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STATUTORY RESERVES FOR NONLEVEL-PREMIUM POLICIES

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ABSTRACT

Currently, two methods for apportioning net premiums over the life of the policy are used for statutory reserves on nonlevel-premium life insurance policies: The unitary method sets net premiums equal to a constant percentage of the gross premiums, and the term method determines level net premiums for each period for which the gross premiums are level. This paper reviews both methods and, upon finding shortcomings, recommends a new method, the unified method, as a method conforming more to the principles of the Standard Valuation Law and accepted actuarial principles.

INTRODUCTION

To determine the reserve for a fixed-premium fixed-benefit life insurance policy by using single decrement net premium methods, the actuary must make at least five choices: mortality table, interest rate, type of functions, initial expense allowance, and method of apportioning net premiums. The mortality and interest rate bases and initial expense allowance are the most prominent features of the reserve valuation, have been well discussed in the literature, and are subject to well-defined minimum standards [4]. The choice of curtate, continuous or semicontinuous functions, though interesting and complex, generally does not materially affect the overall level or pattern of the reserves. This paper focuses on the last choice, the method of apportioning net premiums. For many life policies with nonlevel premiums or death benefits, the method of apportioning the net premiums is critical, dramatically affecting both the level and the pattern of the reserves. The question is then, Which method of apportioning net premiums is consistent with the Standard Valuation Law (SVL) and accepted accounting and actuarial principles? Three principles are used in evaluating the proposed methods. First, from the SVL, principle 1: Net premiums should be a uniform percentage of the gross premiums [6]. (Although the SVL allows a first-year expense allowance, this does not materially alter the principle.) Expressed another way, loadings should be level as a percentage of premium over the life of the policy. This principle is essential to the net premium method. Without this or a similar principle, net premiums could be manipulated to produce very high loadings in the early years and low or nonexistent loadings in the later years, resulting in profits in the early durations and no margin for expenses or experience fluctuations in the later durations (other than those margins implicit in the reserve basis).

Note that although the SVL requires net premiums to be a constant percentage of the gross premiums for level-death-benefit, level-premium policies, this is not an absolute requirement for nonlevel-premium, nonlevel-death-benefit policies. The SVL only directs that they be valued by using a method consistent with principles used in valuing level-premium, level-death-benefit policies [6]. This consistency requires that any method for nonlevel-premium policies when applied to a level-premium policy should produce net premiums that are a constant percentage of the gross premiums.

Also from the SVL is principle 2: Reserves should not be negative. This is implied by the SVL in the phrase "reserves ... shall be the excess, *if any*, ..." of the present value of future benefits over the present value of future net premiums [emphasis added] [6]. This is in accordance with statutory accounting conservatism. A negative policy reserve implies that accumulated net premiums to that point have been less than the accumulated benefits, and hence the company is relying on future net premiums to cover not only future benefits, but also some past benefits as well (that is, post-funding is occurring). In a sense, this amounts to holding questionable assets in the form of future premiums that are not discounted for the probability of lapse.

Principle 2 has been extended to prevent holding of reserves less than the cash surrender value of the policy. Exhibit 8 of the NAIC annual statement blank includes a miscellaneous reserve for "surrender values in excess of reserves otherwise required and carried in this schedule."

Principle 3 arises from conservative accounting principles: A given reserve is not sufficient if projected future profits are negative. *SFAS 60* explicitly provides that GAAP reserves shall be increased if future losses are expected [2]. The NAIC annual statement requires an attached actuarial opinion that the statement reserves make "good and sufficient provision for all unmatured obligations" of the company. Certainly this cannot be done if future losses are expected. Extension of this principle would provide that a reserve *method* is not proper if it is reasonably expected to produce losses in any given year (other than the year of issue). The method should produce higher reserves before the year of expected loss or lower reserves after the year of expected loss, thus releasing more reserves during that year. Support for this can be found in Canadian Statutory practice, in which the Canadian Institute of Actuaries' "Recommendations for Insurance Company Financial Reporting" [1] states

"The assumptions [for valuation] are not reasonable to the extent that projected profits: a: are negative for any year (other than at issue), ... or c: are unstable year-by-year."

A consequence of Principle 3 is the notion of deficiency reserves. If the net premium in a given year is greater than the gross premium (that is, if a "premium deficiency" exists), then a loss would be expected in that year, if experience matches the valuation assumptions. Principle 3 requires that the present value of this loss be reserved in all prior years and be released to offset the premium deficiency when it occurs. For computational purposes, this can be accomplished by limiting the net premiums to the gross premiums, as long as the deficiency reserves are calculated on the same basis as the base reserves.

The rest of this paper analyzes, in light of the above principles, three methods that have been proposed for apportioning net premiums for nonlevel-premium policies. The examples presented assume curtate functions, mean reserves, no expense allowances, and the Commissioners 1980 Standard Ordinary age nearest birthday mortality table for male nonsmokers, with select factors and 5.5 percent interest for both basic reserve and deficiency reserve calculations.

THE UNITARY METHOD

This method is perhaps the easiest to understand and apply. Let G_t equal the gross premium paid at time t. If a level death benefit of 1 is payable at the end of the year of death should this occur before the end of the n year coverage period, then the present value of future benefits is $A'_{x:\overline{n}|}$. The present value of future gross premiums is

$$\sum_{t=0}^{n-1} G_t \times {}_tE_x.$$

If k is defined as

$$\frac{A'_{x:\overline{n}|}}{\sum_{t=0}^{n-1} G_t \times {}_tE_x},$$

then k is the ratio of the present value of benefits to the present value of gross premiums, and net premiums, P_t, can be set equal to $k \times G_t$. If k is greater than 1, P_t is set equal to G_t. Terminal reserves at time t are computed as

$${}_tV_x = A'_{x+t:\overline{n-t}|} - \sum_{j=0}^{n-t-1} (P_{t+j} \times {}_jE_{x+t}).$$

Finally, mean reserves for the policy year beginning at time $t({}_tMV)$ are the greatest of

$$\frac{{}_tV_x + {}_{t+1}V_x + P_t}{2} \tag{1}$$

$$\frac{c_{x+t}}{2} \tag{2}$$

or

$$\frac{{}_tCV_x + {}_{t+1}CV_x + P_t}{2}\,. \tag{3}$$

Expression (1) usually applies in the normal situation in which both ${}_tV_x$ and ${}_{t+1}V_x$ are greater than the respective cash values and is derived as an approximation to the exact reserve by assuming that issues, on average, are halfway between valuation dates. Expression (2) usually applies when the terminal reserves are negative. Expression (3) applies when the terminal reserves are less than the terminal cash values. The effect of Expressions (2) and (3) is to put up extra reserves over the amount determined using the net premiums so that principle 2 is not violated.

