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## GAAP FOR MEDICARE SUPPLEMENT POLICIES

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#### Abstract

Medicare supplement policies are one of an increasing number of products that have emerged since Audits of Stock Life Insurance Companies (Audit Guide) was published and that were not fully anticipated by the American Institute of Certified Public Accountants in its deliberations. This paper examines five alternative valuation methods (prospective, retrospective, global, intermediate, and static) for the proper application of generally accepted accounting principles (GAAP) to Medicare supplement insurance. The focus of this paper is on the changes to expected future claim costs due to changes in the Medicare deductibles. Application of the conclusions of this paper to other insurance products whose anticipated future claim costs change from one year to the next is possible, but will depend on the degree to which those products meet the assumptions of this paper.


## INTRODUCTION

Two features of Medicare supplement policies make them substantially different from other guaranteed renewable health policies. First, claim costs in renewal years are subject to increases because of changes in the Medicare deductibles. Second, many states grant the insurer an automatic rate increase when the deductibles increase. These features raise several questions about the proper benefit reserve and deferred acquisition cost (DAC) amortization methods for this business.

In the following analysis several simplifying assumptions are made:

1. When the Medicare deductibles change, the result is an increase in all future claim costs, which for all ages can be expressed as a level percentage of the claim costs just before the change.
2. Gross premiums are level for all policy years. When the gross premiums are increased, they are increased by the same percentage as in assumption 1 above. The increase applies uniformly to all policy years after the increase.
3. There is no change in anticipated persistency as a result of the increased premiums.

The primary reason for assumptions 2 and 3 is the resulting ease of the mathematical development. Assumption 1 causes the claim cost curve to retain the same general shape after a benefit change and thereby retain more of the original benefit reserve assumptions than a more radical reassessment of the claim costs. Assumption 1 is equivalent to changing the average claim size in proportion to the change in deductible parameters, while keeping fixed the average claim frequency. Medicare supplement policies were chosen as the example for this paper because the indexing of the deductibles causes assumption 1 to hold more generally than would be the case for most other products.

To determine if a specific methodology is good practice, the concept of matching expenses with revenues was taken as of primary importance. In [6, p. 53], Posnak states
"In accounting theory, costs should be allocated to periodic revenue in proportion to the recognition of revenue,"
and
"Costs are not incurred uniformly... . Therefore a reserve, or combination of reserves, must be utilized to control the allocation of costs and, hence, to control the emergence of profits."
So a reserve methodology should produce a pattern of profit recognition that is reasonably constant, taken as a percentage of the revenue that is recognized in the same accounting period.

## BENEFIT RESERVE METHODS

Drawing from standard actuarial practices, we suggest five reserve methods:

- Prospective-adjusting net premiums and reserves for durations after the change
- Retrospective-recalculating reserves from the issue date
- Global-assuming at issue that future changes will be made and assuming the level of those changes, without adjusting for actual changes
- Intermediate - anticipating some future changes, but adjusting to actual (akin to prospective) at the time of change
- Static-using original assumptions at all valuation dates, without anticipating changes and without adjustment.
These methods are explained further below, and their counterparts for DAC amortization are discussed in the following section. They do not produce similar or equally satisfactory results.


## Prospective

The main characteristic of the prospective method is that adjustments are made only to benefit net premiums and reserve factors that apply at durations after the current change. It is similar to aggregate funding methods under pension plans. The currently available fund, the benefit reserve under the most recent set of assumptions, is subtracted from the present value of future benefits, calculated to include the current change in benefits. The balance is the present value of funds needed over and above the current benefit reserve to exactly cover future claim costs. When divided by the present value at the duration of change of a dollar for the remaining premium-paying period, the result is the level benefit net premium for all future years. Note that at the point of implementation of the revised benefit reserves there is no change in the reserve. This approach is essentially the same as indicated in Interpretation 1-I of the American Academy of Actuaries [1]. This method is also discussed by Cloninger [3].

Gross premiums are assumed to increase in proportion to the increase in future claim costs. Because the prospective method does not anticipate future changes, all future gross premiums are level, regardless of the number of benefit changes that have taken place to date. Therefore the slope of the revised gross premiums is the same as that of the original gross premiums. This means the original annuity factors will be applicable to the calculation of benefit net premiums and reserve factors at all subsequent points of time. (This will also be true for the expense net premiums and reserves under the prospective method, as described below.)

The formulas developed in the Appendix and used in the ensuing discussion are based on the following definitions:
$t$ is an integral duration;
$k$ is an integral duration used to indicate the number of benefit changes, some of which could have been zero, which have taken place;
$a(t)$ is the present value at time $t$ of a dollar at time $t$ and on each subsequent anniversary;
$r(t)$ is the increase in benefits that takes place at duration $t$, where $0 \leq r(t)$ and $r(0)=0 ;$
$b(t, k)$ is the present value at time $t$ of all future benefits, which reflects the first $k$ benefit changes, where $t \geq k$;
$N(k)$ is the benefit net premium due at time $k$ and all subsequent anniversaries whose present value at time $k$ when added to $V(k, k-1)$ equals $b(k, k)$;
$V(t, k)$ is the terminal benefit reserve at time $t$, which reflects the first $k$ benefit changes, where $t \geq k$.

The last two definitions can be expressed by formulas as

$$
N(k)=\frac{b(k, k)-V(k, k-1)}{a(k)},
$$

and

$$
V(t, k)=b(\mathrm{t}, \mathrm{k})-N(k) a(t) .
$$

With these definitions the following are true:

$$
\begin{align*}
N(k)= & N(0)+\sum_{t=1}^{k}\left\{\frac{r(t) b(t, 0)}{a(t)} \prod_{s=1}^{t-1}[1+r(s)]\right\}  \tag{I}\\
= & N(0) \prod_{t=1}^{k}[1+r(t)]+\sum_{t=1}^{k}\left\{\frac{r(t) V(t, 0)}{a(t)} \prod_{s=1}^{t-1}[1+r(s)]\right\}  \tag{II}\\
V(t, k)= & V(t, 0) \\
& +\sum_{s=1}^{k}\left\{r(s)\left[V(t, 0)-\frac{a(t) V(s, 0)}{a(s)}\right] \prod_{v=1}^{s-1}[1+r(v)]\right\}  \tag{III}\\
= & V(t, 0) \prod_{s=1}^{k}[1+r(s)] \\
& -a(t) \sum_{s=1}^{k}\left\{\frac{r(s) V(s, 0)}{a(s)} \prod_{v=1}^{s-1}[1+r(v)]\right\} . \tag{IV}
\end{align*}
$$

Formulas (I) and (III) are counterparts, as are (II) and (IV). Dividing both sides of (III) by $V(t, 0)$ expresses the ratio of the new benefit reserve to the original reserve in an interesting symmetric formula

$$
\begin{equation*}
\frac{V(t, k)}{V(t, 0)}=1+\sum_{s=1}^{k}\left\{r(s)\left[1-\frac{a(t) V(s, 0)}{V(t, 0) a(s)}\right] \prod_{v=1}^{s-1}[1+r(v)]\right\} \tag{V}
\end{equation*}
$$

Note that if no increases take place after time $k$, then $r(t)=0$ for all $t>k$. Formula (I) shows $N(t)=N(k)$ and Formula (III) shows $V(t, t)=V(t, k)$ for all $t>k$; both are the anticipated result of no further changes.

A benefit change may be viewed as the purchase of a new policy for an additional premium. The change in the present value of future benefits at time $k$ caused by the increase in benefits at time $k$ is

$$
b(k, k)-b(k, k-1)=r(k) b(k, k-1)=r(k) b(k, 0) \prod_{s=1}^{k-1}[1+r(s)] .
$$

So the additional net benefit premium is

$$
\frac{r(k) b(k, 0) \prod_{s=1}^{k-1}[1+r(s)]}{a(k)}
$$

which, from (I), is $N(k)-N(k-1)$. From (III), the additional benefit reserve at time $t$ for the increase at time $k$ is

$$
V(t, k)-V(t, k-1)=r(k)\left\{V(t, 0)-\frac{a(t) V(k, 0)}{a(k)}\right\} \prod_{s=1}^{k-1}[1+r(s)],
$$

which can be shown to be the benefit reserve based on the net premium and increased benefits above (see the Appendix).

