BOOK REVIEWS AND NOTICES

Greg Dinallo, *Final Answers*, 309 pp., published by Pocket Books, a division of Simon & Schuster Inc., New York, N.Y., 1992, $20.00.

Final Answers is a novel. It is unusual in the sense that an actuary is the main character. Greg Dinallo manages to express subtly the spirit of the Society of Actuaries motto in a real-world setting. The traditional view of an actuary, a dull and uninteresting character concerned mainly with death and statistics, is contrasted with a lively character using death and statistics in a powerful way, solving problems both out of curiosity and out of survival, in the cities of the U.S. as well as in the jungles of Thailand and Laos.

The book talks about how a veteran, now a pension actuary, gets involved in trying to find those servicemen missing in action during the Vietnam War. Somehow this rubs against the nerve of a drug lord, who decides to get rid of him. After some bad experiences, our hero becomes determined to pursue his agenda all the way to Thailand and Laos, and eventually succeeds in obtaining the "final answers." The author manages to make the story seem so real, full of suspense and punch, and to bring to life the pain and repeated disappointment of MIA families. At the same time, it reveals both the ugly side of war and of reality as well as the excitement. In addition, the actuarial nature of the hero is expertly cast as a sharp mind, quick to use statistics and other rules of thumb, and skillful in solving real-life problems through formulating and testing hypotheses based on facts and analyses.

I think this novel will make interesting reading for the general public, and especially for people interested in the actuarial profession or the Vietnam War.

STEPHEN LI

Bart Kling, *Life Insurance, a Non-Life Approach (Tinbergen Institute Research Series no. 60)*, 155 pp., published by Thesis/Tinbergen Institute, Amsterdam, 1993, $25; available from International Book Distributors, 24 Hudson St., Kinderhook, N.Y. 12106.

Writing a research monograph is a challenging task. Unlike a standard textbook, a monograph is not necessarily expected to have numerous examples to communicate the basic principles or exercises to learn by

practice. The success of a monograph depends totally on its presentation and the extensive coverage of the current research of the specific discipline. This book is a monograph. The author is expected to meticulously present the most recent results and show the smooth coherent bridge from past results to current ones. In this review I discuss how the author succeeds in his efforts at effective presentation, but fails on a few occasions to notice some incorrect results.

Actuaries use their statistical and mathematical knowledge to find solutions to actuarial problems. This book uses mathematical statistics, probability theory in particular, throughout. The text is the author's doctoral thesis from the University of Amsterdam. The book has five chapters covering three main topics of current research in actuarial science, namely, (i) ordering of risks, (ii) credibility, and (iii) portfolio models. Chapter One gives a brief motivation and outline of the research, while Chapter Five gives a concise conclusion and direction for further research. Throughout the book, the coverage of those main topics is extensive and up-to-date. University undergraduate mathematics and statistics knowledge is sufficient to understand the materials covered. The book contains good original presentation, results and practical applications. This reviewer has noted a few careless but serious mistakes, and they are pointed out as we go along.

Ordering of Risk

The treatment follows the same line as found in van Heerwaarden [1] and Goovaerts [6]. The concepts of different kinds of ordering, namely, moment order, stochastic order, and stop-loss order, are explained in a heuristic manner. Single-life and multistate applications are considered thoroughly. Actuarial equivalence of premiums and benefits is a common assumption in premium calculation. An optimum premium (benefit) is one that minimizes certain objective functions, namely, the variance of loss at a certain time in the future subject to the constraint of actuarial equivalence at issue. The author deserves special credit for his coherent treatment of optimal premiums as well as optimal benefits.

On page 48, the notion of bivariate dependence is introduced and is attributed to Norberg [9]. Detailed study of the concept of dependence can be found as early as in Blomqvist [2], Lehmann [8], and Esary and Proschan [3]. The application of multistate models is praiseworthy. Unfortunately, Theorem 2.26 and Theorem 2.27 are wrong. However, several subsequent results of inequalities following Norberg [9] are correct.

