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A PRACTICAL C-1

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ABSTRACT

Methods for calculating C-1 risk levels have been somewhat arbitrary, and reliable data upon which to base these methods are elusive. This paper presents a straightforward method of building a consistent framework for C-1 risk reserve calculation. Sample levels of required surplus are derived for fixed income and equity investments.

INTRODUCTION

The contingency reserve C-1 is held against the risk that an asset loses value because the borrower (in the case of debt assets) defaults; the asset is lost or physically damaged or destroyed; or the future earning power of the issuer is perceived to have fallen. These events would be reflected in wider yield spreads and lower equity prices. Price changes as a function of the general level of interest rates are not considered to be C-1 risks.

The required C-1 risk reserve is that amount which protects the assets against loss from these risk sources with a given level of confidence, such as 90 or 95 percent. In order to determine this amount, we must examine the probability distribution function (PDF) of the value of the portfolio of assets. It is convenient to do this separately for fixed-income and equity-type securities.

I. FIXED-INCOME SECURITIES

Securities generally do not deteriorate overnight. While it is possible for some unanticipated event to severely affect an issuer's credit standing over a very short period, it is much more likely that the decline will be gradual. In that case, the issuer's credit standing will go through several different rating classes prior to actual default, should that, in fact, ever occur. Thus, the likelihood of default and the anticipated total losses that result are actually the aggregate effects of this series of intermediate steps. An analysis of this process would then lead to the sum of many small losses due to small decrements in credit standing multiplied by the associated conditional probabilities of falling to the next rating class.

A simpler way to view the process is to examine the overall likelihood of eventual default in any future period as a function of the current rating of the issue. The aggregation of the series of small future losses can be measured today by the one large loss embodied in a default in a single future time period. The timing of the losses is, of course, different in these two approaches. If an impaired security is held in a portfolio and an unrealized loss is carried, this timing difference is of little consequence. If the security is sold for credit reasons, that portion of the loss is advanced in time. But if default is imminent, the length of the time difference will be small. In any case, this approximation is not a major impediment to the utility of the approach.

Since we do not know how the credit worthiness of a particular issuer or the economic environment in general will change over time, we will assume that default probabilities are fixed and level by period. That is, for a given credit rating, the probability of default in any time period (say, semiannual) is known and is constant through time. We will also assume that the expected loss upon default is fixed. This expected loss, expressed as a percentage of par, recognizes that impaired securities have "salvage value" and most often can be disposed of at prices greater than zero.[1]

We will first consider a security that cannot be called, put, or converted. Also, for simplicity and without loss of generality, we will assume that the risk-free yield curve is flat. This next development follows that of Pye.[2]

1. *Expected Value and Yield*

Let A_t be the asset flows at time t for an asset of length n periods, $n \in [1,\infty]$. If there were no chance of default, the value of the cash flows A_t to the portfolio would be measured by the asset's price P_R at the risk-free rate R:

$$P_R = \sum_{K=1}^{n} (1+R)^{-K} \cdot A_K.$$

Suppose the credit rating of the issue indicates a probability of default of α for each period. Suppose further that there is an expected loss rate of λ upon default with no subsequent chance of recovery. Then the probability

[1] This approach omits losses due to downgrades which do not ultimately default, and it overstates losses on defaulting issues which are sold off prior to the event. These two effects do not generally offset, but for this analysis, the net impact is assumed to be small.

[2] See Gordon Pye, "Gauging the Default Premium," *Financial Analysts Journal*, January-February 1974, pp. 49–52.

today of survival through period $t-1$ and default in period $t \in (0, n]$ is given by:

$$\delta_t = \alpha \cdot (1-\alpha)^{t-1}.$$

The probability of no default is $\delta' = (1-\alpha)^n$. The sum of these probabilities is seen to be one, and they are nonnegative, so they form a discrete PDF for periodic default.

Thus, the value today of an asset not currently in default, given default at time t, is:

$$V_t = \sum_{K=1}^{t-1} (1+R)^{-K}. A_K + (1-\lambda)(1+R)^{-t} \cdot \sum_{K=t}^{n} (1+R)^{t-K} \cdot A_K.$$

The expected value of the asset under the PDF is:

$$P_E = \delta' \cdot P_R + \sum_{t=1}^{n} \delta_t \cdot V_t.$$

P_E as a price implies an "expected yield" to maturity of E on the asset. If the flows were to be received with certainty, they should command a market price of P_R and yield R. However, under our assumptions for default and loss for this credit rating, we see that we can expect the flows to be worth only $P_E < P_R$. This price difference implies an expected yield spread $\pi = E-R$ at which these flows should trade in the market. π is called the "risk premium."

2. *Default Premiums and Spreads*

We have just shown that the required *mean* yield for a stream with a given credit risk is E. Since we will sometimes experience default and earn less than the mean, we will require from the market a promised yield greater than E such that the expected return is actually E. We will demand a yield Y at a total spread S to the risk-free rate R such that:

$$E = (1-\alpha) \cdot Y + \alpha \cdot (-\lambda) = R + \pi.$$

Solving for Y:

$$Y = R + S = \frac{R + \pi + \alpha\lambda}{1-\alpha}.$$

Solving for S:

$$S = Y - R = \pi + \frac{\alpha \cdot (\lambda + R + \pi)}{1 - \alpha}.$$

The "default premium" D is then equal to:

$$D = S - \pi = \frac{\alpha \cdot (\lambda + R + \pi)}{1 - \alpha}.$$

D is seen to be that portion of the total spread required from the market to provide the expected fair yield to the holder. The risk premium π is that part which compensates the holder for taking the risk. The theoretical value of the stream is:

$$P_Y = \sum_{Y=1}^{n} (1+Y)^{-K} \cdot A_K.$$

This price may be more or less than the market price at any point in time due to many factors other than credit standing, as well as to differences in perceived credit among market participants. But under this framework, the actual market price should not affect the theoretically fair credit spread unless it reflects a change in α or λ.

When pricing insurance products, generally the actual market yields on available assets less a "charge for credit risk" are used for input to the process. The credit risk charge is needed ostensibly to provide for losses due to default. From this analysis, it is seen that the amount required for this charge varies with the particular asset and is represented by the default spread D. The risk premium π need not be retained and can be passed on to the product via the pricing process.

3. *Required Reserve for C-1 Risk*

The required C-1 reserve is that amount which protects against the C-1 risks in a portfolio with a given level of confidence, such as 95 percent. Stated another way, it is the amount of cash needed such that this cash, plus the actual value of the portfolio's flows (reflecting any defaults), is at least as great as the unimpaired value of the flows 95 percent of the time.

There is a price P_x such that the value of an asset or a portfolio of assets will fall below it with a probability no greater than x. That level can be found by examining the discrete PDF, or by simulation (see section 5). The required reserve for C-1 risk as a percent of current value is the difference between the unimpaired value of the flows P_R and the downside limit P_x, divided by fair value P_Y:

$$\text{C-1 Reserve} = \frac{P_R - P_x}{P_Y}.$$

This methodology applies to fixed-income securities without options attached. It is thus appropriate for public bonds, private placements, preferred stock, and mortgage products not subject to significant prepayment of principal. Exhibit 1 contains sample calculations for a specific bond.

4. *Effects of Diversification*

The framework presented here for a single bond can be extended to a diversified portfolio of holdings. The only change is in the form of the PDF for the portfolio's value. No longer is the PDF a simple table of δ_t's. Depending on the mix of credits and maturities, the specific form for the PDF can become quite complex.

Given our criterion for the C-1 reserve, i.e., to protect with probability of failure no greater than x, the risk factors for an individual bond exhibit an odd pattern as a function of maturity. The factors are zero for short terms so long as α is less than x. Once the maturity gets beyond a certain point, the C-1 requirement rises sharply. This is a reflection of the arbitrary level of x and the high degree of specific risk. See exhibit 2 for an illustration. In this example, a single-A rated 30-year bond would require \$35,000 surplus per \$1,000,000 market value to cover the risk at the 5 percent level, while a 5-year maturity would require none. Both require 21 basis points per year deduction for credit risk in product pricing.

