.87	0
			Average	614.42

The value of z is obtained by inversion from the standard normal table. That is, $u=\Pr(Z\le z)$. The value of x is obtained from $x = 0.75z + 5.6$. The lognormal value is obtained by exponentiating x and the final column applies the deductible.

Question #238
Key: B

$$MSE = E[(X^2 - \theta^2)^2] = E(X^4 - 2X^2\theta^2 + \theta^4)$$
$$= 24\theta^4 - 2(2\theta^2)\theta^2 + \theta^4 = 21\theta^4$$

Question #239
Key: C

The sample mean of $\frac{157(0)+66(1)+19(2)+4(3)+2(4)}{248} = 0.5$ is the maximum likelihood estimate of the geometric parameter β as well as the method of moments estimate of the Poisson parameter λ. Then, $P = (1+0.5)^{-1} = 0.6667$ and $Q = e^{-0.5} = 0.6065$. The absolute difference is 0.0602.

Question #240
Key: D

$\bar{x} = \frac{5000(0)+2100(1)+750(2)+100(3)+50(4)}{8000} = 0.5125$ and
$s^2 = \frac{5000(0.5125)^2 + 2100(0.4875)^2 + 750(1.4875)^2 + 100(2.4875)^2 + 50(3.4875)^2}{7999} = 0.5874$.
Then, $\hat{\mu} = \hat{v} = \bar{x} = 0.5125$ and $\hat{a} = s^2 - \bar{x} = 0.0749$. The credibility factor is
$Z = \frac{1}{1+0.5125/0.0749} = 0.1275$ and the estimate is 0.1275(1) + 0.8725(0.5125) = 0.5747.

Question #241
Key: B

$s = F_n(3000) = 4/8 = 0.5$ because for the *p-p* plot the denominator is *n*+1.
$t = F(3000) = 1 - e^{-3000/3300} = 0.59711$. For the difference plot, *D* uses a denominator of *n* and so *D* = 4/7 – 0.59711 = –0.02568 and the answer is 0.5 – 0.59711 + 0.02568 = –0.071.

Question #242
Key: B

$\pi(q\,|\,2,2) \propto f(2\,|\,q) f(2\,|\,q)\pi(q) = q(q)(q^2/0.039) \propto q^4$. Because $\int_{0.2}^{0.5} q^4 dq = 0.006186$,
$\pi(q\,|\,2,2) = q^4/0.006186$. Given *q*, the expected number of claims is
$E(N\,|\,q) = 0(0.1) + 1(0.9-q) + 2q = 0.9 + q$. The Bayesian estimate is
$E(N\,|\,2,2) = \int_{0.2}^{0.5} (0.9+q)\frac{q^4}{0.006186} dq = 1.319.$

Question #243
Key: E

$$0.689 = F_{500}(1500) = 0.5F_{500}(1000) + 0.5F_{500}(2000) = 0.5\left(\frac{200+110}{500} + \frac{310+x}{500}\right) \Rightarrow x = 69$$

$$0.839 = F_{500}(3500) = 0.5F_{500}(2000) + 0.5F_{500}(5000) = 0.5\left(\frac{310+69}{500} + \frac{379+y}{500}\right) \Rightarrow y = 81$$

Question #244
Key: A

A is false because the test works best when the expected number of observations is about the same from interval to interval.

Question #245
Key: E

$$n\lambda \ge \lambda_0\left[1+\left(\frac{\sigma_Y}{\theta_Y}\right)^2\right]; \theta_Y = \alpha\theta = 10,000\alpha; \sigma_Y^2 = \alpha\theta^2 = 10^8\alpha$$

$$n\lambda \ge \left(\frac{1.96}{0.1}\right)^2\left[1+\frac{10^8\alpha}{10^8\alpha^2}\right] = 384.16(1+\alpha^{-1})$$

Because α is needed, but not given, the answer cannot be determined from the information given.

Question #246
Key: E

With n + 1 = 16, we need the 0.3(16) = 4.8 and 0.65(16) = 10.4 smallest observations. They are 0.2(280) + 0.8(350) = 336 and 0.6(450) + 0.4(490) = 466.
The equations to solve are:

$$0.3 = 1 - \left(\frac{\theta^\gamma}{\theta^\gamma + 336^\gamma}\right)^2 \text{ and } 0.65 = 1 - \left(\frac{\theta^\gamma}{\theta^\gamma + 466^\gamma}\right)^2$$

$$(0.7)^{-1/2} = 1 + (336/\theta)^\gamma \text{ and } (0.35)^{-1/2} = 1 + (466/\theta)^\gamma$$

$$\frac{(0.7)^{-1/2} - 1}{(0.35)^{-1/2} - 1} = \frac{(336/\theta)^\gamma}{(466/\theta)^\gamma}$$

$$0.282814 = (336/466)^\gamma$$

$$\ln(0.282814) = \gamma \ln(336/466)$$

$$\gamma = 3.8614.$$

Question #247
Key: D

Let E be the even of having 1 claim in the first four years. In four years, the total number of claims is Poisson(4λ).

