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THE "PENSION MAX" ELECTION: AN INVESTIGATION OF THE STRUCTURAL AND ECONOMIC DIFFERENCES BETWEEN THE 100% CONTINGENT ANNUITY PENSION BENEFIT OPTION AND THE STRAIGHT LIFE BENEFIT OPTION USED IN CONJUNCTION WITH PENSION MAX

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ABSTRACT

The so-called "pension max" life insurance marketing concept is generating a great deal of excitement among life insurance marketers. It's also received some skeptical reviews in the popular news media, the most recent being a *Newsweek* article by Jane Bryant Quinn in the April 4, 1994 issue. The marketing concept calls for electing an unreduced pension, that is, *maximizing* the pension, in lieu of electing a reduced "contingent annuity" (CA) or "joint and survivor" (J&S) option. Further, the marketing concept calls for the purchase of life insurance on the employee to substitute for the economic exposure created by not electing the CA or J&S option. Although the marketing concept is clear and simple, its economics continue to generate a great deal of discussion. This paper attempts to add some clarity to this issue.

The differences between pension max and the CA option are examined in three parts:

- Differences in the funding pattern
- Differences in the benefit pattern
- Relative economics.

The paper concludes with some generalizations based on this analysis.

1. INTRODUCTION

We start with an immediate straight life annuity, of amount N, payable to an employee age X. The present value for this (in actuarial notation) is $N\ddot{a}_x$. The N stands for "normal" form. The pension plan benefit formula describes benefit accrual amounts in units of the normal form. The benefit amounts under other optional forms of payment are converted to different amounts, usually determined as the actuarial equivalent of the normal form.

All qualified pension plans must offer the option to receive benefits in a joint and survivor (J&S) or contingent annuity (CA) form. A *K%* J&S option refers to an optional annuity form whereby the annuity amount is reduced to *K%* of the initial annuity payment after the first death, regardless of whether the employee or the spouse dies first. This is similar to, but not the same as, the CA option. A *K%* CA option provides for a reduction to *K%* of the initial annuity amount only if the employee dies first. For *K%* equal to 100%, the CA option and the J&S option are the same.

For this discussion, we evaluate the trade-offs between the 100% CA (or J&S) annuity option and the straight-life normal form. An extension of the analysis to other percentages, such as a 50% CA or J&S option, is straightforward [see Equation (3)]. We assume the employee is age X and has a spouse age Y.

The present value of the 100% CA benefit option, for an annuity amount of $(N-R)$, in actuarial notation, is:

$$(N - R)\ddot{a}_x + (N - R)\ddot{a}_{x|y}.$$

(The notation $\ddot{a}_{x|y}$ is used to denote a contingent annuity of \$1 per year payable to Y after the death of X.)

If R is the actuarial reduction, the expression for the 100% CA option says that the present value of an annuity of N payable in the straight-life normal form is equal to the present value of a reduced annuity of $(N-R)$ payable to X plus a contingent annuity of $(N-R)$ to Y if the employee, X, dies first.

In actuarial terms,

$$N\ddot{a}_x = (N - R)\ddot{a}_x + (N - R)\ddot{a}_{x|y}. \tag{1}$$

The left side of this equation can also be expressed as

$$N\ddot{a}_x = (N - R)\ddot{a}_x + R\ddot{a}_x. \tag{2}$$

Equation (2) says N dollars can be split into two pots, one pot having $(N-R)$ dollars and the other pot having R dollars. Equation (2) holds for all values of R. From (1) and (2), it follows that[1]:

$$R\ddot{a}_x = (N - R)\ddot{a}_{x|y}. \tag{3}$$

Finally, the normal form of the annuity can also be expressed as

[1]In order to generalize the discussion to other than 100% CA options, this expression can be interpreted as $R\ddot{a}_x=k(N-R)\ddot{a}_{x|y}$, with $k=1$. For $k=½$, this represents the familiar 50% CA option.

$$N\ddot{a}_x = (N - R)\ddot{a}_x + R\ddot{a}_{xy} + R\ddot{a}_{y|x}. \quad (4)$$

This expression is less obvious, but it follows from Equation (2) by substituting $(R\ddot{a}_{xy}+R\ddot{a}_{y|x})$ for the term $R\ddot{a}_x$. This substitution says that a life annuity of R dollars to X has the same value as an annuity of R while both X and Y are alive, plus a contingent annuity of R payable to X after Y dies. Equation (4) is true for any value of R.

From (1) and (2) it follows that:

$$(N - R)\ddot{a}_x + R\ddot{a}_x = (N - R)\ddot{a}_x + (N - R)\ddot{a}_{x|y}. \quad (5)$$

From (1) and (4) it follows that:

$$(N - R)\ddot{a}_x + R\ddot{a}_{xy} + R\ddot{a}_{y|x} = (N - R)\ddot{a}_x + (N - R)\ddot{a}_{x|y}. \quad (6)$$

If we clear out equal terms from both sides of Equations (5) and (6), we can see the idea behind pension max more clearly. Pension max is essentially an election to take $(R\ddot{a}_{xy}+R\ddot{a}_{y|x})$ instead of $(N-R)\ddot{a}_{x|y}$. That is, instead of $(N-R)$ payable to the spouse after the employee's death, the employee elects to receive the unreduced annuity N, which for simplicity we separate into $(N-R)$ and R. And R is then mathematically separable into two payment streams. The first payment stream is payable while both X and Y are alive, and the second payment stream is payable to the employee X after the death of the spouse Y. *The reduction, R, "pops up" again if the spouse dies first.* These two modes for receiving the pension benefit have the same value but very different characteristics.

2. DIFFERENCE IN THE FUNDING PATTERN BETWEEN PENSION MAX AND CA OPTION

One important reason for the appeal of the pension max election derives primarily from an unattractive aspect of the CA option. The election of a CA option involves a contract to purchase $(N-R)\ddot{a}_{x|y}$ in exchange for $R\ddot{a}_x$. This derives from Equation (5). *One problem with this election is that the obligation to fund the survivor benefit (that is, the reduction to the normal form* N) *often extends beyond the spouse's death.* This is similar to buying an expensive lottery ticket with a purchase price of $(N-R)\ddot{a}_{x|y}$ on the installment plan, with payments of R payable for life, but with the obligation to continue to make payments after the ticket has been declared a loser. That is, the funding obligation period could extend beyond the economic life of the benefit being purchased. The pension max election clearly avoids this problem.

On the other hand, the main problem with the pension max election is that if the employee dies first, the spouse has no retirement income. To cover this financial exposure, the reduction, R, is used to purchase life insurance on the employee. The use of life insurance instead of the CA option avoids the anomaly of funding the survivor annuity beyond the date of death of the spouse. If the spouse dies first, the employee merely lapses the life insurance policy, and the reduction, R, is effectively restored, or pops up again. This is the first importance difference between pension max and the CA or J&S option, although, as we shall see later, some pension plans have CA options with pop-ups that are designed to eliminate this problem.

Examining the trade-off between pension max and the CA option in the context of Equation (6), we note that Equation (6) provides a technique for quantifying the portion of the CA option purchase price that, on average, remains unpaid or unfunded after the spouse has died.

The contingent annuity of $(N-R)$ is purchased by $R\ddot{a}_x$. This purchase price, $R\ddot{a}_x$, as we saw above, equals the contingent piece of the annuity, $R\ddot{a}_{y|x}$, plus the joint life piece, $R\ddot{a}_{xy}$. The value of the contingent annuity, $R\ddot{a}_{y|x}$, relative to the total purchase price, $R\ddot{a}_x$, or relative to $(N-R)\ddot{a}_{x|y}$, which follows from Equation (3), is a measure of the proportion of the obligation remaining to be paid on the losing lottery ticket. Figure 1 gives some sample values for these pieces for a pair of age/sex combinations.