This can be seen more clearly by looking at the event of default for a single bond. Statistically, it is a Bernoulli trial (an event with two mutually exclusive outcomes, like a coin toss) with probability α and outcome values Y and $-\lambda$. This, in fact, is how we derived the algebraic form of the risk and default premiums π and D. If α is less than x (our stated comfort level), there will be no contribution to C-1 from the first exposure period. If $\alpha + (1-\alpha)\cdot\alpha$ is still less than x, there will be no required C-1 for the first two periods because we will have no losses from default at least $100-x$ percent of the time. So shorter terms have zero factors. Once the term is lengthened or α is increased such that the probability of a default is at least as great as x, the severe portfolio effect ($-\lambda$ versus Y) generates a large factor. The expected losses from default are $\alpha\lambda$, and the probability of a loss the size of λ is (trivially) α.

EXHIBIT 1

5-Year Bond, 12% Semiannual Coupon
α = .5% per Period, λ = 40%
Risk-Free Rate = 10%

t	A_t	δ_t	V_t	$\Sigma\delta_t$
1	6	.5000%	64.63	0.5000%
2	6	.4975	66.92 ← (99% level)	0.9975
3	6	.4950	69.10	1.4925
4	6	.4925	71.17	1.9850
5	6	.4900	73.14	2.4750
6	6	.4876	75.02	2.9627
7	6	.4852	76.81	3.4479
8	6	.4828	78.52	3.9307
9	6	.4803	80.14	4.4110
10	106	.4779	81.69 ← (95%)	4.8890

P_R = 107.72 δ' = .9511

P_E = 106.06 E = 10.43%

π = .43% D = .45%

S = .88%

Y = 10.88% P_Y = 104.23

C-1 Reserve

x –	1%	5%
P_x –	66.93	81.73
C-1 –	39.1%	24.9%

EXHIBIT 2

C-1 Risk Factors for Individual Bonds

Quality	1 year	5 years	30 years	Default Spread
U.S. Treasury and GNMA α = 0%/yr.	0%	0%	0%	0%
Agency and AAA α = .05%/yr.	0	0	0	.04
AA α = .15%/yr.	0	0	0	.11
A α = .3%/yr.	0	0	3.5	.21
BAA α = .5%/yr.	0	0	17.5	.35
Below Grade α = 1.6%/yr.	0	46.6	56.3	1.15

12% semiannual coupon
λ = 60%
Risk-free rate = 10%
Confidence level = 95%

Much of this effect can be diversified away. Suppose the portfolio is made up of m independent bonds of the same general credit. The event of a default in the portfolio is a binomial random variable, $b(m,\alpha)$. The expected number of defaults is $m \cdot \alpha$. The probability of at least one default is:

$$1-(1-\alpha)^m \cong 1-(1-m\alpha + m(m-1)/2\ \alpha^2) = m\alpha - m(m-1)/2 \cdot \alpha^2 > \alpha.$$

The expected loss due to any one default is $-\ \lambda/m$. While the expected losses due to all defaults are still $\alpha \cdot \lambda$ ($= m\alpha \cdot \lambda/m$), the probability of a loss the size of λ (which was α in the case of a single bond), is now α^m, the same as the probability of every bond defaulting.

Thus, if a large number of independent credit risks are held, there is a much higher chance of at least one default even in nearby future periods, so C-1 for short maturities is not zero. At the same time, any one default has much less impact on the whole portfolio, so the slope of the factors' increase with maturity is less steep.

5. *Determining the PDF*

The problem of calculating C-1 reserve factors under these conditions amounts to computing the PDF for the portfolio. It is made up of the present values of a series of binomial random variables at times $t \in (0,n]$, $b(n_t,p)$, where $p = \alpha$ and n_t is the number of bonds surviving to time t. The fact that n_t itself is a random variable complicates the process. Rather than arriving at this distribution analytically, we can model it with Monte Carlo methods. The resulting simulation of an empirical PDF for any portfolio can be used to select C-1 factors for any desired level of risk.

For a portfolio with m bonds, the model would proceed as follows for N trials. For each bond, a string of uniform random numbers on (0,1) is generated—one for each period in the bond's term to maturity. These random numbers are compared to α. The first occurrence, if any, of a random number less than α indentifies the period of default. The bond's contribution to the portfolio's value is then determined by discounting, at the risk-free rate, its cash flows reflecting the default. When all bonds have been simulated in this way, the sum of their discounted cash flows is the portfolio's value for that trial. This is repeated N times, where N is large. The resulting array of values represents the PDF of the portfolio with respect to the default risk. The values are sorted in ascending order, and the value in the $x \cdot Nth$ cell is P_x, the level above which the portfolio's value will lie with the desired frequency.

6. *Sample Results*

The method described was applied to a set of assumptions in order to generate the tables of C-1 factors in exhibits 3A, 3B, and 3C. In these simulations, the bonds are assumed to have 12 percent semiannual coupons, the risk-free rate is 10 percent, and $\lambda = 60$ percent. These were run for 1,000 trials by an APL program on an IBM PC AT. As an example of how one might use the tables, consider a fairly large portfolio (>100 issues) with an average maturity of around 5 years and an average quality of BAA. If the maturity range is fairly narrow about the average (as might be the case in a GIC portfolio), exhibit 3C indicates a surplus requirement of 2.4 percent of the portfolio's value for credit risk. In contrast, a much smaller portfolio with longer maturities and higher credit would require more of its assets as surplus funds. A structured settlement annuity portfolio with ten A-rated bonds and 10-year maturities would require 5.3 percent from exhibit 3A.

Probabilities of default and expected losses by credit rating are rather difficult to obtain. The levels I have used here were estimated from two recent studies of default experience for public bonds.[3] Most compilations of default data are not well-suited to this type of analysis. Also, total market experience may be deemed inappropriate for a company that considers its bond analysis staff to contribute significant value through credit selection. Further, the probabilities of default I have inferred from market data indicate no required reserve for credits down to a Standard & Poor's rating of double-A. This may not be appropriate for all portfolios. For those who wish to estimate parameters based on their own company's experience, the method offers a simple and practical way to develop C-1 factors consistently across credit and maturity ranges. For those who choose not to develop their own factors, interpolation in these tables ought to provide a reasonable base from which to start a C-1 risk analysis for bond portfolios. Alternatively, if one's own company data are not suitable, another source for data might be insurance companies' financial statements—though some careful interpretation would be needed.

In any case, no model, however embellished with technical detail, is a substitute for good judgment.

[3] Edward I. Altman and Scott A. Nammacher, "The Default Experience on High Yield Corporate Debt," Morgan Stanley and Co. Incorporated, March 1985, and J. DiDonato, "Bond Default Update," Shearson/Lehman Taxable Fixed Income/High Yield Research, February 1985.

EXHIBIT 3A

C-1 Risk Factors

10-Bond Portfolio	Term			
Quality	1 year	5 years	10 years	30 years
U.S. Treasury and GNMA α = 0%/yr.	0%	0%	0%	0%
Agency and AAA α = .05%/yr.	0	0	0	0
AA α = .15%/yr.	0	0	0	0
A α = .3%/yr.	0	4.9	5.3	6.2
BAA α = .5%/yr.	0	5.8	6.4	7.6
Below Grade α = 1.6%/yr.	6.1	11.7	16.1	20.2

EXHIBIT 3B

C-1 Risk Factors

50-Bond Portfolio	Term			
Quality	1 year	5 years	10 years	30 years
U.S. Treasury and GNMA α = 0%/yr.	0%	0%	0%	0%
Agency and AAA α = .05%/yr.	0	0	0	0
AA α = .15%/yr.	0	0	0	0
A α = .3%/yr.	1.0	2.0	2.3	2.7
BAA α = .5%/yr.	1.2	2.8	3.8	5.0
Below Grade α = 1.6%/yr.	2.4	7.5	10.7	13.9

EXHIBIT 3C

C-1 Risk Factors

100-Bond Portfolio	Term			
Quality	1 year	5 years	10 years	30 years
U.S. Treasury and GNMA α = 0%/yr.	0%	0%	0%	0%
Agency and AAA α = .05%/yr.	0	0	0	0
AA α = .15%/yr.	0	0	0	0
A α = .3%/yr.	0.6	1.3	1.9	2.5
BAA α = .5%/yr.	1.2	2.4	3.1	4.1
Below Grade α = 1.6%/yr.	2.4	6.1	9.1	13.1

II. EQUITY INVESTMENTS

The same general approach for determining C-1 risk levels as used for fixed-income securities can be applied to equities. The essential ingredient is an assumed distribution of total return, most of which is price action since income is usually low and of secondary importance to capital appreciation.