$$\Pr(Type\,I \mid E) = \frac{\Pr(E \mid Type\,I)\Pr(Type\,I)}{\Pr(E)} = \frac{e^{-1}(0.05)}{\Pr(E)} = \frac{0.01839}{\Pr(E)} = 0.14427$$

$$\Pr(Type\,II \mid E) = \frac{e^{-2}(2)(0.2)}{\Pr(E)} = \frac{0.05413}{\Pr(E)} = 0.42465$$

$$\Pr(Type\,III \mid E) = \frac{e^{-4}(4)(0.75)}{\Pr(E)} = \frac{0.05495}{\Pr(E)} = 0.43108$$

$$Note: \Pr(E) = 0.01839 + .05413 + .05495 = 0.12747$$

The Bayesian estimate of the number of claims in Year 5 is:
0.14427(0.25) + 0.42465(0.5) + 0.43108(1) = 0.67947.

Question #248
DELETED

Question #249
Key: C

Because 0.656 < 0.7654 < 0.773, the simulated number of losses is 4. To simulate a loss by inversion, use

$$F(x) = 1 - e^{-(x/\theta)^\tau} = u$$

$$1 - u = e^{-(x/\theta)^\tau}$$

$$\ln(1-u) = -(x/\theta)^\tau$$

$$x = \theta(-\ln(1-u))^{1/\tau} = 200(-\ln(1-u))^{1/2}$$

$$u_1 = 0.2738,\; x_1 = 113.12$$

$$u_2 = 0.5152,\; x_2 = 170.18$$

$$u_3 = 0.7537,\; x_3 = 236.75$$

$$u_4 = 0.6481,\; x_4 = 204.39$$

With a deductible of 150, the first loss produces no payments and 113.12 toward the 500 limit. The second loss produces a payment of 20.18 and the insured is now out-of-pocket 263.12. The third loss produces a payment of 86.75 and the insured is out 413.12. The deductible on the fourth loss is then 86.88 for a payment of 204.29 – 86.88 = 117.51.

The total paid by the insurer is 20.18 + 86.75 + 117.51 = 224.44.

Question #250
Key: A

The density function is $f(x) = \theta x^{-2} e^{-\theta/x}$ and the likelihood function is

$$L(\theta) = \theta(186^{-2})e^{-\theta/186}\theta(91^{-2})e^{-\theta/91}\theta(66^{-2})e^{-\theta/66}(e^{-\theta/60})^7 \propto \theta^3 e^{-0.148184\theta}$$

$$l(\theta) = \ln L(\theta) = 3\ln(\theta) - 0.148184\theta$$

$$l'(\theta) = 3\theta^{-1} - 0.148184 = 0$$

$$\theta = 3/0.148184 = 20.25.$$

The mode is $\theta/2 = 20.25/2 = 10.125$.

Question #251
Key: D

We have $\mu(\theta) = 4\theta$ and $\mu = 4E(\theta) = 4(600) = 2400$. The average loss for Years 1 and 2 is 1650 and so 1800 = Z(1650) + (1 – Z)(2400) which gives Z = 0.8. Because there were two years, Z = 0.8 = 2/(2 + k) which gives k = 0.5.

For three years, the revised value is Z = 3/(3 + 0.5) = 6/7 and the revised credibility estimate (using the new sample mean of 2021), (6/7)(2021) + (1/7)(2400) = 2075.14.

Question #252
Key: B

The uncensored observations are 4 and 8 (values beyond 11 are not needed). The two r values are 10 and 5 and the two s values are 2 and 1. The Kaplan-Meier estimate is $\hat{S}(11) = (8/10)(4/5) = 0.64$ and Greenwood's estimate is $(0.64)^2\left(\frac{2}{10(8)} + \frac{1}{5(4)}\right) = 0.03072$.

Question #253
Key: E

$S_m \mid Q \sim bin(m, Q)$ and $Q \sim beta(1, 99)$. Then

$E(S_m) = E[E(S_m \mid Q)] = E(mQ) = m\dfrac{1}{1+99} = 0.01m$. For the mean to be at least 50, m must be at least 5,000.

Question #254
Key: D

The posterior distribution is

$\pi(\lambda \mid data) \propto (e^{-\lambda})^{90}(\lambda e^{-\lambda})^{7}(\lambda^{2}e^{-\lambda})^{2}(\lambda^{3}e^{-\lambda})\dfrac{\lambda^{4}e^{-50\lambda}}{\lambda} = \lambda^{17}e^{-150\lambda}$ which is a gamma distribution with parameters 18 and 1/150. For one risk, the estimated value is the mean, 18/150. For 100 risks it is 100(18)/150 = 12.