The two incorrect statements are of opposite natures and correct themselves when applied jointly! The corrected version of Theorem 2.26 is as follows:

Theorem 2.26: *If two two-dimensional random vectors,* $\mathbf{X}=(X_1, X_2)$ *and* $\mathbf{Y}=(Y_1, Y_2)$, *from which the real random elements* X_i *and* Y_i $(i=1, 2)$ *have the same marginal distribution* $(X_i \sim Y_i)$, *are such that* $\mathbf{X} \leq_D \mathbf{Y}$, *then*

$$E[f(\mathbf{X})] \geq E[f(\mathbf{Y})],$$

for all quasi-monotone increasing functions for which the expectations exist. For quasi-monotone decreasing functions, the conclusion is the reverse inequality.

The proof of this theorem easily follows from Proposition 1.10.2 in Stoyan [14]. Note that the equality of the marginal distributions of the respective components of $\mathbf{X}$ and $\mathbf{Y}$, as mentioned in the book, is not required in the proof. This two-dimensional result has a similarity with the one-dimensional results as in Theorem 2.4, and as Stoyan [14] pointed out, the resulting inequalities in them should be in the opposite direction. Now the corrected version of the Theorem 2.27 will be as follows.

Theorem 2.27: *The pair of real random variables* (X, Y), *positively quadrant dependent, is ordered by distribution with the pair* (X^i, Y^i),

$$(X, Y) \leq_D (X^i, Y^i),$$

where X^i *and* Y^i *are assumed to be independent and where the marginal distributions of* X *and* X^i *and the marginal distributions of* Y *and* Y^i *are equal.*

The proof of the theorem is straightforward. Also, it can be found in Aly and Kochar [1, p. 212]. The detailed proofs of the corrected versions of the above-mentioned two theorems also can be found in Sharif [13]. A careful reader can easily verify the subsequent results that are direct consequences of the above theorems. In this section, the author successfully gives the initial impetus for multidimensional ordering of risks in life insurance.

Credibility

The credibility literature has grown extensively recently. In this book, only regression in credibility, in particular, nonlinear regression in credibility, is considered. Statistical reasoning is along the line found in De Vylder [15]. The results were applied to graduate the Dutch male life table 1985–1990, and it was graphically shown that the nonlinear regression in credibility produces a better fit than other methods. Hence the premium calculation was shown to be improved.

Portfolio Models

Since Panjer's paper [10], several papers on both collective and individual risk models have been published. A brief overview of the literature is given. A generalized numerical three-step method is proposed to calculate the total claim distribution for the class of compound generalized power series distributions. The stability of the recursions is not considered. The method is practically convenient and computationally better than the brute force method (using convolutions) of calculating total claim distribution. Two numerical methods (which are based on Seal's equation and the renewal equation) with examples are introduced to calculate finite time ruin probabilities for lattice distribution. Bounds (upper and lower) are also obtained for finite time (finite wealth) survival probabilities. In short, the author has done a good job in this section, except that Theorem 4.4 appears to be wrong. A close look at relation (4.12) indicates that it always generates a null sequence, and hence Theorem 4.4 needs correction. According to this reviewer, the corrected version of the theorem is as follows:

Theorem 4.4: *For a GP counting distribution, the coefficients of the second auxiliary power series* $\sum_{h=0}^{\infty} \eta_j u^j$ *can be calculated recursively by*

$$n_j = \frac{1}{j-1}\left\{\alpha \sum_{h=1}^{j-1} \eta_h (j-h)\eta_{j-h} + \sum_{h=1}^{j-1} \epsilon_h \eta_{j-h}\right\} \tag{1}$$

for $j=2, 3, \ldots,$ *where* $\eta_0=0$ *and* $\eta_1=m_1\theta e^{-\alpha\theta}$.

Note that the book considers only the case $r=1$, where $r=\min\{j:m_j>0\}$. A generalized version of Theorems 4.4 and 4.5 can be found in Sharif and Panjer [11].

On page 108, the author claims that

> Our recursion for the generalized Poisson distribution differs slightly from the one of Goovaerts & Kaas (1991), because we start with another, more general, generating function . . .

