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Credibility Theory: An Application to Pension Mortality Assumptions

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Almost all societies, at least in recent history, seem to be concerned with mortality, both current experience and future trends. That's understandable, considering Ben Franklin's observation that mortality is one of life's two certainties (the other being taxes—a topic we won't address here.)

For pension actuaries and plan sponsors, mortality trends are particularly important, in large part because of their impact on future cash flows to retirees and beneficiaries. When the fact that demographic data is becoming increasingly easy to obtain is combined with the large financial impact that mortality assumptions have on determining pension liabilities, many plan sponsors are now strongly motivated to modify commonly accepted (or government prescribed) mortality tables.

In past decades, only a few pension plans have been large enough and had enough data to deem their experience credible. Sponsors for those plans sometimes developed their own tables or modified existing ones. However, those plans were an exception. Until recently, relatively few pension actuaries had a need to use credibility theory in their practice. As a result, there has not been much literature available for use by a practicing pension actuary.

In light of increasing focus on the mortality assumption by regulatory agencies, especially in the United States, Canada and the United Kingdom, credibility theory has become more relevant for pension actuaries. An example of its newfound relevance is found in the final regulations that the U.S. Department of Treasury published in October 2017. For the first time, U.S. pension regulations permit plan sponsors to modify the prescribed pension mortality tables even when the plan data is too small to be fully credible. In other words, plans no longer need to have very large numbers of lives and deaths to modify the tables and to reflect their own plans' mortality experience, at least to some degree.

In order to help pension actuaries better understand and refresh their knowledge of credibility theory, the Retirement Section has published two papers on the topic. The first is an education resource written by Irina Pogrebivsky in August 2017. The second paper, written by Gavin Benjamin in 2008, provides a more theoretical approach to the topic. Both papers can be accessed on the home page of the SOA Retirement Section website and are listed at the end of this article.

OVERVIEW OF THE EDUCATION RESOURCE

Pogrebivsky's Education Resource was written to help practicing actuaries apply credibility theory.

The paper's objectives are

- to provide an overview of credibility theory
- describe and compare the standard approaches
- provide guidance on how to apply the theory with respect to mortality
- show example of situations that can be used as a basis for evaluating relevant application based on a specific scenario
- provide a list of resources on credibility theory that exist within the actuarial profession

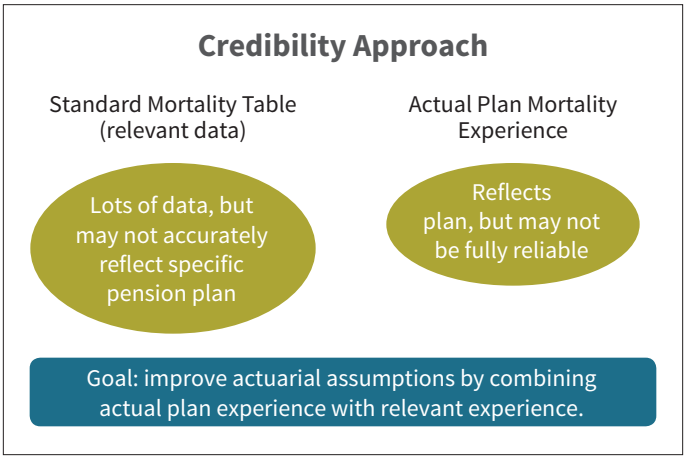
Although this paper was published two months before the U.S. Treasury's final regulations and the supporting Revenue Procedure 2017-55, the concepts presented in the paper are helpful in providing background for the approach described in the regulations and Revenue Procedure.

The remainder of this article summarizes some of the key concepts developed in the paper. For anyone who intends to use credibility theory or to adjust mortality table assumptions in their practice, the paper provides a more thorough background and development of the underlying theory.

WHAT IS CREDIBILITY THEORY?

To quote the paper, "Credibility procedures use statistical approaches to adjust relevant experience-based assumptions." Credibility is a way to combine the experience of one relatively small, stand-alone group, such as the experience of one pension plan, with the relevant experience of a broader group. The goal is to improve the estimate for the smaller group. Life and property and casualty actuaries have historically used credibility in setting premiums for various classes, for instance, groupings of risk with similar characteristics.

Figure 1
Credibility Approach



One of the key aspects of the paper is its focus on setting the base mortality table assumption. Future mortality improvements are not addressed. This is consistent with the final regulations and procedure.

In theory, an actuary could build a mortality table from scratch as follows:

- For each age x, estimate q_x using the plan's experience
- But how much experience would be needed at a given age x for the estimate of q_x to be fully credible?
- If $\hat{q}_x$ is the estimate of q_x, then $\hat{q}_x$ can be considered fully credible when:

  $$\Pr[(1-r)q_x \le \hat{q}_x \le (1+r)q_x] \ge p$$

  where p = confidence level and r = margin of error.

  In other words, there is a p% probability that $\hat{q}_x$ is within r% margin of error.
- Once r and p are selected, you can calculate the minimum number of deaths needed for $\hat{q}_x$ to be considered "fully" credible.

The preceding approach is straightforward, but the number of deaths required for each age would prove impractical in almost all cases. For example if p (the confidence level) were 0.9 and r (the margin of error) were 0.05, the number of deaths for each age x would need to be 1,082. (The paper explains the derivation of the 1,082, which assumes a standard normal distribution for the sample.) Since the probability of death is small at most ages, the amount of data needed is very large. For example, at age 75, if the probability of dying is 2.5%, then that age would need at least 1,082/0.025 or 43,280 life years of experience to claim "full" credibility. Instead, it is more practical to adjust an existing table to reflect a plan's experience using a credibility approach.

