SOCIETY OF ACTUARIES

Article from:

Risk Management

July 2006 – Issue 8

GMWB and the Four Actuaries

by Voltaire

Once upon a time there were four actuaries—the Cash-aware actuary, the Surplus-aware actuary, the Volatility-aware actuary and the Market-aware actuary. One morning at dawn, they all set out to take a walk through the wild forest of insurance products. As they shuffled along and looked around at the many variations of financial services products and riders that were wandering by, they stopped paying attention to where they were walking and became separated, randomly diffused in the forest.

After a while, the Cash-aware actuary came upon a guaranteed minimum withdrawal benefit (GMWB) rider that was lost in the forest looking for its proper premium. The Cash-aware actuary said, "Don't worry young rider. Being the new kid on the block, I will find your proper premium." The Cash-aware actuary priced the snappy GMWB rider reflecting expected benefits and expenses, discounted at the universal discount rate of 15 percent, and told the GMWB rider that his proper premium is $12.

Later that morning, in another part of the wild forest of insurance products, the Surplus-aware actuary was looking for the best-estimate way to get back to the corporate office when she also came upon the GMWB rider. "How do you like my premium?" asked the GMWB rider. The Surplus-aware actuary took a careful look at the GMWB rider and said, "Hmm. This benefit needs to be stochastically tested. Will that hurt?" queried the GMWB rider. "Only if you don't hold still," responded the Surplus-aware actuary. The Surplus-aware actuary then simulated expected benefits and expenses along 10,000 paths, examined various levels of conditional-tail expectation, and said to the GMWB rider: "My, my, you need to hold capital that is four times the premium that you showed me. Given that there is no such thing as a free lunch, there is going to have to be an increase here to pay for that." "Oh, no. I won't be marketable," said the GMWB rider. "It's not that bad," responded the Surplus-aware actuary. "Your premium only needs to increase by about a third to pay for the carrying cost of your capital." So the premium was increased to $16, and the GMWB rider scurried away off in search of his market.

All through the morning, the Volatility-aware actuary tried to build a model of the wild forest that should show him the way back. However, that rascal heteroskedasticity along with her side-kick multicollinearity in a multiple regression context were starting to extract their toll. The Volatility-aware actuary, having wrestled enough with the muscle bound polynomials, finally decided to take a break for lunch and started to conduct a non-linear search for the subsidized cafeteria when he came across the GMWB rider. The GMWB rider was afraid to show his premium to this stranger who kept talking about some mythical person named Vega. However, the rider's outgoing marketing personality got the better of him.

"Look at the shiny new premium that two nice actuaries gave me," said the GMWB rider. The Volatility-aware actuary, upon a quick look, said, "that looks like the right premium for an average-type insurance product—mind if you open your kimono and let me take a look at your volatility?" "My mother told me never to let a stranger look at my volatility," said the GMWB rider. "Do I look strange to you?" replied the Volatility-aware actuary. "Why I

look at volatility all of the time. There is nothing that you are going to show me that I haven't seen before."

The GMWB rider did think that the Volatility-aware actuary looked a little strange, but he seemed to need to look at his volatility very badly. So, the GMWB rider agreed to put his volatility on the table. The Volatility-aware actuary, upon inspection, quickly found there was no charge for volatility in determining the premium and that the projected ROE indeed averaged 15 percent.

However, the standard deviation of the quarterly ROE was 5 percent. "How does my volatility look to you?" asked the GMWB rider. "A material deficiency" answered the Volatility-aware actuary. "The average benefit for other insurance products have half as much volatility as you do." "Is that good?" asked the GMWB rider defensively. "Not very," replied the Volatility-aware actuary. "I will have to risk-adjust your discount rate to about 25 percent. That means your premium will have to double to about $32." The GMWB rider was floored. "But how will I ever face the other benefits and riders? They are all priced at the universal one-size-fits-all discount rate of 15 percent" cried the GMWB rider. "Well, there is another way," replied the Volatility-aware actuary. "I could calculate your premium using the 75th percentile cost instead of the mean cost. It will get us to the same place." The GMWB rider was dazed, confused and very upset with both answers. He was now most certain that he would not look attractive to the market.