To help put these observations into better perspective, it is useful to compare the typical CA option with a true pop-up benefit option that is available in some pension plans. In a true pop-up benefit option, the contingent annuity option, $(N-R)\ddot{a}_x+(N-R)\ddot{a}_{x|y}$ from Equation (1), is modified by adding back the reduction R_p if Y dies first, that is, adding back $R_p\ddot{a}_{y|x}$. Thus R_p is calculated so that

$$N\ddot{a}_x = (N - R_p)\ddot{a}_x + (N - R_p)\ddot{a}_{x|y} + R_p\ddot{a}_{y|x}. \tag{7}$$

Recall from Equation (4) that, for all R,

$$N\ddot{a}_x = (N - R)\ddot{a}_x + R\ddot{a}_{y|x} + R\ddot{a}_{xy}.$$

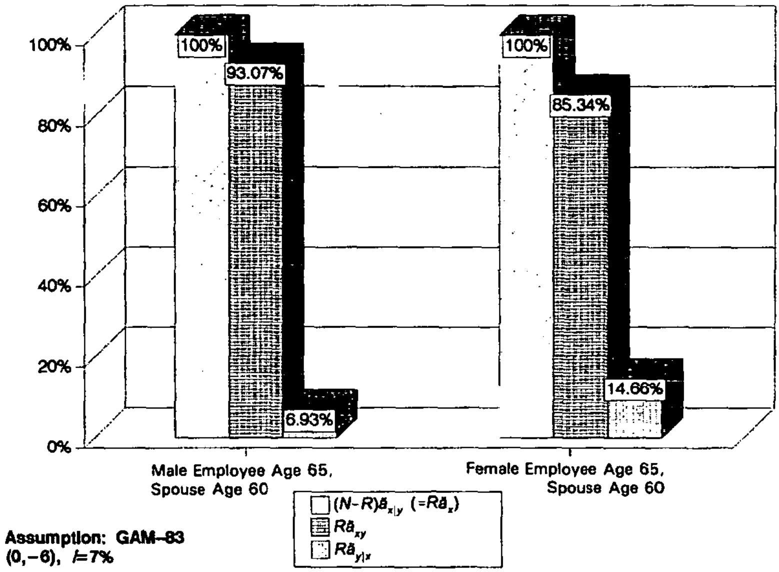
It follows that in a true pop-up, reductions are calculated so that

$$(N - R_p)\ddot{a}_{x|y} = R_p\ddot{a}_{xy}. \tag{8}$$

Equation (8) states that the reduction, R_p, in the pop-up payment mode must be large enough so that, *during the joint lifetime of the employee and spouse*, it will fund the survivor annuity to the spouse. This of course makes

FIGURE 1

DECOMPOSITION OF THE CONTINGENT ANNUITY INTO JOINT LIFE PIECE AND CONTINGENT PIECE FOR A 100% CA OPTION



intuitive sense. The reduction can pop back up only if the contingent annuity has already been paid for at the first death. Because this funding period is shorter on average than the employee's lifetime, the reduction, R_p, to fund the pop-up must be bigger than the reduction to fund the conventional CA option. The reduction factor (that is, the ratio of R_p to N) for a 100% CA option with a true pop-up feature is given by the expression $\ddot{a}_{x|y} \div \ddot{a}_y$. This compares with a reduction factor of $\ddot{a}_{x|y} \div (\ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy})$ for a standard 100% CA (or J&S) election. These ratios are derived from Equations (6) and (8). In the context of the preceding discussion, the election of a true pop-up CA option aligns the funding period with the economic life of the CA benefit option.

Table 1 compares the reductions for a true pop-up election with reductions for a conventional 100% CA option for a small sample of age and sex combinations. Reduction factors increase as the interest assumption decreases. The percentage *extra* cost for the pop-up (or increase to reductions)

TABLE 1

REDUCTION FACTORS FOR 100% CA OPTION*

	Interest = 7%		Interest = 6%		Interest = 5%	
	Regular	Pop-up	Regular	Pop-up	Regular	Pop-up
Male Employee 65/Female Spouse 60	0.2317	0.2447	0.2495	0.2643	0.2688	0.2857
Male Employee 60/Female Spouse 60	0.1591	0.1716	0.1728	0.1874	0.1879	0.2050
Female Employee 65/Male Spouse 60	0.1134	0.1304	0.1215	0.1412	0.1302	0.1532
Female Employee 60/Male Spouse 60	0.0726	0.0864	0.0777	0.0940	0.0832	0.1025

*Assumptions: GAM-83 (0,−6), regular $= \ddot{a}_{x|y} \div (\ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy})$; pop-up$=\ddot{a}_{x|y} \div \ddot{a}_y$; $N=1$.

ranges from just over 5.6% to somewhat more than 23%. The relative cost for the pop-up increases as the interest rate decreases.

As a percentage of the normal form, the extra cost of the pop-up is roughly 2%. In those situations in which the plan does not offer the pop-up annuity form and the CA option is preferred to the pension max option, the purchase of life insurance on the spouse, at a cost roughly equal to 2% of the normal form, would be an alternative way of manufacturing the pop-up.

3. DIFFERENCES IN BENEFIT PATTERNS

Next, we focus the discussion on a comparison of the differences in the benefit streams provided by the CA option and the life insurance benefit in the pension max alternative.

We start with the equivalence, $R\ddot{a}_x=(N-R)\ddot{a}_{x|y}$ from Equation (3). If we now take premium payments of R during the employee's life, with present value $R\ddot{a}_x$, to secure life insurance of F on the employee, with present value FA_x, then it follows from Equation (3) that $FA_x=(N-R)\ddot{a}_{x|y}$. This means that we are mathematically indifferent between the present values produced in the pension max option and the 100% CA option.

This conclusion, however, ignores the mismatch in timing between the benefit pattern provided by the pension plan and the benefit provided by life insurance. The plan provides for life income to the spouse of $(N-R)$, with a present value of $(N-R)\ddot{a}_{y+T}$, if the employee dies at duration T. The life insurance policy provides for level face amount F if the employee dies at duration T, where F is the insurance amount purchased by a premium of R. Figure 2 shows the relative values of these two liabilities at different times, for the special case of a male employee age 65 and a spouse age 60.

All things being equal, the contingent annuity is worth more in the early years, and the life insurance policy is worth more in the later years. The

FIGURE 2
RELATIVE BENEFIT LEVELS AT EMPLOYEE'S DEATH

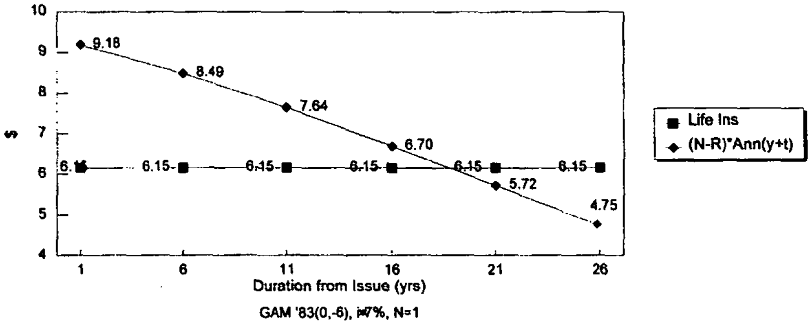


figure demonstrates that even though the present values are the same, the values are not the same at every time. This very crucial difference must be addressed. If this benefit mismatch is acceptable to the customer, then the choice between pension max and the CA comes down to a bet. Pension max wins the bet if the spouse predeceases the employee, *or* if the employee's death occurs after the time, T, when the face amount, F, first exceeds the present value of the surviving spouse's annuity, $(N-R)\ddot{a}_{y+T}$. Pension max loses the bet if the employee predeceases the spouse before time T. The expected value is equal on both sides of the bet. The odds or probability of winning the bet, however, do not have to be equal. The probability of winning the bet is measured by

$$1 - \sum_{t=0}^{T} {}_tp_{xy}q_{x+t}. \tag{9}$$

Table 2 gives some examples of probabilities and crossover years for sample ages. The odds of winning the bet are usually somewhat better than even for couples who are similar in age, and thus appear to be very favorable. To put this observation in perspective, note that for two lives with identical mortality expectations, the odds that one life predeceases the other approach ½ as T in Equation (9) approaches the end of the mortality table. To the extent that the crossover point, T, occurs sooner than that, the probability that one life predeceases the other by time T will be less than ½. The odds of winning the bet, which are equal to 1 minus this probability, thus tend to be better than even.

TABLE 2

100% CA OPTION*

	P†	T‡
Male Employee Age 65/Female Spouse Age 60	0.46	18
Male Employee Age 60/Female Spouse Age 60	0.51	22
Female Employee Age 65/Male Spouse Age 60	0.58	24
Female Employee Age 60/Male Spouse 60	0.66	28

*Assumptions: GAM-83 (0,−6), i=7%.
†Probability that pension max payout exceeds CA payout.
‡Number of years that employee must survive before insurance exceeds CA payout.