Alternatively,

The prior distribution is gamma with $\alpha = 4$ and $\beta = 50$. The posterior will continue to be gamma, with $\alpha' = \alpha + \text{no. of claims} = 4 + 14 = 18$ and
$\beta' = \beta + \text{no. of exposures} = 50 + 100 = 150$. Mean of the posterior is
$\alpha / \beta = 18/150 = 0.12$.
Expected number of claims for the portfolio is 0.12(100) = 12.

Question #255
Key: E

$$0.95 = \Pr(0.95\mu < \bar{X} < 1.05\mu)$$
$$\bar{X} \sim N(\mu, \sigma^2/n = 1.44\mu^2/n)$$
$$0.95 = \Pr\left(\frac{0.95\mu - \mu}{1.2\mu/\sqrt{n}} < Z < \frac{1.05\mu - \mu}{1.2\mu/\sqrt{n}}\right)$$
$$0.95 = \Pr(-0.05\sqrt{n}/1.2 < Z < 0.05\sqrt{n}/1.2)$$
$$0.05\sqrt{n}/1.2 = 1.96$$
$$n = 2212.76.$$

Question #256
Key: B

$$L(q) = \left[\binom{2}{0}(1-q)^2\right]^{5000}\left[\binom{2}{1}q(1-q)\right]^{5000} = 2^{5000}q^{5000}(1-q)^{15000}$$
$$l(q) = 5000\ln(2) + 5000\ln(q) + 15000\ln(1-q)$$
$$l'(q) = 5000q^{-1} - 15000(1-q)^{-1} = 0$$
$$\hat{q} = 0.25$$
$$l(0.25) = 5000\ln(2) + 5000\ln(0.25) + 15000\ln(0.75) = -7780.97.$$

Question #257
Key: C

The estimate of the overall mean, μ, is the sample mean, per vehicle, which is 7/10 = 0.7. With the Poisson assumption, this is also the estimate of v = EPV. The means for the two insureds are 2/5 = 0.4 and 5/5 = 1.0. The estimate of a is the usual non-parametric estimate,

$$\text{VHM} = \hat{a} = \frac{5(0.4-0.7)^2 + 5(1.0-0.7)^2 - (2-1)(0.7)}{10 - \frac{1}{10}(25+25)} = 0.04$$

Then, k = 0.7/0.04 = 17.5 and so Z = 5/(5+17.5) = 2/9. The estimate for insured A is (2/9)(0.4) + (7/9)(0.7) = 0.6333.

Question #258
Key: A

Item (i) indicates that X must one of the four given values.
Item (ii) indicates that X cannot be 200
Item (iii) indicates that X cannot be 400.
First assume X = 100. Then the values of r are 5, 3, 2, and 1 and the values of s are 2, 1, 1, and 1. Then $\hat{H}(410) = \frac{2}{5} + \frac{1}{3} + \frac{1}{2} + \frac{1}{1} = 2.23$ and thus the answer is 100. As a check, if X = 300, the r values are 5, 4, 3, and 1 and the s values are 1, 1, 2, and 1. Then,
$\hat{H}(410) = \frac{1}{5} + \frac{1}{4} + \frac{2}{3} + \frac{1}{1} = 2.12$.

Question #259
Key: B

The estimator of the Poisson parameter is the sample mean. Then,
$E(\hat{\lambda}) = E(\bar{X}) = \lambda$
$Var(\hat{\lambda}) = Var(\bar{X}) = \lambda / n$
$c.v. = \sqrt{\lambda / n} / \lambda = 1/\sqrt{n\lambda}$
It is estimated by $1/\sqrt{n\lambda} = 1/\sqrt{39} = 0.1601$.

Question #260
Key: E

$$\Pr\left(\theta=8\mid X_1=5\right)=\frac{\Pr(X_1=5\mid\theta=8)\Pr(\theta=8)}{\Pr(X_1=5\mid\theta=8)\Pr(\theta=8)+\Pr(X_1=5\mid\theta=2)\Pr(\theta=2)}$$

$$=\frac{0.125e^{-5(0.125)}(0.8)}{0.125e^{-5(0.125)}(0.8)+0.5e^{-5(0.5)}(0.2)}=0.867035.$$

Then,

$E(X_2\mid X_1=5)=E(\theta\mid X_1=5)=0.867035(8)+0.132965(2)=7.202.$

Question #261
DELETED

Question #262
Key: D

$$L(\omega)=\frac{\frac{1}{\omega}\frac{1}{\omega}\frac{1}{\omega}\left(\frac{\omega-4-p}{\omega}\right)^2}{\left(\frac{\omega-4}{\omega}\right)^5}=\frac{(\omega-4-p)^2}{(\omega-4)^5}$$

$$l(\omega)=2\ln(\omega-4-p)-5\ln(\omega-4)$$

$$l'(\omega)=\frac{2}{\omega-4-p}-\frac{5}{\omega-4}=0$$

$$0=l'(29)=\frac{2}{25-p}-\frac{5}{25}$$

$p=15.$

The denominator in the likelihood function is $S(4)$ to the power of five to reflect the fact that it is known that each observation is greater than 4.