Formula (II) indicates that to arrive at $N(k)$, the original net benefit premium $N(0)$ needs to be increased by more than the cumulative product of all past benefit increases. Formula (IV) consequently indicates $V(t, k)$ is less than the original benefit reserve, $V(t, 0)$, increased in the same manner.

At the outset gross premiums were assumed to be increased by the same percentage as benefits. Formula (II) therefore indicates that the ratio of the net benefit premium to the gross premium increases every time benefits increase. There are no new acquisition costs associated with the increased gross premium, so an increase in benefits actually causes the percentage of the gross premium required to cover expenses to decrease, as discussed later in the section on expense reserve methods. Therefore, it is not necessarily true that profit as a percentage of the gross premium has decreased.

Formula (III) can be used in a practical implementation of this approach. Every term on the right-hand side of the equation is known when the original benefit reserves are calculated, except for the magnitude of the benefit changes, $r(t)$. A program can easily be written that will use Formula (III) to calculate revised benefit reserves and construct transaction records for updating the reserve factor files. This program would need to be run once a year when the newest benefit change is known. The only new input to the program each year is the newest benefit change.

One feature of the prospective method is that benefit reserve factors before the current change are not affected by the current change:

$$
V(t, k)=V(t, k-1), \text { for } t<k .
$$

As $t$ increases, the benefit reserves converge to the final set in use. Moreover, the actual sequence of terminal reserves that are used, $V(t, t)$ (those reserves on the diagonal of the rectangular $V(t, k)$ array), is the final set in use.

Figure 1 shows the benefit reserve factors that are in place after $k$ benefit increases, for the given values of $k$. Benefit increases, $r(t)$, are taken to be 15 percent each year. Assumptions and the formula for the $k=0$ reserves are given in the Appendix.

FIGURE 1
Convergence of Prospective Reserves


## Retrospective

The retrospective method always goes back to time zero to recognize a change in benefits. The revised claim cost curve, which reflects all benefit changes from inception to date, is used to calculate benefit reserves as though this claim cost curve was known at issue. The change in gross premiums must also be recognized. The revised benefit reserve factor for the time of the benefit change will be different from the factor being replaced; therefore surplus will change at the point of implementing the new benefit reserve factors.

The retrospective method would not appear to be in compliance with FAS 60. Specifically, paragraph 21 restricts the right to change original assumptions in subsequent accounting periods unless a premium deficiency, as described in paragraph 35, has occurred. However, the first increase in future benefits to be handled by the retrospective method may occur before a premium deficiency has occurred, thus creating a problem relative to compliance with FAS 60. The only support that the author could find for the retrospective method is in FAS 97. Paragraph 32 of FAS 97 not only permits, but also requires, retroactive application of any accounting changes needed to comply with the earlier provisions of FAS 97, including restatement of previously issued financial statements. However, FAS 97 applies only to specific categories of insurance products that do not contain Medicare supplement policies. So FAS 97 does not support using the retrospective method for Medicare supplement policies.

As the number of changes increases, the benefit reserves calculated under the retrospective method also converge to the final set in use. However, there are several important differences from the prospective method. Because the retrospective method adjusts benefit reserves at lower durations, the actual sequence of terminal reserves that are used will not be the final set of reserves. The diagonal of the $V(t, k)$ array has the following properties:

$$
\begin{aligned}
& V(t, t)>V(t, k), \text { for } k<t, \\
& V(t, t)<V(t, k), \text { for } k>t .
\end{aligned}
$$

Thus the curve of benefit reserve factors that are used will cut across each curve that represents the factors at any stage of the process. Figure 2 displays retrospective benefit reserve factors calculated by using the same assumptions underlying the prospective reserves in Figure 1.

## Global

The global method goes farther than the retrospective method by assuming at the outset that some changes will take place. An assumption is made about a pattern of benefit changes by duration. These changes are then reflected in the present value of benefits at issue. The denominator of the net premium is an annuity that recognizes that gross premiums increase in proportion to benefit increases. Formulas for benefit reserves calculated under the global method must increase the net benefit premium described above each year in proportion to the gross premium. Then the ratio of the total GAAP net premium to the gross premium is constant for all durations. The pattern of

FIGURE 2
Convergence of Retrospective Reserves

benefit increases then becomes just another actuarial assumption in the reserve formulas.

The global method anticipates at issue all future benefit changes and does not make any subsequent changes as experience develops. In making the benefit increases an explicit assumption in the reserve calculation, normal GAAP considerations need to be made. Specifically, some provision for adverse deviation from the assumed benefit increases would be necessary. Also the lock-in principle would apply. Thus, once an estimate of premium and benefit increases has been made, any difference in actual experience should fall to the bottom line for the reporting period in which the difference occurs. These differences include both the normal situation in which persistency rates, for example, are not the same as expected and the situation specifically being discussed here, in which more of a structural change in the policy itself has occurred.

## Intermediate

The intermediate method is designed as a compromise between the global and prospective methods. Some assumption about the level of future benefit changes is made, but only for a limited time rather than for the entire policy period as in the global method. When this time expires, benefit reserves are recalculated for future durations in a manner similar to that for the prospective method, except that future benefit changes have again been built into the factors for a limited time. An increase in the gross premium of the same percentage as the increase in benefits is assumed to take place coincidentally with such benefit increase. This means that the annuity factor used to calculate the net benefit premium must reflect the nonlevel gross premiums.

For illustration, this paper uses three years as the limited time and incorporates assumptions about the level of benefit and gross premium increases for the three-year period at each recalculation point. For example, at time $t=0$, the intermediate reserve factors are calculated by assuming $r(0)=0$, $r(t)=0.15$ for $t=1$ and 2 , and $r(t)=0$ for $t>2$.

The intermediate method causes no change in surplus when the benefit reserve factors are recalculated, because the recalculation is done for future durations only. The intermediate method is also similar to the prospective method in that the sequence of reserve factors used during the lifetime of the business converges to the final set in use. The convergence is in some sense less dramatic than that in the prospective method because there are fewer points of recalculation.

## Static

The static method uses benefit reserves based on issue date assumptions for all policy durations. Subsequent benefit or premium changes are never recognized. The benefit reserves for this method are those designated $V(t, 0)$ in the description of the prospective method.

An argument that might be made in favor of the static method is that the Audit Guide and FAS 60 indicate that once reserve factors have been established, it is not appropriate to change those factors until a premium deficiency has occurred (the lock-in principle). This appears to require, rather than permit, the static method. However, the lock-in principle was designed to cover changes in the expectation of future experience, rather than to prevent changes that reflect fundamental differences in the coverage of the policy, as is the case with this product. The Audit Guide and FAS 60 therefore do not appear to support the static method.

The static method causes the net effect of the increased premiums and increased claims to flow directly to the bottom line for each future accounting period. The static method makes no effort to control the incidence of the profit or loss due to changes in the coverage provided by the policy.

## EXPENSE RESERVE METHODS

The five reserve methods discussed above are examined relative to DAC amortization below.

## Prospective

The following definitions are needed to supplement the discussion of benefit reserves above:
$e(t, k)$ is the present value at time $t$ of all future expenses that reflect the first $k$ benefit and gross premium changes, where $t \geq k$;
$E(k)$ is the net expense premium due at time $k$ and all subsequent anniversaries whose present value at time $k$ when added to $S(k, k-1)$ equals $e(k, k)$;
$S(t, k)$ is the terminal expense reserve at time $t$, which reflects the first $k$ benefit and gross premium changes, where $t \geq k$.
This paper has assumed that the only expenses are acquisition expenses. Obviously then, benefit or gross premium changes at any point subsequent to issue do not affect expenses. In terms of the definitions above, $e(t, k)=e(t, 0)$ for all $t$ and $k$. The less restrictive assumption that expenses are ongoing, but not affected by the size of the gross premium, produces the same result.

