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THE PROJECTED UNIT CREDIT METHOD WITH BENEFITS APPORTIONED BY INTEREST-ADJUSTED SALARY

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ABSTRACT

This paper adopts the framework whereby pension benefits are allocated one exit point at a time in some fashion to the years of service of those persons expected to leave at each exit point. Of particular interest is an allocation by interest-adjusted salary. Within the same framework, the paper considers allocations based on the implicitly perceived value of the benefit to employees. While impractical, these perceived value allocations suggest the desirability of something more back-loaded than allocating equal benefits to each year of service. Also considered are the method of combining exit points, the determination of different normal costs for different cost centers, the allocation of costs for persons transferring between cost centers, the problem of negative accruals, and why the current employment benefit formula should not be used to allocate the benefit at an assumed exit point.

INTRODUCTION

There are as many ways to fund a retirement benefit as there are to finance a house. To say that one is wrong is analogous to saying that a house cannot be financed with different payment schedules. To say that one is best presumes a particular criterion or a particular weighting of criteria. From the perspectives of accounting and economic theory, some cost allocation methods make more sense than others. So far, however, the ideal cost allocation method for accrual accounting purposes has not been found. Two accountants (Hall and Landsittel [6]) proposed using the projected unit credit method with the benefit allocated by salary as something of an accounting standard. Their proposal was not particularly well received.[1] With a minor change, however, a rather interesting funding method results. It is not, in my view, the only reasonable alternative; however, its funding philosophy merits further attention.

[1]Reactions by members of the Pension Research Council are given at the end of the 1977 publication.

The rationale of the Hall and Landsittel proposal was simple enough. They reasoned that "since services are rewarded by a wage or salary, which is presumed to reflect the worth of the employee to the firm, the accrual of a pension obligation should be measured in terms of the employee's compensation, irrespective of how the benefit is determined under the terms of the plan."

Among the criticisms made of the Hall and Landsittel proposal are the following:

(1) The proposal did not fully appreciate the fact that it was only one of many possible mappings of the projected benefit onto individual years of service.
(2) Imposing an arbitrary standard causes undue paperwork.
(3) The method has unacceptably high back-loading.
(4) The method would not be permitted by federal tax regulations (1.412(c)(3)-1).

To this list I can add another criticism:

(5) Prorating by nominal salary ignores the effects of inflation.

Since 1977 dollars are not equivalent to 1993 dollars, allocating a benefit in proportion to unadjusted salary leads to peculiar results. An obvious solution is to allocate the projected benefit in proportion to inflation-adjusted salary. Chris Doyle, a colleague at the Department of Defense, wondered, Why not use interest-adjusted salary? This latter suggestion is the one considered here. It would not, at present, be permitted under IRS regulations, but it has an appealing funding philosophy and considerably less back-loading than the original Hall and Landsittel proposal.

Later in the paper, I also discuss what I call perceived value allocations. These assess the employee's perceived value of the retirement benefit and allocate the benefit to individual years of service, taking these perceptions under consideration. While perceived value allocations may be impractical for ordinary uses, they have some implications for what we ought to be doing.

At various places in the paper, I appeal to what makes the best sense for accrual accounting. In doing so, I do not restrict myself to current accounting conventions or regulatory requirements. In fact, what I recommend would not pass muster there. Nevertheless, we can hope that arbitrary standards will be modified and that those not restricted by them may be able to implement some of the ideas presented here.

JUSTIFICATION FOR INTEREST-ADJUSTED SALARY

Philosophically the benefit allocation under the Hall and Landsittel proposal is not without merit. Retirement benefits are almost always related to levels of compensation. So it seems counterintuitive for an allocation scheme to ignore different compensation levels within an individual's career. By eliminating the apples-and-oranges nature of unadjusted dollars, we could get a reasonable salary-based allocation. At the same time, we would eliminate some back-loading. This gives us a projected unit credit method with the benefit allocated by inflation-adjusted salary. In addition, because the Social Security benefit formula is based on an average indexed wage, I think we must consider a related allocation method as a reasonable alternative.

The argument for interest-adjusted salary is similar. We imagine ourselves at an exit point and must somehow apportion the benefit back to individual years of service. In doing this, we want to treat the retirement benefit earned in any year as a form of undistributed compensation that is related to salary. But this situation is similar to a series of investments made by a firm or an individual. The future value is the values brought forward by interest rates. So the benefit at retirement could be apportioned by interest-adjusted salary.

The argument is appealing but not mathematically compelling. However, there are two further arguments in favor of apportioning the benefit by interest-adjusted salary. The first argument is that the benefit builds at a reasonable rate compared to the obvious alternatives. At one extreme the projected benefit is allocated equally to all years of service. This method is required by federal tax regulations (1.412(c)(3)-1) when level dollar amounts or level percentages of pay are not being used for the normal cost and it is not a career-average pay plan.[2] If an employee is expected to work 20 years for a firm, 1/20th of the expected benefit would be allocated to each year. This leads to a relatively fast benefit buildup. At the other extreme would be a benefit apportioned by inflation-adjusted salary.[3] This is essentially the Hall and Landsittel proposal using real dollars rather than nominal dollars.

[2]The regulation states "The numerator of the fraction is the participant's credited years of service. The denominator is the participant's total credited years of service at the anticipated benefit commencement date. Adjustments are made to account for changes in the rate of benefit accrual. An allocation based on compensation is not permitted."

[3]In a later section I describe a perceived value allocation that also seems reasonable. Under some assumptions this could be more heavily back-loaded than an inflation-adjusted salary allocation.

The original Hall and Landsittel scheme, however, leads to a slower benefit buildup than the projected unit credit method with inflation-adjusted dollars. It is not a reasonable alternative because of the apples-and-oranges nature of the nominal dollars used in mapping the benefit.

More extreme in the other direction for plans with substantial early attrition is the entry age normal method. This can lead to a benefit that builds even more rapidly than the level benefit method. Again, however, this is not a reasonable alternative for accounting purposes, since the entry age normal method levies the same normal cost, regardless of the probability of reaching retirement. This disadvantage of the entry age normal method is discussed more fully later in this section.

The second additional argument in favor of apportioning by interest-adjusted salary is the following funding philosophy. The entry age normal method funds by a constant percentage of salary. The projected unit credit method with interest-adjusted salary is similar. A constant is used, but it is applied to the product of salary and the probability of retirement at a particular exit point. The constant is the same for all lengths of service, assuming there is only one point of retirement.[4]

To demonstrate this, we need to develop a few equations. Projected unit credit methods assign the projected benefit at retirement to particular years of service. For the level benefit method, if at the end of 20 years of service, a retirement benefit has a lump-sum value of $300,000, we would assign 1/20th of $300,000 (discounted from retirement back to the service year) to each year of service. If the funding is identical to the accrual amount, the normal cost contribution for one person for one year would pay only for the apportioned piece of the projected benefit. The normal cost contribution for person l in the j-th year of service would then be:

$$NC_{jl} = \frac{\$300{,}000}{20} \times \begin{matrix}\text{interest}\\\text{discount}\end{matrix} \times \text{prob (reaching retirement)}.$$

To simplify matters, I assume that retirement occurs at the end of 20 years of service, that salary and normal cost contributions are paid at the end of each year for the average number working during the year, and that salary increases are given at the beginning of the year. If the number

[4]The extension to multiple exit points is considered later, but the gist of it is that for everyone "expected" to terminate at a particular exit point, we must set aside the same percentage of salary. Different amounts would be set aside for each potential exit point.

reaching the end of the j-th year is n_j and we assume a level distribution of losses during the year, the average number working $(n_{j-1}+n_j)/2$ will equal the mid-year value of $n_{j-0.5}$. The normal cost contribution for each year is the sum of the normal costs for all 20 cohorts. Assuming a constant interest rate i, the normal cost contribution for the cohort in the j-th year of service is:

$$NC_j = n_{j-0.5} \frac{\$300{,}000}{20} \frac{1}{(1+i)^{20-j}} \frac{n_{20}}{n_{j-0.5}}. \tag{1}$$

Equation (1) gives the normal cost contribution in dollars. To get a percentage, Equation (1) and the subsequent equations in this paper need to be divided by the payroll in the j-th service year.