Application of the unitary method to a nonlevel-premium policy is illustrated in Table 1. The policy illustrated is a fairly typical ten-year select and ultimate re-entry term policy. For the first ten years, the insured is charged premiums based upon issue age and duration. At the end of ten years, if the insured does not produce evidence of insurability, premiums are based on attained age only. Due to the antiselection expected when the healthy lives re-enter, these attained-age premiums are typically much higher than the select premiums. One attraction of applying the unitary method to this type of policy is that by raising the ultimate premium scale enough, thereby lowering k below 1, no deficiency reserves need be held, effectively allowing early gross premiums well below the tabular cost of insurance.

A useful tool for analyzing the reserves produced for this policy by the unitary method is the concept of the implied net premiums. The implied net premium is the premium required to fund the increase in reserves and pay expected claims. This implied premium is derived by beginning with the equations

$${}_tMV = \frac{{}_tV + {}_{t+1}V + P_t}{2} \tag{4}$$

$$_{t+1}V = \frac{_tV + P_t - q_{x+t} \times v}{v \times p_{x+t}}. \tag{5}$$

By substituting the value of $_{t+1}V$ from Equation (4) into Equation (5) and solving for P_t,

$$P_t = \frac{2 \times {}_tMV + q_{x+t} / p_{x+t}}{1 + \dfrac{1}{v \times p_{x+t}}} - {}_tV. \tag{6}$$

Given a set of mean reserves ($_tMV$) and q_{x+t}, Equations (4), (5) and (6) allow the determination of P_t (the implied net premiums) and $_tV$ (the implied terminal reserves). Setting $_0V$ to zero allows the computation of P_0 via the use of Equation (6). Using Equation (4) allows for the computation of $_1V$:

$$_1V = 2 \times {}_0MV - {}_0V - {}_0P.$$

This process can be continued until all the $_tV$ and $_tP$ are determined.

Before implied net premiums are used to analyze the unitary method reserves in Table 1, a few properties should be noted. First, in the absence of cash values, if the beginning-of-year terminal reserve is positive and the end-of-year terminal reserve is negative, the implied net premium is greater than the tabular net premium. Second, also in the absence of cash values, if the beginning-of-year terminal reserve is negative and the end-of-year terminal reserve is positive, the implied net premium will be less than the corresponding tabular net premium. Third, assuming experience matches the valuation assumptions and ignoring lapses and expenses, the implied net premium predicts when losses and gains will occur. If the premium needed to maintain reserves and pay claims (the implied net premium) is greater than the gross premium, losses will occur; otherwise, gains will occur. This property will be used throughout the paper.

These properties of implied net premiums are well illustrated in Table 1. First, in year 3, the beginning-of-year terminal reserve is positive; the end-of-year terminal reserve is negative; and the implied net premium is 5.813, which is greater than the tabular net premium of 4.534 and also greater than the gross premium of 5.300. Second, in year 32, the beginning-of-year terminal reserve is negative; the end-of-year terminal reserve is positive; and the implied net premium, 159.756, is less than the tabular net premium of 163.983. Note that between years 3 and 32, there is no fixed relation between the implied net premium and the tabular net premium. Third, in years 3 to 10, the implied net premium is greater than the gross premium, leading us

TABLE 1

COMPARISON OF THE IMPLIED NET PREMIUMS TO THE TABULAR NET PREMIUMS UNDER THE UNITARY METHOD

Male, issue age 55
$1,000 face amount, term to age 100, no cash values
1980 CSO, male nonsmoker, with select factors, 5.5% interest

Time	[1] Gross Premium	[2] Tabular Net/Gross Ratio	[3] Tabular Net Premium	[4] Tabular Terminal Reserve	[5] Tabular Mean Reserve	[6] Total Reserve Held	[7] Extra Reserve Held	[8] Implied Net Premium	[9] Implied Net/Gross Ratio	[10] Implied Expected Loss	[11] *PV* of Expected Loss
0	5.300	0.855	4.534	0.000	2.470	2.470	0.000	4.534	0.855	0.000	24.828
1	5.300	0.855	4.534	0.406	2.487	2.487	0.000	4.534	0.855	0.000	26.319
2	5.300	0.855	4.534	0.033	1.605	2.923	1.319	5.813	1.097	0.513	27.397
3	5.300	0.855	4.534	−1.358	−0.399	3.457	3.856	6.914	1.304	1.614	27.435
4	5.300	0.855	4.534	−3.973	−3.468	3.805	7.273	7.610	1.436	2.310	26.784
5	5.300	0.855	4.534	−7.497	−7.845	4.493	12.338	8.986	1.695	3.686	24.702
6	5.300	0.855	4.534	−12.727	−14.106	5.285	19.391	10.571	1.994	5.271	20.886
7	5.300	0.855	4.534	−20.019	−22.258	5.846	28.105	11.693	2.206	6.393	15.686
8	5.300	0.855	4.534	−29.031	−32.289	6.487	38.776	12.974	2.448	7.674	8.826
9	5.300	0.855	4.534	−40.081	−44.540	7.211	51.752	14.423	2.721	9.123	0.000
10	25.036	0.855	21.417	−53.533	−44.158	10.014	54.172	20.028	0.800	0.000	0.000
11	27.725	0.855	23.717	−56.200	−45.767	11.090	56.857	22.180	0.800	0.000	0.000
12	30.640	0.855	26.211	−59.051	−47.477	12.256	59.733	24.512	0.800	0.000	0.000
13	33.768	0.855	28.887	−62.113	−49.322	13.507	62.829	27.014	0.800	0.000	0.000
14	37.180	0.855	31.805	−65.418	−51.310	14.872	66.182	29.744	0.800	0.000	0.000
15	41.031	0.855	35.100	−69.007	−53.417	16.412	69.830	32.825	0.800	0.000	0.000
16	46.102	0.855	39.438	−72.927	−55.369	18.441	73.809	36.882	0.800	0.000	0.000
17	50.427	0.855	43.137	−77.247	−58.074	20.171	78.244	40.341	0.800	0.000	0.000
18	56.209	0.855	48.084	−82.038	−60.681	22.483	83.164	44.967	0.800	0.000	0.000
19	62.701	0.855	53.637	−87.408	−63.633	25.081	88.714	50.161	0.800	0.000	0.000

