On the Importance of Dispersion Modeling for Claims Reserving: Application of the Double GLM Theory

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Definition

A loss reserve is a provision for an insurer’s liability for claims.

Notes

Introduction

Definition
- A loss reserve is a provision for an insurer's liability for claims.
- A stochastic model uses random variables in a regression framework.

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**Definition**

- A **loss reserve** is a provision for an insurer’s liability for claims.
- A **stochastic model** uses random variables in a regression framework.

**Notes**

- Claims reserves models presented here use GLM theory as introduced in *England, Verrall 2002*. 
Model Definition

Notations

- $C_{i,j}$ Incremental payments
Model Definition

Notations

- $C_{i,j}$ Incremental payments
- $w_{i,j}$ Exposure
Model Definition

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- $C_{i,j}$ Incremental payments
- $w_{i,j}$ Exposure
- $r_{i,j}$ Incremental number of payments

Normalized incremental payments:

$Y_{i,j} = C_{i,j} w_{i,j}$
Model Definition

Notations

- $C_{i,j}$ Incremental payments
- $w_{i,j}$ Exposure
- $r_{i,j}$ Incremental number of payments
- $Y_{i,j}$ Normalized incremental payments
Model Definition

Notations

- $C_{i,j}$: Incremental payments
- $w_{i,j}$: Exposure
- $r_{i,j}$: Incremental number of payments
- $Y_{i,j}$: Normalized incremental payments
- $Y_{i,j} = \frac{C_{i,j}}{w_{i,j}}$
Hypotheses

- \( C_{i,j} \) is a compound Poisson-Gamma distribution
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- Frequency $\sim$ Poisson with mean $\vartheta_{i,j}w_{i,j}$ and variance $\vartheta_{i,j}w_{i,j}$
Model Definition

Hypotheses

- $C_{i,j}$ is a compound Poisson-Gamma distribution
- Frequency $\sim$ Poisson with mean $\vartheta_{i,j} w_{i,j}$ and variance $\vartheta_{i,j} w_{i,j}$
- Severity $\sim$ Gamma with mean $\tau_{i,j}$ and variance $\nu \tau_{i,j}^2$
Model Definition

Hypotheses

- $C_{i,j}$ is a compound Poisson-Gamma distribution
- Frequency $\sim$ Poisson with mean $\vartheta_{i,j}w_{i,j}$ and variance $\vartheta_{i,j}w_{i,j}$
- Severity $\sim$ Gamma with mean $\tau_{i,j}$ and variance $\nu\tau_{i,j}^2$
- Using the following parametrisation

\[
\begin{align*}
p & = \frac{\nu + 2}{\nu + 1}, \quad p \in (1, 2) \\
\mu & = \vartheta \tau \\
\phi & = \frac{\vartheta^{1-p}\tau^{2-p}}{(2 - p)}
\end{align*}
\]
Tweedie Model

\[ Y_{i,j} \sim Tweedie(\mu_{i,j}, p, \phi, w_{i,j}) \]
Tweedie Model

- $Y_{i,j} \sim Tweedie(\mu_{i,j}, p, \phi, w_{i,j})$
- $\mu_{i,j} = e^{X\beta}$
Model Definition

Tweedie Model

- \( Y_{i,j} \sim \text{Tweedie}(\mu_{i,j}, p, \phi, w_{i,j}) \)
- \( \mu_{i,j} = e^{X\beta} \)
- \( E[Y_{i,j}] = \mu_{i,j} \), \( \text{Var}[Y_{i,j}] = \frac{\phi}{w_{i,j}} \mu_{i,j}^p \)
Log-likelihood function

Tweedie Model

\[ I = \sum_{i,j} r_{i,j} \log \left( \frac{(w_{i,j}/\phi)^{\nu + 1} y_{i,j}^\nu}{(p - 1)^\nu(2 - p)} \right) \]

\[ - \log \left( r_{i,j}! \Gamma \left( r_{i,j}^\nu \right) y_{i,j} \right) + \frac{w_{i,j}}{\phi} \left( y_{i,j} \frac{\mu_{i,j}^{1-p}}{1 - p} - \frac{\mu_{i,j}^{2-p}}{2 - p} \right) \]
Parameters

\[ \mu_{i,j} \quad \phi \quad p \quad w_{i,j} \]

Pure premium

Variance

Figure: Parameter main influence
Parameter $p$

Tweedie Model

- Can be estimated only when the number of payments is known.
Parameter $p$

Tweedie Model
- Can be estimated only when the number of payments is known
- Otherwise, it’s supposed fixed and known
Parameter $\rho$