A popular assumption for common stock prices is that they are lognormally distributed. We will make this assumption. Assume further a mean portfolio value $E(X)$ and a variance $V(X)$. Thus, our value X is lognormally distributed, i.e., $\ln(X) \sim N(\mu, \sigma^2)$ where, using the moments of the lognormal distribution,

$$\mu = \ln E(X) - \sigma^2/2$$

$$\sigma^2 = \ln [V(X)/E(X)^2 + 1].$$

If we use the Standard and Poor's 500 index as our measure of the market, the geometric average annual return over the past ten years has been about 10 percent, with an annual standard deviation of about 15 percent. Thus, $E(X) \cong 1.1$, $V(X) \cong .0225$ and

$$\sigma^2 = \ln [.0225/1.21 + 1] = .01842, \text{ so } \sigma = .13574$$

$$\mu = \ln (1.1) - .00921 = .08609.$$

Again using the 95 percent level for C-1 risk, we want to determine the amount of cash needed on hand so that the actual value of the portfolio of stocks plus the cash is at least as great as the expected value of the portfolio of stocks 95 percent of the time. Thus, we want a market value MV_5 such that $P[X \geq MV_5] \geq .95$, or equivalently,

$$P[\ln(X) \geq \ln(MV_5)] \geq .95.$$

Since ln (X) is normal, $\ln(MV_5) = \mu - 1.64\,\sigma = -.136524.$

Thus, $MV_5 = \exp(-.136524) = .872386$. We expect a value of 1.1, so the amount needed for C-1 risk is $1.1 - .8724$ or 22.76 percent of market value.

For stocks whose prices are more or less volatile than the market in general, we can estimate C-1 as a function of the stock's β. Specifically, if a stock's return varies linearly with β times the market return, the standard deviation (SD) of return varies with the absolute value of β times the SD of the market. Then C-1 risk is approximately $E(X) - \exp(\mu - 1.64|\beta|\sigma)$.

III. OTHER INVESTED ASSETS

The previous analyses apply in theory to specifically defined classes of assets with regular characteristics. It often happens that portfolios contain many securities with irregular features that should affect C-1 risk levels. While the effects may be small and the appropriate decision may be to absorb them into the standard calculations, it is useful to develop an understanding of the fundamentals involved.

1. *Futures and Forwards*

Many portfolios will have positions in futures or forward-delivery contracts used in hedges from time to time. While the intent of the hedge is the reduction of risk, presumably C-3 risk, the hedging process itself introduces basis risk which may be significant. (The "basis" in futures is the difference between the futures price and the current or "spot" price of the underlying commodity. "Basis risk" is the risk to the futures market participant due to adverse changes in the basis.) Particularly in cross-hedging (hedging one type of instrument with a contract on a different type, e.g., corporate-bond versus Treasury-bond futures), an estimate of the basis risk should be made, and if large enough, should either be included in C-1 risk or used to lessen the apparent reduction to C-3 risk.

2. *Currency*

A currency exposure (e.g., owning foreign-denominated bonds or investing capital in a foreign subsidiary) presents a risk to surplus which should be reflected in C-1 calculations if unhedged. Estimates of the particular currency's volatility over the holding period would be the basis for C-1 adjustments.

3. *Floaters*

Floating-rate issues can be treated as fixed-income securities for the term of the reset period at the current rate level. If the floating rate is based on an index other than U.S. government issues such as LIBOR (London Inter Bank Offered Rate, a common index for floating-rate issues), an adjustment for changing spreads similar to a currency exchange rate risk is needed.

4. *Swaps*

Interest rate swaps have defined cash flows that look like conventional bonds or floaters. Thus the analysis is consistent with that for bonds, except that there are two sources of credit risk: the credit of the underlying security *and* that of the swap counterparty. For example, if one owns a floating rate note and swaps for a fixed rate, he is at risk to the credit of both the issuer of the note and the fixed rate payer under the swap in the same transaction. Swaps thus require more surplus than ordinary bonds of similar rating and should carry commensurately more yield.

5. *Options*

Positions in options may be held in a number of ways in a portfolio. Aside from outright ownership or short sale of an option, the portfolio may own callable or convertible bonds or preferred stock, pass-throughs, instruments with rate caps or "collars," putable bonds, and so on. The effect on C-1 calculations can be difficult to assess since these options may be held due to insurance product considerations (e.g., in the case of putable municipal bonds in a property-casualty portfolio) or as a means of altering interest rate exposure. Some of the risk may be reflected in C-2 or C-3 calculations and should not be double counted.

The preceding analysis for C-1 factors does not apply easily to options due to the uncertainty of future cash flows and the skewness of the return distribution. Still, some judgments can be made.

Viewing options from the long side, an upper bound on the risk is the premium paid, since in no case can the portfolio lose more than that with respect to the option position.[4] Call options thus can be reserved as premium paid. Put options actually increase in value with falling prices, so from a credit standpoint, upgrades are the concern.

Written options have unlimited risk in the absolute sense but may actually serve to lessen C-1 risk in some cases. The covered call writer has less C-1 risk in his total position than the owner of the stock alone, since if the stock price drops, the written call position will gain value.

Options must be examined for their role on the total portfolio, and C-1 adjustments that fit that role can be made.

6. *Convertibles*

Convertible bonds can be viewed as conventional bonds with a long call option on a stock attached. When the bond is at a very high premium (the option is out-of-the-money, and the convertible trades on its bond value alone), the conventional C-1 factors for fixed income apply. When the bond is at low premium and trades essentially on its equity value, the equity approach is a good approximation (the deep in-the-money option overwhelms the value as a bond). Other times, a weighted sum of the fixed-income factor plus option premium is a reasonable measure.

7. *Callable Bonds*

Callable bonds and pass-throughs have written calls which are a much bigger concern from the C-3 risk standpoint than C-1. One can conservatively ignore the option for C-1 purposes and use a factor based on time to normal maturity if not called.

8. *Real Estate*

Mortgages are handled by the fixed-income techniques, though credit rating and statistics for default rates and losses may be difficult to obtain. Experience of one's own portfolio can serve as the basis for these data.

Equity real estate is treated the same way our common stock example worked, with appropriate estimates for the distribution of total return.

[4] Note that this is not true with respect to C-3 risk; if options are used to hedge a C-3 risk, potential appreciation needed to offset cash losses can exceed premiums paid, and the risk is equally as great if that needed gain does not materialize.

CONCLUSION

In this paper, I have developed a framework for assessing the C-1 risk levels for simple fixed-income and equity investments. Sample results are obtained for various qualities and maturities of fixed-income bonds. Considerations for more complex types of investments were discussed. The framework can serve as the basis for a practical approach to consistent C-1 risk reserve calculations.

Thanks are owed to Ms. L. Herold for technical and computer support in this project.

DISCUSSION OF PRECEDING PAPER

ERROL CRAMER:

Mr. Sega has presented a clear and insightful article which effectively illustrates a practical determination of the C-1 reserve. However, I am not in full agreement with the formula developed in section I.2 of the paper for the price of an asset which has a credit risk. It appears that the loss in yields through defaults is being double-counted in the determination of the fair value price P_Y (all terminology and formulas used in this discussion are the same as those used in Mr. Sega's article). This is best illustrated by a simple example, as follows:

Consider a \$100 bond maturing in six months that is 99 percent certain to default with a 30 percent loss in value.

$$
\begin{aligned}
\text{Let, } \alpha &= 99\% \\
\lambda &= 30\% \\
R &= 5\% \text{ per six months} \\
A_1 &= \$106 \\
\text{then, } P_R &= A_1/(1+R) \\
&= \$100.95 \\
V_1 &= (1-\lambda)A_1/(1+R) \\
&= \$70.67 \\
P_E &= (1-\alpha)P_R + \alpha V_1 \\
&= \$70.97 \\
E &= (A_1/P_E) - 1 \\
&= 49.36\% \\
Y &= (E + \alpha\lambda)/(1-\alpha) \\
&= 7{,}905.99\% \\
P_Y &= A_1/(1+Y) \\
&= \$1.32
\end{aligned}
$$

P_Y of \$1.32 is obviously too low a price for an asset that will yield either \$74.20 (with 99 percent probability) or \$106 (with 1 percent probability) in six months time.