The nature of this benefit mismatch has some product implications. It suggests the use of universal life or current assumption whole life or other low-premium contracts rather than high-premium forms, because low-premium contracts buy more initial face amount per dollar of premium, which minimizes the early-duration benefit mismatch. Furthermore, the risk that constant premium of R will ultimately purchase less insurance than initially illustrated is mitigated by the declining insurance need, measured as the present value of the survivor annuity, $(N-R)\ddot{a}_{y+t}$, as duration from issue goes up.

What happens if we want to reduce or eliminate the benefit mismatch and still elect the pension max option?

To eliminate the mismatch, it is necessary to buy decreasing term insurance on the employee's life for each year as long as both the employee and the spouse remain alive. The amount of term insurance purchased each year must be sufficient to purchase an annuity of $(N-R)$ for the surviving spouse, or $(N-R)\ddot{a}_{y+t}$. The present value of this series of annual term insurance purchases can be expressed as

$$(N - R) \sum_{t=1}^{\infty} v^t + p_y \, {}_{t-1}p_x \, q_{x+t-1} \, \ddot{a}_{y+t}.$$

This present value[2] is precisely equal to $(N-R)\ddot{a}_{x|y}$, which is precisely equal to $R\ddot{a}_x$, which in turn is equal to FA_x. That is, the present value of the required sequence of decreasing-term insurance amounts is equal to the present value of the contingent annuity, is equal to the present value of the employee's annuity reduction, is equal to the present value of the level insurance amount, F. These equivalencies can be expressed as

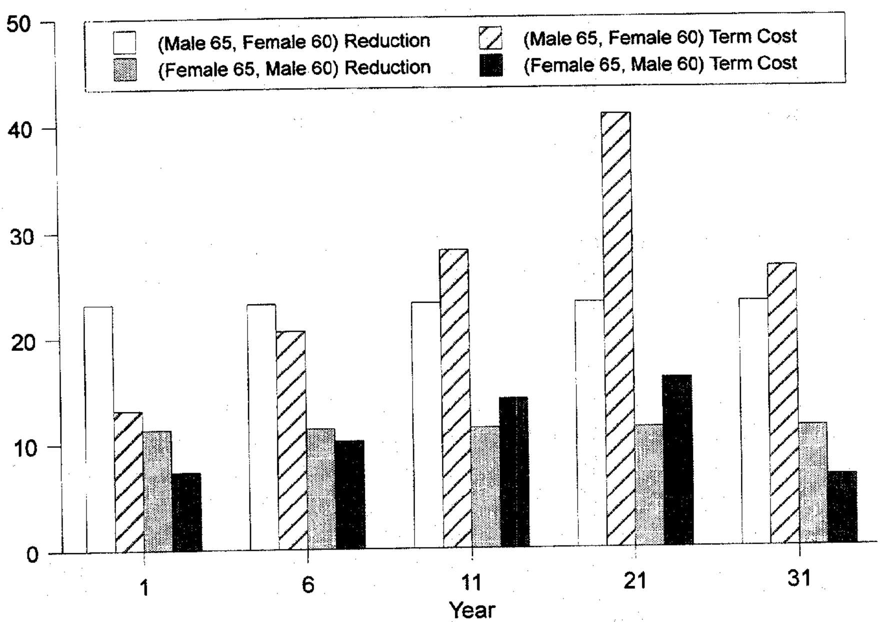
[2]JORDAN, C.W. *Life Contingencies.* Chicago, Ill.: Society of Actuaries, 1952, 231; see also the ensuing discussion by Elias Shiu.

$$R\ddot{a}_x = FA_x = (N - R)\ddot{a}_{x|y} = (N - R) \sum_{t=1}^{\infty} v^t + p_y \, {}_{t-1}p_x \, q_{x+t-1} \, \ddot{a}_{y+t}. \quad (10)$$

We have demonstrated that the benefit pattern under a pension max election with the purchase of annual term insurance of $(N-R)\ddot{a}_{y+t}$ is identical to the benefit provided under the 100% CA option. However, differences in the funding pattern will continue. These funding patterns are illustrated in Figure 3.

FIGURE 3

RELATIVE FUNDING LEVELS BY YEAR FOR TERM COST VERSUS ANNUITY REDUCTION 100% CA OPTION



Assumptions: GAM-83 (0,6), j = 7%, and N = 100

Figure 3 shows that, for these age/sex combinations, term costs start to exceed the reduction, R, in about the eighth year. Note that these term costs are for a decreasing insurance amount equal to $(N-R)\ddot{a}_{y+t}$. Term costs grow significantly higher until about year 20, at which point they decline. The *increase* in the term cost peaks at $1.76 \times R$ for the male employee and at

$1.43 \times R$ for the female employee. This could be problematical if R is the only financial resource available to fund the term insurance. The average outlays for the term insurance appear to be much higher than the reduction, R. A slightly different perspective emerges when the cumulative outlays for these two options are graphed with their associated discounts for mortality and interest.

In Figure 4 we compare the cumulative cost of the reduction R,

$$\sum_{t=0}^{T} R\, v^t\, {}_tp_x,$$

against the cumulative cost of the term insurance,

$$\sum_{t=1}^{T} (N - R)\, v^t + p_y\, {}_{t-1}p_x\, q_{x+t-1}\, \ddot{a}_{y+t}.$$

Another possibility for reducing the benefit mismatch would be to use a contingent first-to-die policy approach.[3] The logic for this thought derives from the following equation:

$$A_x = A^1_{xy} + A^2_{xy}. \tag{11}$$

This equation asserts that life insurance on X is equivalent to the sum of equal amounts of contingent first-to-die insurance on X and contingent second-to-die on X. If we let:

$$A^2_{xy} = rA^1_{xy}, \text{ then } A_x = (1 + r)A^1_{xy}.$$

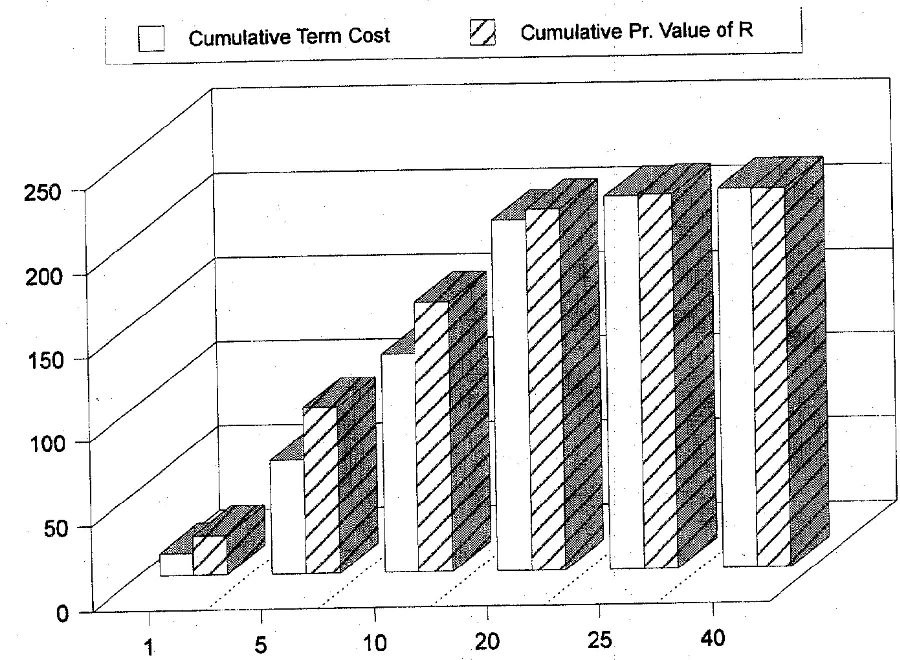
What this tells us is that for each \$1 of life insurance on X, we could purchase \$$(1+r)$ of contingent first-to-die life insurance on X. This would allow us to narrow or perhaps close the early-duration benefit mismatch gap.

For the two combinations, male employee age 65/spouse age 60 and female employee age 60/spouse age 60, the value of r in the expression above is 0.18 and 0.75, respectively, assuming GAM-83 (0,−6), i=7%. If the interest assumption is reduced to 5%, then the value of r increases to 0.21 and 0.96, respectively. For such a product to be useful in practice, nonforfeiture laws might need to be modified to preclude the potential antiselection from selective lapsation in the event death is imminent. The factors above are calculated without regard to such antiselection.

[3]This idea was provided by Cary Lakenbach.