TABLE 1—*Continued*

Time	[1] Gross Premium	[2] Tabular Net/Gross Ratio	[3] Tabular Net Premium	[4] Tabular Terminal Reserve	[5] Tabular Mean Reserve	[6] Total Reserve Held	[7] Extra Reserve Held	[8] Implied Net Premium	[9] Implied Net/Gross Ratio	[10] Implied Expected Loss	[11] *PV* of Expected Loss
20	79.621	0.855	68.111	−93.496	−58.156	27.867	86.023	55.735	0.700	0.000	0.000
21	88.097	0.855	75.362	−90.928	−51.359	30.834	82.193	61.668	0.700	0.000	0.000
22	97.007	0.855	82.984	−87.152	−43.036	33.953	76.989	67.905	0.700	0.000	0.000
23	106.256	0.855	90.896	−81.905	−32.933	37.190	70.123	74.379	0.700	0.000	0.000
24	116.073	0.855	99.294	−74.858	−20.562	40.626	61.188	81.251	0.700	0.000	0.000
25	124.177	0.855	106.226	−65.560	−7.674	44.393	52.067	88.787	0.715	0.000	0.000
26	133.117	0.855	113.874	−56.014	5.823	48.588	42.765	97.175	0.730	0.000	0.000
27	143.160	0.855	122.465	−46.215	20.053	53.327	33.274	106.654	0.745	0.000	0.000
28	154.390	0.855	132.072	−36.143	35.077	58.668	23.592	117.336	0.760	0.000	0.000
29	166.470	0.855	142.406	−25.776	50.753	64.507	13.754	129.014	0.775	0.000	0.000
30	179.015	0.855	153.137	−15.124	66.893	70.711	3.818	141.422	0.790	0.000	0.000
31	191.693	0.855	163.983	−4.227	83.308	83.308	0.000	159.756	0.833	0.000	0.000
32	204.358	0.855	174.817	6.859	99.875	99.875	0.000	174.817	0.855	0.000	0.000
33	216.693	0.855	185.369	18.073	116.392	116.392	0.000	185.369	0.855	0.000	0.000
34	228.927	0.855	195.834	29.342	132.892	132.892	0.000	195.834	0.855	0.000	0.000
35	241.284	0.855	206.405	40.608	149.415	149.415	0.000	206.405	0.855	0.000	0.000
36	254.028	0.855	217.307	51.818	166.026	166.026	0.000	217.307	0.855	0.000	0.000
37	267.680	0.855	228.985	62.927	182.903	182.903	0.000	228.985	0.855	0.000	0.000
38	282.985	0.855	242.078	73.894	200.324	200.324	0.000	242.078	0.855	0.000	0.000
39	302.959	0.855	259.164	84.675	219.553	219.553	0.000	259.164	0.855	0.000	0.000
40	332.742	0.855	284.642	95.266	242.805	242.805	0.000	284.642	0.855	0.000	0.000
41	381.678	0.855	326.504	105.703	274.133	274.133	0.000	326.504	0.855	0.000	0.000
42	469.243	0.855	401.411	116.058	321.961	321.961	0.000	401.411	0.855	0.000	0.000
43	633.175	0.855	541.646	126.453	402.559	402.559	0.000	541.646	0.855	0.000	0.000
44	947.867	0.855	810.847	137.020	473.934	473.934	0.000	810.847	0.855	0.000	0.000

to predict a loss. Column 10, labeled implied expected loss, is the amount of the expected loss. As shown in column 11, the present value of these losses is very high, over 10 times the reserves held in the first year. This amounts to a large "hidden" premium deficiency.

Upon review, the unitary method does not appear to produce reserves consistent with the above-listed principles when applied to the policy in Table 1. First, although the tabular net premiums are a constant percentage of the gross premiums, these tabular net premiums have virtually no effect on the actual reserves held. The net premiums required to reproduce the actual reserves held (the implied net premiums) are not a level percentage of the gross premiums, in violation of principle 1. Second, the reserves produced by the unitary method for the illustrated policy are expected to produce losses, in violation of principle 3.

THE TERM METHOD

This method, recommended by *Actuarial Guideline IV* [3], has its basis in the treatment of renewable term policies as a series of separate policies, each for one renewal period of the policy. The use of the term method has been extended beyond renewable term policies to term-like policies with low early cash values. Net premiums for each period of level gross premiums are level and are just sufficient to cover the benefits payable over the level premium period. Hence, reserves are for the present term only, plus the present value of any future deficiencies. An application of this method to a ten-year renewable term policy is shown in Table 2.

This method has a distinct advantage over the unitary method. Because the net premiums for each level-premium period are calculated separately, large premiums in the later durations do not avoid deficiency reserves if premiums are too low in the early durations. In the same way, this method does not allow large hidden deficiencies to develop.

However, this method has two major drawbacks. First, negative reserves, hidden deficiency reserves, and uneven net premiums can occur when mortality rates are decreasing over time. For example, during the first level-premium period in Table 2, the mortality rates decrease from 1.68 per thousand (at age 20) to 1.44 per thousand (at age 29). Because of this, the net premium 1.483 plus the terminal reserve at time 0 is less than the cost of insurance in year 1, and terminal reserves are negative at the end of year 1. Hence, extra reserves are held, and the implied net premium is greater than the tabular net premium. With more steeply decreasing mortality rates, hidden deficiency reserves could develop, making this method inconsistent with

TABLE 2

Comparison of the Implied Net Premiums to the Tabular Net Premiums under the Term Method

Male, issue age 20
$1,000 face amount, 10-year renewable term to age 60, no cash values
1980 CSO, male nonsmoker, without select factors, 5.5% interest