Question #263
Key: B

$$\mu(\lambda)=v(\lambda)=\lambda$$

$$\mu=v=E(\lambda)=0.1\Gamma(1+1/2)=0.088623$$

$$a=Var(\lambda)=(0.1)^2\Gamma(1+2/2)-0.088623^2=0.002146$$

$$Z=\frac{500}{500+0.088623/0.002146}=0.92371.$$

The estimate for one insured for one month is 0.92371(35/500) + 0.07629(0.088623) = 0.07142. For 300 insureds for 12 months it is (300)(12)(0.07142) = 257.11.

Question #264
Key: D

With no censoring the *r* values are 12, 9, 8, 7, 6, 4, and 3 and the *s* values are 3, 1, 1, 1, 2, 1, 1 (the two values at 7500 are not needed). Then,

$$\hat{H}_1(7000) = \frac{3}{12} + \frac{1}{9} + \frac{1}{8} + \frac{1}{7} + \frac{2}{6} + \frac{1}{4} + \frac{1}{3} = 1.5456.$$

With censoring, there are only five uncensored values with *r* values of 9, 8, 7, 4, and 3 and all five *s* values are 1. Then,

$\hat{H}_2(7000) = \frac{1}{9} + \frac{1}{8} + \frac{1}{7} + \frac{1}{4} + \frac{1}{3} = 0.9623.$ The absolute difference is 0.5833.

Question #265
Key: A

The simulated paid loss is $\exp[0.494\Phi^{-1}(u) + 13.294]$ where $\Phi^{-1}(u)$ is the inverse of the standard normal distribution function. The four simulated paid losses are 450,161, 330,041, 939,798, and 688,451 for an average of 602,113. The multiplier for unpaid losses is $0.801(2006-2005)^{0.851}e^{-0.747(2006-2005)} = 0.3795$ and the answer is 0.3795(602,113) = 228,502

Question #266
This question has been moved to become Question 306

Question #267
Key: E

$$\Pr\left(\lambda=1 \mid X_1=r\right)=\frac{\Pr(X_1=r\mid\lambda=1)\Pr(\lambda=1)}{\Pr(X_1=r\mid\lambda=1)\Pr(\lambda=1)+\Pr(X_1=r\mid\lambda=3)\Pr(\lambda=3)}$$

$$=\frac{\dfrac{e^{-1}}{r!}(0.75)}{\dfrac{e^{-1}}{r!}(0.75)+\dfrac{e^{-3}3^r}{r!}(0.25)}=\frac{0.2759}{0.2759+0.1245(3^r)}.$$

Then,

$$2.98=\frac{0.2759}{0.2759+0.1245(3^r)}(1)+\frac{0.1245(3^r)}{0.2759+0.1245(3^r)}(3)=\frac{0.2759+0.3735(3^r)}{0.2759+0.1245(3^r)}.$$

Rearrange to obtain

$$0.82218+0.037103(3^r)=0.2759+0.03735(3^r)$$

$$0.54628=0.00025(3^r)$$

$$r=7.$$

Because the risks are Poisson:

$$\mu=v=E(\lambda)=0.75(1)+0.25(3)=1.5$$

$$a=Var(\lambda)=0.75(1)+0.25(9)-2.25=0.75$$

$$Z=\frac{1}{1+1.5/0.75}=1/3$$

and the estimate is (1/3)(7) + (2/3)(1.5) = 3.33.

Question #268
Key: E

The uniform kernel spreads the probability of 0.1 to 10 units each side of an observation. So the observation at 25 contributes a density of 0.005 from 15 to 35, contributing nothing to survival past age 40. The same applies to the point at 30. For the next 7 points:
35 contributes probability from 25 to 45 for 5(0.005) = 0.025 above age 40.
35 contributes probability from 25 to 45 for 5(0.005) = 0.025 above age 40.
37 contributes probability from 27 to 47 for 7(0.005) = 0.035 above age 40.
39 contributes probability from 29 to 49 for 9(0.005) = 0.045 above age 40.
45 contributes probability from 35 to 55 for 15(0.005) = 0.075 above age 40.
47 contributes probability from 37 to 57 for 17(0.005) = 0.085 above age 40.
49 contributes probability from 39 to 59 for 19(0.005) = 0.095 above age 40.
The observation at 55 contributes all 0.1 of probability. The total is 0.485.