FIGURE 4

CUMULATIVE OUTLAYS FOR 100% CA OPTION, MALE EMPLOYEE AGE 65 SPOUSE AGE 60 DISCOUNTED FOR MORTALITY AND INTEREST



Assumptions: GAM-83 (0,6), i = 7%

Another approach to reducing the benefit mismatch, *often used in practice*, involves the purchase of life insurance five years or more prior to retirement. This allows for the purchase of higher face amounts for the same outlay, R, *from the retirement date forward.* The increase in face amount that is achieved through a prefunding program could be made sufficiently large to close the benefit mismatch. For example, if a male age 65 (spouse age 60) anticipates retirement at age 65 and uses the anticipated reduction (R=\$23.17) to purchase and begin funding life insurance at age 60, the face amount will be 49% greater than the face amount purchased at age 65 with the same premium outlay from age 65 forward.[4]

[4]Assuming GAM-83 (0,−6), i=7%; $23.17\ \ddot{a}_{60}/A_{60}=1.49(23.17\ \ddot{a}_{65}/A_{65})$.

4. DIFFERENCE IN ECONOMIC VALUE

The discussion to this point has ignored expenses and taxes. In addition, all calculations of the trade-offs between pension max and the CA option to this point have been made with identical interest and mortality assumptions.

A. Expenses

The pension max application involves loads for acquisition costs, administrative expenses, and issuer profit margins that the CA option does not. All things being equal, this would give the economic advantage to the CA option rather than the pension max election. The expense loading for life insurance policies can be as high as 35% or more for fully commissioned products. For low-load policies and high-performance units or riders, the expense load can drop to 10% or less.[5] Assume, for the sake of argument, that the fully loaded premium for a specific application can be represented as 110% of the net premiums calculated on an appropriately chosen mortality table. This might be a small price to pay to retain control over the funding pattern or amortization period for the contingent annuity. This will be all the more true in those situations in which the value of the pop-up is large in relation to the entire present value of the annuity R or, as we shall see next, in which the plan reduction, R, exceeds the theoretically correct value for R. On the other hand, where the value of the pop-up is small, the loads may represent a disproportionate price to pay for the additional control and flexibility in the pension max option.

B. Assumptions Used to Calculate Actuarial Equivalence

In practice, the relationship between R and N is governed by actuarial reduction factors defined in the retirement plan document. In qualified plans this relationship is typically independent of sex, health, or the current economic environment. In direct contrast, the insurance amount, F, which is purchased by premiums of R, as well as the present value of the contingent survivor annuity of $(N-R)$, will depend on *exact* age and sex characteristics, as well as prevailing interest rates and the health of the employee and spouse. Table 3 compares correct values for R with plan factors for R, which are

[5]Richard Schwartz estimated the expense component for a variable life policy (nonsmoker, age 45, 12% earned rate, retained for 20 years), as a 1.9% reduction to the earned rate ["The Scoop on Variable Life," *Probate & Property* (Jan.-Feb. 1993): 28].

TABLE 3

100% CA OPTION CALCULATION FOR R USING PLAN ASSUMPTIONS VERSUS CORRECT ASSUMPTIONS*

	Correct R	Plan R
Male Employee 65/Female Spouse 60	0.2317	0.2142
Male Employee 60/Female Spouse 60	0.1591	0.1610
Female Employee 65/Male Spouse 60	0.1134	0.2142
Female Employee 60/Male Spouse 60	0.0726	0.1610

*Assumptions: Correct: GAM-83 (0,−6); 7%; $N=1$
Plan: UP-84; 7%; $N=1$

derived from assumptions not too dissimilar from those used in a typical corporate plan. We use the GAM-83 (0,−6) Table as a proxy for correct economic assumptions. Plan factors are calculated using the UP-84 Table.

In general, if the reduction R defined by the plan provisions is larger than the R that would be calculated under correct assumptions, then the pension max alternative becomes relatively more attractive, because the overstated reduction can be applied to purchase insurance at a fair price. An exception to this rule occurs if an employee is in bad health. In this case, paying a fair (that is, high) price for the life insurance will erode the advantage gained from using the overstated reduction R to fund the life insurance.

In practice, the variance between theoretically correct actuarial reductions and the actual reductions defined according to plan provisions is quite large. In the ten largest annuity contracts administered at John Hancock, plan reduction factors for a 100% CA option range from a high of 0.2796 to a low of 0.1277 for an employee age 65 and a CA age 60, and from 0.1959 to 0.1060 for an employee age 60 and a CA age 60. The interest assumption used to calculate the actuarially equivalent reductions in this sample varies from 5% to 8%. And *the most commonly used convention* for the mortality assumption is to use a larger setback for the CA than for the employee. This means that when the employee and the CA are equal in age, the CA is assumed to have better mortality than the employee. And, *in the special case where the spouse is the CA*, this is comparable to assuming that the spouse is longer-lived than the employee. Another common convention, used in three of the ten plans, is to assume the UP-84 Table, which makes no distinction between the employee and the CA.

In Table 4, we illustrate how differences between the plan R and the correct R can be used as a tool for choosing between pension max and the CA option.

TABLE 4

DECISION TOOL FOR MAKING A CHOICE BETWEEN THE 100% CA OPTION AND THE PENSION MAX OPTION*

	(1) Plan R	(2) $R\ddot{a}_x$	(3) $(N-R)\ddot{a}_{x\|y}$	(4) A_x	(5) F=(3)/(4)	(6) R'=(3)/$\ddot{a}_x$
	Interest Rate = 7%					
Male Employee 65/Female Spouse 60	0.2142	2.0778	2.2985	0.3654	6.2907	0.2370
Male Employee 60/Female Spouse 60	0.1610	1.7450	1.7206	0.2909	5.9144	0.1587
Female Employee 65/Male Spouse 60	0.2142	2.3658	1.1103	0.2774	4.0019	0.1005
Female Employee 60/Male Spouse 60	0.1610	1.9245	0.7852	0.2180	3.6023	0.0657
	Interest Rate = 5%					
Male Employee 65/Female Spouse 60	0.2142	2.3869	3.2202	0.4694	6.8606	0.2890
Male Employee 60/Female Spouse 60	0.1610	2.0458	2.4671	0.3949	6.2473	0.1942
Female Employee 65/Male Spouse 60	0.2142	2.7847	1.5293	0.3809	4.0148	0.1176
Female Employee 60/Male Spouse 60	0.1610	2.3096	1.0927	0.3169	3.4482	0.0762
	i = 7%, Pref. Mort					
Male Employee 65/Female Spouse 60	0.2142	2.3217	1.9412	0.2909	6.6724	0.1791
Male Employee 60/Female Spouse 60	0.1610	1.8977	1.4507	0.2289	6.3380	0.1231
Female Employee 65/Male Spouse 60	0.2142	2.5605	0.9691	0.2180	4.4458	0.0811
Female Employee 60/Male Spouse 60	0.1610	2.0439	0.6891	0.1695	4.0655	0.0543

*Assumptions: Column (1) UP-84, i=7%; column (2)–column (6) first two blocks assume GAM-83 (0,−6); column (2)–column (6) third block assumes GAM-83 (−5,−11); N=1.

In column (1), the reduction R is calculated in accordance with gender-neutral (UP-84) plan provisions, as defined in Table 3. In columns (2) and (3), we use the R from column (1) in conjunction with correct economic assumptions to calculate the present value of the pension max election ($R\ddot{a}_x$) and the CA option $(N-R)\ddot{a}_{x|y}$. (Recall that if we used "correct" economic assumptions to calculate the reduction R, then columns (2) and (3) would have the same values.) In column (4), we show the present value of \$1 of life insurance. In column (5), we solve for the life insurance face amount, F, that is required to deliver the same economic benefit as would be provided by the plan if the CA option in column (3) had been selected. In column (6), we solve for the premium that will be required to buy F.