From the definitions,

$$
E(k)=\frac{e(k, k)-S(k, k-1)}{a(k)}
$$

and

$$
S(t, k)=e(t, k)-E(k) a(t)
$$

we have

$$
\begin{aligned}
E(1) & =\frac{e(1,1)-S(1,0)}{a(1)} \\
& =\frac{e(1,0)-S(1,0)}{a(1)} \\
& =E(0)
\end{aligned}
$$

and

$$
\begin{aligned}
S(t, 1) & =e(t, 1)-E(1) a(t) \\
& =e(t, 0)-E(0) a(t) \\
& =S(t, 0) .
\end{aligned}
$$

In general, $E(t)=E(0)$ and $S(t, k)=S(t, 0)$. For the prospective method, the expense net premiums and reserve factors do not change as a result of increases in benefits or gross premiums, and the ratio of the GAAP net expense premium to the gross premium decreases, which was alluded to in the discussion of prospective benefit reserves. The continued use of the same expense reserve factors causes no discontinuity at the time of a benefit change, which is consistent with the effect of the prospective method on benefit reserves.

## Retrospective

Under the retrospective method, the recalculation of benefit reserves and net premiums begins at the time of issue, so the expense reserves and net premiums are recalculared in a similar way. Because the gross premiums can vary by duration, the net premium for any given duration is a fraction of the gross premium for that duration. The numerator of the fraction is the present value at issue of all expenses. The denominator is the present value at issue of gross premiums.

As noted under the prospective expense reserve discussion, the assumptions used in this paper cause the present value of expenses measured at issue to be independent of the number and magnitude of any subsequent changes in benefits or gross premiums. Therefore, under the retrospective method, the numerator of the fraction is always the same. The assumption that persistency does not change causes the denominator to increase in value every time the gross premium increases due to a benefit increase. So each time an increase is made to the gross premium, the fraction decreases.

Remember that under the retrospective method the new fraction applies to all durations, in the past as well as in the future. However, the gross premiums for all durations in the past do not change, so the contribution to amortization of deferred acquisition costs from policy years prior to the implementation of the most recent expense reserve factors must be lower than it was before implementation of these factors. The result is higher
expense reserve factors for all durations. Changing to the new expense reserve factors produces a gain to surplus that offsets part of the loss caused by switching to new benefit reserve factors.

## Global

The global method anticipates all benefit and premium changes at issue, so the calculation of the expense reserve factors must do so as well. The calculations described in the retrospective method are used. The difference is that the global method makes an assumption at issue about the level of the gross premium for each policy year and does not subsequently change this assumption.

## Intermediate

The benefit reserves for the intermediate method assume a limited number of benefit increases. Consistent treatment of the expense reserves for this method then requires that they be calculated based on gross premiums, which increase for a limited period and then remain level. When this time expires, then future expense reserve factors are recalculated based on an assumption of future gross premium increases for another limited period.

The expense reserves under the intermediate method differ from the expense reserves under the prospective method. By assuming some level of increases for the gross premium in the factors calculated at time $t=0$, the initial set of expense reserve factors do not match the initial factors under the prospective method. By assuming additional future gross premium increases at each recalculation, the future expense reserve factors for the intermediate method change, whereas expense factors for the prospective method do not change.

## Static

The static method uses the expense reserve factors calculated based on the assumptions known at issue. As with the benefit reserve factors, these factors are not changed.

## COMPARISON OF METHODS

By using the assumptions at issue listed in the Appendix and using $r(t)$ equal to 0.15 in all years, Tables A2-A6 in the Appendix show the income and expense items for each of the five methods. As explained in the Appendix, each item of income and expense in these tables is discounted to
the beginning of the policy year. Investment income is calculated as one year's interest earned on the prior year's ending benefit reserve less one year's interest on the unamortized policy acquisition costs at the beginning of the policy year, after receipt of the net expense premium for that year. The amount of investment income varies from method to method, but the present value of the net gain over the entire policy period is the same for all methods, when the present value is calculated at the rates of interest in the Appendix.

Figure 3 shows the sequence of benefit reserve factors that would be used by each of the five methods. There would appear to be substantial differences between methods, but when expense reserves are included in the comparison, these differences moderate.

FIGURE 3
Comparison of Reserve Factors


Each table in the Appendix develops the excess of premiums and investment income over the sum of claims, increase in benefit reserves, and decrease in the expense reserves. The net gain expressed as a ratio of premiums
is constant for the global method. This result is to be expected because actual results equal the original expectation. This ratio is not constant for the other methods.

Under the prospective method the net gain ratio increases and then decreases. This result is to be expected based on the observation made above about the deterioration of the net to gross ratio caused by the additional benefits each year. The overall net gain ratio increases until the deterioration of the net to gross ratio overcomes the lack of additional acquisition costs and decreases thereafter.

Under the intermediate method the net gain ratio is similar to that under the prospective method in that it increases and then decreases. However, because some benefit and gross premium changes are anticipated under the intermediate method and because for this illustration actual results equal expected results, the intermediate method resembles the global method in that the net gain ratio is constant during each period between recalculations. So the increases and decreases experienced under the intermediate method are less pronounced than those under the prospective method.

Under the static method the net gain ratio also increases and then decreases. The ratio is greater than that under the prospective method ratio in the early years and then becomes less than that under the prospective method. The differences under the prospective method are caused by the smaller reserves utilized by the static method. In the early years the use of smaller reserve factors allows a greater recognition of profit than the prospective method. In the later years the release of the smaller reserves creates a smaller offset to the increased level of claims than the prospective method.

Under the retrospective method the net gain ratio displays a more complicated pattern, which is best examined by looking at the net gain in two pieces. The global method is the limiting case for the retrospective method. That is, if the increases assumed in the global method are what actually occur, then the sequence of benefit and expense reserve factors under the retrospective method converges to those under the global method. As shown by the tables in the Appendix, the financial results for the final policy year are the same for these two methods.

Amortization charges are considered first. The retrospective method has a higher dollar amount of amortization in the early years than the global method. This result occurs because the global method anticipates all gross premium increases, which delays the amortization, while the retrospective method only reacts to gross premium increases as they occur. The total dollars amortized must be equal, so eventually the retrospective method must
amortize less than the global. Finally, in the last year both methods amortize the same amount.

Because expense reserves have been calculated with interest, the ratio of the amortization charge to earned premium is not constant, even for the global method. However, the sum of the amortization charge and one year's interest on the remaining deferred acquisition cost has a constant relationship with earned premium for the global method. Figure 4 displays this relationship to stress the level percentage charge of the global method.

FIGURE 4
Ratio of Amortization to Premiums


For comparison, Figure 4 includes the results for the static and prospective methods. The expense reserve factors used by both methods assume a level gross premium throughout the policy period. Adjusted for interest, the amortization charges created by these factors are a constant percentage of the level gross premium, when actual persistency equals the expense reserve assumptions. So the amortization charges under these methods constitute a
decreasing percentage of premiums when premiums are increased as a result of the benefit increases.

Figure 4 also displays the results for the intermediate method. The ratios for the intermediate method behave generally like the ratios for the prospective method. The intermediate method's ratios move in steps and are generally more level over the entire policy period than those of the prospective method.

The remainder of the net gain is considered next. For the balance of this paper, the term "restricted gain" is used to designate earned premiums plus investment income on the benefit reserve less the sum of incurred claims and the change in benefit reserves. Claims and earned premiums are the same under all methods. Only the change in benefit reserves is examined because the balance of the difference between the methods is caused by interest on the benefit reserve.