Time	[1] Gross Premium	[2] Tabular Net/Gross Ratio	[3] Tabular Net Premium	[4] Tabular Terminal Reserve	[5] Tabular Mean Reserve	[6] Total Reserve Held	[7] Extra Reserve Held	[8] Implied Net Premium	[9] Implied Net/Gross Ratio
0.....	2.000	0.742	1.483	0.000	0.684	0.796	0.112	1.592	0.796
1.....	2.000	0.742	1.483	−0.115	0.571	0.791	0.221	1.583	0.791
2.....	2.000	0.742	1.483	−0.227	0.471	0.777	0.306	1.555	0.777
3.....	2.000	0.742	1.483	−0.315	0.396	0.763	0.367	1.526	0.763
4.....	2.000	0.742	1.483	−0.378	0.351	0.744	0.393	1.488	0.744
5.....	2.000	0.742	1.483	−0.404	0.349	0.720	0.371	1.441	0.720
6.....	2.000	0.742	1.483	−0.382	0.392	0.701	0.309	1.403	0.701
7.....	2.000	0.742	1.483	−0.318	0.467	0.692	0.225	1.384	0.692
8.....	2.000	0.742	1.483	−0.231	0.567	0.682	0.115	1.365	0.682
9.....	2.000	0.742	1.483	−0.119	0.682	0.682	0.000	1.365	0.682
10.....	2.050	0.773	1.584	0.000	0.907	0.907	0.000	1.584	0.773
11.....	2.050	0.773	1.584	0.231	1.130	1.130	0.000	1.584	0.773
12.....	2.050	0.773	1.584	0.445	1.335	1.335	0.000	1.584	0.773
13.....	2.050	0.773	1.584	0.642	1.512	1.512	0.000	1.584	0.773
14.....	2.050	0.773	1.584	0.799	1.644	1.644	0.000	1.584	0.773
15.....	2.050	0.773	1.584	0.905	1.713	1.713	0.000	1.584	0.773
16.....	2.050	0.773	1.584	0.937	1.706	1.706	0.000	1.584	0.773
17.....	2.050	0.773	1.584	0.891	1.604	1.604	0.000	1.584	0.773
18.....	2.050	0.773	1.584	0.733	1.381	1.381	0.000	1.584	0.773
19.....	2.050	0.773	1.584	0.445	1.014	1.014	0.000	1.584	0.773
20.....	3.250	0.925	3.006	0.000	1.945	1.945	0.000	3.006	0.925
21.....	3.250	0.925	3.006	0.884	2.764	2.764	0.000	3.006	0.925
22.....	3.250	0.925	3.006	1.638	3.450	3.450	0.000	3.006	0.925
23.....	3.250	0.925	3.006	2.256	3.981	3.981	0.000	3.006	0.925
24.....	3.250	0.925	3.006	2.699	4.332	4.332	0.000	3.006	0.925
25.....	3.250	0.925	3.006	2.959	4.474	4.474	0.000	3.006	0.925
26.....	3.250	0.925	3.006	2.983	4.364	4.364	0.000	3.006	0.925
27.....	3.250	0.925	3.006	2.739	3.967	3.967	0.000	3.006	0.925
28.....	3.250	0.925	3.006	2.189	3.246	3.246	0.000	3.006	0.925
29.....	3.250	0.925	3.006	1.297	2.152	2.152	0.000	3.006	0.925
30.....	7.010	1.000	7.008	0.000	4.752	4.752	0.000	7.008	1.000
31.....	7.010	1.000	7.008	2.495	7.102	7.102	0.000	7.008	1.000
32.....	7.010	1.000	7.008	4.701	9.120	9.120	0.000	7.008	1.000
33.....	7.010	1.000	7.008	6.531	10.697	10.697	0.000	7.008	1.000
34.....	7.010	1.000	7.008	7.855	11.757	11.757	0.000	7.008	1.000
35.....	7.010	1.000	7.008	8.651	12.214	12.214	0.000	7.008	1.000
36.....	7.010	1.000	7.008	8.769	11.930	11.930	0.000	7.008	1.000
37.....	7.010	1.000	7.008	8.084	10.793	10.793	0.000	7.008	1.000
38.....	7.010	1.000	7.008	6.494	8.683	8.683	0.000	7.008	1.000
39.....	7.010	1.000	7.008	3.864	5.436	5.436	0.000	7.008	1.000

principle 3. The second major drawback is also illustrated in Table 2. Because the entire policy is not taken into account when net premiums are determined, the gross premiums can be manipulated so that loading is uneven and skewed to the early durations. These drawbacks imply that the term method is sometimes not consistent with principle 1.

THE UNIFIED METHOD

This method, first described by Sarnoff [7] and more recently described by Olson [5], attempts to blend the best features of the term method with those of the unitary method. As such, the unified method can be viewed either of two ways: the unitary method modified to eliminate negative terminal reserves (and terminal reserves less than cash values), or the term method modified to merge level-premium periods when necessary to provide for net premiums being a level percentage of gross premiums.

When the latter approach is taken, net premiums are first calculated by the term method, and then consecutive level-premium periods are merged when the ratio of net to gross premiums in the prior period is less than the ratio of net to gross premiums in the subsequent period. This approach results in a level ratio of net to gross premiums over the combined period, increases net premiums in the prior period (hence also terminal and mean reserves), and thereby prevents loading from being skewed to the early durations.

As a calculating approach, however, this view is impractical. The testing to determine whether level-premium periods should be merged is tedious. Also, when the mortality rates are decreasing with duration within a given level-premium period, there may be negative terminal reserves in the initial calculation of reserves, leading to implied net premiums that are not a level percentage of gross premiums.

A more practical view is that unified method net premiums are unitary method net premiums for segments of the policy, where each segment is as long as possible without generating negative terminal reserves or terminal reserves less than the cash value. This method is somewhat analogous to the approach of the Commissioners Annuity Reserve Valuation Method (CARVM). In that method, the reserve is the greatest excess of the present value of future benefits over the present value of future guaranteed premiums. In the unified method, net premiums are based on the greatest ratio of future benefits to future gross premiums. As such, both methods use "worst case" scenarios and do not allow future factors more favorable to the company to lower current reserves.

To derive this method, note that (ignoring deficiency reserves)

$$ {}_tV = \frac{\left(\sum_{j=0}^{t-1} P_j \times {}_jE_x\right) - A'_{x:\overline{t}|}}{{}_tE_x}. \tag{7} $$

Therefore, to ensure that ${}_tV$ is greater than the cash value (and hence also non-negative), it is sufficient to ensure that

$$ \sum_{j=0}^{t-1} P_j \times {}_jE_x \geq A'_{x:\overline{t}|} + {}_tE_x \times {}_tCV \tag{8} $$

for each year t.

To do this, we divide the policy into segments and set net premiums for each segment as a constant percentage of the gross premiums. If k is the ratio of net to gross premiums over the segment, t the length of the segment, and m the beginning of the segment, then

$$ {}_mk_t = \frac{A'_{x+m:\overline{t}|} + {}_tE_{x+m} \times {}_{t+m}V - {}_mV}{\sum_{j=0}^{t-1} G_j \times {}_jE_{x+m}}. \tag{9} $$

The terminal reserves used in Equation (9) should be the minimum terminal reserves acceptable, either zero or the appropriate cash values. When defined in this way, the terminal reserves produced at the end of each segment are equal to the minimum acceptable reserves.

Segments of the policy are determined in such a way as to reproduce the unitary method, if appropriate. The first segment includes the first policy year and extends as far as possible. This is done by computing ${}_0k_t$ for all possible values of t and defining the first segment as being t years long, where k_t is the maximum of the k_i's. The following segments start at the end of the previous segments and are determined similarly. After the ratio of net to gross premiums is found, net premiums are the lessor of gross premiums or gross premiums times the appropriate ratio. Terminal reserves and mean reserves are then computed as with the unitary method.