Question #269
Key: A

$X \sim Exp(\theta)$

$\sum_{i=1}^{n} X_i \sim \Gamma(n,\theta)$

$\bar{X} \sim \Gamma(n,\theta / n)$

$E(\bar{X}^2) = (\theta / n)^2 (n)(n+1) = (n+1)\theta^2 / n.$

The second line follows because an exponential distribution is a gamma distribution with $\alpha = 1$ and the sum of independent gamma random variables is gamma with the "α" parameters added. The third line follows because the gamma distribution is a scale distribution. Multiplying by 1/*n* retains the gamma distribution with the "θ" parameter multiplied by 1/*n*.

Question #270
Key: C

The sample means are 3, 5, and 4 and the overall mean is 4. Then,

$$\hat{v} = \frac{1+0+0+1+0+0+1+1+1+1+1+1}{3(4-1)} = \frac{8}{9}$$

$$\hat{a} = \frac{(3-4)^2+(5-4)^2+(4-4)^2}{3-1} - \frac{8/9}{4} = \frac{7}{9} = 0.78.$$

Question #271
DELETED

Question #272
Key: C

$\pi(q \mid 2) = 6q^2(1-q)^2 6q(1-q) \propto q^3(1-q)^3$

The mode can be determined by setting the derivative equal to zero.

$\pi'(q \mid 2) \propto 3q^2(1-q)^3 - 3q^3(1-q)^2 = 0$

$(1-q) - q = 0$

$q = 0.5.$

Question #273
Key: B

For the severity distribution the mean is 5,000 and the variance is $10,000^2/12$. For credibility based on accuracy with regard to the number of claims,

$$2000 = \left(\frac{z}{0.03}\right)^2, \quad z^2 = 1.8$$

Where z is the appropriate value from the standard normal distribution. For credibility based on accuracy with regard to the total cost of claims, the number of claims needed is

$$\frac{z^2}{0.05^2}\left(1+\frac{10000^2/12}{5000^2}\right) = 960.$$

Question #274
Key: C

$$\hat{S}(10) = e^{-\hat{H}(10)} = 0.575$$

$$\hat{H}(10) = -\ln(0.575) = 0.5534 = \frac{1}{50}+\frac{3}{49}+\frac{5}{k}+\frac{7}{12}.$$

The solution is $k = 36$.

Question #275
Key: A

The annual dental charges are simulated from

$$u = 1-e^{-x/1000}$$

$$x = -1000\ln(1-u).$$

The four simulated values are 356.67, 2525.73, 1203.97, and 83.38. The reimbursements are 205.34 (80% of 256.67), 1000 (the maximum), 883.18 (80% of 1103.97), and 0. The total is 2088.52 and the average is 522.13.

Question #276
Key: B

$$L(\theta)=\left(1-\frac{\theta}{10}\right)^9\left(\frac{\theta}{10}-\frac{\theta}{25}\right)^6\left(\frac{\theta}{25}\right)^5\propto(10-\theta)^9\theta^{11}$$
$$l(\theta)=9\ln(10-\theta)+11\ln(\theta)$$
$$l'(\theta)=-\frac{9}{10-\theta}+\frac{11}{\theta}=0$$
$$11(10-\theta)=9\theta$$
$$110=20\theta$$
$$\theta=110/20=5.5.$$

Question #277
Key: A

The maximum likelihood estimate is $\hat{\theta}=\bar{x}=1000$. The quantity to be estimated is $S(\theta)=\exp(-1500/\theta)$ and $S'(\theta)=1500\theta^{-2}\exp(-1500/\theta)$. For the delta method,
$$Var[S(\hat{\theta})]\cong[S'(\hat{\theta})]^2 Var(\hat{\theta})=[1500(1000)^{-2}\exp(-1500/1000)]^2(1000^2/6)=0.01867.$$
This is based on $Var(\hat{\theta})=Var(\bar{X})=Var(X)/n=\theta^2/n$.

Question #278
Key: A

Based on the information given
$$0.21=\frac{36}{n}+\frac{0.4x}{n}$$
$$0.51=\frac{36}{n}+\frac{x}{n}+\frac{0.6y}{n}$$
$$n=200+x+y.$$
Then,
$$0.21(200+x+y)=36+0.4x$$
$$0.51(200+x+y)=36+x+0.6y$$
and these linear equations can be solved for x = 119.37.