In a comparison of the retrospective with the global method, essentially the same conditions hold as for amortization. For both methods the sum of the benefit reserve changes is the same. The benefit reserve change starts out lower for the retrospective method because the global method anticipates all benefit increases. The benefit reserve changes for both methods are the same in the final policy year. The result is a sequence of ratios with the same general shape as found for the amortization ratios (Figure 5). For comparison, Figure 5 also includes the ratios of the restricted gain to gross premiums for the prospective, intermediate and static methods.

Figure 6 displays the net gain ratios found in the Appendix. Each curve in Figure 6 also equals the corresponding curve in Figure 5 minus the corresponding curve in Figure 4. The resulting pattern is not what might be expected.

At the outset the gain under the retrospective method from the reduced amortization charge more than offsets the loss from the higher benefit reserve. After a few years this relationship reverses and the net gain ratio decreases. So far this sounds like the results for the prospective method. However, after a few more years the relationship reverses again because of the convergence of the retrospective method to the global method. This answer, while correct mathematically, does not make the final result more understandable. The overall shape of the other four curves in Figure 6 has been commented on above.

Note that in the global method one assumption is the sequence of annual increases in the claim costs. Results presented so far for the global method have been prepared under the assumption that the original expectations were

FIGURE 5
Ratio of Restricted Gain to Premiums


FIGURE 6
Ratio of Net Gain to Premiums

exactly achieved; this caused the results to look unnecessarily reasonable. To demonstrate the effect of making an inaccurate assumption about the sequence of increases, consider Figure 7.

FIGURE 7
Ratio of Net Gain to Premiums
Global Method
Actual $r(t)=0.15$


Here three sets of benefit and expense reserve factors were developed by using constant expected values of $r(t)$ equal to $0.10,0.15$ and 0.20 . Figure 7 displays the sequence of the ratio of the net gain to earned premium under the circumstances that each $r(t)$ is in fact 0.15 . The varying net gain ratios that resulted from a comparatively small difference from the assumptions underlying the global method are an indication of the problems that can arise when initial assumptions turn out to have badly estimated future actual results. The intermediate method has the same kind of potential for fluctuation, although this potential is more limited than that for the global method due to the limited period for which benefit increases are assumed.

The static method considers the increased premium from each rate increase as income, but sets up no additional benefit reserve for the corresponding increase in future benefits. This would appear to be correct only in the event that all future claim costs increased by the same dollar amount, rather than the assumptions that apply here. That portion of the increased gross premium that would be put into reserves under the prospective method is, under the static method, allowed to become part of the recognized gain. As future claim payments exceed the expected claims being released by the static benefit reserve factors, the excess serves to reduce the recognized gain. It thus appears that the static method affects the proper incidence of the recognition of profit. This can provide the company's management with conflicting signals: A large rate increase will artificially increase the next accounting period's recognized gain, thereby giving the appearance that conditions have improved more than they actually have.

The intermediate method requires more work than the global method, because the benefit reserve factors must be updated periodically, but less work than the prospective method, because the benefit reserve factors are not updated annually.

The Audit Guide requires each company to periodically verify that it retains the ability to recover its unamortized acquisition costs from its future revenues. This is called a loss recognition test and can be performed as follows:
a. Using the current best estimates of future experience, with no margins, calculate the present value of future gross premiums.
b. Using the current best estimates of future experience, with no margins, calculate the present value of future claims, expenses and commissions.
c. Determine the excess of the benefit reserve over the unamortized policy acquisition cost. (This is the net GAAP liability.)
d. The company fails if $a-b+c$ is negative and passes otherwise.

For this test items a and b can be calculated on a basis that is independent of whatever reserve basis is in use. So to the extent that the choice of reserve method can affect the recoverability test, item c measures that effect. By using the tables in the Appendix, the prospective method has the highest net GAAP liability for the first 14 policy years. Thereafter, the intermediate method has the highest net GAAP liability. For the specific example being considered here, the only circumstances that produce a negative loss recognition test occur at the end of policy years 16 and 17 for the static method.

These results are, of course, dependent on the level of the acquisition costs per policy. If acquisition costs per policy are increased, then the number of years in which the intermediate method has the highest net GAAP
liability decreases. In fact, a modest increase in the acquisition cost is all that is required for the prospective method to have the highest net GAAP liability in all policy years. If the acquisition cost is decreased, the global method replaces the intermediate method as having the highest net GAAP liability at the later policy durations. Continued decreases in the acquisition cost result in the global method first having the highest net GAAP liability at correspondingly earlier durations. For example, if the acquisition costs are cut in half, then the prospective method has the highest net GAAP liability for the first three policy years, and the global method has the highest net GAAP liability for the remainder of the policy period.

The various methods were compared by using assumptions that matched actual experience. This was done to avoid a circumstance in which the maximum net GAAP liability would come from a combination of method and assumptions that failed to produce a useful pattern of earnings during the policy period.

The results are also dependent on the level of the annual increase in the benefits. For example, if benefits are increasing at a rate of 50 percent per year, then for a given level of acquisition cost the global method will have the highest net GAAP liability at an earlier duration than when the benefits increase at a 15 percent annual rate.

In all five methods, the policies of each issue year must have a separate set of benefit and expense reserve factors because each has different past benefit increases. Even the static approach requires different benefit reserves because of revised claim expectations at issue for each successive issue year.

## CHANGES TO MEDICARE

Congress recently reduced the coverage provided by Medicare supplement policies (for example, adding Medicare catastrophic benefits). Thus we should consider which of the conclusions above can be extended to a benefit reduction that meets the assumptions stated at the beginning of this paper.

As noted in the Appendix, the validity of the proof does not depend on the sign of the terms in the formula. Therefore Formulas (I) through (V) apply to benefit decreases as well as to benefit increases.

If $r(t) \leq 0$ for all $t$, then certain comments made above continue to apply when properly modified. Under the prospective method reserve factors before the current change are not affected by the current change. The actual sequence of terminal reserves that are used is the final set in use. Corresponding to Figure 1 is a graph of reserve factors that converge downward to the final set in use. In addition, if the parameters are chosen so that the
global method produces a reasonable level of profit, then the graph of the ratio of the net gain to earned premium looks like Figure 6, except that the graph has been revolved around the line that is the graph of the global method ratios. Similarly, the general shape of the graphs that correspond to Figures 4 and 5 can be found by performing a revolution around the graph of the global method ratios.

Formulas (I) through (V) also apply when benefit increases and decreases are interspersed. However, mixing of increases and decreases produces sequences of reserve factors that, when graphed, have corners and are generally not as smooth as when mixing does not occur. The same is true for the graph of the net gain as a ratio to the earned premium, except that the global method always produces a horizontal line for this graph. The vast range of possible sequences of increases and decreases makes a more detailed discussion impractical.

One assumption at the outset was that the benefit increases could be expressed as a level percentage of the claim costs before the increase for all attained ages. I suspect that the effect of the legislative changes on the claim costs for a Medicare supplement policy cannot be expressed as a constant percentage decrease across all attained ages. The formulas given in the prospective approach can therefore yield some clues about the result of recalculating the reserve factors, but the results will not be exactly correct.

Although the legislative changes to Medicare did not provide full firstdollar coverage or complete catastrophic coverage, they did reduce the coverage that a Medicare supplement policy can provide. An ongoing process of reducing Medicare supplement benefits may before long cause the policyholders to question the need to continue their policies. This calls into question the applicability of the assumption in the prospective method about the constancy of persistency rates in the face of the changes in future benefits. So again, there appears to be more reason to question the use of Formulas (I) through (V) when evaluating the effect of an ongoing sequence of benefit reductions.