Table 3 provides an illustration of this method. First, an attempt is made to apply the unitary method to the entire policy. The ratio, k_t, of policy benefits to gross premiums through year t is computed for all years. Note that although the overall ratio is 0.855, this is less than the ratio after ten years, 1.552. Therefore, use of the ratio 0.855 would produce negative

TABLE 3

COMPUTATION OF UNIFIED METHOD NET PREMIUMS BY THE MODIFIED UNITARY METHOD

Male, issue age 55
$1,000 face amount, term to age 100, no cash values
1980 CSO male nonsmoker, with select factors, 5.5% interest

			Maximum k_t = 1.552			Maximum k_t = 0.800			Maximum k_t = 0.747					
Year	G_t*	$A'_{x:\overline{t}\rceil}$	$\Sigma G_j \times {}_jE_x$	$A'_{x:\overline{t-1}\rceil}$	k_t	$\Sigma G_j \times {}_{j-10}E_{x+10}$	$A'_{x+10:\overline{t-9}\rceil}$	k_t	$\Sigma G_j \times {}_{j-20}E_{x+20}$	$A'_{x+20:\overline{t-19}\rceil}$	k_t	k	Net Premium	Reserve Held
0.....	5.300	4.151	5.300	4.151	0.783							1.000	5.300	26.553
1.....	5.300	4.908	10.302	8.783	0.853							1.000	5.300	28.806
2.....	5.300	5.847	15.018	13.986	0.931							1.000	5.300	30.320
3.....	5.300	6.914	19.461	19.782	1.016							1.000	5.300	30.892
4.....	5.300	7.610	23.642	25.785	1.091							1.000	5.300	30.589
5.....	5.300	8.986	27.572	32.449	1.177							1.000	5.300	29.195
6.....	5.300	10.571	31.263	39.810	1.273							1.000	5.300	26.171
7.....	5.300	11.693	34.722	47.441	1.366							1.000	5.300	21.533
8.....	5.300	12.974	37.961	55.369	1.459							1.000	5.300	15.313
9.....	5.300	14.423	40.988	63.608	1.552							1.000	5.300	7.211
10.....	25.036	20.028	54.338	74.287	1.367	25.036	20.028	0.800				0.800	20.028	10.014
11.....	27.725	22.180	68.054	85.261	1.253	50.760	40.608	0.800				0.800	22.180	11.090
12.....	30.640	24.512	82.087	96.487	1.175	77.076	61.661	0.800				0.800	24.512	12.256
13.....	33.768	27.014	96.366	107.910	1.120	103.856	83.085	0.800				0.800	27.014	13.507
14.....	37.180	29.744	110.844	119.493	1.078	131.008	104.807	0.800				0.800	29.744	14.872
15.....	41.031	32.825	125.514	131.228	1.046	158.519	126.815	0.800				0.800	32.825	16.412
16.....	46.102	36.882	140.596	143.294	1.019	186.804	149.444	0.800				0.800	36.882	18.441
17.....	50.427	40.341	155.624	155.316	0.998	214.989	171.991	0.800				0.800	40.341	20.171
18.....	56.209	44.967	170.827	167.479	0.980	243.500	194.800	0.800				0.800	44.967	22.483
19.....	62.701	50.161	186.139	179.728	0.966	272.216	217.773	0.800				0.800	50.161	25.081

*For ease of exposition, gross premiums for this hypothetical policy have been set to 125% of the cost of insurance in years 11–20, 143% in years 21–25, and thereafter grading to 100% in year 45.

TABLE 3—*Continued*

Year	G_t*	$A'_{x:\overline{t}\rceil}$	Maximum k_t = 1.552: $\Sigma G_j \times {}_jE_x$	$A'_{x:\overline{t-1}\rceil}$	k_t	Maximum k_t = 0.800: $\Sigma G_j \times {}_{j-10}E_{x+10}$	$A'_{x+10:\overline{t-9}\rceil}$	k_t	Maximum k_t = 0.747: $\Sigma G_j \times {}_{j-20}E_{x+20}$	$A'_{x+20:\overline{t-19}\rceil}$	k_t	k	Net Premium	Reserve Held
20	79.621	55.735	203.593	191.946	0.943	304.951	240.687	0.789	79.621	55.735	0.700	0.747	59.441	31.798
21	88.097	61.668	220.823	204.007	0.924	337.264	263.306	0.781	158.216	110.751	0.700	0.747	65.770	39.621
22	97.007	67.905	237.636	215.777	0.908	368.796	285.378	0.774	234.910	164.437	0.700	0.747	72.421	48.729
23	106.256	74.379	253.842	227.121	0.895	399.188	306.653	0.768	308.833	216.183	0.700	0.747	79.326	59.353
24	116.073	81.251	269.305	237.945	0.884	428.188	326.953	0.764	379.369	265.559	0.700	0.747	86.655	71.926
25	124.177	88.787	283.642	248.195	0.875	455.074	346.177	0.761	444.765	312.317	0.702	0.747	92.705	84.921
26	133.117	97.175	296.844	257.833	0.869	479.834	364.252	0.759	504.990	356.281	0.706	0.747	99.379	98.410
27	143.160	106.654	308.923	266.832	0.864	502.487	381.128	0.758	560.088	397.329	0.709	0.747	106.877	112.494
28	154.390	117.336	319.881	275.160	0.860	523.038	396.746	0.759	610.073	435.317	0.714	0.747	115.261	127.208
29	166.470	129.014	329.694	282.765	0.858	541.441	411.009	0.759	654.836	470.008	0.718	0.747	124.279	142.416
30	179.015	141.422	338.335	289.591	0.856	557.646	423.811	0.760	694.251	501.147	0.722	0.747	133.645	157.949
31	191.693	154.313	345.797	295.598	0.855	571.640	435.076	0.761	728.289	528.547	0.726	0.747	143.109	173.644
32	204.358	167.573	352.110	300.775	0.854	583.479	444.784	0.762	757.085	552.160	0.729	0.747	152.564	189.406
33	216.693	180.938	357.333	305.136	0.854	593.275	452.963	0.763	780.910	572.054	0.733	0.747	161.773	205.075
34	228.927	194.588	361.564	308.733	0.854	601.211	459.709	0.765	800.214	588.462	0.735	0.747	170.906	220.702
35	241.284	208.711	364.924	311.639	0.854	607.512	465.160	0.766	815.540	601.719	0.738	0.747	180.132	236.346
36	254.028	223.545	367.539	313.940	0.854	612.416	469.475	0.767	827.467	612.215	0.740	0.747	189.646	252.095
37	267.680	239.573	369.535	315.726	0.854	616.158	472.825	0.767	836.570	620.362	0.742	0.747	199.838	268.141
38	282.985	257.517	371.029	317.086	0.855	618.961	475.375	0.768	843.386	626.565	0.743	0.747	211.264	284.772
39	302.959	280.237	372.133	318.108	0.855	621.032	477.291	0.769	848.424	631.225	0.744	0.747	226.175	303.143
40	332.742	312.777	372.943	318.869	0.855	622.550	478.718	0.769	852.118	634.697	0.745	0.747	248.410	325.215
41	381.678	364.502	373.533	419.432	0.855	623.657	479.775	0.769	854.809	637.267	0.746	0.747	284.944	354.528
42	469.243	455.166	373.956	319.843	0.855	624.450	480.544	0.770	856.739	639.139	0.746	0.747	350.316	398.391
43	633.175	623.678	374.237	320.120	0.855	624.978	481.064	0.770	858.022	640.403	0.746	0.747	472.700	470.895
44	947.867	947.867	374.374	320.256	0.855	625.234	481.320	0.770	858.645	641.026	0.747	0.747	707.635	473.934

*For ease of exposition, gross premiums for this hypothetical policy have been set to 125% of the cost of insurance in years 11–20, 143% in years 21–25, and thereafter grading to 100% in year 45.

terminal reserves at time 10, resulting in extra reserves held and post-funding. To correct this, the ratio 1.552 is used for the first ten years. Beginning at time 10, the largest $_{10}k_t$ is 0.800, and it is used for the next ten years. Finally, beginning at time 20, the largest $_{20}k_t$ is 0.747, which is used for the rest of the policy.