COMMON CREDIBILITY APPROACHES

The paper describes two main approaches: the Greatest Accuracy Credibility Theory (GACT) and the Limited Fluctuation Credibility Theory (LFCT). The paper reviews the merits of the two approaches and how the two differ. The LFCT is not as theoretically rigorous as GACT, but it has the advantage of requiring far less data and therefore being more practical. Most of the paper describes how to apply the LFCT in real-life situations.

SHIFTING THE BASE TABLE

The LFCT approach "shifts" a standard mortality table up or down based on a plan's experience; in other words it adjusts mortality rates for all ages by the same ratio.

The amount of the shift depends on the ratio of actual to expected deaths (using the aggregate experience of all ages) and the credibility factor is assigned to that ratio.

PENSION AMOUNTS VERSUS LIVES

Mortality experience studies can be conducted using either lives or pension amounts. Most studies are conducted using amounts, and the new regulations also require using amounts. In addition, most standard mortality tables such as RP 2014 are based on amounts-weighted analysis.

Amounts-weighted mortality rates are often viewed as a proxy for weighting mortality rates by pension liabilities and are often lower than the rates produced by lives-based analyses.

- For amounts-weighted analyses, the estimate of q_x =

  $$\frac{Sum\ of\ pension\ amounts\ for\ actual\ deaths\ age\ x}{Sum\ of\ pension\ amounts\ for\ exposures\ age\ x}$$

THE CREDIBILITY THEORY MODEL USING LFCT

For plans that do not have enough data to have fully credible experience, but have enough experience to be partially credible, the mortality rate at each age is a blend of actual plan experience and the expected experience of the standard table. The amount of weighting assigned to the actual plan experience is the credibility factor, called **Z**.

$$q_x^F = Z \times \left[\hat{f} \times q_x^S\right] + [1 - Z] \times q_x^S$$

where

- $\hat{f}$ = $\frac{Sum\ of\ pension\ amounts\ for\ actual\ deaths\ across\ all\ ages}{Sum\ of\ pension\ amounts\ for\ expected\ deaths\ across\ all\ ages}$
- q_x^S = mortality rate at age x based on the standard table
- Z = credibility assigned to the plan experience
- q_x^F = final mortality rate at age x, which reflects the results of the experience study

HOW TO DETERMINE Z

First, the actuary needs to determine how many lives are required to achieve full credibility. To do that, the actuary needs to decide on ***p*** (the confidence level, often decided to equal 0.9) and ***r*** (the margin of error, often decided to equal 0.05).

> *For a lives-based analysis*, ***Z*** = 1, or $\hat{f}$ is assigned full credibility, if the total number of study deaths is at least equal to 1,082 (for p = 0.9 and r = 0.05)
>
> *For a amounts-weighted analysis*, ***Z*** = 1, or $\hat{f}$ is assigned full credibility, if the total number of study deaths is at least equal to 1,082 (for p = 0.9 and r = 0.05) × ***Benefit Dispersion Factor***

Where

> ***Benefit Dispersion Factor*** =
> *[Expected number of deaths during study period]* ×
> *[Sum of the mortality-weighted square of the benefits]* ÷
> *[Square of the sum of mortality-weighted benefits]*

If there are not enough total study deaths to assign full credibility to $\hat{f}$:

$$Z = \sqrt{\frac{total\ number\ of\ study\ deaths}{number\ of\ study\ deaths\ needed\ for\ full\ credibility}}$$

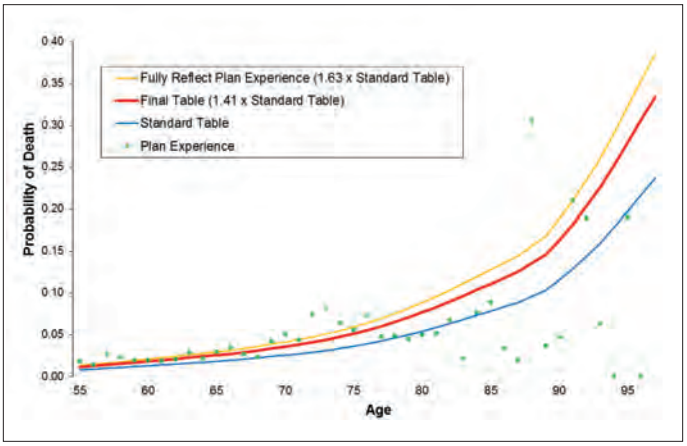
Once ***Z*** is determined, the actuary can construct the customized table using the preceding key formula:

$$q_x^F = Z \times \left[\hat{f} \times q_x^S\right] + \left[1 - Z\right] \times q_x^S$$

EXAMPLE OF A CUSTOMIZED MORTALITY TABLE

After completing the credibility analysis, the result is often a table that better reflects a plan's experience. Figure 2 shows an example of a typical plan whose experience differed substantially from the standard table and whose data was large enough to be partially credible.

Figure 2
Adjusted Mortality Rates



The figure shows a comparison of four sets of data:

- The expected mortality rates based on the standard table (blue line)
- The actual mortality experience of the plan (green dots)
- The mortality rates if the standard table were fully adjusted to reflect the actual mortality experience of the plan (yellow line)
- The final, blended rates based on partial adjustment (red line)

The preceding example demonstrates how partial credibility can improve the fit between expected and actual mortality rates. For further reading on the subject, please see the references at the end of this article. ■

REFERENCES

Credibility Educational Resource for Pension Actuaries by Irina Pogrebivsky, FSA, *https://www.soa.org/Files/static-pages/sections/pension/credibility-resource-pension.pdf*

Selecting Mortality Tables: A Credibility Approach by Gavin Benjamin, FSA, *https://www.soa.org/Files/Research/Projects/research-2008-benjamin.pdf*

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