In fact, both derivations of the recursion for the compound generalized Poisson distribution start with the mathematically equivalent function. After the correction has been made in Theorem 4.4 (as pointed out above), the two sequences, $\{r_j\}$ and $\{\theta\alpha_j\}$, in Goovaerts and Kaas [5] are identical with the two sequences, $\{\epsilon_j\}$ and $\{\eta_j\}$, in the book, respectively. A detailed explanation is found in Sharif and Panjer [11].

Another minor error appears on page 109, where the author refers to Gerber [4],

> . . . who considers a recursive formula for the generalized negative binomial distribution. His result is a special case of our method, which includes the recursive evaluation of the compound generalized negative binomial probabilities.

His claim is not strictly true, because what Gerber [4] considers as *the generalized negative binomial distribution* is different from the GNB considered in the book, even though they seem to use the same technical name. Gerber's GNB is a mixture of Poisson with generalized gamma distribution, while GNB in the book is a member of the Lagrange family and they are different distributions.

Even though there are several typographical errors throughout the book, the presentation is coherent and original. This reviewer also spotted a couple of isolated mistakes in formulas in the book. The first one is on page 14 (last line); the summation should range over a finite countable set and $F_X^s(x)$ should be replaced by $f_X^s(x)$. The second one is on page 29: the equality results in the second line are false and need correction. In fact, the corrected right-hand side would be the ratio of two single premiums calculated at interest rate 2δ and δ, respectively. The symbols T_y and T_{x-k} on page 30 may cause some confusion; the symbols $T_Y(y)$ and $T_X(y-k)$ instead would be more meaningful.

In spite of these errors, a careful reader who has great interest in insurance research will welcome this book. This detailed review of the book may help enthusiastic readers ply through the book comfortably and not get hung up on some omission errors. Finally, this reviewer is convinced that all actuarial students and researchers in the field of

actuarial science will find this text a very useful one to have on their bookshelf.

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A. HOQUE SHARIF

C.D. Daykin, T. Pentikainen, and M. Pesonen, *Practical Risk Theory for Actuaries* (Monographs on Statistics and Applied Probability 53), 546 pp., published by Chapman & Hall, London, 1994, $35.

Practical Risk Theory for Actuaries is intended to become the standard actuarial textbook on risk theory in the U.K. and other countries and to replace *Risk Theory* by Beard et al., which has become obsolete as a result of new research and the advance of information technology. While many works are available that treat risk theory along strictly theoretical lines, few books are available that develop the theories into forms suitable for practical purposes or demonstrate their application. This work does just that. It makes an important contribution to the field by significantly narrowing the gap between practical actuaries and the proponents of risk theory.

The book is divided into two parts to make it useful to a wider range of readers. Part One is intended to be used as a primary textbook. Familiarity with the basic concepts and elementary techniques of probability calculus is assumed. Most proofs have been moved to appendixes or covered by references to other publications, to keep the main text as brief as possible and focused on principles and practical applications. Exercises are included after each section, and solutions are given in the appendixes. Part Two gives guidance in the solution of more complex problems and in more extended applications of the theory. This is designed to help the reader who is interested only in a particular problem, for example, the evaluation of capital at risk, net retention in reinsurance, run-off phenomena, asset risks, and so on. The book is useful for reference because it provides an index of topics that links related subjects and because it includes an extensive bibliography. Diagrams and graphical representations, often based on simulation techniques, make the chapters seem more interesting and alive.

For calculating compound aggregate claim distribution functions, the book considers various recursion formulas as well as certain approximate formulas, such as the Normal Power method, the Haldane method, and the Wilson-Hilferty formula, which is adopted as a standard for many applications given. Simulation methods are considered, including use of the Wilson-Hilferty generator for compound Poisson and compound mixed Poisson random numbers. Application of the methods introduced in Part One are considered for short and long time horizons. For example, the

book evaluates the fluctuation range of claim amounts, assesses the corresponding capital requirements, and analyzes the effect of reinsurance and the level of net retention, as well as the basic mathematics of rating reinsurance contracts.