Although this method may seem complex, use of a modern high-speed computer renders application of this method quite tractable. A sample APL program to compute reserves by the unified method is given in the Appendix.

A few properties of net premiums and reserves produced by the unified method should be pointed out. First, by construction, terminal reserves are always at least as great as the cash value or 0. Second, net premiums as a percentage of gross premiums will be monotonically nonincreasing over the life of the policy.

An interesting problem can arise when the unified method is applied to a level-premium, level-death-benefit policy. If the mortality rates are increasing over the life of the policy, unified method net premiums will be identical to unitary method net premiums, as required by the SVL. However, if mortality rates over some interval are decreasing, the unified net premiums may not be a level percentage of the gross premiums. An example of this is shown in Table 4.

Although the results of the unified method are not in exact conformance with the requirements of the SVL, the results may nevertheless be acceptable. First, unified method reserves will always be greater than those required by the SVL. The SVL reserves are not the exact reserves that must be held but a minimum floor, and as such, unified method reserves are acceptable. Second, whenever unified method reserves are not equal to unitary method net premiums, SVL minimum reserves are not consistent with the broader principles outlined above. Whenever negative terminal reserves arise, implied net premiums will not be a level percentage of gross premiums, violating principle 1. In extreme cases, this can lead to implied net premiums being greater than the gross premiums and the expectation of future losses, in violation of principle 3.

SUMMARY

The Standard Valuation Law directs the valuation actuary to apply principles consistent with the valuation of level-premium, level-death-benefit policies to the valuation of nonlevel-premium, nonlevel-death-benefit policies. This paper has shown that in certain cases unitary and term methods currently in use are not consistent with these principles. The unified method

TABLE 4

COMPARISON OF UNIFIED METHOD AND UNITARY METHOD RESERVES FOR LEVEL-PREMIUM LEVEL-DEATH-BENEFIT POLICY WITH DECREASING MORTALITY RATES

Male, issue age 0
$1,000 face amount, term to age 20, no cash values, gross premium $5.00 per year
1980 CSO male, 5.5% interest

			Unified Method			Unitary Method				Unified
Time	$1000\times Q$	Minimum Reserve	Net Premium	Terminal Reserve	Mean Reserve	Net Premium	Terminal Reserve	Mean Reserve	Actual Reserve	Extra Reserve
0....	4.180	1.981	3.962	0.000	1.981	1.212	0.000	−0.851	1.981	0.000
1....	1.070	0.507	1.014	0.000	0.507	1.212	−2.914	−2.285	0.507	0.000
2....	0.990	0.469	0.969	0.000	0.500	1.212	−2.868	−2.198	0.469	0.031
3....	0.980	0.464	0.969	0.032	0.538	1.212	−2.740	−2.062	0.464	0.074
4....	0.950	0.450	0.969	0.076	0.598	1.212	−2.595	−1.897	0.450	0.148
5....	0.900	0.427	0.969	0.152	0.701	1.212	−2.411	−1.683	0.427	0.275
6....	0.860	0.408	0.969	0.282	0.855	1.212	−2.167	−1.412	0.408	0.448
7....	0.800	0.379	0.969	0.460	1.068	1.212	−1.869	−1.076	0.379	0.689
8....	0.760	0.360	0.969	0.708	1.343	1.212	−1.495	−0.671	0.360	0.983
9....	0.740	0.351	0.969	1.009	1.663	1.212	−1.059	−0.213	0.351	1.312
10....	0.730	0.346	0.969	1.348	2.016	1.212	−0.579	0.285	0.346	1.670
11....	0.770	0.365	0.969	1.715	2.373	1.212	−0.063	0.796	0.796	1.577
12....	0.850	0.403	0.969	2.063	2.691	1.212	0.443	1.276	1.276	1.415
13....	0.990	0.469	0.969	2.350	2.916	1.212	0.896	1.672	1.672	1.244
14....	1.150	0.545	0.969	2.514	3.005	1.212	1.236	1.941	1.941	1.064
15....	1.330	0.630	0.969	2.527	2.928	1.212	1.434	2.054	2.054	0.874
16....	1.510	0.678	0.969	2.361	2.668	1.212	1.463	1.995	1.995	0.673
17....	1.670	0.730	0.969	2.006	2.222	1.212	1.314	1.761	1.761	0.461
18....	1.780	0.758	0.969	1.470	1.617	1.212	0.997	1.380	1.380	0.237
19....	1.860	0.787	0.969	0.794	0.882	1.212	0.551	0.882	0.882	0.000

conforms to the principles listed and should be considered for adoption as a standard for the United States statutory valuation of all nonlevel-premium policies.

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APPENDIX

The following is a sample APL program for computing reserves according to the unified method.

```
    UNIFIEDΔMETHOD
⍝THIS FUNCTION CALCULATES STATUTORY RESERVES ACCORDING TO THE
⍝UNIFIED METHOD.  THE FOLLOWING VARIABLES ARE ASSUMED TO BE
⍝KNOWN:
⍝  N      -- DURATION OF POLICY IN YEARS
⍝  Q      -- N LENGTH VECTOR OF MORTALITY RATES
⍝  GPREM  -- N LENGTH VECTOR OF GROSS PREMIUMS
⍝  CV     -- N+1 LENGTH VECTOR OF CASH VALUES (0 IF NONE)
⍝  I      -- ANNUAL INTEREST RATE
V←(1+I)*-0,⍳N-1 ⍝                         INTEREST DISCOUNT
P←¯1↓1,×\1-Q ⍝                            MORTALITY DISCOUNT
PVPREM←GPREM×V×P ⍝                        PRESENT VALUE OF PREMIUMS
PVDB←1000×Q×V×P÷1+I ⍝                     PRESENT VALUE OF DB
PVCV←CV×(V,V[N]÷1+I)×(P,P[N]×1-Q[N]) ⍝    PRESENT VALUE OF CASH VALUE
KT←0⍴0 ⍝                                  RATIO OF NET TO GROSS PREMIUM
LOOP:
RATIO←((+\PVDB)+(1↓PVCV)-PVCV[1])÷+\PVPREM ⍝ RATIO OF BENEFITS TO PREM
⍝                                            IN SEGMENT SINCE BEGINNING
K←⌈/RATIO ⍝                               RATIO TO USE IN SEGMENT
T←¯1↑(K=RATIO)/⍳⍴RATIO ⍝                  DURATION OF SEGMENT
KT←KT,T⍴K
PVPREM←T↓PVPREM ⍝                         GET READY FOR NEXT SEGMENT
PVDB←T↓PVDB
PVCV←T↓PVCV
→(0≠⍴PVDB)⍴LOOP ⍝                         IF NO MORE YEARS LEFT, STOP
NPREM←GPREM×1⌊KT ⍝                        COMPUTE NET PREMIUMS
CUMPVNPREM←⌽+\⌽NPREM×V×P ⍝                COMPUTE PV OF NET PREMIUMS
CUMPVDB←⌽+\⌽1000×Q×V×P÷1+I ⍝              COMPUTE PV OF DEATH BENEFITS
TRES←(CUMPVDB-CUMPVNPREM)÷P×V ⍝           COMPUTE TERMINAL RESERVES
MRES←(TRES+NPREM+1↓TRES,CV[N+1])÷2 ⍝      COMPUTE MEAN RESERVES
```