Question #279
Key: B

Pays 80% of loss over 20, with cap of payment at 60, hence u = 60/0.8 + 20 = 95.

$$E(Y \text{ per loss}) = \alpha[E(X \wedge 95) - E(X \wedge 20)] = 0.8\left[\int_0^{95} S(x)dx - \int_0^{20} S(x)dx\right]$$

$$= 0.8\int_{20}^{95} S(x)dx = 0.8\int_{20}^{95}\left(1 - \frac{x^2}{10,000}\right)dx = 0.8\left(x - \frac{x^3}{30,000}\right)\Bigg|_{20}^{95} = 37.35$$

$$E(Y \text{ per payment}) = \frac{E(Y \text{ per loss})}{1 - F(20)} = \frac{37.35}{0.96} = 38.91$$

Question #280
Key: D

Let S = aggregate claims, I_5 = claims covered by stop loss

$E(I_5) = E(S) - 5 - 5\Pr(S = 0)$

$E(S) = 5[0.6(5) + 0.4k] = 15 + 2k$

$\Pr(S = 0) = e^{-5}$

$E(I_5) = 15 + 2k - 5 - 5e^{-5} = 28.03$

$10.034 + 2k = 28.03$

$k = 9$

Question #281
DELETED

Question #282
Key: A

Let S = aggregate losses, X = severity
Since the frequency is Poisson,
$Var(S) = \lambda E\left(X^2\right)$

$E(X^2) = \dfrac{2^2\Gamma(3)\Gamma(1)}{\Gamma(3)} = 4$ (table lookup)

$Var(S) = 3(4) = 12$
You would get the same result if you used
$Var(S) = E(N)Var(X) + Var(N)E(X)^2$

Question #283
Key: D

For each member $P(z)=\left[1-1.5(z-1)\right]^{-1}$
so for family of 4, $P(z)=\left[1-1.5(z-1)\right]^{-4}$ which is negative binomial with $\beta=1.5, r=4$

k	p_k
0	0.026
1	0.061
2	0.092
3+	0.821

$E(N\wedge 3)=0(0.026)+1(0.061)+2(0.092)+3(0.821)=2.71$
$E(N)-E(N\wedge 3)=6-2.71=3.29$
3.29(100 per visit) = 329

Alternatively, without using probability generating functions, a geometric distribution is a special case of the negative binomial with r = 1. Summing four independent negative binomial distributions, each with $\beta=1.5, r=1$ gives a negative binomial distribution with $\beta=1.5, r=4$. Then continue as above.

Question #284
Key: E

$$E(X\wedge 2)=1f(1)+2[1-F(1)]=1f(1)+2[1-f(0)-f(1)]$$
$$=1(3e^{-3})+2(1-e^{-3}-3e^{-3})=2-5e^{-3}=1.75$$

Cost per loss with deductible is
$E(X)-E(X\wedge 2)=3-1.75=1.25$

Cost per loss with coinsurance is $\alpha E(X)=3\alpha$
Equating cost: $3\alpha=1.25\Rightarrow\alpha=0.42$

Question #285
Key: A

Let N be the number of clubs accepted
X be the number of members of a selected club
S be the total persons appearing

N is binomial with m = 1000 q = 0.20

$$E(N) = 1000(0.20) = 200, \quad Var(N) = 1000(0.20)(0.80) = 160$$

$$E(S) = E(N)E(S) = 200(20) = 4000$$

$$Var(S) = E(N)Var(S) + E(X)^2 Var(N) = 200(20) + 20^2(160) = 68,000$$

Budget $= 10E(S) + 10\sqrt{Var(S)} = 10(4000) + 10\sqrt{68,000} = 42,610$

Question #286
Key: C

Insurance pays 80% of the portion of annual claim between 6,000 and 1,000, and 90% of the portion of annual claims over 14,000.

The 14,000 breakpoint is where Michael reaches 10,000 that he has paid:
1000 = deductible
1000 = 20% of costs between 1000 and 6000
8000 = 100% of costs between 14,000 and 6,000

$$E(X \wedge x) = \theta\left(1 - \frac{\theta}{x+\theta}\right) = \frac{5000x}{x+5000}$$

x	$E(X \wedge x)$
1000	833.33
6000	2727.27
14000	3684.21
∞	5000

$$0.80[E(X \wedge 6000) - E(X \wedge 1000)] + 0.90[E(X) - \mathrm{E}(X \wedge 14000)]$$
$$= 0.80[2727,27 - 833.33] + 0.90[5000 - 3684.21]$$
$$= 1515.15 + 1184.21 = 2699.36$$

Question #287
Key: D

We have the following table:

Item	Dist	$E(\ )$	$Var(\ )$
Number claims	$NB(16,6)$	16(6) = 96	16(6)(7) = 672
Claims amounts	$U(0,8)$	(8 – 0)/2 = 4	$(8-0)^2/12 = 5.33$
Aggregate		4(96) = 384	$96(5.33)+672(4^2)=11,264$

Premium $= E(S)+1.645\sqrt{Var(S)} = 384+1.645\sqrt{11,264} = 559$

Question #288
Key: E

With probability p, $\Pr(N=2)=0.5^2=0.25$. With probability $(1-p)$,
$\Pr(\text{N=2})=\binom{4}{2}0.5^4=0.375$. $\Pr(N=2)=p(0.25)+(1-p)(0.375)=0.375-0.125p$

Question #289
Key: D

600 can be obtained only 2 ways, from $500+100$ or from 6(100).