In general, if column (3) exceeds column (2), then the CA option is a better value than the pension max option, and vice versa. A good measure of the relative advantage or disadvantage between the CA option and pension max is to compare column (6) with column (1). Column (6) is the premium that is required to purchase life insurance with the same economic value as the CA annuity in column (3). *If column (6), after suitable adjustment for expenses and taxes, is less than column (1), then pension max is cheaper.*

The Table 4 calculation is developed for two different correct economic scenarios: 7% and 5%. In addition, a third scenario tests for the sensitivity in the results to lower mortality assumptions, in this case a five-year setback. The plan reduction factor, R, is the same for all three scenarios, UP-84 and $i=7\%$. *In practice, plan reduction factors do not change very often because it is administratively inconvenient and because the accrued benefit anticutback rules preclude reductions to accrued benefits, including optional forms.*

Table 4 allows us to make the following generalizations:

1. All things equal, pension max has a greater chance of success in a high-interest-rate scenario. This can be observed by noting the increase in the ratio of column (6) to column (1) as interest rates drop from 7% to 5%.
2. The tendency, or convention, in a qualified plan to assume that the CA has equal or better mortality experience than the employee will normally result in plan reductions for female employees being larger than the theoretically correct reduction calculation. In such a case, pension max will have a greater tendency to be a good value for female employees than for male employees.
3. All things equal, as mortality rates decline, the present value of $(N-R)\ddot{a}_{x|y}$ declines relative to $R\ddot{a}_x$. Thus, the relative economics of pension max will be improved to the extent that the employee qualifies for preferred underwriting.

C. Taxes

The present value analysis to this point has ignored taxes. To replicate the CA's annuity on a post-tax basis, we need to address three different tax considerations:

1. We need to ensure that the life insurance face amount, F, is not subject to estate or income taxes.
2. In practice, the option to receive the unreduced annuity subjects the entire pension payment, including R_1, to taxes. Thus only $(1-T)R$ is available to buy life insurance.
3. If we want to replicate the periodic income stream of the CA annuity, we need to *invest* the proceeds from the life insurance policy. The investment income that is generated from investing the proceeds may be taxable.

The first issue is easily taken care of. Life insurance proceeds are exempt from income taxes, and the ownership of a policy can be structured to avoid estate taxes.

The second issue is that we only have $(1-T)R$ to spend on life insurance. On the other hand, the contingent annuity, $N-R$, is also subject to taxes. If the tax rate for the contingent annuity, $N-R$, is also T, then the appropriate target amount of life insurance need only be sufficient to provide for an after-tax survivor annuity of $(1-T)(N-R)$. That is, on an after-tax basis, we need to provide $(1-T)(N-R)\ddot{a}_{x|y}$. Since

$$R\ddot{a}_x = (N - R)\ddot{a}_{x|y} = FA_x,$$

it follows that

$$(1 - T)R\ddot{a}_x = (1 - T)(N - R)\ddot{a}_{x|y} = (1 - T)FA_x.$$

This means that the conclusions drawn from the pre-tax analysis above will carry over for the post-tax scenario, provided that the tax rates before and after the employee's death are not too dissimilar.

To replicate the CA annuity, we need to *invest* the proceeds from the life insurance policy. While proceeds from life insurance are not subject to income tax, *the investment income* that is generated from investing the proceeds *is taxable*. Therefore, we need to find an investment with an after-tax rate of return equal to that assumed in the present-value calculation of the CA annuity. If we cannot find such an investment, we need to increase the premium commitment above, R, to maintain the target CA benefit amount. Stated a different way, to avoid increasing the premium commitment, proceeds would have to be invested aggressively. Conversely, a more conservative investment strategy will typically require a higher premium commitment or a lower target annuity amount.

We examine this issue by first looking at a special case with a very conservative investment policy. We assume that the proceeds from the life insurance are invested in tax-free municipal bonds, in which investment income is sheltered from further income taxes. We use Equation (10):

$$R\ddot{a}_x = (N - R)\ddot{a}_{x|y} = \sum_{t=1}^{\infty} v^t + p_y \, {}_{t-1}p_x \, q_{x+t-1} \, (N - R)\ddot{a}_{y+t}.$$

In the pre-tax environment, all calculations are assumed to be done at 7%. If the term insurance proceeds of $(N-R)\ddot{a}_{y+t}$ in the expression above were invested in municipal bonds at 5%, then we would avoid income taxes, but we would need to reduce the CA benefit to maintain the equivalencies in Equation (10). We can see this if we rewrite the last expression in Equation (10) as follows:

$$\sum_{t=1}^{\infty} v^t_{(7\%)} + p_y\ {}_{t-1}p_x\ q_{x+t-1} \left[(N - R) \frac{\ddot{a}_{y+t(7\%)}}{\ddot{a}_{y+t(5\%)}} \right] \ddot{a}_{y+t(5\%)}.$$

The reductions to the CA benefit will range from 15% (17%) for male (female) CAs if the employee's death occurs at the CA's age 60 to 8% (10%) for male (female) CAs if the employee's death occurs at the CA's age 80.

Alternatively, we could increase the funding levels to maintain the same benefit to the CA. If, in the expression above, we use a tax-free municipal bond investment rate of 5% to calculate the present value of an annuity of $(N-R)\ddot{a}_{y+t}$ to the spouse Y at every duration t, this defines a conservative estimate of the term insurance amount, F_t, that will be required at each of those durations to replicate the survivor annuity that would be provided by the plan. We use the expression,

$$\sum_{t=1}^{\infty} v^t_{(7\%)} + p_y\ {}_{t-1}p_x\ q_{x+t-1}\ (N - R)\ddot{a}_{y+t(5\%)} = R^1\ddot{a}_{x(7\%)},$$

to calculate the required premium, R^1, for this series of term insurance amounts. We compare this with the results of a similar calculation,

$$\sum_{t=1}^{\infty} v^t_{(7\%)} + p_y\ {}_{t-1}p_x\ q_{x+t-1}\ (N - R)\ddot{a}_{y+t(7\%)} = R\ddot{a}_{x(7\%)},$$

where we use the pre-tax rate of 7% throughout, to calculate the required premium, R, for the term insurance of $(N-R)\ddot{a}_{y+t(7\%)}$. The ratio of these two calculations applied to R will give a conservative estimate of the additional percentage outlay required under pension max to replicate the qualified plan's benefits on an after-tax basis.

Similarly, we could apply this ratio to the face amount, F, previously calculated, if we wanted to adjust the level-target life insurance face amount to a tax-adjusted basis.

Table 5 quantifies this discussion with some examples. The 15% increase for the female spouse on account of the tax effect makes intuitive sense if we think of the average price sensitivity or duration of a straight-life annuity at these ages as approximately equal to 7.5. This combined with an assumed difference of 2% in the post-tax interest rate would suggest this additional cost and similarly for the male spouse at an average duration of 6.

The discussion above presumes that the employee's pre-tax hurdle rate is equal to the market-based fixed-income rate. This may not be true. To generalize the analysis of the after-tax effect, we need to consider the possibility

TABLE 5

ADJUSTMENT FOR TAXATION OF INVESTMENT INCOME ON LIFE INSURANCE PROCEEDS SET ASIDE TO FUND THE SURVIVOR ANNUITY*

$$(N-R)\sum_{t=1}^{\infty} v^t + p_y\,{}_{t-1}p_x\,q_{x+t-1}\,\ddot{a}_{y+t}$$

100% CA Option	(1) v@7%, $\ddot{a}$@5%	(2) v@7%, $\ddot{a}$@7%	(1)/(2)
Male Employee 65/Female Spouse 60	2.5925	2.2474	1.15
Male Employee 60/Female Spouse 60	1.9779	1.7245	1.15
Female Employee 65/Male Spouse 60	1.4088	1.2527	1.12
Female Employee 60/Male Spouse 60	0.9740	0.8680	1.12

*Assumptions: GAM-83 (0,−6), N=1.

that the employee's pre-tax hurdle rate is different from the pre-tax fixed-income rates used in the analysis so far. The employee's after-tax rate is then determined as a function of this pre-tax hurdle rate. (In this discussion, the calculations have centered around 7%, where 7% is assumed to represent prevailing fixed-income returns.)

Selecting the employee's hurdle rate is not always an obvious exercise. Many individuals would prefer to have cash-in-hand (that is, the investable proceeds from life insurance) to the alternative of a greater present value, payable in periodic installments, as measured by the tax-adjusted market-based fixed-income rate. One reason for such a preference is the greater flexibility of cash-in-hand. Such flexibility could be manifested by investing the cash in opportunities such as a family business, at much higher rates than fixed-income rates, for example. The logic for selecting the employee's hurdle rate is beyond the scope of this paper.

In Table 6 we develop a generalized logic for making a decision on the relative merits of pension max versus the CA option on an after-tax basis. We start with the assumption that the employee's after-tax hurdle rate is 5% and market-based fixed-income returns are 7%. These rates are not necessarily linked, although they can and often will be linked.