Part Two introduces many new elements that are important in managing the insurance business but are usually ignored in classical theory. For inflation and investment return, first-order autoregressive processes similar to those developed by Wilkie are used for stochastic modeling of future changes in the consumer price index, investment prices and returns, and the composition of the asset portfolio. Simulation techniques are used to model aggregate incurred claim amounts for a year and accumulated aggregate claim amounts over a period of years that take into account long-term trends, irregular cyclical patterns, and random variations in risk propensity. The run-off process used to determine claim reserves is also modeled by using simulation. Some insurance and investment terms may be unfamiliar to U.S. readers, but they are not an obstacle to understanding the concepts presented.

The causes and mechanisms generating insurance business cycles are explained from a mathematical point of view. Examples from the U.S. and other countries are used to provide some insights into the significant features that may underlie cyclical variations and the mechanisms inducing them. A basic model is presented and studied to show how the insurance process can be simulated with decomposition into lines of business to include investment return, inflation, claims, premiums, and so forth. Later chapters treat how simulation of the insurance process can assist an insurer's strategic planning and how interactions with the insurance market might be taken into account.

While the main focus of the book is on general insurance (property/casualty insurance), there also are chapters on life insurance and pensions because of their unique problems. In general, the book is written for all actuaries without distinction between casualty and life actuaries.

As actuaries are called upon to play more important roles in the financial management of insurance companies and pension funds, the need to explore the consequences of uncertainty and variability through stochastic models is inescapable. Over the next few years, the material is this book will be essential reading for every actuary who wants to keep

abreast of these developments. You should expect to see this type of practical textbook on the actuarial syllabus both in the U.K. and the U.S.

THOMAS J. LIVORSI

William H. Aitken, *A Problem-Solving Approach to Pension Funding and Valuation,* 358 pp., published by Actex Publications, Winsted, Connecticut, 1994, $40.

Professor Aitken believes that the best way to learn a mathematical topic is to do numerical examples, especially if the material presents the fundamentals of a topic, as opposed to issues of regulation and taxation. This text emphasizes the calculation of normal costs, actuarial liabilities and experience gains and losses for a wide variety of cost methods.

Numerical examples form the basis of the book, five chapters of which are on the SOA Course 210 syllabus. Typical is Chapter 5, which has 35 worked examples and 33 exercises, the answers to which are given. Only about a quarter of the book comprises discussion and algebraic analysis. Some would consider this to be less than one would wish, but the numerical examples clarify and make any necessary extensions to the theoretical material.

The subject text forms an intriguing contrast with *Pension Mathematics for Actuaries* (A.W. Anderson, Actex Publications), which is on the syllabus for Courses P-359C, P-360U (EA-1, Segment B), P-461U, and P-462C. Anderson gives a thorough and very useful algebraic development of gain and loss formulas but no numerical examples and a mere handful of numerical exercises. Students will find Anderson's book difficult if they use it as an introductory pensions text. One wonders also about a beginning pension consultant's step from mastering the algebra to actually performing a pension plan valuation. The two books, each excellent in its own way, complement each other well. Aitken generally uses notation consistent with Anderson's, and the Society syllabus effectively follows the policy of regarding Aitken's as the introductory text.

Professor Aitken worked on his book for the first year of his formal retirement from the University of Waterloo. It was read in draft form by several people, and a large number of improvements and corrections were incorporated. One or two points of confusion remain. Withdrawal gain is given on page 154 as

$$^{\text{with}}G_1 = \sum_{w} (AL_1 - {}^{i}WB) - \sum_{A_0} q_x^{(w)}(AL_1 - WB)$$

and again on page 186 as

$$^{\text{with}}G_1 = \sum_{w} (AL_1 - {}^{i}WB) - \sum_{x} q_x^{(w)}(AL_1 - {}^{i}WB).$$

The use of WB in one formula and ${}^{i}WB$ in the other is the result of somewhat inconsistent treatment of assumptions about the date of withdrawal and the crediting of interest. The second formula also highlights the use of a summation over ages x to indicate what is given in the first formula as a summation over the set of active members. The latter notation is consistent with that used by Anderson and is preferred by this reviewer. This inconsistency is emphasized here not as an exercise in advanced pedantry; rather it is mentioned because actuarial students studying in isolation are more troubled by such difficulties than, for example, university students. Who are they gonna call? In general, though, Aitken's book is unusually and blessedly free of such flaws.