To make Equation (1) more general, assume a benefit worth B_z at the end of z years and an apportioning ratio of f_j. Then

$$NC_j = n_{j-0.5}\, B_z f_j \frac{1}{(1+i)^{z-j}} \frac{n_z}{n_{j-0.5}}.$$

The generalization to different possible years of retirement with differing probabilities and benefits is straightforward and need not concern us here. Similarly, we might use months rather than years or assume different interest rates in each year.

What is of interest here is the apportioning ratio, f_j. The first two equations assumed 1/20th, or $1/z$. The apportioning could also be done by salary, interest-adjusted salary, and so on.

By using interest-adjusted salary in the f ratio, the normal cost for service year j becomes:

$$NC_j = n_{j-0.5}\, B_z \frac{Sal_j(1+i)^{z-j}}{\sum_{k=1}^{z} Sal_k(1+i)^{z-k}} \frac{1}{(1+i)^{z-j}} \frac{n_z}{n_{j-0.5}}. \tag{2}$$

where Sal_j=the salary of a typical individual paid at the end of the j-th year.

If inflation-adjusted salary were wanted, the apportioning ratio in Equation (2) is all that would have to be modified. To get the formula for the entry age normal method, we proceed as follows. In Equation (2) we apportioned the benefit by the work (as measured by interest-adjusted salary) of those who retire. For the entry age normal method, we want to include the work of those who will not make it to retirement. Each individual who makes it to retirement in this formulation could be thought of as having a leverage in year j such that the contribution is effectively increased by the ratio $n_{j-0.5}/n_z$. The normal cost contribution then becomes

$$NC_j^{ean} = n_{j-0.5}\, B_z \frac{\frac{n_{j-0.5}}{n_z} Sal_j(1+i)^{z-j}}{\sum_{k=1}^{z} \frac{n_{k-0.5}}{n_z} Sal_k(1+i)^{z-k}} \frac{1}{(1+i)^{z-j}} \frac{n_z}{n_{j-0.5}}. \tag{3}$$

By simplifying and repositioning $n_{j-0.5}$ and n_z, Equation (3) becomes

$$NC_j^{ean} = n_z B_z \frac{n_{j-0.5}\, Sal_j}{\sum_{k=1}^{z} n_{k-0.5}\, Sal_k(1+i)^{z-k}}. \tag{4}$$

If the $n_z B_z$ in Equation (4) is divided by $n_0(1+i)^z$, it represents the present value of future benefits at the point of entry. When the summation is divided by the same term, it becomes the present value of future salaries at the point of entry. Thus we have

$$NC_j^{ean} = \frac{PV(\text{future benefits})}{PV(\text{future salary})}\, n_{j-0.5}\, Sal_j. \tag{5}$$

Equation (5) is a more familiar version of the entry age normal formula with an assumed exit point. It is distinguished from the projected unit credit method in Equation (2) only by the benefit-apportioning ratio. This relationship between the entry age normal and the projected unit credit methods has also been demonstrated by Anderson [1, p. 153]. The term with the summation in the denominator in Equation (3) is the benefit-apportioning ratio for the entry age normal method.

The entry age normal method has the peculiar interpretation that it apportions the benefit to people who are not expected to retire, as well as those who will. This may be seen more directly by rearranging the terms of Equation (3) as follows:

$$NC_j^{ean} = n_z B_z \frac{n_{j-0.5}\, Sal_j(1+i)^{z-j}}{\sum_{k=1}^{z} n_{k-0.5}\, Sal_k(1+i)^{z-k}} \frac{1}{(1+i)^{z-j}}.$$

Because of the above peculiarity, the entry age normal method apportions too much benefit to the early years of service. Consequently, for accrual accounting purposes, it leaves something to be desired.

Of particular interest here is an alternative development of Equation (2). It begins with the entry age normal method. However, instead of a uniform percentage contribution on all salaries, a uniform contribution is made on the product of salary and the probability of reaching retirement. This is given by

$$NC_j = \frac{PV(\text{benefit})}{PV[\text{salaries} \times \text{prob (retirement)}]} \, n_{j-0.5} \, Sal_j \, \frac{n_z}{n_{j-0.5}}$$

or

$$NC_j = \frac{\dfrac{n_z B_z}{(1 + i)^z}}{\displaystyle\sum_{k=1}^{z} \frac{n_{k-0.5} \, Sal_k}{(1 + i)^k} \, \frac{n_z}{n_{k-0.5}}} \, n_{j-0.5} \, Sal_j \, \frac{n_z}{n_{j-0.5}}. \tag{6}$$

Simplification shows that Equation (6) is equivalent to (2). So an alternative interpretation of (2) is that it uses a uniform percentage applied to the product of salary and the probability of reaching retirement. Another way of looking at this is to say that we contribute a uniform percentage of salary for the n_z persons in a cohort who will reach retirement. This avoids a problem of the entry age normal method, that is, making those who leave pay for those who stay.

A COMPARISON OF BENEFIT ALLOCATIONS FOR EQUATION 2 WITH OTHER METHODS

Table 1 presents the benefit-apportioning ratio by service year for regular enlistees in the military services of the U.S. under the five different cost methods. The first three columns show the benefit allocation under the projected unit credit method with the benefit being allocated by unadjusted salary, inflation-adjusted salary, and interest-adjusted salary, respectively. The fourth column gives a level benefit allocation. The final column shows the benefit allocation under the entry age normal method.

All methods assume the net loss rates for regular enlistees in the U.S. Armed Forces given in the valuation report [3] and ignore other losses or transfers to other statuses. The five columns also assume retirement at 20 years of service, an entry age to the nearest birthday of 19, an interest rate of 7 percent, an inflation rate of 5 percent, an annual pay scale increase of 5.75 percent, and the promotion and longevity increases given for regular enlistees. The annual pay scale increase is in addition to promotions and length-of-service increases.

Table 1 looks at a cohort retiring at the 20th year of service and asks in which year various pieces of the retirement benefit should have been accounted for. Alternatively, it looks at those expected to retire in an entry cohort and apportions their retirement benefit to a year of service.

TABLE 1

PROPORTION OF RETIREMENT BENEFIT EARNED BY REGULAR ENLISTEES IN THE MILITARY SERVICES BY YEAR OF SERVICE AND COST METHOD

	Projected Unit Credit Method with Benefit Allocated by				
Year of Service	Salary	Inflation-Adjusted Salary	Interest-Adjusted Salary	Level-Benefit Method	Entry-Age Normal Method
1	0.015	0.026	0.032	0.05	0.107
2	0.017	0.029	0.035	0.05	0.106
3	0.020	0.032	0.038	0.05	0.096
4	0.023	0.035	0.041	0.05	0.074
5	0.026	0.038	0.044	0.05	0.057
6	0.029	0.040	0.045	0.05	0.050
7	0.032	0.043	0.047	0.05	0.046
8	0.035	0.045	0.048	0.05	0.041
9	0.039	0.047	0.049	0.05	0.038
10	0.042	0.049	0.050	0.05	0.036
11	0.047	0.051	0.052	0.05	0.035
12	0.051	0.053	0.053	0.05	0.034
13	0.056	0.056	0.054	0.05	0.034
14	0.061	0.058	0.055	0.05	0.034
15	0.067	0.061	0.057	0.05	0.034
16	0.073	0.063	0.058	0.05	0.034
17	0.080	0.065	0.059	0.05	0.035
18	0.087	0.067	0.060	0.05	0.035
19	0.095	0.070	0.061	0.05	0.036
20	0.103	0.072	0.062	0.05	0.036
Total	1.00	1.00	1.00	1.00	1.00

Table 2 shows the normal cost percentages for each of these cost methods, using the same assumptions as for Table 1.