A more meaningful value is P_E of \$70.97 which is the price that provides an expected yield equal to the risk-free rate of 5 percent. This is shown as follows:

$$\text{yield if no default} = (\$106/\$70.97) - 1 = 49.36\%$$

$$\text{yield if default} = [(.7)(\$106)/(\$70.97)] - 1 = 4.55\%$$

$$\text{expected yield} = 49.36\% \times .01 + 4.55\% \times .99 = 5.00\%$$

The actual market price likely would be less than P_E, to compensate for risk taking, but clearly would be higher than P_Y of \$1.32.

This technical critique of P_Y does not detract from the results illustrated by Mr. Sega. P_Y is used only in the C-1 reserve factor formula, $(P_R - P_X)/P_Y$. For the examples in Mr. Sega's article, P_Y and P_E are not too different, and therefore, the impact on the C-1 reserve is not significant. Nevertheless, the formula for P_Y is clearly inappropriate in circumstances where the risk of default is large.

ELIAS S.W. SHIU:

A review of the *Record of the Society of Actuaries* will show that in the past few years our Society has held many sessions discussing the problem of C-3 risk. However, there has not been as much emphasis on C-1 risk. This paper is certainly a welcome addition to the actuarial literature.

I was surprised when I first studied sections I.1 and I.2. It seemed that, given R, α, and λ, the author was able to derive a formula for the price of a bond subject to default, without reference to investors' risk preference. The sentences which puzzled me are:

> We have just shown that the required *mean* yield for a stream with a given credit risk is E. Since we will sometimes experience default and earn less than the mean, we will require from the market a promised yield greater than E such that the expected return is actually E. We will demand a yield Y at a total spread S to the risk-free rate R such that:
>
> $$E = (1 - \alpha)\cdot Y + \alpha\cdot(-\lambda) = R + \pi.$$

Suppose that we live in a *risk-neutral* world, where the utility function of each investor is linear. Then, the risk premium π would be zero, and the equation quoted would read:

$$R = (1 - \alpha)Y + \alpha(-\lambda).$$

However, in the paper the value of π is calculated using α, λ, R, and $\{A_k\}$, and it is always positive. Thus I looked up Gordon Pye's paper [2].

Pye [2, page 49] wrote:

> If an investor is willing to purchase a bond subject to default, he will require that its expected return exceed the return available on a default-free security. The excess compensates him for the uncertainty or risk he accepts about the return he will receive on the security subject to default. The difference between the expected return on a bond subject to default and the return on a default-free security will be called the "risk premium."

Pye's risk premium π depends on the risk preference (utility function) of the investor and cannot be calculated with α, λ, R, and $\{A_k\}$ alone. It is determined by the marketplace. On the other hand, the risk premium π in the paper is defined by the equation

$$\pi = E - R,$$

where E is the solution to the equation

$$\delta' \cdot P_R + \Sigma\, \delta_t \cdot V_t = \Sigma\, (1 + E)^{-k} \cdot A_k. \quad (1)$$

Pye [2, p. 51] uses the approximate formula

$$D \approx \alpha(\lambda + R)$$

to estimate the default premium D and then calculates the risk premium with the formula

$$\pi = Y - R - D,$$

where the yield rate Y is the actual yield rate observed in the marketplace and not a theoretical rate as presented in the paper.

As the left side of equation 1 is the expected value of the cash flows $\{A_k\}$ discounted at rate R, Mr. Sega uses the term *expected yield* or *mean yield* to describe E. However, if what is wanted is the interest rate ϵ for which the *statistical expectation* of the rate of return of the investment is the risk-free rate R, then the analysis given by Pye [2, pp. 50–51] shows that ϵ can be computed from the relation:

$$R = (1 - \alpha)\epsilon + \alpha(-\lambda).$$

I feel uneasy about Pye's model because I do not believe that α and λ can be independent of time. Pye [2, p. 51] has pointed out that his analysis can be generalized to the case where R, π, α, and λ are functions of time; however, such a model will not be simple to use. Perhaps, it would be useful to point out that chapter 8 of the book [4] deals with the default-risk structure of interest rates.

My last comment is on the lognormal assumption used in section II. In the 1960s, B. Mandelbrot, E. Fama, and others tested whether the ratios of stock prices were lognormally distributed. They found that actual stock-price ratios had many more outliers than could be accounted for by a lognormal distribution. A lognormal distribution does not have a fat tail. An elegant discussion on stock prices and the lognormal distribution can be found in section 6 of [3]. I conclude this discussion by quoting the abstract of a recent paper [1]:

> Using 55 years of data separated into five-year intervals, this study demonstrates that, in general, security and portfolio variances are dependent on stock price levels and the relationship is a function of portfolio size. The relationship is unstable over time. The results suggest possible detrimental effects of diversification and financial models based on log-normality are questionable.

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DONALD R. SONDERGELD:

I congratulate the author on providing some tools to help quantify the C-1 (asset default) risk. Risk is a four-letter word that is often misunderstood and can be used to mean different things. It seems that life insurance companies are not generally in the risk taking business, but in the risk sharing business—as they are not compensated for taking large risks. And, if an actuary cannot quantify the financial effect of the risk—it is prudent not to take the risk. This paper is, therefore, a good addition to actuarial literature as it provides a method risk quantification.

The purposes of my discussion are to ask a question, to provide a few comments, and to make some suggestions.

It would seem logical that the pricing technique should be modified to include utility theory. My question is: Why did the author not utilize utility theory concepts?

My suggestions and comments relate to statutory reporting of the asset default risk. It is certainly easy to criticize statutory reporting—even recognizing the fact it is balance sheet, rather than income statement, oriented—and I would like to add my name to the list of critics.

If there is a profit and risk charge included in the premium that policyholders pay, or if that charge is netted out of investment income before interest is credited to policyholders—that charge generates income in the income statement. When a C-2 or C-3 risk materializes, it is also reflected in the income statement—yet, a C-1 risk is handled differently by an adjustment being made directly to the surplus account. This is not consistent accounting.

My suggestions are that:

1. assets be valued at market, with liabilities valued on a consistent basis,
2. MSVR be eliminated and be replaced by an actuarial determination of "risk surplus" (to cover the C-1, C-2, and C-3 risks),
3. interest on risk surplus be allocated to those lines of business that generate the risk, and
4. realized and unrealized capital gains and losses be reflected in both the balance sheet and the income statement.

These changes would provide an improvement in statutory reporting and also force management awareness of market values.

BENJAMIN W. WURZBURGER*:

In the first section of my discussion I compare the Sega bond-loss model with the Merton [2] formulation. The next two sections emphasize the importance of statistical dependence and risk aversion for pricing and reserve calculations. Finally, I conclude with an observation about reserves for equity investments.

*Dr. Wurzburger, not a member of the Society, is in the Financial Research and Risk Analysis division at the John Hancock.

MR. SEGA'S MODEL OF BOND DEFAULT

Mr. Sega begins with an important observation about the default process: "Securities generally do not deteriorate overnight. ... the decline will be gradual. ... the issuer's credit standing will go through several different rating classes prior to default."

The notion that individual securities evolve gradually rather than abruptly deteriorating overnight has major implications. Merton's pathbreaking development of a theory for pricing bonds when there is a significant probability of default does rely heavily on a gradual continuous evolution. (Merton describes the assumption of continuity as one of his "critical assumptions.") Based on this continuity assumption, Merton ([1], page 459) derives the result that the default premium (in basis points) is characteristically an increasing function of maturity, and the premium goes to zero as the time to maturity goes to zero.[1]

The continuous Merton model readily lends itself to numerical implimentation on real-world data. I, for one, have constructed such a model (see [3]). The calculations indeed indicate that, for high-grade securities, the expected annualized loss is very low for very low maturities—a typical result is that the expected annualized loss (in basis points) on a 3-year Aa is only about a third of the annualized loss on a 20-year Aa. Casual empiricism also shows that the market agrees with this assessment, as the credit spreads (corporate versus corresponding Treasury) are normally an increasing function of maturity: intermediate corporate bonds display lower default spread than long term, and commercial paper spreads are even lower.

Unfortunately, after having provided a realistic description of a gradual default process, Mr. Sega proceeds to present a simpler model: "A simpler way to view the process is ... default probabilities are fixed and level by period. ... the expected loss upon default is fixed." In contradiction to the continuous Merton formulation, Mr. Sega's discontinuous Bernoulli-type formulation allows bonds to deteriorate abruptly, and it is this discontinuous model that Mr. Sega incorporates in his subsequent calculations. Mr. Sega's specification theory fails to capture the important phenomenon that the annualized default probability on a short-term Aa is much lower that the annualized probability on a long-term Aa.