The GMWB rider turned and ran away without even stopping to thank the Volatility-aware actuary for correcting his premium. As he made a left turn to avoid running smack into a term rider, he instead tripped over the Market-aware actuary who was looking for signs of where everyone else had gone . Why? So that she could follow them! The Market-aware actuary had found that if she followed the path that most everyone else had taken before, she would always get … where everyone else was going! Anyhow, the GMWB rider, being so flustered and upset, blurted out rudely: "Look at what those actuaries did to my premium! I will never sell." The Market-aware actuary took a long and careful look at the GMBD rider and finally said: "You are very different from anything that I have seen traded in the financial markets. To replicate you, I would have to devise a complicated dynamic hedge embedded within a well defined hedging strategy." The GMWB rider just starred at the Market-aware actuary. No one had ever said anything like that in his presence before. "What would you do with the dynamic hedge and why would you want to replicate me?" asked the GMWB rider.

"Why everything needs to be replicated," said the Market-aware actuary. "How else would you know what it you are worth!" The GMWB benefit didn't take that for an answer and asked "then would you have to hedge me?" The Market-aware actuary grinned and said: "No, silly benefit. You do not have to execute the hedge. But knowing how much the hedge costs, tells you how much your benefit should cost. You can test your premium for market consistency against thousands of trades that make up the financial markets."

The GMWB rider thought that was a good idea, so he asked the Market-aware actuary to build the replicating hedge. The Market-aware actuary pulled out her portable Bloomberg terminal, tapped into the forest's enterprise computing grid and got to work. After a while, she woke the GMWB rider and told him: "I've got it. I couldn't find any single set of hedges that will do, but if you maintain this set of delta, gamma, vega and primrose hedges, then you will cost $30 if the market stays just like it is today." The GMWB rider started to cry. "Why that is almost the same as what the Volatility-aware actuary said." "Come back tomorrow" said the Market-aware actuary "and I will tell you what your replicating hedge cost is on that day." So off ran the GMWB rider even more perplexed than ever.

continued on page 26 ◗

" You do not have to execute the hedge. But knowing how much the hedge costs, tells you how much your benefit should cost. "

GMWB and the Four Actuaries

◗ continued from page 25

Just before dinner-time, the four actuaries encountered an extreme-tail event and all haphazardly and coincidentally managed to wander into the same small glen at the same time. They all cordially greeted each other and began to all talk at once about their adventures. It soon became apparent that they had all met the GMWB rider and given him a price.

They started to argue about who had given the best price. "But you cannot sell the GMWB rider at a price higher than mine" said the Cash-aware actuary. "But your company will notice that things are amiss as soon as the new C-3 Phase II RBC requirements come into effect and your ROE will stink to high heaven" said the Surplus-aware actuary. "Before too long, you will have other problems" said the Volatility-aware actuary. "Your company's earnings will have so much volatility that the stock price will take a big hit." "And worst of all, if your company wants to lay-off the risk, they will find that there is not enough premium there to pay for a hedge" says the Market-aware actuary. Their discussion went on and on in a recursive loop format until the argument almost turned into an open brawl.

Suddenly, the four actuaries were surrounded by the GMWB rider and many other bells and whistles that were all looking for their premiums. "Please, please, give us premiums," cried the benefits in high shrill voices. "Okay, come with me," said the Cash-aware actuary. Most of the benefits followed along eagerly, knowing that they would quickly sell with the premiums that the Cash-aware actuary would give them. "Who is coming with me?" asked the Surplus-aware actuary. Almost all of the rest of the benefits followed him home.

There was only one benefit left—the GMWB rider. He said: "I cannot decide with whom to go with. You are both so close together." "Well" said the Volatility-aware actuary to the Market-aware actuary. "Maybe we should we should work together. Our GMWB rider will probably sell after the others burn out. We can both calculate the premiums." "Agreed" said the Market-aware actuary. "And whenever we disagree, we can put our heads together. We might find that it is a market opportunity or maybe it is a time when your model needs recalibrating." The GMWB followed along unnoticed as the two actuaries went on and on about the benefits of market consistent pricing and the calibration of stochastic scenario generators.

And at the end of the day, they all got what they deserved! ✦