Since $\lambda=5$ and $p(100) = 0.8$, $p(500) = 0.16$.

$$\Pr\left(6 \text{ claims for } 100\right)=\frac{e^{-5}5^6}{6!}\left(0.8\right)^6=0.03833 \text{ or } 3.83\%$$

$$\Pr\left(\overset{\text{2 claims}}{500+100}\right)=\frac{e^{-5}5^2}{2!}\left[(0.8)^1(0.16)^1(2)\right]=0.02156=2.16\%$$

The factor of 2 inside the bracket is because you could get a 500 then 100 or you could get a 100 then 500. Total is 3.83% + 2.16% = 5.99%.

Question #290
Key: D

The mixture can be written as
$$f(x)=\frac{1}{4}\frac{1}{2}e^{-x/2}+\frac{3}{4}\frac{1}{3}e^{-x/3},\ x\ge 0$$
and so $\Pr(J = 1) = 0.25$. Because $0.2 < 0.25$, the observation is from the first distribution in the mixture. Then by inversion,
$1-e^{-x/2}=0.6 \Rightarrow x=-2\ln(0.4)=1.83.$

Question #291
Key: D

The value of m is 500 – 23 – 59 = 418. The value of q is 0.15/(1 – 0.05 – 0.10) = 0.18.

Question #292
Key: A

Policy 1 turns 82 on 4-1-2012 and has 6 months exact exposure (time to death) and 12 months actuarial exposure (deaths exposed to next birthday).

Policy 2 turns 82 on 7-1-2012 and has 6 months exposure by both methods.

Policy 3 turns 82 on 8-1-2012 and has 5 months exposure by both methods.

Policy 4 turns 82 on 9-1-2012 and has 2 months exposure by both methods.

The difference of the probabilities is $\left|[1-e^{-1/(19/12)}]-[1/(25/12)]\right| = 0.012$.

Question #293
Key: E

Policy 1 turns 82 on 3-1-2012 and has 7 months exact exposure (time to death) and 12 months actuarial exposure (deaths exposed to next birthday).

Policy 2 turns 82 on 10-1-2012 and has 3 months exposure by both methods.

Policy 3 turns 82 on 1-1-2013 and has no exposure.

Policy 4 turns 82 on 8-1-2012 and has 3 months exposure by both methods.

The difference of the probabilities is $\left|[1-e^{-1/(13/12)}]-[1/(18/12)]\right| = 0.064$

Question #294
Key: C

Policy 1 turns 82 on 3-1-2012 and has 7 months exact exposure (time to death) and 12 months actuarial exposure (deaths exposed to next birthday).

Policy 2 turns 82 on 10-1-2012 and has 7 months exposure by both methods.

Policy 3 turns 82 on 1-1-2013 and has no exposure.

Policy 4 turns 82 on 8-1-2012 and has 3 months exposure by both methods.

The difference of the probabilities is $\left|[1-e^{-1/(17/12)}]-[1/(22/12)]\right| = 0.039$.

Question #295
Key: A

$e = 13/12,\ d = 1,\ b-a = 1$. The estimate is $\hat{q} = 1-e^{-12/13} = 0.603.$

$V\hat{a}r(\hat{q}) = (1-0.603)^2(1)(1)(12/13)^2 = 0.134.$

Question #296
Key: D

$$P_{61} = (10,000-100-800+1000)+8000 = 18,100$$
$$d_{61} = 120+80 = 200$$
$$w_{61} = 700+700 = 1400$$
$$n_{61} = 0$$
$$q = \frac{200}{18,100+(0-1400)/2} = \frac{200}{17,400} = 0.0115.$$

Question #297
Key: D

$$P_{61} = (10,000-100-800+1000)+8000 = 18,100$$
$$d_{61} = 120+80 = 200$$
$$w_{61} = 700+700 = 1400$$
$$n_{61} = 0$$
$$q = 1-\exp\left(\frac{200}{18,100+(0-1400-200)/2}\right) = 1-e^{-200/17,300} = 0.0115.$$

Question #298
Key: E

For loss amount: $X = e^{5.2+1.4Z}$

For expenses: $F(y\,|\,x) = u = 1 - e^{-y\sqrt{x}/2}$ and $y = \dfrac{2}{\sqrt{x}}[-\ln(1-u)]$

Thus,

Z	*X* = Loss	*Y*	*E*=*XY*
1.53	1543.797	0.0289	44.587
0.03	189.048	0.0515	9.730
– 0.58	80.479	0.8509	68.480

Average = 40.93

Question #299
Key: C

The time intervals to the occurrence of the first claim, and between claims follow the exponential distribution with mean 1/5. Thus

The first claim occurs at [–ln(1 – 0.605)]/5 = 0.186

The second claim occurs at 0.186+ [–ln(1 – 0.529)]/5 = 0.337

The third claim occurs at 0.337+ [–ln(1 – 0.782)]/5 = 0.642

0.642x12 = 7.7 months

Since month 1 is January, month 7 = July, month 8 = August, so this means the date will be in late August.