DISCUSSION OF PRECEDING PAPER

ROY C. OLSON:

It is a pleasure to express my gratitude to Mr. Beach for writing this paper, which restates and clarifies reserve methodology conforming to Standard Valuation Law and accepted actuarial principles. The point is simple and clear and backed up by accurate detail. The unification of the "unitary method" and the "term method" is based on the following three principles:

1. Net premiums should be a uniform percentage of gross premiums.
2. Reserves should not be less than cash values.
3. Reserves should not generate future losses.

Some life insurance products that have been developed focus on principle 1, using the unitary method in a manner that produces negative reserves. The most obvious example is the first example in the paper, "competitive" ten-year term insurance, converting to "expensive" annual renewable term insurance. One property of a consistent reserve methodology is monotonicity: If plan A represents a liability as great as or greater than plan B, then the same relationship should hold with respect to reserves. If one level of reserves is appropriate for a plain ten-year term plan, then the opportunity to select against the insurer after ten years (by renewing) should not decrease reserves (to less than zero). In fact, such negative reserves do not comply with principle 1 because the implied net premiums are then not a uniform percentage of gross premiums, as noted by Mr. Beach.

In the case of uniform premiums and amounts of insurance, the SVL defines modified net premiums to produce non-negative reserves and fully fund the benefits provided, plus an expense allowance, as defined in the law. Principle 1 is not a principle that overrides other considerations. As noted, where terminal reserves are equal to zero, the customary mean reserves are one-half the annual cost of insurance on the valuation basis, that is, one-half the implied net premium.

The unified method, which Mr. Beach has presented so well, conforms to the objectives of the proposed Actuarial Guideline XXX, but in a manner that facilitates the calculations. According to proposed Actuarial Guideline XXX, the valuation net premiums would satisfy the following conditions:

1. Reserves are non-negative.
2. Net premiums are a uniform percentage of gross premiums unless negative reserves would be produced.
3. Net premiums are replaced by gross premiums, if less.

The unified method satisfies these conditions. For each policy anniversary, net premiums are calculated as a uniform percentage of gross premiums were the policy to terminate on that anniversary. This percentage defines a ratio associated with the particular policy anniversary. The unified method compares these ratios, defining the initial segment as the one from issue to the policy anniversary that produces the highest such ratio. The procedure is repeated for subsequent segments. Thus, if in later policy years the gross premiums increase more than proportionately to the tabular cost of benefits, new segments are created. Otherwise, prefunding exists and is recognized in the unified method.

The net premiums are based on the *pattern* of gross premiums. Once defined, the net premiums are compared to the gross premiums, so as not to exceed them. As stated in Actuarial Guideline IV, this is in accordance with the principle of anticipating no future profits but providing for all future losses. The SVL also provides for this substitution.

The term method permits a degree of simplicity for plans in which gross premiums are based on attained age. This method is justified by reference to the provision, in Standard Nonforfeiture Law for Life Insurance, that defines the issue date to be "the date as of which the rated age of the insured is determined." Transferring this concept to the SVL permits nonrecognition of some prefunding, as well as a new expense allowance when a new premium applies. It would seem that technically, with the unified method, the term of a life insurance policy runs from the issue date until termination by the policyholder. Also, the only expense allowance under the SVL would occur when the policy is issued.

Regarding the expense allowance described in the SVL, there seems to be no reason not to add it to the present value of benefits on the issue date (but not on renewal dates) and apply the unified method to calculate CRVM reserves. The expense allowance will be amortized over the first segment created by the unified method. I will provide a sample APL program for calculating the expense allowance to interested readers. One worthwhile observation is that high gross premiums in later durations tend to reduce or eliminate the expense allowance, as do gross premium deficiencies.

The last example of this paper is a level-premium 20-year term plan issued at age 0. The unified method produces two-year preliminary term reserves. I submit that this is the correct result. Alternatives based on negative reserves at the end of the first policy year are *not* more consistent with the SVL than the unified method. Mr. Beach recommends the unified method for nonlevel-premium policies. I believe the unified method is appropriate for level-premium policies also.

Considerable controversy surrounds the unified method. Many companies have developed low-priced life insurance products, to the benefit of the public. However, reserves under the unified method could be many times higher than the reserves held under the unitary method or annual term method. The primary argument against the unified method is that it produces reserves that are too high for many insurers with low levels of surplus. Another argument against the unified method, which recognizes gross premium deficiencies, is that the mortality standard required by the SVL is unreasonable in light of current risk classification practices. It seems that this question is one of the proper mortality *standard* and should be separated from questions about the method of apportioning net premiums. Both arguments fail to recognize that a function of statutory reserves is to control earnings available to stockholders and management, so as not to put the public and the guaranty associations at risk needlessly. If a company believes its underwriters can beat the statutory standard, let it support that belief with its surplus! Surplus should not be created by weakening reserves to a level that is not consistent with the principles of the SVL.

Mr. Beach is to be congratulated for his fine exposition on a subject of keen interest to so many life actuaries and their employers and clients.

WILBUR M. BOLTON:

Mr. Beach has produced an interesting analysis of methods of applying the SVL to current issues of nonlevel-premium policies. This subject is currently of wide interest within the life insurance industry. A joint ACLI/NAIC industry task force on precisely this topic submitted a lengthy report to the NAIC Life & Health Actuarial Task Force in December 1990. (See *ACLI General Bulletin No. 4310*, dated December 21, 1990.)

The joint industry task force, broadly representative in its membership, engaged in a number of discussions concerning proper reserve levels *and* the reasons for development of the variety of life products covered by this paper. Although I am a member of the task force, the views here are mine and are not necessarily shared by other task force members.