R, in row (1), is calculated as before. The values in rows (2) and (3) are calculated at 5%, which is now meant to represent an arbitrary after-tax hurdle rate, specifically selected by our employee. *The ratio of rows* (2) *and* (3) *calculated at 5% is* entered in row (4). This ratio is *a measure of the relative value of the CA option and the pension max option on the after-tax basis selected by the employee.* For comparison, the results of a similar calculation at 7% (from Table 4) are entered in row (5). A quick comparison of rows (4) and (5) demonstrates that the use of lower discount rates (or

TABLE 6

DECISION TOOL FOR MAKING A CHOICE BETWEEN THE 100% CA OPTION AND THE PENSION MAX OPTION USING EMPLOYEE AFTER-TAX HURDLE RATES

	Assumption Key*	Male Employee 65/ Female Spouse 60	Male Employee 60/ Female Spouse 60	Female Employee 65/ Male Spouse 60	Female Employee 60/ Male Spouse 60
(1) Plan R	a	0.2142	0.1610	0.2142	0.1610
(2) $R_{\ddot{a}_x}$	b	2.3869	2.0458	2.7847	2.3096
(3) $(N-R)\ddot{a}_{x\mid y}$	b	3.2202	2.4671	1.5293	1.0927
(4) (2)/(3)	b	0.7412	0.8292	1.8208	2.1137
(5) (2)/(3)@7%	c	0.9040	1.0142	2.1307	2.4510
(6) $F=(3)/A_x$	b	6.8606	6.2473	4.0148	3.4482
(7) $R^1=(6)A_x/\ddot{a}_x$	d	0.2584	0.1677	0.1009	0.0629
(8) $R^2=(6)A_x/\ddot{a}_x$	e	0.1956	0.1298	0.0809	0.0515
(9) $R^3=F\dagger A_x/\ddot{a}_x$	d	0.2317	0.1519	0.1134	0.0726
(10) $R^4=R^1 1.2$	d	0.3101	0.2012	0.1210	0.0755

*Assumptions: $N=1$.
a=UP-84; reduction factor as defined in the plan.
b=GAM-83 (0,−6), i=5%; this represents the employee after-tax hurdle rate.
c=GAM-83 (0,−6), i=7%; this represents the pre-tax fixed income rate.
d=GAM-83 (0,−6), i=7%; this represents the insurer's assumptions.
e=GAM-83 (−5,−11), i=7%; same as d with lower mortality assumption.
†From Table 4, column (5).

hurdle rates) usually causes the present value of the CA benefit to increase by a greater amount than the present value of the annuity reduction, because lower discount rates give more weight to later cash flows. *This implies, all other things equal, that the higher the employee's after-tax hurdle rate, the more likely it is that pension max will be the more economically viable option.*

If $R\ddot{a}_x$ is still greater than $(N-R)\ddot{a}_{x|y}$, assuming the employee's individually selected after-tax hurdle rates, then the previous conclusions about the economic advantage of pension max over the CA annuity do not change. In Table 6, this occurs when the ratio in row (4) is greater than one and applies to the two scenarios in which the employees are female.

If $R\ddot{a}_x$ is less than $(N-R)\ddot{a}_{x|y}$, assuming the employee's after-tax hurdle rates, then a further determination needs to be made to determine whether premiums of R are sufficient to purchase an insurance amount, F, where F satisfies $(N-R)\ddot{a}_{x|y}=FA_x$, calculated using the *employee's hurdle rate.*[6] In other words, F is calculated to provide the same economic benefit as the CA option at the employee's after-tax hurdle rate. This value of F is entered in row (6).

In row (7), we solve for the value of the premium, R^1, which is required, using the *insurer's assumptions*, to purchase the insurance amount, F, which was calculated in row (6). Row (8) is similar to row (7), except that it tests for the premium required, R^2, to purchase F, assuming the insurer uses a better (preferred) mortality assumption. Row (8) is designed merely to test the sensitivity of R to the insurer's mortality assumption. *If* R^1 (or R^2, *if applicable*) *is less than* R [*from row (1)*], *then pension max is cheaper than the CA option on an after-tax basis, and vice versa.*

In row (7) the proxy for the insurer's assumptions is assumed to be GAM-83 (0,−6), $i=7\%$.[7] Life insurance mortality assumptions are often based on multiples of the 1975–80 Basic Table. The GAM-83 (0,−6) Table, which is used for most of this discussion, produces mortality rates roughly equivalent to the ultimate rates from the 1975–80 Basic Table at ages 60 and beyond. Policies sold with preferred mortality are commonly priced at less than 100% of the ultimate rates in the 1975–80 Basic Table. This logic would establish the GAM-83 (0,−6) Table as a conservative standard for judging the relative

[6]Theoretically, this calculation of F also needs to reflect the customized mortality assumption for the employee and the CA, to the extent that it differed from the insurer's mortality assumption.

[7]To put this in perspective, these assumptions produce premiums of \$39.85, \$26.84, \$25.98, and \$18.24 per \$1,000 for insurances on a male 65, male 60, female 65, and female 60, respectively.

economics of pension max for individuals who qualify for preferred mortality rates.

Row (9) is similar to row (7) except that it tests for the required premium R^3 to purchase F assuming a higher employee after-tax hurdle rate (7%). Row (10) adds a 20% loading to the life insurance premiums calculated in row (7). The loading can be tailored to any specific situation.

Each of R^1, R^2, R^3, *and* R^4 *is designed to be tested against the* R *of row (1). The superscripted* R*'s represent the premiums required to purchase life insurance for an amount that provides present value precisely equal to the present value of the CA annuity. As a general rule, when* R^n *is less than* R, *the employee is economically better off using the pension max approach.*

In practice, we would use an actual premium from an actual insurance illustration in making the pension max decision. This requires of course that we have some knowledge about the assumptions underlying the insurer's illustration. By choosing a set of calculation parameters from the illustration system that reflect the employee's (or an advisor's) view of the appropriate mortality, expense and interest assumptions, any desired degree of conservatism can be introduced into the algorithm. In this connection, most illustration systems calculate premiums to reflect any desired interest rate assumption. In the context of Table 6, if the reduction, R, from row (1) is sufficient to purchase, or fund, the equivalent insurance face amount, F, from row (6), as determined through such a unit cost calculation within the insurer's illustration system, then pension max is clearly a viable alternative.

We can also evaluate the tax effect in terms of Table 2, which was used to illustrate the benefit mismatch. The effect of the lower tax-adjusted discount rate is to increase the early-duration benefit mismatch and to extend the time before the face amount of insurance, F, exceeds the value of the survivor annuity. The results of a sample calculation are shown in Table 7.

For the Table 7 calculation, the target insurance amount is the survivor annuity, $(N-R)\ddot{a}_{y+t}$, calculated at the employee's after-tax hurdle rate. The face amount, F, is purchased by annual premiums of R. The reduction, R, is determined by the plan, and F is calculated by using the insurer's assumptions. The insurer's interest assumption in Table 7 is assumed to remain constant at 7%. The employee's hurdle rate is assumed to be 5% post-tax and 7% pre-tax. A comparison of the respective entries at 5% and 7% is a fair measure of the tax effect under these assumptions.

TABLE 7

EFFECT OF POST-TAX INTEREST*

		Insurer Interest Rate 7%; Employee Hurdle Rate 5%		Insurer Interest Rate 7%; Employee Hurdle Rate 7%	
R		T†	P‡	T†	P‡
0.214	Male Employee 65/Female Spouse 60	24	0.31	21	0.38
0.161	Male Employee 60/Female Spouse 60	25	0.44	22	0.51
0.214	Female Employee 65/Male Spouse 60	6	0.92	1	0.98
0.161	Female Employee 60/Male Spouse 60	7	0.94	2	0.98

*Assumptions: The plan reduction factor, R, is calculated assuming UP-84, $i=7\%$, other entries assume GAM-83(0,−6)

†Number of years that employee must survive before insurance exceeds CA payout.

‡Probability that pension max payout exceeds CA payout.

5. WHAT CONCLUSIONS CAN WE DERIVE FROM THIS ANALYSIS?

Before we develop any conclusions, it is important to point out that this analysis has ignored certain other relevant factors that could influence the choice between pension max and the CA annuity option. Cost-of-living provisions were ignored. The analysis is easily extended to include them. Some plans extend eligibility for post-retirement health insurance to contingent annuitants, but do not extend eligibility to surviving spouses of employees who have elected an unreduced benefit. This factor is also ignored. Special plan provisions like these need to be analyzed separately before an informed decision on the pension max election can be made.