Tweedie Model
- Can be estimated only when the number of payments is known.
- Otherwise, it's supposed fixed and known.
- $\rho$ and $\phi$ need both to be estimated at the same time.
Optimizing $\phi$ using the **likelihood** principle

$$\hat{\phi}_p = -\sum_{i,j} w_{i,j} \left( y_{i,j} \frac{\mu_{i,j}^{1-p}}{1-p} - \frac{\mu_{i,j}^{2-p}}{2-p} \right) \frac{1}{(1 + \nu) \sum_{i,j} r_{i,j}}$$
Figure: Optimizing $p$ using the likelihood principle for $\phi$
Optimizing $\phi$ using the **deviance** principle

$$
\hat{\phi}_p = \sum_{i,j} \frac{2}{N - Q} \left( y_{i,j} \frac{y_{i,j}^{1-p} - \mu_{i,j}^{1-p}}{1 - p} - \frac{y_{i,j}^{2-p} - \mu_{i,j}^{2-p}}{2 - p} \right)
$$
**Figure:** Optimizing $\phi$ using the deviance principle for $\phi$
Parameter $w_{i,j}$

Note

- The exposure has been incorporated in the beginning, within the initial hypothesis
Parameter $w_{i,j}$

Note

- The exposure has been incorporated in the beginning, within the initial hypothesis.
- Different ways of incorporating the exposure within the initial hypothesis would lead to different models.
Frequency vs Severity

Compound Poisson Model

\[ Y = \sum_{k=1}^{N} X_i \]

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**Table:** Mean and variance of total costs for various situations
Typical situation in a long-tail business

- Decreasing average frequency
Typical situation in a long-tail business

- Decreasing average frequency
- Increasing average severity
Frequency vs Severity

Typical situation in a long-tail business
- Decreasing average frequency
- Increasing average severity
- Increasing variance in the severity
Impact of the Distribution

**Figure:** Fitted curve for Normal, Poisson and Gamma models

**Figure:** Fitted curve for Normal, Poisson and Gamma models
Model Definition

\[ Y_{i,j} \sim Tweedie(\mu_{i,j}, p, \phi_{i,j}, w_{i,j}) \]
Dispersion Models

Model Definition

- \( Y_{i,j} \sim \text{Tweedie}(\mu_{i,j}, p, \phi_{i,j}, w_{i,j}) \)
- \( \mu_{i,j} = e^{X_{i,j}\beta} \)
Dispersion Models

Model Definition

- $Y_{i,j} \sim \text{Tweedie}(\mu_{i,j}, p, \phi_{i,j}, w_{i,j})$
- $\mu_{i,j} = e^{X\beta}$
- $E[Y_{i,j}] = \mu_{i,j}$, $\text{Var}[Y_{i,j}] = \phi_{i,j} \mu_{i,j}^p$
Dispersion Models

Model Definition

- $Y_{i,j} \sim \text{Tweedie}(\mu_{i,j}, p, \phi_{i,j}, w_{i,j})$
- $\mu_{i,j} = e^{X_\beta}$
- $E[Y_{i,j}] = \mu_{i,j}$, $\text{Var}[Y_{i,j}] = \phi_{i,j}\mu_{i,j}^p$
- $\phi_{i,j} = e^{V_\gamma}$
Dispersion Models

Log-likelihood

\[ l = \sum_{i,j} r_{i,j} \log \left( \frac{(w_{i,j}/\phi_{i,j})^{\nu+1} y_{i,j}^\nu}{(p-1)^\nu (2-p)} \right) - \log \left( r_{i,j}! \Gamma \left( r_{i,j} \nu \right) y_{i,j} \right) \]

\[ + \frac{w_{i,j}}{\phi_{i,j}} \left( y_{i,j} \frac{\mu_{i,j}^{1-p}}{1-p} - \frac{\mu_{i,j}^{2-p}}{2-p} \right) \]
Notes

- The $p$ parameter is optimized at the same time as all other parameters using implicitly the likelihood principle.
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Accident years do not have a significant impact on the dispersion parameter.
Dispersion Models

Notes

- The $p$ parameter is optimized at the same time as all other parameters using implicitly the **likelihood** principle.
- Accident years do not have a significant impact on the dispersion parameter.
- Due to lack of data in the last column, the model was built so that the last two columns have the same dispersion parameter.
Notes

- The $\rho$ parameter is optimized at the same time as all other parameters using implicitly the likelihood principle.
- Accident years do not have a significant impact on the dispersion parameter.
- Due to lack of data in the last column, the model was build so that the last two columns have the same dispersion parameter.
- Possibility to incorporate trends in the dispersion parameter by using the Hoerl's curve parametrisation.
Double GLMs