Question #300
Key: A

Solution:
The kernel density estimate represents a mixture:

$$\hat{F}(x) = \sum_{j=1}^{n} p(y_j) K_{y_j}(x)$$

In the general form, because the sample mixes the kernel distribution we can add the means and (population) variances of the sample and kernel.

Here the kernel is a uniform distribution with bandwidth 1.5, unlike the standard uniform, this kernel is not centered around the observation.

	Mean	Population Variance	Coefficient of Variation
Sample	***5.00***	6.00	0.4899
Kernel	0.50	0.75	1.7321
KDE	5.50	6.75	**0.4724**

With the variance of a uniform being the square of its width divided by 12.

Alternatively, we can compute the KDE, which gives a uniform with equal probability between [1,10], which gives mean=5.5 and variance = (10 – 1)^2/12

Question #301
Key: B

j	0	1	2	3	4
(n – w – d) from preceding interval	0	50	52	20	-6
P = running total of preceding row	0	50	102	122	116
$0.7n_j - 0.3w_j$	54	50	40	19	
r_j = Row 3 + Row 4	54	100	142	141	
d_j	10	8	20	16	

Thus, $\hat{S}(24) = \frac{44}{54}\frac{92}{100}\frac{122}{142}\frac{125}{141} = 0.571$, and $\hat{F}(24)$ = 1 – 0.57 = 0.43.

Question #302
Key: **A**

$F = 1 - e^{-x/20}$ and $x = -20\ln(1-F)$
Thus

	u	amount x	deductible	payment
One claim	0.237	5.41	3.41	3.41
Two claims	0.661, 0.967	21.64, 68.22	19.64, 38	50

Expected value = 0(0.7) + 3.41(0.2) + 50(0.1) = 5.682

Question #303
Key: D

Solution:
Let d_{30} represent the number of deaths in the interval from exact age 30 to exact age 31.

Let e_{30} represent the exact exposure in this interval.

Let $\hat{q}_{30}$ represent the maximum likelihood estimate of the conditional probability of death in this interval.
We have a = 30 and b = 31.

From the given data and the assumption of a constant hazard rate in this age interval, we have

$$0.1 = \hat{q}_{30} = 1 - \exp\left[-\frac{d_{30}(b-a)}{e_{30}}\right] = 1 - \exp\left(-\frac{98}{e_{30}}\right) \text{ and so } e_{30} = -\frac{98}{\ln(0.9)}.$$

The estimate of the variance of this maximum likelihood estimator is

$$V\hat{a}r(\hat{q}_{30}) = (1-\hat{q}_{30})^2(b-a)^2\frac{d_{30}}{e_{30}^{\ 2}} = (1-0.1)^2(31-30)^2\frac{98}{\left[-\frac{98}{\ln(0.9)}\right]^2}.$$

$$= (0.9)^2\frac{\left[\ln(0.9)\right]^2}{98} \approx 0.000092$$

Question #304
Key: **E**

Solution:

Since the sum of αs is one, $\alpha_1 = 0.1, \alpha_2 = 0.2, \alpha_3 = 0.3, \alpha_4 = 0.4$.
The given random numbers for J results in αs of 2, 3, and 4. Thus the three simulated loss amounts are:

–ln(1 – 0.435)/0.2 = 2.85
–ln(1 – 0.298)/0.3 = 1.18
–ln(1 – 0.678)/0.4 = 2.83

The mean is 2.29

Question #305
Key: A

Solution:
Conditional survival function that loss is over 750

Interval 750-1000, risk set = 120 – 8 = 112, losses = 10 + 10 = 20

$$\hat{p}_j = \frac{112-20}{112} = 0.8214$$

Interval 1000-1500, risk set = 112 – 20 = 92, losses = 12 + 15 = 27

$$\hat{p}_j = \frac{92-27}{92} = 0.7065$$

Interval 1500-5000, risk set = 92 – 27 – 6 = 59, losses = 14 + 18 = 32

$$\hat{p}_j = \frac{59-32}{59} = 0.4576$$

Probability = (0.8214)(0.7065)(0.4576) = 0.2656 = 0.27

Question #306
Key: A

(This question was formerly Question 266.) The deduction to get the SBC is $(r/2)\ln(n) = (r/2)\ln(260) = 2.78r$ where r is the number of parameters. The SBC values are then –416.78, –417.56, –419.34, –420.12, and –425.68. The largest value is the first one, so model I is to be selected.

Question #307
Key: D

(This question is effective with the October 2016 syllabus.) The deduction to get the AIC is r, where r is the number of parameters. The AIC values are then –415, –414, –414, –413, and –415. The largest value is the fourth one, so model IV is to be selected.