[1]This result is analogous to the pricing of an out-of-the-money option. The default premium represents the value of the stockholders' option to put the assets of the firm to the bondholders rather than satisfy the bondholders' nominal claim. Interestingly, for an Aa firm to run into default, it must fall through several rating classes. Such an event is highly unlikely over a short maturity, since the probability is of a "higher order" than the maturity. For a recent textbook-like exposition on the analogy between options and defaults, see Bookstaber and Jacob [1].

To summarize this section of my discussion, Mr. Sega has presented a discontinuous default model that is distinctly inferior to the continuous Merton formulation. While the Merton model recognizes that bond-price sample paths are continuous, as the gradual arrival of new information alters the estimated probability of a specific bond default, the Sega model allows for an Aa bond to abruptly default. Nevertheless, this shortcoming does not by itself invalidate the paper—modelers are permitted to judiciously simplify reality. The section on "expected value and yield" which I shall now discuss is much more problematic.

EXPECTED VALUES, YIELDS, DEFAULTS, PREMIUMS, AND SPREADS

Mr. Sega values the bonds and the spreads under the assumption that assets are priced according to their expected payouts. This assumption would be legitimate provided one of the following conditions holds: (1) The asset holder is risk neutral. (2) The risk on the asset is independent (in particular, uncorrelated) with the asset holder's aggregate portfolio risk (no systematic risk), so the asset does not incrementally contribute to the overall portfolio risk. The fundamental breakthrough in modern finance, the Capital Asset Pricing Model (CAPM) revolution, has emphasized the importance of systematic risk.

Neither of these conditions, however, holds in practice. As far as risk neutrality, it is generally accepted that financial intermediaries, among them life insurance companies, are in fact risk averse. As far as the independence condition, bond defaults are affected by a common factor, and a recession, for example, will clearly raise the default probability for almost[2] all bonds. The addition of a default-prone bond to a portfolio therefore will raise the portfolio overall risk exposure. A risk averse intermediary therefore will value a default-prone bond at less than the present discount value of the expected payout.

By constructing a model where bonds are priced according to expected payout, Mr. Sega is able to derive conclusions such as "the risk premium can be passed on to the product via the pricing process." This sort of conclusion is a direct artifact of his particular assumption. The problem of

[2]The caveat "almost" is intended to take care of firms whose equity enjoys a negative beta. Aggregate bad news is good for their stocks and, hence, (presumably) also favorable for the debt. Consider, for example, the proverbial red ink manufacturer. A recession should favorably impact this manufacturer's stock price and also reduce the default probability.

Why do I bother noting the atypical case of a negative beta? In part, I mention this case because Mr. Sega deals, at least implicitly, with this situation when he constructs a formula that involves the absolute value of beta.

how to price products in the presence of default risk does represent a difficult and critical question. Since he neglects the fundamental issues of risk aversion and statistical dependence, Mr. Sega generates conclusions that are unfortunately not relevant for the real world.

REQUIRED RESERVE FOR C-1 RISK

Mr. Sega calculates this reserve as "the amount of cash needed such that this cash, plus the actual value of the portfolio's flows (neglecting any default), is at least as great as the unimpaired value of the flows 95 percent of the time." I find his treatment unsatisfactory on two counts: (1) The criterion of concentrating on a single point (the 95th percentile) on the probability distribution function (p.d.f.) rather than examining a statistic such as the standard deviation and, more disturbing, (2) the independence assumption used to generate the p.d.f. The percentile criterion generates a perverse result for small portfolios, while the problematic independence assumption invalidates the results for large multibond portfolios.

C-1 RESERVE FOR SMALL PORTFOLIOS

The percentile intention generates perverse results for small portfolios. Mr. Sega reports that the required C-1 reserve for a 1-bond, 5-year Baa portfolio is 0, while the associated reserve for a corresponding 10-bond portfolio is 5.8 percent. (See his exhibits 2 and 3A. These numerical estimates are based on Monte Carlo stochastic simulations.) The results therefore are implicitly claiming that, for small portfolios, diversification raises the required percentage reserve. This perversity arises because the probability of default on a single bond is under 5 percent.

It is generally recognized, however, that the standard deviation tends to provide a better criterion than does a percentage criterion. Had Mr. Sega utilized a standard deviation criterion, he would have avoided this perversity since diversification (e.g., from 1 to 10 bonds) does reduce the standard deviation.

C-1 RESERVE FOR LARGE PORTFOLIOS

The fundamental source of risk on a large portfolio is the nondiversifiable risk; bond defaults are not statistically independent, and a recession will raise the default probability for all bonds. A bond reserve calculation that neglects default dependence is incongruous.

From a computational standpoint (as opposed to conceptual), the independence assumption does have its attractions. If we are interested in calculating, say, the 95th percentile of the p.d.f., the independence assumption allows us to invoke the law of large numbers from probability theory (for a large number of bonds) and approximate this percentile by the mean. Since it is computationally much easier to calculate an expected value than to run stochastic Monte Carlo simulations, the independence assumption can reduce the computer bill. Interestingly enough, Mr. Sega assumed independence (the conceptually weak, computationally cheap approach) but nevertheless evaluated the C-1 reserve using the expensive Monte Carlo technique (1,000 simulations on 100 bonds) instead of analytically evaluating the expected value.

I will conclude this section of my discussion on a more positive note. I very much like the idea of introducing probabilistic notions into a default reserve calculation and thus, at least implicitly, recognizing the role of risk aversion. (Absent risk aversion, the expected value would provide a valid summary measure.) Once one develops an adequate probabilistic model for calculating this reserve, the model ought to be integrated into the pricing process.

C-1 RESERVE FOR EQUITY INVESTMENTS

Mr. Sega assumes that the C-1 reserve for equity is an increasing function of the stock's *absolute* beta value. This approach differs from the standard assumption that a negative beta stock requires lower reserves than a zero beta stock inasmuch as the negative beta stock reduces the overall portfolio risk.

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DONALD D. CODY:

Mr. Sega has drawn attention to some of the rich literature on modern portfolio theory and has given valuable insight into the quantification of C-1 risk. I want to suggest a desirable extension of his findings to the determination of risk surplus needed beyond reserves on junk bonds, including implications on pricing.

My approach is described in detail in my paper "Contingency Surplus Needed for C-1, C-2, and C-3 Risks (Capacity Utilized)" on pages 697–713 of *RSA* 8:2 (1982). The quantifications are of a magnitude nature only, awaiting the completion of a planned study by the Society's C-1 Risk Task Force. The thrust of my comments is that the risk surplus needed for C-1 risk should be based on plausible very adverse economic environments where correlation of defaults of individual issues is close to 100 percent and where experience data based on normal environments are inapplicable. Also, risk surplus needed is capital advanced from corporate surplus and should be charged for in pricing.

Even in normal economic environments, we should be skeptical as to whether experience to date on junk bonds will prove to be applicable to those issued in recent years in the rash of leveraged buy-outs. Furthermore, there is an unknown level of correlation between C-1 risk and C-3 risk on junk bonds, which would increase surplus needed considerably in companies not controlling C-3 risk by optimum coordination of asset/liability cash flows.

With this background, I will now offer some magnitude quantifications, using the assumptions and approaches in *RSA* 8:2 and some of the derived data in Mr. Sega's paper.

The appropriate plausible adverse economic environments for determining risk surplus needed for C-1 risk are either a deflationary episode like the Great Depression of the 1930s or a deflationary episode worse than the serious stagflation of the 1970s. In such episodes, there is 100 percent correlation among defaults by issue on all securities and investments, other than government bonds, because they are all subject to the same catastrophic environment. Thus, Mr. Sega's Exhibit 2 for individual bonds may be illustrative, indicating perhaps a gross risk surplus needed of 50 percent of reserves at the 5 percent level of ruin probability on junk bonds. This figure is not inconsistent with figures in *RSA* 8:2, considering the equity nature of junk bonds in difficult environments. Such a level of risk surplus needed represents a serious call for capital advanced from corporate surplus.

Credit against this 50 percent gross is available from margins in pricing and net income, but this credit will be ignored because of the narrow margins in interest sensitive products.