## OTHER CONSIDERATIONS

This discussion has assumed that benefit increases take place on policy anniversaries. These increases actually take place on January 1 each year, so the increases occur at various non-integral durations. The implementation of a reserve methodology should recognize that increases do not occur on the anniversary or at a constant fraction of a policy year after an anniversary.

Consideration should be given as to how the results derived here would change if the three original assumptions are relaxed. The key assumption appears to be the third (no change in anticipated persistency as a result of the increased premium). Removing the first two assumptions renders the formulas of this paper inapplicable, but leaves the other conclusions in place. For the prospective method in particular, the benefit increase may still be viewed as a new policy with a separate benefit net premium and reserve and with no acquisition cost. The prospective expense reserves remain unchanged. Although the benefit reserves are harder to calculate, they display the same kind of convergence shown above for both prospective and retrospective methods.

Relaxing the persistency assumption causes more severe problems. As with the first two assumptions, the formulas in this paper no longer hold. But now the change in the present value of future benefits is partly the result of increasing the benefits and partly the result of the persistency changes. That is, $b(k, k)-b(k, k-1)$ does not equal the present value of the benefits added to the policy at time $k$. Under these circumstances the general relationships developed above cannot be retained. Benefit net premiums and reserves are not a sum of terms, each of which equals the net premium or reserve for the appropriate benefit increase.

To this point the possibility of rate increases different from the increase in benefits has not been considered. The calculation of benefit and expense reserve factors under the prospective method relies upon the gross premiums being the same in all future years, rather than having any specific value. Suppose profitability considerations dictate that the gross premiums must be increased by an amount greater than that called for due to increases in the benefit structure. If this additional increase is implemented so that all future gross premiums are level, then the formulas for the prospective method will continue to hold in the presence of such rate increases. If the additional increase is phased in over time, then the prospective method formulas do not hold. In either event, any comments about the ratio of GAAP net premiums to gross premiums will need to be modified.

The global and retrospective methods can clearly be applied when the increase to the gross premium is different than the increase in benefits. The resulting pattern of profits will then be other than presented here, although if actual experience matches expected, the global method will continue to have profits equal a constant percentage of premium and the same general observations made above will continue to hold.

## CONCLUSION

Benefit reserves for Medicare supplement business must be adjusted periodically to reflect changes in future benefits due to increases in Medicare deductibles. Of the choices considered in this paper, the global method is least preferable. Because all increases are predicted at issue, this method produces the greatest chance of a subsequent mismatch between revenue and expense.

The prospective method is easy to understand and requires less effort to implement, because only the benefit reserve factors need to be changed. The prospective method is in compliance with the American Academy of Actuaries' recommendations and does not require a change in surplus when implemented. While not producing, in total, a net gain that is a level percentage of revenue in all years, it does produce results that are much more reasonable than the retrospective method. Finally, in the early years when substantial amounts of the business remain on the books, the prospective method gives the most favorable result for the loss recognition test.

The intermediate method, being a combination of the global and prospective methods, has some attributes of each method. It is only a little more complicated than the prospective method and requires changes less often than the prospective method. There is no effect on surplus when a change is implemented by the intermediate method. Figure 6 implies that the pattern of earnings, expressed as a percentage of premiums, is flatter over the lifetime of the business than that for the prospective method, although not as flat as that for the global method. As with the global method, the reasonableness of the results above for the intermediate method is in part due to allowing the actual experience to equal the assumptions in the intermediate reserve factors. When actual experience differs from expected, the results for the intermediate method will be less appealing, as Figure 7 indicated for the global method.

In my opinion, the prospective method is the best choice in this illustration, with the intermediate method still having much to recommend it. My preference is based on the potential for variation in the results with the intermediate method and on the results of the loss recognition comparison.

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## APPENDIX

The following formula will be needed later. The proof is by induction.

$$
\begin{equation*}
\prod_{t=1}^{k}\left(1+x_{i}\right)-\sum_{t=1}^{k}\left\{x_{i} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\}=1 \tag{A}
\end{equation*}
$$

Note that when the upper index for a product of a sequence of terms is less than the lower index for that product, the product is taken to be 1 . For $k=1$ in Formula (A), the only value that $t$ takes on is 1 . Thus the upper index of the product, which is $t-1$ or zero, is less than the lower index, specified by $s=1$, and the value of the product for $t=1$ is 1 . Then the left-hand side of the formula is $\left(1+x_{1}\right)-x_{1}$, which equals 1 .

Assume the formula is true for $k=n$, and let $k=n+1$ :

$$
\begin{aligned}
\prod_{t=1}^{n+1}\left(1+x_{t}\right) & -\sum_{t=1}^{n+1}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\} \\
= & \left(1+x_{n+1}\right)\left[1+\sum_{t=1}^{n}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\}\right] \\
& -\sum_{t=1}^{n+1}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&=\left(1+x_{n+1}\right)+x_{n+1} \sum_{t=1}^{n}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\} \\
&+\sum_{t=1}^{n}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\}-\sum_{t=1}^{n+1}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\} \\
&= 1+x_{n+1}\left[1+\sum_{t=1}^{n}\left\{x_{t} \prod_{s=1}^{t-1}\left(1+x_{s}\right)\right\}-\prod_{s=1}^{n}\left(1+x_{s}\right)\right] \\
&=1
\end{aligned}
$$

since the term in brackets is 0 by the induction hypothesis. Note that there is no restriction of the magnitude of the $x_{t}$.

From the definition of $V(t, k)$, we get

$$
N(k)=\frac{b(t, k)-V(t, k)}{a(t)}
$$

for all $t>k$. From the definition of $b(t, k)$, we get

$$
\begin{aligned}
b(k, k) & =b(k, k-1)[1+r(k)] \\
& =b(k, 0) \prod_{t=1}^{k}[1+r(t)] .
\end{aligned}
$$

By definition,

$$
\begin{aligned}
N(k) & =\frac{b(k, k)-V(k, k-1)}{a(k)} \\
& =\frac{b(k, k-1)-V(k, k-1)+r(k) b(k, k-1)}{a(k)} \\
& =N(k-1)+\frac{r(k) b(k, 0)}{a(k)} \prod_{s=1}^{k-1}[1+r(s)] \\
& =N(0)+\sum_{t=1}^{k}\left\{\frac{r(t) b(t, 0)}{a(t)} \prod_{s=1}^{t-1}[1+r(s)]\right\},
\end{aligned}
$$

the last step being by repeated application of the first three steps or by an induction hypothesis. This is Formula (I) in the text.

Using Formula (A), we get

$$
\begin{aligned}
N(k)= & N(0)\left[\prod_{t=1}^{k}[1+r(t)]-\sum_{i=1}^{k}\left\{r(t) \prod_{s=1}^{t-1}[1+r(s)]\right\}\right] \\
& +\sum_{t=1}^{k}\left\{r(t) \frac{b(t, 0)}{a(t)} \prod_{s=1}^{t-1}[1+r(s)]\right\} \\
= & N(0) \prod_{t=1}^{k}[1+r(t)] \\
& +\sum_{t=1}^{k}\left\{r(t)\left[\frac{b(t, 0)}{a(t)}-N(0)\right] \prod_{s=1}^{t-1}[1+r(s)]\right\} \\
= & N(0) \prod_{t=1}^{k}[1+r(t)]+\sum_{t=1}^{k}\left\{\frac{r(t) V(t, 0)}{a(t)} \prod_{s=1}^{t-1}[1+r(s)]\right\}
\end{aligned}
$$