WEIGHTED NORMAL COST PERCENTAGE

As the bottom row of Table 2 suggests, the normal cost percentage at each length of service can be weighted to get a weighted normal cost percentage. This is defined as the normal cost percentage, which if applied to all salaries would be equivalent to the length-of-service specific

TABLE 2

NORMAL COST PERCENTAGES FOR REGULAR ENLISTEES BY YEAR OF SERVICE AND COST METHOD

Year of Service	Projected Unit Credit Method with Benefit Allocated by: Salary	Inflation-Adjusted Salary	Interest-Adjusted Salary	Level-Benefit Method	Entry-Age Normal Method
1	6.4	11.3	13.9	21.7	46.4
2	7.5	12.7	15.3	21.9	46.4
3	9.6	15.5	18.2	24.1	46.4
4	14.4	22.2	25.7	31.3	46.4
5	21.3	31.1	35.4	40.6	46.4
6	26.5	36.9	41.2	46.0	46.4
7	32.9	43.6	47.7	50.9	46.4
8	39.7	50.1	53.8	56.2	46.4
9	47.1	56.6	59.7	60.3	46.4
10	54.4	62.3	64.4	64.1	46.4
11	61.7	67.3	68.3	66.1	46.4
12	69.0	71.7	71.3	67.5	46.4
13	76.4	75.6	73.9	67.8	46.4
14	83.7	78.9	75.7	68.2	46.4
15	91.1	81.8	77.0	67.5	46.4
16	98.7	84.4	77.9	67.4	46.4
17	106.6	86.8	78.6	66.5	46.4
18	114.8	89.0	79.1	65.9	46.4
19	123.2	91.0	79.4	65.0	46.4
20	131.8	92.7	79.4	64.1	46.4
Weighted	49.3	48.5	48.1	47.6	46.4

normal cost percentages applied to the salaries at each length of service. In this case the salaries used are what we would see after 20 successive equally sized cohorts, assuming all assumptions were met. The weighted normal cost would be expected to vary, however, as force composition changed. The formula for the weighted normal cost percentage is as follows:

$$\text{Weighted } NCP_t = \frac{\sum_{j=1}^{z} (NCP_j \, Sal_{j,t})}{\sum_{j=1}^{z} Sal_{j,t}}$$

where

j = length of service

t = period for which a normal cost contribution is being made

$Sal_{j,t}$ = salaries earned for all persons in their j-th year of service during period t.

The weighted normal costs in Table 2 differ by funding method, since the more front-loaded methods will build a larger fund prior to reaching equilibrium. Although the present value of future benefits is the same in all the cost methods, the present value of future normal cost contributions differs. In effect, when we choose a cost method, we assume a present value of past normal cost contributions. If we pick a method with smaller assumed past normal cost contributions, the future normal cost contributions must be larger.

The motivation for getting a weighted normal cost is to simplify matters administratively. Ultimately, our goal is to have weighted normal costs that differ for different cost centers. For example, we might have a different normal cost for each branch of the military services, with separate normal costs for officers and enlistees. To do this, we have to extend Equation (2) to more than one point of retirement and to allow transfers between cost centers.

EXTENSION TO MULTIPLE EXIT POINTS

The extension to multiple points of retirement is straightforward. For each possible point of retirement, we apply Equation (2). The n_z in Equation (2) now means those who retire at z years, a smaller number than those who complete z years. Then for each length of service we add the appropriate normal costs. For example, if a person with 20 years of service could retire at any time in the next 10 years, there would be 10 normal costs, one for each of the 10 possible points of retirement. So the normal cost for the person with 20 years of service would be a sum of 10 numbers. The normal cost for a person with 19 years of service would be a sum of 11 numbers, and so forth. The normal cost equation for period j is then

$$NC_j = \sum_z \frac{f_{z,j} B_z n_z}{(1 + i)^{z-j}}$$

where $f_{z,j}$=part of B_z attributable to period j.

Administratively, this is not prohibitively complicated, since for each cost center we can use a weighted normal cost percentage, which is just one number.

The unfunded liability is given by

$$UFL = \sum_z \sum_j \left(\frac{n_{z,j} B_z \left(\sum_{k=1}^{j} f_{z,k} \right)}{(1+i)^{z-j}} \right) - \text{Assets}$$

where $n_{z,j}$=number at length of service j expected to retire at exit point z.

The normal costs here are assumed to be paid at the end of the period and included in the assets. Otherwise there is nothing special in the treatment of gains. If, for example, we assume an amortization payment on an earlier unfunded liability is paid at the beginning of period t, then the gain at the end of the period is given by

$$Gain_t = UFL_{t-1}(1+i) - UFL_t - \text{am. payment}_t(1+i).$$

Note also that the allocation of benefits within exit points should have no effect on pension benefits guaranteed for particular exit points.

ALLOWING TRANSFERS BETWEEN COST CENTERS

Transfers between cost centers are more complicated. To do this, we must imagine a network of nodes. There would be one node for each completed length of service at each cost center. The nodes represent the end of each length of service. The connecting segments represent the periods of service.

A procedure for allocating benefits by cost center is available for any allocation scheme. This can be done by separating the network into all possible paths. For example, in Figure 1, AH is a path; AI is a path; and so forth. But each path can be handled by Equation (2). An allocation can be applied to the path to get the f_j's for the path. So for each of these paths we know not only f_j's but also the cost center for each f_j. To get the normal cost for cost center r and length of service j, we must sum the normal costs for all exit points and paths, but include only normal costs associated with a cost center.

The normal cost is

$$NC_{r,j,z} = \sum_p \left[\frac{f_{r,j,z,p} B_z n_{r,j,z,p}}{(1+i)^{z-j}} \right]$$

where

$NC_{r,j,z}$ = the current normal cost for cost center r, length of service j, for the benefit at exit point z

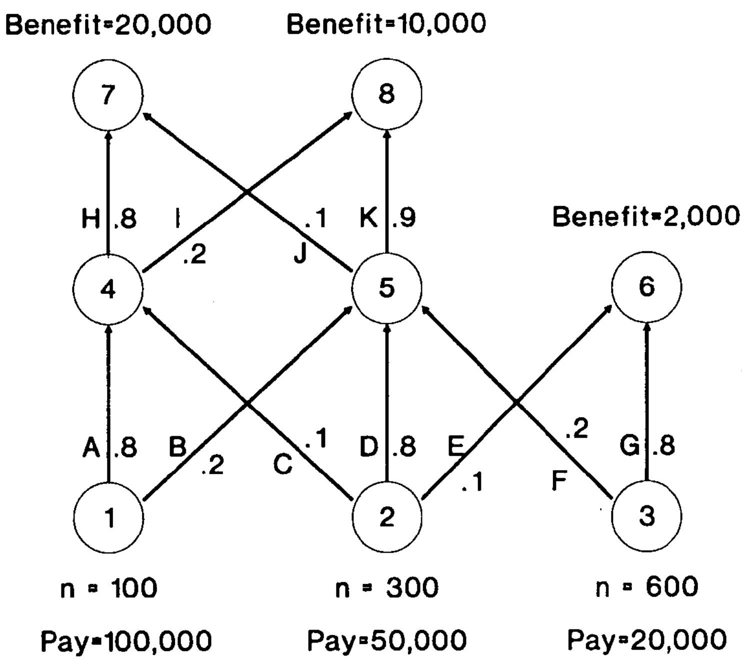
$f_{r,j,z,p}$ = the allocation formula for cost center r, length of service j, exit point z, and path p

$n_{r,j,z,p}$ = the number currently at cost center r, length of service j, who reached where they are by path p and will exit at z by path p

p = all paths that end at z.

The subscript r in $f_{r,j,z,p}$ and $n_{r,j,z,p}$ is added for clarity. Once we are given exit point z and path p, j will either be in cost center r or not. The formula assumes that $f_{r,j,z,p}$ and $n_{r,j,z,p}$ will be zero if path p does not pass through cost center r at length of service j.

FIGURE 1
NETWORK ALLOCATION



The total normal cost for a cost center is then

$$NC_r = \sum_z \sum_j NC_{r,j,z}.$$

For an example of a network allocation, let's do an interest-adjusted salary allocation for the network in Figure 1. The number entering at nodes 1, 2, and 3 is given, as well as the starting pay. Each segment is identified by a letter and shows the probability of moving along that segment. We assume salary and normal costs are paid at the end of the year and that transfers occur at the end of the year after salary and normal costs have been paid.