The wide use of nonlevel-premium policies (characterized by early policy duration contract premiums less than the "implied net premium" as defined in Mr. Beach's paper) has several causes:

- Mortality—both insured and U.S. population—has improved significantly, particularly at attained ages 45 and above, since the current statutory standards were developed.
- Increasing evidence shows that the smoker/nonsmoker versions of the 1980 CSO Mortality tables (*1982 TSA Reports*, pp. 376–379) contain margins relatively thin for smokers, but redundant for nonsmokers.
- With the introduction of advanced underwriting tools to screen for substance (including alcohol) abusers, death claims from accidents can be greatly reduced, particularly in early policy durations.

Blocks of business underwritten in recent years using these newer risk classification tools are developing actual mortality results substantially below the experience underlying statutory tables. Companies trying to reflect these mortality improvements in their contract premiums for new issues (of such plans as ten-year term) found themselves impeded by reserve requirements in the SVL, particularly Section 7. Review of anticipated year-by-year cash flows of some of these products under reasonably realistic assumptions for mortality, expense, interest and lapsation, using natural benefit reserves, show positive margins (for adverse deviation and profit) consistently emerging in renewal years to policy maturity/expiry.

However, when Section 7 of the SVL is applied to these policies at issue, the deficiency reserves generated may be far in excess of the natural benefit reserve and, in fact, several times the annual premium. Mr. Beach illustrates this property beautifully in Table 3 of his paper. In the absence of an infinite capital and surplus account, companies cannot afford to issue straight term policies at the level of guaranteed premiums otherwise justified.

However, actuaries confronted with this problem discovered that a level term policy issued at age (x) for n years, followed by a whole life policy issued at age $(x+n)$, if combined and analyzed under the unitary method—Mr. Beach's principle 1—did *not* develop premium deficiency reserves *if* the whole life premium was "large enough."

The contract premium guaranteed for the first n years could be less than the statutory net premium for a stand-alone n-year term policy. For the successor whole life policy issued at $(x+n)$, the guaranteed contract premium must exceed the statutory net premium for a stand-alone whole life policy issued at age $(x+n)$. The SVL defines statutory net premiums as a uniform percentage of the respective gross premiums. Under this interpretation, "later sufficiencies could offset earlier deficiencies." By this means, it appeared that deficiency reserves otherwise mandated under SVL Section 7 could be legally avoided.

In effect, then, the nonlevel-premium policies analyzed in Mr. Beach's paper may be regarded as a subterfuge, assuming the unitary method of the SVL, developed to minimize the surplus strain from issuing level term policies at justifiably low guaranteed rates.

Mr. Beach has proposed three principles: the first two based on the SVL and the third on "conservative accounting principles." Perhaps another principle also applies: the method used to apply Section 7 of the SVL to various products should meet the test of common sense.

In order for the premium deficiency reserve statutes to meet the test of common sense, the mortality assumptions mandated in the SVL must be kept reasonably current. Otherwise, as a consequence of the unreasonably high level of "unified method" reserves illustrated in Mr. Beach's paper, Section 7 of the SVL becomes a legal device to raise the prices paid by consumers above the level of contract premiums otherwise justified.

The report of the joint industry task force notes that, "The Standard Valuation Law, and its supporting regulations, do not:

- adequately deal with many products currently in the marketplace;
- reflect lapses;
- dynamically allow for mortality changes over time and for differences in risk classification;
- reflect current expense levels, including products which contain no acquisition expense;
- recognize the interrelationships of the above factors."

The joint industry task force also tested several variations in concept of reserve methodology. Its report includes other findings and conclusions and makes a series of recommendations dealing with diagnosed shortcomings in the current SVL. Readers interested further may obtain copies of this report from Stanton L. Cole, at his *Yearbook* address.

Mr. Beach is to be congratulated on opening a dialog about possible shortcomings in the SVL and proposals to improve it. Although neither this paper nor the report of the joint industry task force should be regarded as the "last word" on this subject, both point the way toward a constructive resolution of the problems diagnosed.

(AUTHOR'S REVIEW OF DISCUSSION)

A. STEPHEN BEACH:

I appreciate the interest shown by the two discussants. Judging by the tones of their discussions, I think they are on opposite sides of the issue. This issue is controversial, but before commenting on the opposing philosophies, I would like to comment on the technical aspects of the discussion.

First, I thank Mr. Olson for his derivation of a CRVM expense allowance. This expense allowance, taken only at the beginning of the first segment, fits neatly with the rest of the paper. Like the unified method, it acts as an extension of the reserving methods applicable to level-premium policies, but does not result in negative terminal reserves.

Second, I discuss the ACLI/NAIC task force report referred to by Mr. Bolton. Essentially, the report makes two recommendations: that the reserve mortality table be modified to bring it up-to-date and that a method very similar to the unified method be used. The task force method first defines the unitary method reserve slightly differently than I do in this paper and then applies the revised unitary-method mean reserves as a floor to the unified-method reserve. In defining the unitary-method mean reserve, the task force first sets all negative terminal reserves to zero and then applies Expressions (1), (2) and (3).

Practically, the differences between the unified-method reserves and the task-force-method reserves are small, because they only appear at the later durations for negligible amounts. Theoretically, I question whether the extra reserves are needed. If negative terminal reserves are not set to zero, it can be shown that unified-method mean reserves are always greater than or equal to unitary-method mean reserves. As demonstrated in this paper, the unified-method reserves by themselves are sufficient to prevent future expected losses. Adding extra reserves to meet an arbitrary unitary-method floor would be redundant.

Now that I have taken care of the technical matters, I comment on the opposing philosophies represented by Mr. Olson and Mr. Bolton. To paraphrase Mr. Bolton, the combination of the unified method and the 1980 CSO tables produces reserves that are ridiculously high. Holding reserves in the first year of more than five times the first-year premium (as in Table 1), when the premiums are otherwise justifiable, is clearly ridiculous. But to paraphrase Mr. Olson, the alternative of the unitary method is also flawed. Statutory reserves should have some minimum, and I suggest that the only reasonable functional definition of a proper statutory reserve is that it prevent future expected losses. Deficiency reserves are the means of preventing future expected losses. Avoiding deficiency reserves by using the subterfuge outlined by Mr. Bolton is also ridiculous. The only way of reconciling these two positions is to have both a reasonable method and a reasonable valuation standard. Rather than settling for two wrongs (an overly conservative table and an easily abused method), let us set the method aright and begin looking for the right mortality table to use with it.

In closing, I acknowledge the contributions of Hank Hansen; Donald Leapman and Jerry Enoch of the Editorial Board of the *Transactions*; and Barbara Simmons, technical editor of the *Transactions*. Their many comments vastly improved the paper.