Suppose the charge in pricing for capital advanced is 15 percent, after federal income tax (FIT) and that the investment income rate, after FIT, on surplus is 5 percent. (Surplus earnings are naked to FIT, and in mutual companies the surplus tax is an additional burden.) Then, the net charge for capital advanced is 10 percent (15% minus 5%). This translates into a charge against investment earnings in pricing on a contract supported by junk bonds equal to 5 percent of reserves (10% times 50%). This is well in excess of the default spread (D) of 1.15 percent quoted in Exhibit 2. This charge in pricing would become higher after the effects of the correlated C-3 risk are introduced.

Once my approach is accepted, it appears that the charge in pricing for capital advanced for risk surplus needed for C-1 risk from junk bonds will not reduce below 3 percent of reserves or so, regardless of changes in the detailed assumptions. Whether one agrees or not with these rough magnitude quantifications, I would recommend that this extended approach be used in considering the use of junk bonds as an appreciable percentage of assets.

JAMES C. HICKMAN:

I would like to thank Mr. Sega for writing his paper. Actuarial science needs research on contingency reserves for asset losses.

In formulating my questions and comments, I will use the author's notation except that I will use $P(X)$ to represent an asset valued at interest rate X.

I wonder about the first displayed equation in Section I.2,

$$E = (1 - \alpha)Y + \alpha(-\lambda).$$

Clearly this can be interpreted as an expected rate. Why is the constant period default probability α used? I had expected that the probabilities δ' and $1 - \delta'$ would be used. A consequence of this equation is a nonstandard definition of the risk premium.

In section I.3, I had difficulty understanding the formula $[P(R) - P(X)] \div P(Y)$, for the C-1 reserve factor. This factor is to be applied to what accounting number to produce the C-1 contingency reserve?

If the bond market is efficient, why not use $[P(R) - P(B)]/P(B)$, applied to book values, as a C-1 reserve factor? In this expression, $P(B)$ denotes asset values taken at the book (amortizing) rate of interest. This factor would capture default risk as measured by the market. The total C-1 contingency reserve would be

$$\Sigma\, V(B) \left[\frac{P(R)}{P(B)} - 1\right],$$

where $V(B)$ is the total book value of bonds with similar maturity and risk characteristics valued at book rate B. The summation is over maturity and risk classes.

Section I.4, third paragraph, second sentence is a statement which is valid for a single coupon period when gains and losses are valued at the end of the coupon period. It is not true in general. In the business world, the events of default of a bond, when viewed as a time series, are undoubtedly positively autocorrelated. Likewise, in any period the events associated with the default of several bonds are undoubtedly correlated. Perhaps the empirical factor suggested in item 3 of this discussion would capture these correlations as they are viewed by the market.

Section I.4, fourth paragraph, third sentence is valid if we are concerned with one period and assume independence. Following the displayed equation, I am confused by the discussion of expected losses. Why divide the loss ratio, $0 < \lambda < 1$, by m? It appears as if the definition of λ has shifted.

In reviewing the sample results in section I.6, the reader may want to refer to Chapter 2, "Value Losses of Bonds" in *Valuation of Securities Holdings of Life Insurance Companies* by H. G. Fraine, published by R. D. Irwin in 1962. Fraine discusses the massive study of bond defaults by W. B. Hickman.

In the next-to-last sentence of Section II, the author should say, "The variance of returns is β^2 Var(X) + Var($Y|X$)." One must not overlook the variance of random deviations from the line.

I express my thanks once more to Mr. Sega for working on this problem.

JOHN A. MEREU:

Mr. Sega has written an interesting paper on the timely subject of quantifying the C-1 risk on the risk of asset loss.

For fixed-income securities, he postulates a simple but appealing model in which the chance of default is a constant α in each interval with expected loss rate λ on defaults. The model might be enhanced if λ were treated as a random variable on the (0,1) range.

The author shows how the present value of the asset P_R is reduced to P_E when the expected defaults are recognized. An alternate but equivalent equation for P_E would be $P_E = \Sigma\,(1 + R)^{-K} A_K\,[(1 - \lambda) + \lambda\,(1 - \alpha)^K]$.

The author then acknowledges that the apparent yield of riskier investments on the marketplace is higher than for a risk-free investment for two reasons: first to compensate the investor for the expected losses on default and second an amount π to compensate him for assuming the risk.

The higher yield Y is established in the marketplace, but the author does not show how the two components of extra yield can be deduced. The values of π, α, and λ appear to be arbitrary.

The author then computes Y using a one-year horizon recognizing the realized yield as a weighted average of Y when there is no default and a negative yield $(-\lambda)$ when there is a default. His approach gives a reasonable approximation to Y. For an exact value of Y, we would find that $P_E = P_Y$ when $\pi = 0$. A better approximation can be obtained by replacing $(-\lambda)$ by $[R - \lambda(1 + R)]$ in the one-year horizon equation. This results in the following equation for Y

$$Y = \frac{R + \pi + \alpha\lambda}{1 - \alpha\lambda}.$$

In going from the single security to a portfolio, the author uses a Monte Carlo technique assuming that the risks on the individual securities are independent.

Unfortunately this is probably not true in the real world. In times of recession or severe inflation, the risk of default becomes more prevalent. The reserve computed by the author could therefore significantly understate the risk.

BARRY PAUL:

Mr. Sega is to be commended for developing a practical tool for actuaries to use in the calculation of C-1 risk reserves. The theoretical development is insightful, and the numerical exhibits are enlightening. I second Mr. Sega's observation that good judgment must be exercised, though, in using the exhibits. The results must be tailored to the circumstances of a given company.

While Mr. Sega's paper provides excellent guidance in the calculation of a C-1 risk reserve, the paper does not address the purpose and uses for such a reserve. The following discussion addresses these points.

STATUTORY FINANCIAL STATEMENTS

Under current proposals to implement the valuation actuary concept in the United States, the valuation actuary will be directly responsible for overseeing the evaluation of the C-1 risk and expressing an opinion on the adequacy of a C-1 risk reserve. (The valuation actuary will be permitted to express reliance on investment officers to develop information as to the quality and distribution of invested assets.) Current U.S. statutory accounting rules require life insurance companies to establish a mandatory securities valuation reserve (MSVR) as a statutory balance sheet liability. This liability is generally recognized as a form of C-1 risk reserve. However, the MSVR is quite limited in its scope as a C-1 risk reserve. As the rest of this section demonstrates, it would not be prudent for the valuation actuary to blindly rely on the MSVR as a proxy for an adequate C-1 risk reserve.

The MSVR has two components, a bond and preferred stock component and a common stock component. The bond and preferred stock component functions as a reserve against asset depreciation and default. The common stock component serves to stabilize statutory surplus against swings in market values. Each component, in fact, generally has served its intended statutory purpose satisfactorily. It is not the purpose of this discussion to discredit the usefulness of the MSVR for the statutory statement. However, the MSVR has the following major shortcomings when viewed solely as a C-1 risk reserve:

- It does not adequately reflect the specific circumstances of a given company.
- It excludes certain categories of assets.
- It can be depleted even while C-1 risk remains.

COMPANY CIRCUMSTANCES

The MSVR is based upon a single set formula for all companies. This over-simplified approach is obviously necessary for practical reasons. However, the appropriateness of the formula must be evaluated for a given company's portfolio. For example, on the surface the bond and preferred stock component looks very much like a C-1 risk reserve. The formula for this component specifies required annual additions which vary with the National Association of Insurance Commissions quality ratings for each security. However, this generic approach is similar only in form, but not necessarily in substance, to a C-1 risk reserve. It is important for a given company to consider variables such as portfolio size, extent of investment underwriting, term structure of the portfolio, and other elements of management judgment in evaluating the adequacy of the bond and preferred stock component as an appropriate C-1 risk reserve.

ASSET EXCLUSIONS

The MSVR entirely ignores the risk of potential default on real estate and mortgage loans on real estate. Yet, these asset categories comprise over 25 percent of the life insurance industry's total assets! If a company has significant investments of this nature, such investments should be considered in a thorough evaluation of C-1 risk.

Financial options and futures are a new and growing element of the investment strategies of many life insurance companies. The value of futures and other financial options is not considered when establishing the required contribution to MSVR. However, the capital gains and losses that result from trading in these securities can affect the MSVR under certain conditions. These investments should also be considered when evaluating the C-1 risk.