The normal costs for segments E and G are simplest. At the end of the first year, the normal cost for E is given by 300(0.1)(2000).

No interest adjustment is needed for segments E and G. For other segments we need to allocate by the interest-adjusted salary. Let's assume 7 percent interest and a 10 percent increase in the second period. The number of employees, the salaries, benefits, and normal cost percentage for each of the remaining paths are as shown in Table 3.

TABLE 3

NORMAL COST PERCENTAGES IN A NETWORK ALLOCATION

Path	Number	1st-Year Salary	2nd-Year Salary	Exit Benefit	Normal Cost Percentage
AH	64	100,000	110,000	20,000	9.22
AI	16	100,000	110,000	10,000	4.61
BJ	2	100,000	55,000	20,000	12.35
BK	18	100,000	55,000	10,000	6.17
CH	24	50,000	110,000	20,000	12.23
CI	6	50,000	110,000	10,000	6.12
DJ	24	50,000	55,000	20,000	18.43
DK	216	50,000	55,000	10,000	9.22
FJ	12	20,000	55,000	20,000	26.18
FK	108	20,000	55,000	10,000	13.09

The normal cost percentage for path AH, for example, is computed from

$$20{,}000/[(100{,}000)(1.07) + 110{,}000].$$

To get the normal cost, we would apply the 9.22 percent to the salaries of persons believed to be on path AH.

While this method can be done with any funding philosophy, shortcuts would be needed for most applications, since the number of paths can become large rather quickly. For a level benefit allocation, a nice simplification results when all who reach a particular node have the same number of years of service, n. The proportion of the individual person's benefit paid for by the segment that feeds that node is $1/n$, while $(n-1)/n$ is passed down to comprise the benefit at lower nodes. The lower node benefit equals the passed-down benefits from the connecting next higher nodes, with each passed-down benefit reduced by interest and multiplied by the segment probability.

NEGATIVE OR SMALL ACCRUALS

In many retirement systems, when a person reaches advanced age and service, the present value of the benefit declines from year to year. Increased salary and length-of-service multipliers do not offset the declining number of years over which the benefit is payable. However, none of the funding methods given in Tables 1 and 2 would show a negative accrual. The traditional unit credit method would. If we want our accounting to reflect negative or small accruals, it makes good sense to switch to the unit credit method at some point. For example, the projected unit credit method using interest-adjusted salary might be used for the first 25 years, using the false assumption that all who complete 25 years would then retire. Thereafter, the unit credit method would be used to show the small or negative accruals beyond the 25th year.

But at what point should we switch? To resolve this, we need to consider perceived value allocations, which are discussed in the next section.

PERCEIVED VALUE ALLOCATIONS

Except for negative or small accruals, mentioned in the previous section, we have followed a single idea for allocating costs. Each exit point allocates its costs among the people who are expected to reach that exit point. Perceived value allocations adopt this same framework. We assume we are at one of the exit points and want to allocate the benefits at that exit point back to individual years of service of those people who are expected to reach the exit point. How should this be done?

If we could show that the retirement benefit has a motivational equivalent of cash in particular years, then it would make good accounting sense to allocate each year's cash equivalent to that year. To do this, we need to know the employee's implicit allocation of the benefit v_j, at an assumed exit point z to each year of service and the employee's perceived value of the benefit at z. The perceived value is the present value of $(B_z \times v_j)$, discounted back to time j using personal discount rates. The personal discount rates and the employee's allocation are clarified below.

Personal discount rates are like interest rates, which tend to be higher for young people than market interest rates (Gilman [5]). To find a personal discount rate, we must first find the cash amount at which a person is indifferent to whether he or she receives the cash now or a deferred amount later. Then we use the cash amount as the present value in a present value equation and solve for the personal discount rate.[5]

To find the employee's allocation, we ask how much it would take to lure an employee away from a company now, assuming no retirement benefits were retained. An interest-adjusted difference between the payoff amounts for two successive periods will reveal the part of the benefit the employee implicitly allocates to a given period. This is given by

$$v_j = \frac{\text{Payoff}_j(1 + i)^{z-j} - \text{Payoff}_{j-1}(1 + i)^{z-j+1}}{B_z} \tag{7}$$

where

payoff$_j$ = the amount an employee leaving at the end of period j would accept in lieu of receiving a retirement benefit

v_j = the employee's implicit allocation of the benefit to period j. It's like the f_j shown earlier, but the sum of the v_j's need not equal 1.

i = the employer's investment rate of return.

A complete set of v_j's is the perceived value allocation. Each v_j is the difference between two interest-adjusted payoff amounts, divided by the benefit at exit point z. If the benefit at z, B_z, is a lump sum, then (ignoring tax considerations) payoff$_z$ would equal B_z and the v_j's would sum to 1. However, payoff$_z$ need not always equal B_z.

Finding the payoff amounts would require a good deal of empirical research. However, if we assume an employee at the margin is contemplating outside employment, we can develop the relationship between the v_j's, the personal discount rates, and the characteristics of the retirement benefit in outside employment. We assume the salary and benefits in outside employment are the same as that of current employment. We

[5]Gilman in his 1976 paper actually found the personal discount rates by fitting probit regressions to joiners and nonjoiners in pension plans and getting maximum likelihood estimates for the coefficients of the independent variables, including the personal discount rates.

do not require that the outside retirement plan be identical to that of the current plan. However, we do require that persons retiring at z in outside employment have the same benefit at z, B_z, if they had entered outside employment at time 0.

Let us next define $B^o_{z,j}$ to be the benefit that accumulates in outside employment for a person who leaves current employment after j and stays in the outside employment until z. Then the part of the benefit attributable to potential outside employment during period j is given by

$$f^o_j = \frac{B^o_{z,j-1} - B^o_{z,j}}{B_z}. \tag{8}$$

If j is 1, $B^o_{z,j-1}$ is identical to B_z. If j is z, then $B^o_{z,j}$ is zero. So the f^o_j's sum to 1.

If we know the payoff amounts, we can solve for personal discounts in the following equation, by starting at high values of j and working down. Or knowing the personal discount rates, we can solve for the payoff amounts.

$$\text{Payoff}_j = \frac{K_j(B_z - B^o_{z,j})}{(1 + d_j)^{z-j}} = \frac{K_j\left(\sum_{k=1}^{j} f^o_j\right) B_z}{(1 + d_j)^{z-j}}. \tag{9}$$

where K_j is a scaling factor giving 1 if B_z is a lump sum and otherwise is the ratio of two annuities at the age at z, the numerator evaluated with d_j and the denominator evaluated with i.

Substitute this back into the definition of v_j. We obtain

$$v_j = \frac{K_j\left(\sum_{k=1}^{j} f^o_k\right)B_z(1 + i)^{z-j}}{B_z(1 + d_j)^{z-j}} - \frac{K_{j-1}\left(\sum_{k=1}^{j-1} f^o_k\right)B_z(1 + i)^{z-j+1}}{B_z(1 + d_{j-1})^{z-j+1}}.$$

This can be rearranged as follows:

$$v_j = f^o_j \frac{K_j(1 + i)^{z-j}}{(1 + d_j)^{z-j}} + \left(\sum_{k=1}^{j-1} f^o_k\right)\left[\frac{K_j(1 + i)^{z-j}}{(1 + d_j)^{z-j}} - \frac{K_{j-1}(1 + i)^{z-j+1}}{(1 + d_{j-1})^{z-j+1}}\right]. \tag{10}$$

Equation (10) shows that the v_j has two parts. The first part is the f^o_j discounted by the ratio of $1+i$ over $1+d_j$ and the ratio K_j. The second part is a correction for the sum of earlier f_j^o's, which now must be discounted for one less year with K_j and d_j instead of K_{j-1} and d_{j-1}. Also, if d_j is always equal to i, then $v_j = f_j^o$.