Other relevant factors were also deemed to be beyond the scope of this analysis, including the risk of changing tax rates and changing tax laws, as well as the investment risks associated with investing the proceeds of the life insurance policy.

The following conclusions are subject to the above-mentioned caveats:

1. The pension max application is a more flexible funding vehicle and does a better job of retaining control of the retirement annuity asset than the CA option. The design of the funding pattern as well as the benefit pattern can be fully controlled by the employee.
2. The present value of the life insurance purchased by annual premiums equal to the actuarial reduction, R, is equal to the present value of the CA benefit except for expenses and taxes. However, the expenses may be a small price to pay for the additional flexibility and control afforded by the pension max election.

3. There is a tax inefficiency in the pension max election. The inefficiency occurs because investment income from proceeds of life insurance is taxable. This inefficiency is measurable and is quantified in the text. Tax losses due to this inefficiency can be mitigated or eliminated by using aggressive investment policies. (See Tables 5 and 6.)
4. Close attention should be paid to the calculation of the actuarial reduction factors in the plan, to determine how the reduction compares with the correct actuarial reduction determined without regard to qualified-plan nondiscrimination rules, particularly those pertaining to unisex mortality rates. (See Table 4.) The unisex convention for calculating the plan reduction factors tends to make the pension max option more viable for female employees than male employees.
5. A good proxy for deciding whether the pension max election is a good bet or a bad bet is to use the ratio of the contingent annuity, $R\ddot{a}_{y|x}$, to the contingent annuity, $(N-R)\ddot{a}_{x|y}$. (See Figure 1.) This is a measure of the funding obligation that remains on average after the spouse has died. Another tool for deciding whether pension max is a good bet or a bad bet is to calculate probabilities directly from Formula (9). (See also Table 2.)
6. A fundamental challenge in implementing a successful pension max application is to determine how important it is to match the liability exposure created by opting out of the CA option. Decreasing term or universal life with target term capability is a better fit for solving this problem than high-premium/high-dividend policies. *It is fundamentally important, with regard to this objective, that the policy stays in force.*

In conclusion, the pension max option bears some additional costs, but these costs may be a fair price to pay for the additional advantages. One key driver of the relative economics of pension max versus the CA option is the set of actuarial assumptions that are used in the qualified plan to calculate the equivalencies between the benefit options. These assumptions can cause dramatic differences between the economic value of the pension max and the CA option, in both directions.

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APPENDIX

1983 GAM MORTALITY TABLE

Age	Tabular Mortality	Scale H	Age	Tabular Mortality	Scale H
10	0.000293	0.00750	60	0.009158	0.01500
11	0.000298	0.00500	61	0.010064	0.01500
12	0.000304	0.00250	62	0.011133	0.01500
13	0.000310	0.00240	63	0.012391	0.01500
14	0.000317	0.00230	64	0.013868	0.01500
15	0.000325	0.00220	65	0.015592	0.01500
16	0.000333	0.00210	66	0.017579	0.01500
17	0.000343	0.00200	67	0.019804	0.01500
18	0.000353	0.00180	68	0.022229	0.01450
19	0.000365	0.00160	69	0.024817	0.01400
20	0.000377	0.00140	70	0.027530	0.01350
21	0.000392	0.00120	71	0.030354	0.01300
22	0.000408	0.00100	72	0.033370	0.01250
23	0.000424	0.00100	73	0.036680	0.01250
24	0.000444	0.00100	74	0.040388	0.01250
25	0.000464	0.00100	75	0.044597	0.01250
26	0.000488	0.00100	76	0.049388	0.01250
27	0.000513	0.00100	77	0.054758	0.01250
28	0.000542	0.00230	78	0.060678	0.01250
29	0.000572	0.00360	79	0.067125	0.01250
30	0.000607	0.00490	80	0.074070	0.01250
31	0.000645	0.00620	81	0.081484	0.01250
32	0.000687	0.00750	82	0.089320	0.01250
33	0.000734	0.01000	83	0.097525	0.01150
34	0.000785	0.01250	84	0.106047	0.01050
35	0.000860	0.01500	85	0.114836	0.00950
36	0.000907	0.01750	86	0.124170	0.00850
37	0.000966	0.02000	87	0.133870	0.00750
38	0.001039	0.02000	88	0.144073	0.00700
39	0.001128	0.02000	89	0.154859	0.00650
40	0.001238	0.02000	90	0.166307	0.00600
41	0.001370	0.02000	91	0.178214	0.00550
42	0.001527	0.02000	92	0.190460	0.00500
43	0.001715	0.01950	93	0.203007	0.00420
44	0.001932	0.01900	94	0.217904	0.00340
45	0.002183	0.01850	95	0.234086	0.00260
46	0.002471	0.01800	96	0.248436	0.00180
47	0.002790	0.01750	97	0.263954	0.00100
48	0.003138	0.01750	98	0.280803	0.00067
49	0.003513	0.01750	99	0.299154	0.00033
50	0.003909	0.01750	100	0.319185	0.00000
51	0.004324	0.01750	101	0.341086	0.00000
52	0.004755	0.01750	102	0.365052	0.00000
53	0.005200	0.01700	103	0.393102	0.00000
54	0.005660	0.01650	104	0.427255	0.00000
55	0.006131	0.01600	105	0.469531	0.00000
56	0.006618	0.01550	106	0.521945	0.00000
57	0.007139	0.01500	107	0.586518	0.00000
58	0.007719	0.01500	108	0.665268	0.00000
59	0.008384	0.01500	109	0.760215	0.00000
			110	1.000000	0.00000

DISCUSSION OF PRECEDING PAPER

CHARLES L. TROWBRIDGE:

Mr. Shigley brings to our attention some of the intricacies of the J&S and CA optional forms offered under most defined-benefit pension plans. He goes on to describe another way of accomplishing the same general purposes, through what he calls pension max.

Until 1980 I considered myself a pension, social security, and life insurance actuary, with a special knowledge of defined-benefit pension plans. Since then I have followed pension matters less closely, and Mr. Shigley's paper is my first exposure to the pension max concept. I hope readers view the following as comments from an intellectually interested retired actuary with adequate background but with no direct pension max experience.

There is general agreement among students of pensions that the election of some form of J&S annuity, to replace the normal single-life form, should be encouraged whenever the employee reaches retirement with a living spouse. As Mr. Shigley states in his second paragraph, all qualified pension plans must offer at least one J&S or CA alternative, and many offer several. There is no obligation on the retiring employee, however, to choose any such option. The normal form of straight life annuity may be more appropriate when the spouse has an adequate retirement income independent of that of the former employee.

The J&S and CA Options—How Alike and How Different?

As a starting point, consider the full J&S (or CA) option, whereby each \$1 of the normal pension is replaced by a reduced pension of \$$(1-r)$, payable for as long as either x or y is alive; r is easily shown to be

$$r = \frac{\ddot{a}_{x|y}}{\ddot{a}_{xy} + \ddot{a}_{x|y} + \ddot{a}_{y|x}}.$$

Depending on the age and sex of x and y and on both the interest rate and the mortality tables assumed in the calculation of the annuities $\ddot{a}_x$, $\ddot{a}_y$, and $\ddot{a}_{xy}$, Mr. Shigley shows values of r as high as 0.27 and as low as 0.07. Of course the actual range is wider.

Next recognize that many married couples prefer that the pension benefit after the first death be a fraction (¾, ⅔, or ½) of the pension during their joint lifetime. The pension while both are alive can be larger, and living expenses for one person are presumably less than for two. This thinking gives rise to what Mr. Shigley calls the $X\%$ J&S option, of which the most popular is "joint and ⅔." The single-life annuities after the death of either x or y are ⅔ of the amount while both are alive.

Readers should be aware that OASI retirement benefits under present law are *exactly* joint and ⅔ to a retiring couple, *if* the larger PIA is more than twice the smaller. If the PIAs are not that far apart, the result lies somewhere between joint and ⅔ and joint and ½. Note in particular that the benefit after the first death does not depend on which of the couple dies first. In either case it is the larger of the two PIAs.

Finally, there is the CA option, under which the percentage reduction on the first death occurs only if (x) dies first. Other things being equal, a ⅔ CA option will require a greater reduction during y's lifetime, in exchange for a larger pension to the former employee after y has died.