In line with the book's philosophy, the algebraic development is somewhat skeletal in parts. Some readers may wish for more extended and less intuitive derivations. Page 150 consists entirely of 14 formulas. They are justified partly by analogy with results in life insurance; this format and logic will not be to everyone's taste. But such reservations are minor.

Aitken's text has already been tested in action. Hundreds of students have used it. The concentration on numerical examples has proved very successful. Students work through the examples until they feel comfortable with the material. They can determine for themselves the number of examples and exercises to work. It is unlikely that anyone will find Aitken's supply to be anything less than abundant.

The text is a valuable addition to the pension literature. Readers will find that it gives them a genuine feel for the material that will serve them well in practical applications.

KEITH P. SHARP

C. Eugene Steuerle and Jon M. Bakija, *Retooling Social Security for the 21st Century*, 332 pp., published by The Urban Institute Press, Washington, D.C., 1994, $18.95.

Retooling Social Security for the 21st Century is a comprehensive analysis of the options for reform of our Social Security system. The authors identify all the options that have received serious consideration in the national debate, as well as some other options that should be considered. Just as importantly, they lay out a framework for evaluating these or other options, so that policymakers will know whether a suggested reform will actually achieve its intended ends.

The book is divided into three parts. Part One lays out the principles that are used to judge suggested reforms: (1) redistribution from the better-off to the less well-off, (2) assurance of a fair return on contributions, (3) equal treatment of equals, (4) attainment of maximum benefit to society from available resources, and (5) opportunity cost (the need to make trade-offs among competing programs and priorities). These principles often compete against each other. The debates over suggested reforms often center on how each debater weighs the importance of each principle.

Part Two examines the current Social Security system. Because the demographic bulge will be approaching retirement age beginning about 2010, costs will increase dramatically, yet there is no scheduled increase in the tax rate. Income will be insufficient to pay benefits (with the exact year of insufficiency depending on assumptions about inflation and wage growth). Thus change is inevitable; either it will be well thought out and planned for, or it will be hurried and ad hoc.

Part Three examines options for reform, grouped into payroll tax options, benefit options, spousal benefits, and rules for early and delayed retirement, and Medicare reform. The authors do not favor increasing the limit on taxable earnings, for what appear to be weak reasons. They do, however, favor expanding the taxability of Social Security benefits. They recommend saving tax rate increases to be used as a balancing item, after changes in benefit provisions are made.

The authors recommend against a decrease in the annual cost-of-living adjustment (COLA), unless it is part of a revision of the CPI to better reflect actual cost of living. A preferred option would be to reduce initial benefit levels, since it would preserve more of the principles presented in Part One. While they recommend against the often-suggested COLA

cutback, they favor a most obscure change (which has been favored by this reviewer for some time): eliminating the dropout years in the benefit computation. This insightful analysis should be required reading for all our policymakers.

The authors' biases (they are economists, not actuaries) show through at times, such as when they devote too much space to a detailed presentation of "money's-worth" examples. Some people get back more in Social Security benefits than they paid in taxes; others get back less. But this facet of Social Security should not be a design issue; it is a consequence of the proper design of Social Security as a social insurance system.

There is so much of the book that is good that a poorly analyzed section is all the more surprising. With minimal analysis, the authors recommend increasing the special minimum benefit—this benefit is an abomination that violates almost every one of the authors' principles. This and a few other cases of insufficient analysis mar an otherwise well-written book.

This is an excellent book for those people interested in the future design of Social Security, though in some places it takes an expert eye to spot weaknesses in the authors' arguments.

STEVEN F. MCKAY