To illustrate, consider the interest-adjusted salary allocation shown in Table 1. This column can be constructed by accumulating the interest-adjusted salary for a person expected to reach exit point z and dividing the accumulated sum from each period by the total accumulation. As a consequence, the numbers in the column will give the f_j^o's for the case in which the outside benefit is a defined-contribution plan. Similarly, the level benefit column in Table 1 would give the f_j^o's when the alternative in outside employment is a final-pay plan that uses a constant length-of-service multiplier. Table 4 shows the v_j's for each of these, assuming a constant market rate of return of 7 percent and the employee discount rates shown. Table 4 also assumes the same salary growth as Table 1.

TABLE 4

TWO PERCEIVED VALUE ALLOCATIONS FOR YOUNG EMPLOYEES EARNING $25,000 (BASED ON INTEREST OF 7% AND OTHER ASSUMPTIONS OF TABLE 1)

Year of Service	Personal Discount Rate (%)	Outside Benefit Is a Final-Pay Plan		Outside Benefit Is a Defined-Contribution Plan	
		f_j^o	v_j	f_j^o	v_j
1	16.3	0.05	0.006	0.032	0.004
2	15.6	0.05	0.009	0.035	0.006
3	14.9	0.05	0.012	0.038	0.009
4	14.3	0.05	0.016	0.041	0.013
5	13.6	0.05	0.022	0.044	0.018
6	13.0	0.05	0.027	0.045	0.023
7	12.4	0.05	0.034	0.047	0.030
8	11.9	0.05	0.039	0.048	0.034
9	11.4	0.05	0.045	0.049	0.041
10	10.9	0.05	0.053	0.050	0.048
11	10.4	0.05	0.060	0.052	0.057
12	10.0	0.05	0.062	0.053	0.060
13	9.6	0.05	0.067	0.054	0.066
14	9.2	0.05	0.072	0.055	0.073
15	8.8	0.05	0.077	0.057	0.079
16	8.4	0.05	0.080	0.058	0.085
17	8.2	0.05	0.067	0.059	0.074
18	7.9	0.05	0.076	0.060	0.084
19	7.6	0.05	0.076	0.061	0.086
20	7.4	0.05	0.068	0.062	0.079
Total		1.00	0.969	1.00	0.969

In both examples in Table 4, the perceived value allocation is more back-loaded than the f_j^o's. Using the discount rates of Table 4, a level-benefit allocation is insufficiently back-loaded, even when the outside benefit is a final-pay plan. This assumes the perceived value allocation is used as an ideal standard. The projected unit credit method using interest-adjusted salary is also insufficiently back-loaded here, although it's closer to the perceived value allocation.

Also, note in Table 4 that the v_j's do not sum to 1. This occurs because d_{20} was equal to 7.4 percent, somewhat higher than the market interest rate of 7.0 percent. As mentioned earlier, the K's would each equal 1, if the benefit were a lump sum. For purposes of determining K_{20} in this example, I assumed the benefit was paid in the form of a 25-year annuity due.

The extent of back-loading in perceived value allocations is particularly sensitive to the personal discount rates. Higher discount rates lead to more back-loading. The discount rates used here are numbers that are not out of the question. They were taken from Gilman's estimate for persons 20 to 39 earning \$25,000 per year, in 1975 dollars. Gilman's estimates are expressed as continuous rates of real interest. To get the rates used here, I converted using the formula

$$1 + d_j = (1.05)e^{r_j}$$

where

r_j = Gilman's estimate of the continuous real interest rate for the age associated with length of service j

d_j = the rate appearing for certain lengths of service in Table 4.

Gilman's results gave r_j's for all ages below 20, for even ages between 20 and 30, and thereafter only for ages evenly divisible by 5. The remaining d_j's were obtained by interpolation.

Gilman also gave discount rates for persons earning \$5,000 and \$15,000 per year. Table 5 was constructed by using the discount rates for persons 20–39 who earned \$5,000 per year.

The v_j's in Table 5 are also back-loaded. But what is most remarkable about Table 4 is how little of the benefit gets allocated by the v_j's. This suggests that the perceived value allocation acts more like a constraint on the employer's allocation. To be consistent with the perceived value allocation, each f_j should equal or exceed the corresponding v_j. The employer might satisfy this by setting the f_j's to v_j's that have been rescaled so they sum to 1.

As I mentioned, the v_j's will equal the f_j^o's when the personal discount rates equal the market interest rates. In this event a level benefit allocation will be the perceived value allocation when the outside benefit is

TABLE 5

TWO PERCEIVED VALUE ALLOCATIONS FOR LOW-AGE, LOW-INCOME EMPLOYEES
(BASED ON INTEREST OF 7% AND OTHER ASSUMPTIONS OF TABLE 1)

Year of Service	Personal Discount Rate (%)	Outside Benefit Is a Final-Pay Plan		Outside Benefit Is a Defined-Contribution Plan	
		f_j^o	v_j	f_j^o	v_j
1	31.8	0.05	0.000	0.032	0.000
2	31.0	0.05	0.001	0.035	0.000
3	30.2	0.05	0.001	0.038	0.001
4	29.5	0.05	0.001	0.041	0.001
5	28.8	0.05	0.002	0.044	0.002
6	28.1	0.05	0.003	0.045	0.003
7	27.4	0.05	0.005	0.047	0.004
8	26.8	0.05	0.006	0.048	0.005
9	26.2	0.05	0.008	0.049	0.007
10	25.6	0.05	0.011	0.050	0.010
11	25.1	0.05	0.014	0.052	0.013
12	24.6	0.05	0.018	0.053	0.017
13	24.2	0.05	0.022	0.054	0.021
14	23.7	0.05	0.028	0.055	0.027
15	23.3	0.05	0.034	0.057	0.033
16	22.8	0.05	0.042	0.058	0.042
17	22.6	0.05	0.046	0.059	0.047
18	22.3	0.05	0.057	0.060	0.058
19	22.0	0.05	0.067	0.061	0.070
20	21.7	0.05	0.078	0.062	0.083
Total		1.00	0.446	1.00	0.446

a constant multiplier final-pay plan. Under the same assumption of i_j equals d_j, projected unit credit using interest-adjusted salary will be the perceived value allocation when the outside benefit is a defined-contribution plan.

Normally, however, there will be a mixture of outside alternatives. This problem can be solved by restating Equation (9) probabilistically:

$$\text{Payoff}_j = \frac{\sum_{l_j} P_{l_j}\left(\sum_{k=1}^{j} f_{kl}^o\right) K_j B_z}{(1 + d_j)^{z-j}} \tag{11}$$

where

l_j indexes all the outside alternatives available at j

Pl_j gives the probability of each outside alternative at j

f_{kl}^o is defined like f_j^o in Equation (8) but for alternative l.

The probability in Equation (11) deserves some discussion. We assume we are allocating the benefits of someone who will leave the current job at exit point z. However, at point j this person has the perceptions of similarly situated persons who may leave and accept employment elsewhere. So Pl_j gives the probability of accepting employment along path l_j at length of service j for a person who temporarily is planning to leave at j. The Pl_j's sum to 1. Note also that the probability of being offered employment may be smaller along the more back-loaded paths. However, the probability of accepting employment, given that it is offered, may be increased for the more back-loaded plans.

Substituting Equation (11) back into Equation (7), we have

$$v_j = \sum_{l_j} P_{l_j} \left[\frac{K_j \sum_{k=1}^{j} f_{kl}^{o}(1+i)^{z-j}}{(1+d_j)^{z-j}} \right] - \sum_{l_{j-1}} P_{l_{j-1}} \left[\frac{K_{j-1} \sum_{k=1}^{j-1} f_{kl}^{o}(1+i)^{z-j+1}}{(1+d_{j-1})^{z-j+1}} \right].$$

This can be rearranged as

$$\begin{aligned} v_j = {} & \left(\sum_{l_j} P_{l_j} f_{jl}^{o} \right) \frac{K_j(1+i)^{z-j}}{(1+d_j)^{z-j}} \\ & + \left[\sum_{l_j} P_{l_j} \left(\sum_{k=1}^{j-1} f_{kl}^{o} \right) \right] \frac{K_j(1+i)^{z-j}}{(1+d_j)^{z-j}} \\ & - \left[\sum_{l_{j-1}} P_{l_{j-1}} \left(\sum_{k=1}^{j-1} f_{kl}^{o} \right) \right] \frac{K_{j-1}(1+i)^{z-j+1}}{(1+d_{j-1})^{z-j+1}} \end{aligned} \tag{12}$$

Equation (12) again has two parts. The first part, in line 1, gives the expected fraction of the benefit (based on outside employment characteristics) for period j, multiplied by a discounting factor. The remainder of Equation (12) is a correction term giving year j's expected amount of the benefit accumulated by the end of $j-1$ and year j's discounting, less year $j-1$'s expected amount of benefit accumulated by the end of $j-1$ and year $j-1$'s discounting.