POTENTIAL FOR DEPLETION

A key distinction between MSVR and a C-1 risk reserve is the different manner in which each is built up and taken down. The MSVR is a surplus stabilization fund which can and does fluctuate widely. It is gradually built up over time, but can be entirely depleted while a company remains a going concern, thus leaving surplus unprotected against further fluctuations until the MSVR can eventually be replenished. On the other hand, a C-1 risk reserve is essentially an allocation of surplus which is directly related to the asset depreciation risk inherent in the asset portfolio as of a valuation date. As an example, it is conceivable that two companies with identical investment portfolios and with the same degree of C-1 risk could have completely different MSVRs. Given different historical development, one company could have a minimal MSVR while the other company could be near the maximum. This disparity is a significant drawback of the MSVR as a C-1 risk reserve.

INDUSTRY DATA

As a final note on MSVR, it is interesting to look at some industry statistics and to compare the magnitude of the industry's total MSVR with that of the C-1 risk factors developed in Mr. Sega's paper.

Table A shows the size of the MSVR for the life insurance industry as a percentage of invested assets. For 1985, the total industry MSVR was 1.5 percent of invested assets. Over 50 percent of the companies had less than 0.6 percent of invested assets in MSVR.

Table B shows the close correlation during the past several years between the stock market and the MSVR for the total industry. The annual growth

TABLE A

U.S. LIFE INSURANCE INDUSTRY
MSVR VERSUS INVESTED ASSETS

	MSVR ($ Billions)	Invested Assets ($ Billions)	Ratio of MSVR to Invested Assets: Mean %	Median %	Number of Companies
1980	6.4	429	1.5	.7	1,326
1981	5.5	463	1.2	.6	1,419
1982	6.7	510	1.3	.6	1,493
1983	8.1	563	1.4	.6	1,517
1984	7.3	629	1.2	.5	1,560
1985	10.5	712	1.5	.6	1,570

SOURCE: A.M. Best Company's Industry Data Tapes

TABLE B

HISTORICAL GROWTH RATES
MSVR VERSUS DOW JONES INDUSTRIAL AVERAGE

	MSVR—Total Industry		Dow Jones Industrial Average	
	$ Billions	% Annual Increase	Index Value	% Annual Increase
1980	6.4	...	946	...
1981	5.5	−14	878	−7
1982	6.7	22	1,033	18
1983	8.1	21	1,258	22
1984	7.3	−10	1,189	−6
1985	10.5	44	1,550	30

SOURCE: A.M. Best Company's Industry Data Tapes

in MSVR for the industry clearly parallels the trend for the stock market (using the Dow Jones Industrial Average as a proxy for the market). Despite the fact that less than 10 percent of the industry's total assets are invested in common stock, it is evident that the common stock component exerts significant influence on the direction of the MSVR for the industry. In fact, this table provides further evidence that, at least on a macro level, the MSVR is primarily serving its intended statutory purpose, that of a stabilization fund against swings in market values.

Table C shows the distribution of MSVR by size of company for 1985. Smaller companies typically have little or no common stock investments and invest in government securities (which have no MSVR requirement). Not surprisingly, the median MSVR was smallest (0.4 percent) for the smaller

TABLE C

U.S. LIFE INSURANCE INDUSTRY
1985 MSVR RANKED BY COMPANY SIZE

COMPANY SIZE (Amount of Invested Assets) $ Millions	MSVR $ Billions	INVESTED ASSETS $ Billions	RATIO OF MSVR TO INVESTED ASSETS Mean %	Median %	NUMBER OF COMPANIES
Less than 50	0.1	11.8	1.2	0.4	1,002
50 to 99	0.1	9.7	1.3	1.0	135
100 to 499	1.0	61.9	1.6	1.1	265
500 to 999	0.7	43.8	1.5	1.2	62
1,000 to 1,999	1.2	64.1	1.9	1.3	47
2,000 to 4,999	1.9	119.1	1.6	1.3	41
More than 5,000	5.5	401.9	1.4	1.2	18
All	10.5	711.5	1.5	0.6	1,570

SOURCE: A.M. Best Company's Industry Data Tapes

companies which constitute a majority of companies in the industry, i.e., those with assets less than $50 million.

Although the average industry MSVR percentage (1.5%) lies within the range of the C-1 risk factors presented in Exhibit 3C of the paper (0.6 to 13.1%), it is misleading (for reasons already outlined) to conclude that the magnitude of the MSVR is generally in line with that of a C-1 risk reserve or that the MSVR is an acceptable "rule-of-thumb" proxy for a C-1 risk reserve for a given company. Such a conclusion is likely to be unsatisfactory under the proposed valuation actuary concept. It will be incumbent upon the U.S. valuation actuary in the near future to explicitly evaluate the adequacy of the MSVR as a C-1 risk reserve. The techniques presented in Mr. Sega's paper should provide a good starting point for one approach to the necessary analysis, with simulation and scenario testing of projected cash flows as other possible approaches.

GENERALLY ACCEPTED ACCOUNTING PRINCIPLES (GAAP) FINANCIAL STATEMENTS

For certain products, it is currently possible to hold an implicit C-1 risk reserve by reducing the interest rate used to value the products' liabilities by an assumed margin (i.e., by providing a margin for adverse deviation due to default). However, this implicit reserve will be released only as a function of the products' liabilities. This approach does not produce satisfactory results on the GAAP income statement, since the timing of the reserve release is completely unrelated to an actual event of default.

There is currently no provision for an explicit C-1 risk reserve in U.S. GAAP financial statements. The MSVR is eliminated entirely in adjusting from statutory to GAAP. The rationale for this accounting treatment is that items in the nature of general contingency reserves are not appropriate for a GAAP balance sheet. In fact, Federal Accounting Standards Board (FASB) Statement No. 5, "Accounting for Contingencies," effectively prohibits such a reserve. FASB No. 5 only permits an accrual of a loss contingency reserve if both of the following conditions are met:

(a) Information available prior to issuance of the financial statements indicates that it is probable that an asset had been impaired or a liability had been incurred at the date of the financial statements. It is implicit in this condition that it must be probable that one or more future events will occur confirming the fact of the loss.
(b) The amount of loss can be reasonably estimated.

Despite such a restrictive standard, I believe that from an actuarial perspective the holding of an explicit C-1 risk reserve as a GAAP liability is prudent and makes good economic sense. Clearly, the timing of a future default is unknown as of a current balance sheet date. However, Mr. Sega's paper effectively demonstrates that the probability of default can be ascertained based upon studies of historical data and an explicit C-1 risk reserve can be reasonably estimated. Given the economic reality of default, coupled with the increased popularity of high-yielding "junk bonds" as investments for life insurance companies, in my opinion, it is time now to reevaluate current GAAP principles which preclude C-1 risk reserves.

INTERNAL FINANCIAL STATEMENTS

In the absence of an explicit C-1 risk reserve on the published GAAP balance sheet, it may be prudent and desirable for stock companies to establish a C-1 reserve for internal financial statements for reporting to management. For example, the management of a company which invests heavily in junk bonds as a strategic business decision is in particular need of establishing such a memo account. However, using this approach only for internal purposes is likely to be entirely unsatisfactory in the event of default. The existence of the memo account will provide little consolation for management and stockholders when the full amount of the actual loss hits the bottom line of the published GAAP income statement.

In lieu of setting up a liability, a C-1 value can be established as an internal allocation of surplus. This allocation plus other internal allocations of surplus have been called target, benchmark, required, or earmarked surplus, among other names (herein called target surplus). Among other uses, target surplus formulas can be used to allocate surplus among business units within a company. The allocated surplus serves as the denominator in calculating return on equity (ROE) for each business unit. The performance of each business unit can then be evaluated based upon these ROEs.

Target surplus formulas are usually developed to cover C-1, C-2, C-3, and C-4 risks in aggregate. A few companies have developed distinct formulas for each C risk. The C-1 risk factors shown in Exhibit 3 of Mr. Sega's paper should prove to be quite helpful for companies to establish (or reevaluate) the C-1 component of such formulas.

ALLEN ELSTEIN:

I wish to commend Richard Sega for an excellent paper. He has done an outstanding job of presenting a very complex topic clearly and concisely.

I did have some difficulty in getting a feel for how C-1 risk (such as the table shown in Exhibit 3C) would look under interest rates such as 6 or 8 percent. I wonder how additional tables for 90, 95, and 99 percent confidence levels would appear.