Algorithm

1. Start with the initial exposure and find the normalized incremental payments

Find the deviance between fitted and observed values

Model the deviance

Establish the new exposure and start over
Double GLMs

Algorithm

1. Start with the initial exposure and find the normalized incremental payments
2. Find the deviance between fitted and observed values
Double GLMs

Algorithm

1. Start with the initial exposure and find the normalized incremental payments
2. Find the deviance between fitted and observed values
3. Model the deviance
Double GLMs

Algorithm

1. Start with the initial exposure and find the normalized incremental payments
2. Find the deviance between fitted and observed values
3. Model the deviance
4. Establish the new exposure and start over
Double GLMs

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<th>Component</th>
<th>Mean modeling</th>
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<td>$\phi$</td>
</tr>
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<td>Variance</td>
<td>$\phi V(\mu)$</td>
<td>$2\phi^2$</td>
</tr>
<tr>
<td>Link</td>
<td>$\eta = g(\mu)$</td>
<td>$\eta_d = g_d(\phi)$</td>
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<tr>
<td>Linear predictor</td>
<td>$\eta = X\beta$</td>
<td>$\eta_d = V\gamma$</td>
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<tr>
<td>Deviance</td>
<td>$2 \int_X \frac{y-\mu}{V(\mu)} d\mu$</td>
<td>$2(- \log(d/\phi) + (d - \phi)/\phi$</td>
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<tr>
<td>Exposure</td>
<td>$w/\phi$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Figure: Inter-relationship between the two sub-models
Algorithm additional specifications

- Analogous to Fisher’s weighted scoring method for optimization
Iterative Weighted Least Squares

Algorithm additional specifications

- Analogous to Fisher’s weighted scoring method for optimization
- IWLS implicitly uses the deviance principle for estimating $\phi$
Maximum likelihood estimators are biased downwards when the number of estimators is large compared to the number of observations.
Maximum likelihood estimators are biased downwards when the number of estimators is large compared to the number of observations.

REML produces estimators which are approximately and sometimes exactly unbiased.
Restricted Maximum Likelihood

Notes

- Maximum likelihood estimators are biased downwards when the number of estimators is large compared to the number of observations.
- REML produces estimators which are approximately and sometimes exactly unbiased.
- Approximately maximizes the penalized log-likelihood

\[
I_{*p}(y; \gamma; p) = I(y; \beta, \gamma; p) + \frac{1}{2} \log \left| X^T W X \right|
\]
Optimizing $p$

**Algorithm**

1. Suppose $p$ fixed and known
Optimizing $p$

Algorithm

1. Suppose $p$ fixed and known
2. Evaluate all the other parameters using the DGLM
Optimizing $p$

Algorithm

1. Suppose $p$ fixed and known
2. Evaluate all the other parameters using the DGLM
3. Evaluate the penalized log-likelihood
Optimizing $p$

### Algorithm

1. Suppose $p$ fixed and known
2. Evaluate all the other parameters using the DGLM
3. Evaluate the penalized log-likelihood
4. Start over with different values for $p$ and compare the penalized log-likelihood
Reserve Variability

Mean Square Error of Prediction

- Dispersion Models: overdispersion included *implicitly* in the parameter covariance matrix
Reserve Variability

Mean Square Error of Prediction

- Dispersion Models: overdispersion included *implicitly* in the parameter covariance matrix
- GLMs: overdispersion is included *manually* in the parameter covariance matrix
## Incremental Payments

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Swiss Motor Industry (Wütrich 2003)
Data Analyzed

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<td>92 326</td>
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Total Volume per Accident Year $i$
### Data Analyzed

#### Number of Payments

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**Figure**: Restricted log-likelihood for various $\rho$ in a DGLM
## Analysis of Results

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## Analysis of Results

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<th>Estimation</th>
<th>Process</th>
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**Figure:** Adjusting the most deviant observations has a bigger influence on the deviance principle
Analysis of Results

**Figure:** Contribution of each cell to the dispersion. \( p = 1.1741 \), \( w_{i,j} \equiv 1 \)

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Further Discussion

- Lack of observations and abundance of parameters is a hostile environment for DGLMs.
Conclusion

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- The "bounds" of convergence need to be explored further.
**Stochastic Claims Reserving in General Insurance.**  
*Istitute of Actuaries and Faculty of Actuaries*

**Fitting Tweedie’s Compound Poisson Model to Insurance Claims Data : Dispersion Modelling.**  
*Astin Bulletin, 1, 143–157*

Wüthrich, MV (2003).  
**Claims Reserving Using Tweedie’s Compound Poisson Model.**  
*Astin Bulletin, 331–346*
"But this is the simplified version for the general public."