Table 6 illustrates Equation (12) for middle-aged (35–54), higher-than-average income ($25,000) employees. The P_l's are constant, giving equal probabilities of the outside benefit being a final-pay or a defined-contribution plan. The personal discount rates above age 40 are set to 7 percent. Gilman's results showed lower rates at those ages, but here I'm assuming a 2 percent spread between inflation and the interest rates. To be consistent with this assumption, I have restricted real interest rates to no lower than the difference between 5 percent inflation and 7 percent interest. To use a lower spread here would be comparable to assuming that someone would be willing to borrow at the market rate and loan at a lower rate. In this example, the v_j's sum to 1, because d_{20} is equal to the market interest rate.

Using the perceived value allocation in Table 6 as a standard, a level-benefit allocation would be slightly too front-loaded and an interest-adjusted salary allocation would be slightly too back-loaded.

TABLE 6

PERCEIVED VALUE ALLOCATIONS FOR MIDDLE-AGED EMPLOYEES EARNING $25,000 (BASED ON 7% INTEREST AND OTHER ASSUMPTION OF TABLE 1)

Year of Service	Personal Discount Rate (%)	Outside Benefit Has an Equal Chance of Being a Final-Pay or a Defined-Contribution Plan	
		Weighted f_j^o	v_j
1	8.4	0.041	0.029
2	8.2	0.043	0.033
3	7.9	0.044	0.041
4	7.6	0.046	0.048
5	7.4	0.047	0.051
6	7.1	0.048	0.060
7	7.0	0.049	0.054
8	7.0	0.049	0.049
9	7.0	0.050	0.050
10	7.0	0.050	0.050
11	7.0	0.051	0.051
12	7.0	0.052	0.052
13	7.0	0.052	0.052
14	7.0	0.053	0.053
15	7.0	0.054	0.054
16	7.0	0.054	0.054
17	7.0	0.055	0.055
18	7.0	0.055	0.055
19	7.0	0.056	0.056
20	7.0	0.056	0.056
Total		1.00	1.00

But the big question is whether we should treat the perceived value allocation as a standard. To answer this question, reconsider Equation (7). It is hard to quarrel with the payoff amounts in Equation (7), at least not until we give more details. A payoff amount, in effect, tells us how much an employee has allocated to the past. Also, the interest adjustment and division by B_z in Equation (7) are straightforward. If an employee invested adjacent payoff amounts, the difference between their accumulations at time z as a ratio to B_z yields the v_j's.

A potential problem emerges when we consider the payoff amounts for an employee who is considering employment elsewhere, but who we nevertheless assume will remain until the assumed exit point with the current employer. The hypothetical employee at the margin is different from an employee who has no thought of leaving. But this is generally true in pricing. Employees who are not at the margin are paid more than is necessary to retain them. So it seems reasonable to base our "prices" on this hypothetical person at the margin.

I also see no disadvantages (other than practical ones) in using personal discount rates. If we accept Equation (7), we must ask how it should be evaluated. The use of personal discount rates seems quite reasonable. Gilman's equations were developed by examining the extent of participation in pension plans in which there were matching employer contributions as well as tax advantages. This examination of actual behavior seems a sounder basis than would a survey eliciting respondents' stated payoff amounts under hypothetical circumstances.

Equation (9) makes three further assumptions, however. First, it assumes the current and outside salaries are equal. Second, it assumes the pension benefits at z are equal for current and alternative employment, given entry to either at time 0. Third, it assumes our hypothetical person at the margin evaluates the payoff amount by using the alternative of staying in outside employment until exit point z. While these are strong assumptions, they are not quite so strong for a person we already assume will be staying in current employment until z. If the outside benefits were larger, then the employee should already have left, other things being equal. If the outside benefits were less, then the employee is not at the margin. If the employee is considering outside employment but not staying until z, it still seems reasonable to evaluate the alternative as if the employee would stay until z in outside employment.

But to find a payoff for a hypothetical employee considering outside employment, why not use a hypothetical internal payoff? The internal payoff would remove the accumulated years and salary from the benefit at z, but there would still be a benefit at z based on subsequent years and salary. The difference between the external and internal payoffs is that a person really could leave, while an internal payoff is entirely hypothetical. If a person considers leaving, he or she will consider what is being lost by doing so. This leads back to estimating payoff amounts.

What about the person who considers leaving, when the overall benefits outside equal the internal benefits (in present value terms using market interest rates), but the pension/salary mix is different? Consider, for example, a larger salary but a smaller retirement benefit. I suggest that the extra salary increments be treated as an early retirement benefit. Negative salary differentials could be treated as an early negative pension stream. This generalization would complicate our equations, but would not erode the underlying rationale. A similar comment could be made about employment paths that include periods of unemployment but nevertheless yield the same overall benefit.

There is also a question about how free we are to set the f_j^o's, if we fix B_z and require that the internal and external salaries be equal. Let us define an internal allocation, f_j^I, corresponding to f_j^o and based on the internal benefit formula. Next, let us define g_j^I's to correspond to the f_j^I's but based on salaries to date, rather than projected salaries. For a career-average plan, the g_j^I's would equal the f_j^I's, but for a final-pay plan, they would not. Now we may ask, What happens if

$$\sum_{k=1}^{j} g_k^I > \sum_{k=1}^{j} f_k^o?$$

This would imply that an employee could leave current employment at j and get a total benefit at z larger than B_z based on the current employment until j and outside employment after j. For such an employee, the perceived value would be more likely to be based on the sum of the g_j^I's than on the sum of the f_j^o's. So we should require that

$$\sum_{k=1}^{j} g_k^I \leq \sum_{k=1}^{j} f_k^o. \tag{13}$$

Where the current employment has a career-average plan, this is not a difficult requirement. Because of the back-loading inherent in career-average plans, the sum of the early g_j^I's will be small. For a final-pay plan, this is normally not an imposing constraint, since the early g_j^I's will be less than the f_j^I's. In any case Inequality (13) requires that we treat outside alternatives where the inequality is not met in a special way, namely, to substitute the g_j^l's for the f_j^o's in Equation (11) for those lengths of service not meeting the inequality.[6]

But then why not have a constraint that prevents someone from leaving to get a smaller total benefit? In other words, why not make Inequality (13) an equality? Well, that would discard one of the advantages of perceived value allocations. During the early years of employment, an employee may feel he or she has earned not only benefits dictated by the formula, but also some share of later benefits, since the later years' benefits are not possible without the earlier ones. The question is, How much? Equations (9) and (11) answer the question by asking how much an employee would have to be paid off to jump into alternative employment at mid-career.

There are two theoretical disadvantages of the perceived value allocation, however. First, the reader may have noticed a shift in the meaning of "payoff" from Equation (7) to Equation (9). In Equation (7), it isn't clear what "payoff" means. In Equation (9) we operationally define "payoff," but in a way that may not entirely correspond to the employees' feelings on the matter. For example, if we computed the Equation (9) payoffs when the current employment had a defined-contribution plan, the payoff need not correspond to the accumulated investment value. The employee may see the accumulated investment value (less discounts based on the fact that the employee cannot immediately get the money) as closer to what the person thinks he or she has. Second, the allocation depends on the mix of outside alternatives. As a consequence, the "perceived value" can change as the mix varies. While the resulting allocation has these two theoretical weaknesses, there is still likely to be an overlap between the payoff amounts under Equations (7) and (9). It is this overlap that provides the justification for perceived value allocations.