which is Formula (II).
From the definitions,

$$
\begin{aligned}
V(t, k-1)= & b(t, k-1)-N(k-1) a(t) \\
= & b(t, 0) \prod_{s=1}^{k-1}[1+r(s)] \\
& -\left[N(0)+\sum_{s=1}^{k-1}\left\{\frac{r(s) b(s, 0)}{a(s)} \prod_{v=1}^{s-1}[1+r(v)]\right\}\right] a(t) \\
= & b(t, 0)\left[1+\sum_{s=1}^{k-1}\left\{r(s) \prod_{v=1}^{s-1}[1+r(v)]\right\}\right] \\
& -\left[N(0)+\sum_{s=1}^{k-1}\left\{\frac{r(s) b(s, 0)}{a(s)} \prod_{v=1}^{s-1}[1+r(v)]\right\}\right] a(t) \\
= & {[b(t, 0)-N(0) a(t)] } \\
& +\sum_{s=1}^{k-1}\left\{r(s)\left[b(t, 0)-\frac{b(s, 0) a(t)}{a(s)}\right] \prod_{v=1}^{s-1}[1+r(v)]\right\}
\end{aligned}
$$

$$
\begin{aligned}
= & V(t, 0)+\sum_{s=1}^{k-1}\{r(s)[b(t, 0)-N(0) a(t) \\
& \left.\left.+\frac{a(t)[N(0) a(s)-b(s, 0)]}{a(s)}\right]_{v=1}^{s-1}[1+r(v)]\right\} \\
= & V(t, 0)+\sum_{s=1}^{k-1}\left\{r(s)\left[V(t, 0)-\frac{a(t) V(s, 0)}{a(s)}\right]_{v=1}^{s-1}[1+r(v)]\right\}
\end{aligned}
$$

which is Formula (III).

$$
\begin{aligned}
= & V(t, 0)\left[1+\sum_{s=1}^{k-1}\left\{r(s) \prod_{v=1}^{s-1}[1+r(v)]\right\}\right] \\
& -a(t) \sum_{s=1}^{k-1}\left\{\frac{r(s) V(s, 0)}{a(s)} \prod_{v=1}^{s-1}[1+r(v)]\right\} .
\end{aligned}
$$

Using Formula (A), we get

$$
=V(t, 0) \prod_{s=1}^{k-1}[1+r(s)]-a(t) \sum_{s=1}^{k-1}\left\{\frac{r(s) V(s, 0)}{a(s)} \prod_{v=1}^{s-1}[1+r(v)]\right\}
$$

which is Formula (IV).
From Formula (III), the additional benefit reserve at time $t$ for the increase at time $k$ is

$$
V(t, k)-V(t, k-1)=r(k)\left[V(t, 0)-\frac{a(t) V(k, 0)}{a(k)}\right] \prod_{s=1}^{k-1}[1+r(s)] .
$$

But we have

$$
V(t, 0)=b(t, 0)-a(t) N(0),
$$

for all $t$. Hence,

$$
\begin{aligned}
V(t, k)-V(t, k-1)= & r(k)[\{b(t, 0)-a(t) N(0)\} \\
& \left.-\frac{a(t)}{a(k)}\{b(k, 0)-a(k) N(0)\}\right] \prod_{s=1}^{k-1}[1+r(s)]
\end{aligned}
$$

$$
\begin{aligned}
= & r(k) b(t, 0) \prod_{s=1}^{k-1}[1+r(s)] \\
& -a(t)\left[\frac{r(k) b(k, 0)}{a(k)} \prod_{s=1}^{k-1}[1+r(s)]\right] \\
= & {[b(t, k)-b(t, k-1)]-a(t)[N(k)-N(k-1)] . }
\end{aligned}
$$

The items in the first set of parenthesis equal the present value at time $t$ of all future benefits due to the benefit increase at time $k$. The remainder equals the present value at time $t$ of the future net premiums due to the benefit increase at time $k$. The difference is then the standard presentation of a terminal reserve: the present value of future benefits less the present value of future net premiums.

The assumptions used in calculating reserves in this paper are as shown in Table A1. Withdrawal allocation factors are used to allocate the $q(t)$ terminations to the beginning and end of the policy year. No premiums are earned or claims paid on the $w(t) q(t)$ terminations allocated to the beginning of the policy year. A full premium is earned and a full year's claims are paid on the $[1-w(t)] q(t)$ terminations allocated to the end of the year.

TABLE A1

| Year | Claim Cost, $c c(t)$ | $\begin{aligned} & \text { Temmination } \\ & \text { Rate, } q(t) \end{aligned}$ | Interest Rate, i(f) | Withdrawal Allocation Factor, w(r) |
| :---: | :---: | :---: | :---: | :---: |
| 1... | 167.16 | 0.27 | 7.5\% | 0.60 |
| 2. | 206.09 | 0.22 | 7.5 | 0.50 |
|  | 235.13 | 0.20 | 7.5 | 0.40 |
|  | 241.43 | 0.19 | 7.5 | 0.40 |
|  | 247.93 | 0.19 | 7.5 | 0.40 |
|  | 254.47 | 0.20 | 7.5 | 0.40 |
| 7. | 260.65 | 0.20 | 7.5 | 0.40 |
| 8. | 266.84 | 0.21 | 7.5 | 0.40 |
| 9. | 273.17 | 0.22 | 7.5 | 0.40 |
| 10. | 279.74 | 0.23 | 7.5 | 0.40 |
| 11. | 286.12 | 0.24 | 7.3 | 0.40 |
| 12. | 292.88 | 0.25 | 7.1 | 0.40 |
| 13. | 299.69 | 0.26 | 6.9 | 0.40 |
| 14... | 306.59 | 0.28 | 6.7 | 0.40 |
| 15. | 313.59 | 0.30 | 6.5 | 0.40 |
|  | 320.68 | 0.32 | 6.4 | 0.40 |
| 17. | 327.94 | 0.34 | 6.3 | 0.40 |
| 18. | 353.31 | 1.00 | 6.2 | 0.40 |

Then in terms of the symbols defined in the text,

$$
\begin{aligned}
V(t, 0)= & (\{V(t-1,0)+N(0)[1-w(t) q(t)]\}[1+i(t)] \\
& -c c(t) \sqrt{1+i(t)}[1-w(t) q(t)]) /[1-q(t)]
\end{aligned}
$$

The assumptions in Table A1 were used to produce Tables A2-A6. Other assumptions are that deferred acquisition costs are $\$ 1,000,000$; the new paid annualized premium is $\$ 1,000,000$; and the premium per unit at issue is $\$ 325$. The increase in claim costs and gross premiums, $r(t)$, is taken as 0.15 in all years.

Premiums are received at the beginning of each policy year. Investment income is received at the end of each policy year and equals one year's interest earned on the prior year's ending benefit reserve less one year's interest on the unamortized policy acquisition costs at the beginning of the policy year, after receipt of the net expense premium for the year. Claim payments are made in the middle of each policy year. The change in reserve takes place at the end of the policy year. All values have been discounted to the beginning of the policy year. As a consistency check among the illustrations, the present value of the net gains was calculated and found to equal $\$ 508,556$.

TABLE A2
Prospective Method

| Year | Premiums | Investment Income | Cleims | Change in Reserve | Amortization | Net Gain | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 838,000 | -55,258 | 415,708 | 133,657 | 207,973 | 25,404 | 0.030 |
| 2 | 747,155 | -34,072 | 456,962 | 48,426 | 159,987 | 47,709 | 0.064 |
|  | 692,789 | -21,406 | 483,416 | -2,797 | 129,488 | 61,276 | 0.088 |
| 4 | 640,137 | -14,329 | 458,645 | -6,119 | 104,437 | 68,845 | 0.108 |
| 5 | 596,288 | -8,839 | 438,730 | -9,228 | 85,265 | 72,682 | 0.122 |
|  | 553,037 | -4,691 | 417,641 | -11,745 | 69,383 | 73,067 | 0.132 |
|  | 508,794 | -1,663 | 393,561 | -13,069 | 56,017 | 70,621 | 0.139 |
| 8 | 466,056 | 503 | 369,064 | - 13,967 | 45,105 | 66,358 | 0.142 |
|  | 421,563 | 1,958 | 341,749 | -14,467 | 35,859 | 60,378 | 0.143 |
| 10 | 376,483 | 2,837 | 312,545 | -14,706 | 28,152 | 53,330 | 0.142 |
| 11 | 331,907 | 3,183 | 282,086 | -14,719 | 21,934 | 45,789 | 0.138 |
| 12 | 288,803 | 3,174 | 251,486 | - 14,624 | 16,879 | 38,235 | 0.132 |
|  | 247,986 | 2,908 | 221,170 | -14,166 | 12,837 | 31,051 | 0.125 |
| 14 | 209,152 | 2,481 | 191,009 | -13,376 | 9,605 | 24,394 | 0.117 |
| 15 | 171,617 | 1,968 | 160,459 | -12,206 | 7,004 | 18,327 | 0.107 |
| 16 | 136,896 | 1,457 | 130,951 | - 10,682 | 4,978 | 13,105 | 0.096 |
| 17 | 106,071 | 967 | 103,810 | -9,043 | 3,464 | 8,807 | 0.083 |
| 18 | 55,908 | 487 | 58,977 | -7,851 | 1,632 | 3,636 | 0.065 |
| Present Value |  |  |  |  |  | 508,556 |  |