I do have some difficulty with the notion that many of the better-rated bonds have 0 percent C-1 risk, although if one uses Mr. Sega's model, that is clearly the result.

For purposes of illustration, I decided to arbitrarily assume a very simple model which could produce lower bounds for C-1 tables. In essence, I took the default spreads of Exhibit 2, and took the present value at 12 percent simple interest (to match Mr. Sega's 12 percent assumption) to get a lower bound for Exhibit 3C. The following table shows where I used .01 percent for U.S. Treasuries.

EXHIBIT 3C
C-1 RISK FACTORS
(Lower Bounds in Parenthesis)

100-BOND PORTFOLIO (Quality)	1 YEAR		5 YEARS		10 YEARS		30 YEARS	
U.S. Treasury and GNMA α = 0%/yr.	0.0%	(.01%)	0.0%	(.04%)	0.0%	(.06%)	0.0%	(.08%)
Agency and AAA α = .05%/yr.	0.0	(.04%)	0.0	(.14%)	0.0	(.23%)	0.0	(.32%)
AA α = .15%/yr.	0.0	(.10%)	0.0	(.40%)	0.0	(.62%)	0.0	(.89%)
A α = .5%/yr.	0.6	(.19%)	1.3	(.76%)	1.9	(1.19%)	2.5	(1.69%)
BAA α = .5%/yr.	1.2	(.31%)	2.4	(1.26%)	3.1	(1.98%)	4.1	(2.82%)
Below Grade	2.4	(1.03%)	6.1	(4.15%)	9.1	(6.50%)	13.1	(9.26%)

As can be seen, the lower bounds dominate for issues graded at least AA. I would suggest to actuaries setting C-1 risk that statistical models can give a starting point for setting risk, and indeed for identifying magnitudes, but those models may need modification before actually being used.

In closing, I very much enjoyed Mr. Sega's presentation. Expecially helpful was the discussion relating spread, risk premium, and default premium.

(AUTHOR'S REVIEW OF DISCUSSION)

RICHARD L. SEGA:

I would like to thank all those who took the time to think about this problem and submit their comments. Any paper's value (to both its readers and its author) is enhanced by the critical thinking and review of interested parties.

I want to say at the outset that this paper was never intended to provide a theoretical underpinning for the dynamics of bondholders' contingent claims on debt-issuing corporations. Others have already done that, and at least one of them is mentioned in the discussions. However, the application of such theory is, for a number of reasons, often intractable to the practitioner. I have attempted to construct a framework for analysis that would be very simple to understand and apply, that could be flexibly altered to individual situations, and that would give consistent comparisons of maturities and rating classes so practitioners could make risk-reward decisions.

Most of the discussion centered around two main points: the role of utility and the assumption of independence. Messrs. Cramer, Shiu, Sondergeld, and Wurzburger all addressed the role of the investor's utility in the model's formulation. Dr. Shiu in particular correctly pointed out that in Pye's original formulation, the risk premium π is observed in the marketplace rather than derived. Thus it represents a measure of "consensus" utility, at least among investors who have traded the observed bonds in the recent period prior to observation. This, of course, changes constantly as different investors are in and out of markets, and as investors change their own utility preferences. The possibilities are too many for one model to cope with. Not wanting to assume risk neutrality where $\pi = 0$, I attempted to select a method for arriving at a plausible level for π that had some intellectual appeal. I may or may not have succeeded in that, but either way I agree with the comments. My derivation for the risk premium was arbitrary and in fact not essential to the rest of the model. Various investors will express preferences generating π larger or smaller than mine, as they may be larger or smaller than the observed "consensus" π at any time. That is, as they say, what makes markets.

Messrs. Cody, Hickman, Mereu, and Wurzburger made the valid point that the assumption of independent risks is inapplicable to the real world. It is true the risks are correlated, but how bad is it and what is the alternative given the aims of the project at hand? The runaway-inflation or depression scenarios are usually called up when thinking about this aspect of the model, and they present real problems. By and large though, away from these extremes, the assumption of independence is not terribly inappropriate. Institutional investors act daily on just such an assumption when implementing portfolio guidelines with respect to position limits in an issuer, sector diversification, asset class allocation, and the like. If such an assumption truly cannot be made, and the likely result of making it is calamitous, I then have two observations regarding our industry: (1) We will buy nothing but Treasuries and agencies because available spreads on rated paper are inadequate to provide a reasonable return on the required surplus. (2) We need to take a hard look at the businesses we are in given that the trend for life insurers has been steadily away from insurance risk-pooling and toward the role of credit intermediator. As for alternatives, there are several, none of which is likely to satisfy everyone. Correlations can be estimated and built into the model. The full benefits of diversification as measured by the model can be understated by using C-1 factors for portfolio sizes much smaller than one actually holds. One can use a very severe ruin probability, say 99 or 99.9 percent. None of these are really satisfactory, nor are they necessary for the model to be useful.

Dr. Wurzburger expresses serious concerns about the whole approach, and not surprisingly finds problems with my conclusions. Suffice it to say that if one refuses to accept the Parallel Postulate, then one will find fault with much of Euclidean geometry. It is not then necessary to engage in the redundancy of invalidating each conclusion separately. I would like to point out, however, that although we might agree that in the "real world" parallel lines really do meet, Euclidean geometry has proven to be a rather useful model anyway. In drawing this parallel, I do not mean to put Euclidean geometry and my C-1 risk model on the same plane. But rather, I only mean to say that models are by design an imperfect and simple representation of a more complex reality. If this model can help an actuary clarify the issues and make better decisions than before, then the fact that it might not satisfy a purist's notion of rigor does not trouble me.

I very much want to thank Mr. Paul for his excellent complement to the paper, bringing the perspective of the valuation actuary and insight into the nature of MSVR. The ideas in this discussion are worth a paper in themselves.

I also thank Mr. Elstein for providing an appealing solution to the troublesome zeroes in the tables for higher-rated categories. Of course, the discounting of default spreads amounts to a recasting of the objective. Whereas we began with the 95 percent rule, the use of lower bounds is a different rule. My motivation for using a ruin-probabilistic objective rather than a risk measure related to say, standard deviation, is that I think the former is more relevant for two reasons. First, insurance companies are surplus-bound and cannot bear short-term catastrophes well. We will give up expected return to protect against a disastrous hit to surplus. Keying on that probability of ruin is more relevant to me than just the volatility of results. Second, the default risk can be viewed as an option-like risk, and options skew return distributions. Standard deviation as a risk measure is only relevant for symmetric distributions of return, not skewed ones. Thus, I feel a specified loss point is a better measure of risk. It does cause anomalies, though, and I thank Mr. Elstein for fixing one.

Mr. Mereu and Mr. Cramer have uncovered another problem caused by my attempts to keep the model simple. They question the form of P_Y, and while it does not affect the results in a major way, it is an interesting exercise to examine Mr. Cramer's example. The root of the problem lies in the definition and subsequent usage of λ as a percentage of par. Strictly speaking, the formula for Y is only valid if it produces a price of par. It is not valid for premium or discount purchases. For example, a 30 percent loss of *par value* (equaling \$30) for a bond costing \$1.32 is not a -30 percent return. It is more like $-2{,}230$ percent. Y then really depends on λ/P_Y, and the formula is recursive, too impractical for a paper entitled "A Practical C-1." However, an adjustment should be made for large blocks of zero-coupon or deep discount bonds. I thank both gentlemen for pointing out the problem.

Last, Dean Hickman poses several questions which I will try to answer. The equation for E is meant to be a year-by-year return, and thus the yearly default probability α rather than the total probability δ is used. The C-1 factor is meant to be applied to market value, represented approximately by P_Y. If one chooses not to keep factors current, and not to specify the downside limit for loss, and to accept the bond market's efficiency, and to accept the market consensus for risk, then I believe the factor he suggests is fine. In going from a single bond to a diversified portfolio, the definition of λ does change somewhat. It goes from a single bond's loss as a percent of par, to the aggregate portfolio's loss as a percent of the total portfolio par value. As a percent of the total, a single bond's loss is $1/m \cdot \lambda$ for the whole m-bond portfolio. I owe thanks to Dean Hickman for pointing out that I have omitted from my equity formula a component for contribution to systematic risk when $\beta \neq 1$.