The perceived value allocation is designed to answer this question: "If a particular retirement system were eliminated, with what cash outlays could the employer replace it so that an employee does not feel a gain or a loss?" For the person who really is at the margin and is really considering outside employment until z, a perceived value allocation approximates this ideal. For others, it seems reasonable to base "prices" on this hypothetical person at the margin.

[6]When no outside alternative satisfies the inequality, then the g_j^l's are all that are needed for that length of service.

There are, however, some practical problems. The parameters needed for a perceived value allocation are substantial. Personal discount rates are needed for all ages and income levels as well as for other characteristics that may be important. In addition, the mixture of benefits available in outside employment and the probabilities associated with each must be known. Because all these can change, the number of parameters necessary becomes unwieldy.

Because of this parameter estimation problem, perceived value allocations cannot be used routinely. I suggest that they be used only in a broad way to determine how to allocate benefits. In particular, Tables 4 and 5 suggest that something more back-loaded than a level benefit allocation is appropriate for young or relatively poor employees. While a level benefit allocation is appropriate for older, somewhat wealthier groups when the outside benefit alternative is a final-pay plan, this represents one extreme. In other cases, something more back-loaded is more appropriate.

NEGATIVE OR SMALL ACCRUALS RECONSIDERED

In the section on negative accruals, there is an apparent contradiction between my advocacy of the projected unit credit method with multiple exit points and my advocacy of switching to the traditional unit credit method when negative or small accruals appear for long-tenure employees. How do perceived values help resolve this seeming contradiction?

Suppose a person is retirement-eligible and earning no additional benefit under the formula. Then this person should not accept a payoff that is less than what he or she would get by leaving. So let's expand Equation (9) as follows

$$\text{Payoff}_j = \text{Max}\left[\frac{K_{z,j}(B_z - B^o_{z,j})}{(1 + d_j)^{z-j}}, K_{j,j}B_j\right]$$

where

$K_{z,j}$ is the same as our earlier K_j, the ratio of two annuities at z, the numerator evaluated using the personal discount rate at j and the denominator evaluated using the market interest rate

$K_{j,j}$ is defined similarly but for annuities beginning at j.

To simplify the argument, assume the employer wants to rescale the v_j's so that they sum to 1. In this case, the K's would equal 1. When the B_j's begin to decline, B_j will be larger than the discounted $B_z - B^o_{z,j}$.

So the payoff is simply B_j, and the difference between two successive interest-adjusted payoff amounts is

$$B_{j+1} - B_j(1 + i).$$

But this is the normal cost under the traditional unit credit method, without the mortality and continued service factors.[7] So switching to unit credit at some point more closely follows the perceived value of the benefit to an employee.

Most fundamental is to allocate to the past the amount needed to lure an employee away from a company now, assuming no pension benefits are preserved. If an employee has reached retirement eligibility, the amount allocated to the past should not be less than the full value of the benefit. It could, however, be more in the case of a plan that is back-loaded by virtue of the formula, by virtue of late career salary growth, or by an interaction of the two.

So the switch to unit credit occurs when two tests have been met. First, the employee must be retirement-eligible. Second, the present value of future normal cost contribution under the traditional unit credit method must have reached the point at which it is less than the present value of future normal cost under the projected unit credit method. Not switching to unit credit at this point implicitly allocates an insufficient part of the benefit to the past.

This can lead to negative normal costs. Some may question whether these should be permitted. An alternative would be to pay no normal cost at that point and later recognize a gain. However, ignoring any regulatory constraints, it seems better to recognize now what will occur if all our assumptions are met. That would imply that it is better to recognize a negative normal cost.

USE OF CURRENT EMPLOYMENT BENEFIT FORMULA

The preceding sections have already presented the main ideas of this paper. However, an interesting related issue is the use of current employment benefit formulas for allocating the benefit to individual years of service. Is this use of the benefit formula here deceptive?

[7]For perceived value allocations, we allocate the benefit for persons expected to reach z, so the mortality and continued service factors are present indirectly.

The chief reason for ignoring the characteristics of the plan is that they are largely irrelevant.[8] We are already allocating one exit point at a time and using the benefit formula for the benefit at each exit point. This approach takes into account the probabilities of leaving at each exit point. When exit points are combined, some other allocation could reemerge, but that would depend on the probability of leaving at each exit point and on the benefit at each exit point. A similar view on combining separate exit points was expressed by Atteridg et al. [2],[9] who suggested that we "allocate equal portions of the benefit associated with each exit age to the employee's period of service up to that age." It is possible that, when using a single assumed or typical exit point, the benefit formula may be a good proxy for a reasonable accumulation under multiple exit points. However, this would have to be demonstrated.

A second reason for not using the benefit formulas within exit points goes back to perceived value allocations. The employee's perception of the benefit accumulation need not correspond to that of the benefit formula. In the notation of the perceived value section, the v_j's need not equal the g_j^{I}'s. For an employer, the employee's perceptions become important when trying to find a cash value for the motivating influence of the pension plan.

Another approach to the same question is to ask, What would happen in a divorce pension case? Consider the first column of Table 1. Because this allocation is based on salary unadjusted by inflation, its allocation is the same as the benefit formula under a career-average plan. But could you convince a judge that only 1.5 percent of the benefit was earned in the first year, as shown by Table 1? (An ex-spouse married to someone for the last 19 years of a 20-year career is someone who could benefit from such an allocation.)

The superficial answer is that the benefit formula dictates this. The peculiar result is caused by the peculiar retirement benefit. We should not blame the accounting procedure for the benefit design.

The counterargument is that during the first year the retiree earned not only a share of the first-year's benefit, as dictated by the benefit formula,

[8]It is, of course, relevant for determining the benefit at each exit point and for satisfying Inequality (13), as discussed under perceived value allocations.

[9]Atteridg et al. were not, however, recommending that any consideration be given to perceived values or that the allocation take into account anything but service time.

but also a share of subsequent years' benefits, since a second year would not be possible without a first year.

Suppose we have cliff vesting at 20 years? Surely we wouldn't suppose that spouses who precede the spouse at the time of vesting have no rightful share? Similarly, we can see strict application of the benefit formula as leading to a series of small cliffs, or steps. We should, I think, give ex-spouses some share of those later steps, which would not be possible without the earlier steps.

But the benefit allocation for an employer should not be all that different from that used in divorce. Perhaps a divorce court should go further than an employer in apportioning part of the benefit to lean years during which a business or skill was being developed. For a divorce court, perceived values may not be relevant. But both the divorce court and the employer should ideally consider exit points one at a time. Once we assume a particular exit point, the operation of the benefit formula plays only a small role.

DISCUSSION

Perceived value allocations provide a justifiable basis for allocating the benefit at an assumed exit point. As discussed earlier, a switch to unit credit may be needed for high-tenure employees, but in that case unit credit is consistent with the underlying rationale for perceived value allocations.

Because of the difficulties in carrying out a perceived value allocation, however, I doubt whether this method will be used except in the rarest of circumstances. We may, however, work backwards. We may find a perceived value allocation that is appropriate for a comparable situation. Then we may find some other allocation that is reasonably consistent with the perceived value allocation in terms of back-loading.

An allocation based on inflation-adjusted dollars has some appeal in this respect. While the back-loading for this method seemed high in Table 1, it is really quite reasonable in view of the perceived value allocations shown in Tables 4 and 5.

An allocation based on interest-adjusted salary was insufficiently back-loaded at times, but did a reasonably good job for the examples given in Tables 4, 5, and 6. Its back-loading is perfect when the outside alternative is a defined-contribution plan and the personal discount rates equal the market rates. An added selling point is the funding philosophy.

All who are expected to reach a particular exit point by a particular path pay the same percentage of salary. While this method is more complicated than the level-benefit method, few additional inputs would be required once we are dealing with packaged software. In most situations this method would not be legally permitted; however, we hope that this could change.