TABLE A3
Retrospective Method

| Year | Premiums | Investment Income | Claims | Change in Reserve | Amorization | Net Gain | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 838,000 | -57,222 | 415,708 | 138,409 | 179,812 | 46,847 | 0.056 |
| 2 | 747,155 | -38,021 | 456,962 | 58,610 | 126,436 | 67,127 | 0.090 |
| 3 | 692,789 | -26,718 | 483,416 | 9,090 | 99,001 | 74,564 | 0.108 |
| 4 | 640,137 | -20,444 | 458,645 | 6,233 | 80,156 | 74,660 | 0.117 |
| 5 | 596,288 | -15,204 | 438,730 | 2,512 | 68,407 | 71,435 | 0.120 |
| 6 | 553,037 | -10,836 | 417,641 | -1,465 | 59,906 | 66,120 | 0.120 |
| 7 | 508,794 | -7,233 | 393,561 | -4,821 | 53,221 | 59,600 | 0.117 |
| 8 | 466,056 | -4,227 | 369,064 | -8,254 | 48,263 | 52,756 | 0.113 |
|  | 421,563 | -1,790 | 341,749 | -11,509 | 43,805 | 45,728 | 0.109 |
| 10 | 376,483 | 118 | 312,545 | -14,588 | 39,716 | 38,927 | 0.103 |
| 11 | 331,907 | 1,514 | 282,086 | -17,527 | 36,243 | 32,618 | 0.098 |
| 12 | 288,803 | 2,417 | 251,486 | -20,282 | 33,012 | 27,004 | 0.094 |
| 13 | 247,986 | 2,891 | 221,170 | -22,521 | 30,051 | 22,175 | 0.089 |
| 14 | 209,152 | 3,008 | 191,009 | -24,108 | 27,189 | 18,070 | 0.086 |
| 15 | 171,617 | 2,829 | 160,459 | -24,731 | 24,169 | 14,549 | 0.085 |
| 16 | 136,896 | 2,479 | 130,951 | -24,441 | 21,207 | 11,657 | 0.085 |
| 17 | 106,071 | 2,003 | 103,810 | -23,720 | 18,580 | 9,402 | 0.089 |
| 18 | 55,908 | 1,133 | 58,977 | -18,267 | 10,825 | 5,505 | 0.099 |
| Present Value |  |  |  |  |  | 508,556 |  |

TABLE A4
Global Method

| Year | Premiums | Investment Income | Claims | Change in Reserve | Amorization | Net Gain | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 838,000 | -62,831 | 415,708 | 177,522 | 99,423 | 82,515 | 0.099 |
| 2 | 747,155 | -43,379 | 456,962 | 85,272 | 87,971 | 73,570 | 0.099 |
| 3 | 692,789 | -31,175 | 483,416 | 26,727 | 83,253 | 68,217 | 0.099 |
| 4 | 640,137 | -23,692 | 458,645 | 16,230 | 78,538 | 63,032 | 0.099 |
| 5 | 596,288 | -17,221 | 438,730 | 6,321 | 75,302 | 58,715 | 0.099 |
| 6 | 553,037 | -11,727 | 417,641 | -2,734 | 71,947 | 54,456 | 0.099 |
| 7 | 508,794 | -7,179 | 393,561 | -10,179 | 68,134 | 50,099 | 0.099 |
| 8 | 466,056 | -3,453 | 369,064 | -16,700 | 64,349 | 45,891 | 0.099 |
| 9 | 421,563 | -525 | 341,749 | -22,135 | 59,914 | 41,510 | 0.099 |
| 10 | 376,483 | 1,654 | 312,545 | -26,504 | 55,024 | 37,071 | 0.099 |
| 11 | 331,907 | 3,093 | 282,086 | - 30,024 | 50,257 | 32,682 | 0.099 |
| 12 | 288,803 | 3,880 | 251,486 | - 32,511 | 45,270 | 28,438 | 0.099 |
| 13 | 247,986 | 4,128 | 221,170 | -33,721 | 40,246 | 24,418 | 0.099 |
| 14 | 209,152 | 3,959 | 191,009 | -33,637 | 35,144 | 20,595 | 0.099 |
| 15 | 171,617 | 3,479 | 160,459 | -32,110 | 29,849 | 16,899 | 0.099 |
| 16 | 136,896 | 2,855 | 130,951 | -29,338 | 24,658 | 13,480 | 0.099 |
| 17 | 106,071 | 2,142 | 103,810 | -25,938 | 19,896 | 10,444 | 0.099 |
| 18 | 55,908 | 1,133 | 58,977 | -18,267 | 10,825 | 5,505 | 0.099 |
| Present Value |  |  |  |  |  | 508,556 |  |

TABLE A5
Intermediate Method

| Year | Premiums | Invesiment Income | Claims | Change in Reserve | Amortization | Net Gain | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 838,000 | -59,638 | 415,708 | 149,158 | 145,194 | 68,302 | 0.082 |
| 2 | 747,155 | -39,227 | 456,962 | 57,856 | 132,213 | 60,898 | 0.082 |
| 3 | 692,789 | -25,968 | 483,416 | -906 | 127,843 | 56,466 | 0.082 |
| 4 | 640,137 | - 19,986 | 458,645 | 5,644 | 86,718 | 69,144 | 0.108 |
| 5 | 596,288 | -13,686 | 438,730 | -4,762 | 84,226 | 64,407 | 0.108 |
| 6 | 553,037 | -8,346 | 417,641 | -14,356 | 81,670 | 59,736 | 0.108 |
| 7 | 508,794 | - 5,454 | 393,561 | -6,662 | 56,883 | 59,559 | 0.117 |
| 8 | 466,056 | -2,171 | 369,064 | $-13,950$ | 54,216 | 54,556 | 0.117 |
| 9 | 421,563 | 345 | 341,749 | - 20,252 | 51,063 | 49,348 | 0.117 |
| 10 | 376,483 | 1,317 | 312,545 | $-13,159$ | 35,701 | 42,714 | 0.113 |
| 11 | 331,907 | 2,551 | 282,086 | - 18,046 | 32,761 | 37,657 | 0.113 |
| 12 | 288,803 | 3,170 | 251,486 | - 21,962 | 29,683 | 32,766 | 0.113 |
| 13 | 247,986 | 2,935 | 221,170 | -16,739 | 21,170 | 25,319 | 0.102 |
| 14 | 209,152 | 2,894 | 191,009 | -18,828 | 18,511 | 21,354 | 0.102 |
| 15 | 171,617 | 2,548 | 160,459 | - 19,565 | 15,749 | 17,522 | 0.102 |
| 16 | 136,896 | 1,964 | 130,951 | $-16,413$ | 11,754 | 12,568 | 0.092 |
| 17 | 106,071 | 1,463 | 103,810 | -15,499 | 9,484 | 9,738 | 0.092 |
| 18 | 55,908 | 780 | 58,977 | -12,582 | 5,160 | 5,133 | 0.092 |
| Present Value |  |  |  |  |  | 508,556 |  |