On the other hand, a level allocation has the advantage of simplicity and a longer history. It did a reasonably good job for the example given in Table 6. Its back-loading is perfect when the outside benefit is a final-pay plan with a constant length-of-service multiplier and the personal discount rates equal the market interest rates. In addition, a simplified node-by-node approach can be used for network allocations.

SUMMARY

1. Perceived value allocations can be developed by using personal discount rates and the characteristics of alternative pension plans in outside employment. This results in a justifiable basis for allocating pension benefits at an assumed exit point. Such allocations may be impractical because of their unwieldy parameter estimation requirements, but they may be used in a crude way to evaluate other allocations. They do not always allocate all the benefit, but in that case they give a minimum percentage of the benefit to be allocated to various lengths of service.
2. The projected unit credit method with the benefits allocated by interest-adjusted salary has a particularly appealing funding philosophy. A constant percentage of salary is applied to all persons expected to retire at a particular exit point. In terms of back-loading, it does a reasonably good job in many situations. Its allocation is ideal when the outside benefit alternative is a defined-contribution plan and personal discount rates are identical with market interest rates.
3. The projected unit credit method with the benefits allocated by inflation-adjusted salary also has a somewhat appealing funding philosophy. In terms of back-loading, it seems superior to the interest-adjusted salary allocation for young or low-income employees.
4. The projected unit credit method with the benefits allocated equally to all lengths of service has a straightforward funding philosophy. It gives a reasonably good allocation for employees who are not too

young or poor. Its allocation is ideal when the outside benefit alternative is a constant-multiplier final-pay plan and personal discount rates are identical with market interest rates. In addition, a simplified network allocation is possible.

5. Each of the four projected unit credit methods discussed here allocates one exit point at a time. The allocations must be combined to get a single normal cost for each length of service. However, a single percentage of pay, a weighted normal cost, can be developed for different cost centers.
6. When there are transfers between cost centers, a method is available to allocate benefits by using the same funding philosophy.
7. Most funding methods will not show negative accruals for employees at high lengths of service. Switching to unit credit at high lengths of service makes it possible to recognize negative or small accruals.
8. There are good reasons for not using the current employment benefit formula for allocations within exit points.

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DISCUSSION OF PRECEDING PAPER

INGER M. PETTYGROVE:

I found Mr. Giesecke's paper both interesting and informative. In reading it, however, a question occurred to me. The section on network allocations mentions that shortcut methods would be needed if projected unit credit with interest-adjusted salary were used. Does the author have any suggestions along these lines? I look forward to reading the response.

KENNETH A. STEINER:

Mr. Giesecke has written an interesting paper that illustrates his point that "there are as many ways to fund a retirement benefit as there are to finance a house." I agree with his statement that "to say that one is wrong is analogous to saying that a house cannot be financed with different payment schedules." However, I do believe that criteria do exist for selecting whether one actuarial cost method may be better than another for meeting a plan sponsor's funding objectives for a particular plan. For example, one criterion I use is whether the method produces a reasonable relationship between the plan's actuarial accrued liability (or "asset target") and the plan's actuarial present value of accrued benefits when consistent assumptions are used to develop both values. In the discussion that follows, I refer to the ratio of these two items as the "asset target ratio."

Assuming the plan sponsor's funding objective is to accumulate sufficient plan assets to cover plan termination liability at all times with reasonable certainty but without overfunding the plan, a reasonable asset target ratio might be 1.25–1.75. A narrower reasonable asset target ratio range for a plan sponsor with this objective could be developed by examining such factors as the relationship between the ongoing plan investment return assumption and the interest rate used to determine plan termination liability, volatility of plan assets and liabilities, plan demographics, and so on. If an actuarial cost method does not produce a reasonable asset target ratio, it is probably a good time for the actuary to initiate discussions with the plan sponsor about whether the sponsor's funding objectives can be better accomplished by using a different actuarial cost method.

Given current Internal Revenue Service regulations and Financial Accounting Standards Board interpretations, Mr. Giesecke's inflation-adjusted and interest-adjusted methods appear to have limited potential application in the U.S. (funding for final average pay plans not governed by Section 412 of ERISA and possibly career average plans). I do believe that his cost methods can, under certain circumstances, produce reasonable asset target ratios. However, since his methods fail to consider the actual pattern of benefit accruals, they are much more likely to produce unreasonable asset target ratios for specific situations than the projected unit credit methods currently anticipated by the Internal Revenue Service and the Financial Accounting Standards Board.

(AUTHOR'S REVIEW OF DISCUSSIONS)

GERALD LEE GIESECKE:

I thank the referees and Ms. Pettygrove and Mr. Steiner for their helpful comments.

Inger Pettygrove

Ms. Pettygrove asks about shortcut methods for network allocations under the projected unit credit cost method with interest-adjusted salary. One shortcut would be to do something similar to what I suggested for level benefit network allocations. This essentially divides the benefit at each node into two pieces. Part is allocated to the segment that precedes the node. The rest is passed down to the feeding node where that segment originates.

In a level benefit allocation, the fraction of the benefit $1/n$ is allocated to the feeding segment, while $(n-1)/n$ is passed down the feeding node. The benefit for reaching the feeding node equals the passed-down benefit from the connecting next higher nodes, with each passed-down benefit reduced by interest and multiplied by segment probability.

The same procedure can be applied to the projected unit credit cost method with the benefit apportioned by interest-adjusted salary. In place of 1 and $(n-1)$, we have the interest-adjusted salary of the preceding segment versus the expected interest-adjusted salary accumulated at the feeding node.

As an example of the shortcut method, consider Figure 1 in the paper. For a person on segment H, we would pay:

$$\frac{20{,}000 \times 110{,}000}{110{,}000 + 1.07 \times \{[80/(80 + 30)] \times 100{,}000 + [30/(80 + 30)] \times 50{,}000\}}$$

The numerator is the amount needed (20,000) multiplied by segment H pay (110,000), which is the numerator of the apportioning fraction. The 110,000 is not multiplied by an interest adjustment, since it is paid at the end of the year. The denominator is the 110,000 plus the expected accumulated interest-adjusted salary from node 4. The shortcut method gives a normal cost of $10,869.08 for segment H. This differs from the weighted all-path normal cost of segment H, which equals $11,043.

As a variant of the shortcut method, certain mainstream paths could be separated and calculated independently. This may be overkill, but the result should more closely approximate the all-path method. With a level benefit allocation, the shortcut method would equal the all-path method, so isolating certain mainstream paths would make no difference.

Kenneth Steiner

Mr. Steiner suggests we use any latitude we have in selecting the actuarial cost method to get a desirable "asset target ratio." This seems like a reasonable goal to me; however, it was not my goal.

My goal was to develop a scheme that makes pricing sense. In allocating pension cost to years of service, we change the price for using an employee's time. This can affect decisions on whether we have more generals or more privates as well as on a host of other tradeoffs. One can of course use a funding method that tries to isolate pension costs from such considerations. But if you let pension costs influence personnel decisions, some actuarial cost methods make more sense than others.

By looking at perceived value allocations, I have tried to formalize the notion that pension benefits act as more of an incentive to older workers than they do to younger workers. This suggests that our accounting be more back-loaded so that these incentives are allocated just as they are perceived. More work is needed, however, since I assumed the preferences of a hypothetical employee who was at the margin between leaving and staying. Where this is not true, the right side of Equation (9) in the paper would no longer hold. Equation (7)—which still holds—might then suggest a more front-loaded benefit, particularly if the salary, pension, and other benefits available in outside employment are less than those available in current employment.

Perceived value allocations still leave considerable latitude in allocating pension costs to years of service, in part because the perceived value allocation does not always allocate all the benefit, in part because the perceived value allocation can allocate some of the benefit to the time of entry, and in part because we do not know the details of the perceived value allocation. I have advocated an interest-adjusted salary allocation as a reasonably good proxy for perceived value allocations in many—but not all—situations. I am also tempted to use an interest-adjusted salary allocation to distribute any residual, since it proportionately raises the costs of all persons expected to leave at a particular exit point by a particular path. But this point needs further study. So for the time being, at least, I agree that there should be considerable latitude in how we allocate the pension benefit.