TABLE A6
Static Method

| Year | Premiums | Investment Income | Claims | Change in Reserve | Amortization | Net Gain | Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 838,000 | -55,258 | 415,708 | 133,657 | 207,973 | 25,404 | 0.030 |
| 2 | 747,155 | -34,072 | 456,962 | 38,588 | 159,987 | 57,547 | 0.077 |
| 3 | 692,789 | -22,143 | 483,416 | -9,196 | 129,488 | 66,937 | 0.097 |
| 4 | 640,137 | -15,547 | 458,645 | $-13,409$ | 104,437 | 74,918 | 0.117 |
| 5 | 596,288 | -10,604 | 438,730 | -16,120 | 85,265 | 77,809 | 0.131 |
| 6 | 553,037 | -6,972 | 417,641 | -17,374 | 69,383 | 76,416 | 0.138 |
| 7 | 508,794 | -4,367 | 393,561 | -17,234 | 56,017 | 72,083 | 0.142 |
| 8 | 466,056 | -2,513 | 369,064 | -16,466 | 45,105 | 65,841 | 0.141 |
| 9 | 421,563 | -1,246 | 341,749 | -15,192 | 35,859 | 57,900 | 0.137 |
| 10 | 376,483 | -421 | 312,545 | -13,642 | 28,152 | 49,007 | 0.130 |
| 11 | 331,907 | 84 | 282,086 | -12,022 | 21,934 | 39,993 | 0.121 |
| 12 | 288,803 | 346 | 251,486 | -10,394 | 16,879 | 31,178 | 0.108 |
| 13 | 247,986 | 446 | 221,170 | -8,784 | 12,837 | 23,209 | 0.094 |
| 14 | 209,152 | 448 | 191,009 | -7,239 | 9,605 | 16,224 | 0.078 |
| 15 | 171,617 | 391 | 160,459 | -5,771 | 7,004 | 10,316 | 0.060 |
| 16 | 136,896 | 315 | 130,951 | -4,442 | 4,978 | 5,724 | 0.042 |
| 17 | 106,071 | 236 | 103,810 | -3,330 | 3,464 | 2,363 | 0.022 |
| 18 | 55,908 | 121 | 58,977 | $-1,954$ | 1,632 | -2,627 | -0.047 |
| Present Value |  |  |  |  |  | 508,556 |  |

## DISCUSSION OF PRECEDING PAPER

## ERIC SEAH AND ELIAS S. W. SHIU:

We wish to remark that Formula (A) in the Appendix of the paper can be viewed as a generalization of the compound interest formula

$$
(1+i)^{k}-i s_{k}=1
$$

by considering $x_{k}$ as the first-period interest rate, $x_{k-1}$ as the second-period interest rate, $\ldots$, and $x_{1}$ as the $k$-th period interest rate. Similarly, by considering $-x_{1}$ as the first-period discount rate, $-x_{2}$ as the second-period discount rate, $\ldots$, and $-x_{k}$ as the $k$-th period discount rate, we see that formula (A) also generalizes the formula

$$
v^{k}+d \vec{a}_{k 1}=1
$$

Formula (A) can also be proved by applying the formula

$$
\sum_{j=a}^{b} \Delta f(j)=f(b+1)-f(a) .
$$

Here, $a=0, b=k-1, f(0)=1$ and, for $j=1,2,3, \ldots, k$,

$$
f(j)=\prod_{i=1}^{j}\left(1+x_{i}\right) .
$$

We also wish to note that, in terms of $A P L$, the left-hand side of (A) can be written as

$$
(\times / 1+X)-+/ X \times-1 \downarrow \times \backslash 1,1+X
$$

For each vector $X$, the value of this expression is 1 .

## CHARLES D. FRIEDSTAT:

I thank and congratulate Mr. Raws for writing such a fine paper on an area that has been discussed only briefly in actuarial literature-GAAP for accident and health insurance. I hope this paper serves as a catalyst for more discussions relating to:

1. Further work related to Medicare supplement reserving outside of the cases of the simplifying assumptions. Premium increases for adverse
experience over and above that related to the increase in Medicare deductible, effects of shock lapses, and other changes in assumptions could be explored from a GAAP statement point of view.
2. GAAP accounting for other coverages including individual major medical and small group. For these accident and health coverages in particular, I think it would be very valuable to have additional guidance for the practicing actuary in an environment in which substantial rate increases may be required quite unexpectedly, lapse rates are relatively high, and it may be difficult to anticipate the effect of high rate increases on future claim costs, shock lapses and adverse selection.
Because I had the privilege of reviewing this paper prior to final submission, all my prior observations appear to have been addressed in this published version. My remaining comments deal with the practical aspects of implementing a methodology that not only is consistent with GAAP but also can be implemented reasonably from a practical point of view.

Mr. Raws demonstrates that in the case in which the three simplifying assumptions are prevalent, reserves under the prospective approach can be adjusted rather easily. Unfortunately, this will almost never be the case. That is why I suggested to Mr. Raws that he consider an approach somewhere between the global approach (with its obvious problems from a GAAP earnings point of view) and the prospective approach (which appears to be the technically correct GAAP answer and produces a desirable pattern of earnings). The prospective approach simply takes too much effort to make the changes annually, especially after you have been issuing the policy for many years. This attempt at a more practical approach has been designated by Mr. Raws as the intermediate approach.

According to GAAP, prospective unlocking of assumptions should generally occur when current estimates of future premium revenue and benefit increases are materially different from those in the current GAAP assumptions. In the current health insurance environment, predicting future claim costs, the need for future rate increases, the level of future Medicare deductible increases, and so on is virtually impossible other than for a short time period. I believe that a practical approach for a company might be to project future premium and benefit increases for, say, a three-year period before unlocking prospectively (unless developing experience indicates earlier unlocking is required). This might overcome the obvious problems with the global method and the practical problems of revising factors each year under the prospective method. The intermediate method may then serve to
put a company back on course (to levelize the pattern of future GAAP earnings) on a timely basis. Based on the examples in the paper and the implementation of similar approaches at some insurers, the intermediate method may have merit.

In conclusion, I thank Mr. Raws for a well-written paper on a topic and area of GAAP accounting in which little has been written. I hope this paper leads to additional research and papers in the health insurance GAAP financial reporting area.

## (AUTHOR'S REVIEW OF DISCUSSION)

ALFRED RAWS, III:
The author wishes to thank Dr. Seah and Dr. Shiu for their observations concerning Formula (A) in the appendix. I had made no connection between that formula and any standard interest formulas.

The author wishes to thank Mr. Friedstat for his continuing thoughts on my paper. I agree with him in the hope that the paper will serve to provoke additional thought in the application of GAAP to accident and health products.

The author has had phone conversations with two actuaries who chose not to submit written discussions. One suggested that the reserve factors could be calculated as a ratio to premiums. Then when the gross premiums are increased by the same percentage as future claim costs, the valuation system would automatically adjust the reserve without having to update any factor files. This amounts to a reserve formula per unit of coverage, which is as follows:

$$
V(t, k)=V(t, 0) \prod_{s=1}^{k}[1+r(s)]
$$

This should be compared to Formula (IV) in the paper. The suggested formula has some of the problems associated with the prospective method described in the paper. Specifically, there would be a change in the carried reserve at the time of a change in the gross premium. This discontinuity does not seem to be desirable. The author does, however, see the appeal of this idea and certainly appreciates the benefit of not having to perform annual updates to the factor file.

The second conversation related that the formulas in the paper associated with the prospective method had been adopted by a client company. The author is gratified that his ideas have been found sufficiently sensible as to be put into use somewhere on a day-to-day basis.