The rationale for CA, rather than J&S, is that the normal single-life annuity (1) arose through the employment of (x) and hence (2) in some sense "belongs" to x, and (3) y should therefore be satisfied with a smaller after-the-first-death pension than x. Were the employee to take a community property view of the normal form pension, the choice would be expected to be J&S rather than CA. I have no information on the popularity of J&S versus CA, but I admit to a personal preference for the former. Of course Mr. Shigley is correct when he points out that the 100% CA and the 100% J&S are identical.

Actuarial Bases and Antiselection

The table of reductions when an optional form is elected can be found in the pension plan itself. It will invariably be based on some mortality table or tables and an interest rate. The mortality assumption will likely be conservative from an annuity point of view (such as GAM-83). The difference between male and female mortality may be recognized, likely through an age setback for females, but as Mr. Shigley has noted, some plans have chosen to use a sex-combined table such as UP-84.

The interest rate is theoretically the rate that the pension plan (or the insurance company if the retired life portion of the pension liability is insured) expects to earn on its investments. The higher the interest rate, the smaller the reduction factor r, and hence the cheaper the reversionary annuities, because the election of any of these options causes the pension to be paid later, and the value of this delay is greater when interest rates are high.

Note that mortality antiselection exists. Employees reaching retirement in poor health can be expected to maximize the pension payable to their spouses after their death. This antiselection is especially powerful if an employee, in poor health but not yet retired, simultaneously elects a 100% CA option and early retirement. At one time it was common to require any J&S or CA form to be elected at least three (or five) years prior to retirement to avoid some of the obvious antiselection. Despite such provisions, some antiselection must be expected. When the spouse is in poor health, as one example, it is very likely that the normal straight-life form will be the choice. Other than setting back the age for (y) and hence calculating the reduction on the assumption that y is in especially good health, there is no practical way of counteracting this less obvious form of antiselection.

The foregoing paragraphs suggest that there is no correct answer on the size of the reductions when an option is selected. The formulas are clear, but the assumptions behind them are not. The variations noted by Mr. Shigley (in Section 4-C) seem large, but they may be par for the course. We could wish that it were otherwise, but this is an imperfect world.

The Rationale behind Pension Max

Unacquainted as I am with the origin of pension max, I can only guess the motivation behind it. It seems to be based on the recognition that the conversion of a part of each r unit of straight-life pension $({}_{x|y}\ddot{a}_x)$ to a survivor income in reduced amount, $(1-r)\ddot{a}_{y+t}$, is technically the same as selling insurance on the life of (x) in a decreasing amount that will provide $(1-r)\ddot{a}_{y+t}$ where death occurs at age $x+t$. Perhaps one of the policies in the life insurance ratebook can be adapted to this use. If so, there is an additional opportunity to sell life insurance.

Two Important Differences between CA and Pension Max

In Sections 2 and 3 of the paper, Mr. Shigley tells us that CA and pension max, while similar in many ways, have important differences. He categorizes these as funding pattern differences and benefit pattern differences. The former revolve around the introduction of a "pop-up" option, the latter around the use of life insurance on x as a source for the reduced retirement income to y.

These two differences are only partially independent. The pop-up can exist outside of pension max, but pension max seems to be impossible without it. This discussion follows the order in which Mr. Shigley presents the two differences and hence treats the pop-up first.

Differences in Benefit Patterns

The formula for r, the dollar amount of reduction (for each \$1 of normal straight-life pension) required to provide a contingent annuity to y after the death of x, can be recast in the form

$$r(\ddot{a}_{xy} + \ddot{a}_{y|x}) = (1 - r)\ddot{a}_{x|y}.$$

In this form we see clearly that a reduction of r, imposed over the (xy) period and any period (y/x) will exactly fund $(1-r)\ddot{a}_{x|y}$, the reduced annuity to y after the death of x. In Shigley's Table 1, r turns out to be 0.2317 for the case of male employee 65/female spouse 60 and 7% interest.

Some of the rationale that Mr. Shigley presents for pension max revolves around the idea that there is something inherently defective with the traditional CA option *if* the contingent annuitant is the first to die. The reduction in pension during the joint life period (xy) continues for the period (y/x), leading Mr. Shigley to state, "One problem with this election is that the obligation to fund the survivor benefit (that is, the reduction to the normal form) often extends beyond the spouse's death." His facts are true enough—actually this will always occur whenever x lives longer than y. *Part of the reversionary annuity to y is being paid for by x's whose y's have already passed away.*

The question remains whether this is a real deficiency or simply one of the characteristics that needs to be recognized. It is true that the spouse annuity is being paid for throughout the life of x, not just the period (xy), and it is tempting to compare this with a losing wager whose loss is spread over a period after the die has been cast. It could just as well be argued that the pop-up feature (no reduction during the period when x is the survivor) is paid for (by the spouse) after x's death has made the pop-up valueless. There are other examples of this phenomenon that we take for granted. One that comes to mind is the loser of a political campaign, who has to pay campaign debts long after the election has been lost.

However one argues the desirability of the pop-up, one need not quarrel with the fact that some employees will prefer it. Any pension plan that chooses to do so can provide the pop-up among its range of options. There is a price, however. The formula for r', the reduction during the periods xy and $x|y$, but not during $y|x$, can be expressed as $r'\ddot{a}_{xy}=(1-r')\ddot{a}_{x|y}$. To avoid the reduction in the $y|x$ period, Mr. Shigley's Table 1 shows us that the reduction during y's lifetime must be 0.2447, instead of 0.2317. For a larger pension during the period $y|x$, the pension during xy and $x|y$ must be smaller.

Incidentally, the natural assumption that the pop-up will prove to pay out more dollars whenever y dies before x is not necessarily true. The critical factor is the ratio of the length of the period xy to the period $y|x$. If this ratio is as much as about 10, the r' modification to the CA option will pay less in total even if x dies first. As the extreme case, if x and y die in a common accident, the pop-up provision will prove to have been an unfortunate choice and especially so if the period xy is long.

There is another form of the pop-up, however, that ensures that if y dies first, the total payout will always be higher than under the usual form of the CA. Instead of a reduction of r' during the period xy, let it be r. The pop-up requires that the reduction during $y|x$ be zero. We can then calculate the necessary reduction r'' during $x|y$, such that $r\ddot{a}_{xy}=(1-r'')\ddot{a}_{x|y}$. As one would expect, r'' turns out to be greater than r', which in turn is greater than r. For the same example that we have illustrated before, r=0.2317, r'=0.2447, and r''=0.2849.

This third pattern of reductions is a continuation of the asymmetrical pattern seen before. With each step from J&S to CA and from r to r' to r'', the pattern is better for x, but worse for y. No pension plan of which I am aware provides a CA option in this r'' form, so why introduce it here? The reason becomes apparent by looking more carefully at how Mr. Shigley formulates pension max.

Differences in Benefit Patterns

Section 3 describes the second matter that distinguishes pension max. An insurance policy on the life of x, with premium r (per \$1 of normal annuity) payable throughout the life of x and with a level face amount F congruent with the insurer's premium structure, can substitute for the $(1-r)\ddot{a}_{x|y}$ that would have been provided under the CA. Presumably the contingent annuitant is the designated beneficiary, and a life income settlement option will be elected.

Mr. Shigley relies on Equation (10) when he states that "the benefit pattern under a pension max election ... is identical to the benefit provided under the 100% CA option." To paraphrase what Equation (10) tells us, "the death benefits from all of the insurance bought and maintained on the lives of all the x's are just sufficient to provide a $(1-r)$ contingent annuity for all the y's who survive their respective x's." Note that the insurance cannot lapse, even upon the death of y, and further that if x dies without a surviving y, the insurance proceeds must somehow be transferred to add to the CAs of those y's who have survived. This is clearly an impossible condition, but one that is necessary if the contingent annuities are to be in an amount $(1-r)$.

One quickly comes to the conclusion that pension max, as applied in the real world, *necessarily* incorporates a pop-up. The premium r may well be paid over the xy period but surely not over $y|x$. As shown earlier, if there is no premium over the $y|x$ period and if the premium during the xy period is r, the present value of the resulting CAs will be only $(1-r'')\ddot{a}_{x|y}$, not the $(1-r)\ddot{a}_{x|y}$ illustrated in Figure 2. If Mr. Shigley were to draw a graph of $(1-r'')\ddot{a}_{y+t}$, in addition to the annuity line that is already shown, we could tell at once how much of the gap between the insurance and annuity lines is caused by the fact that pension max is truly a pop-up coverage.

The rest of the gap between the insurance and annuity lines is simply the result of providing a level insurance coverage (F) to meet a decreasing need $(1-r'')\ddot{a}_{y+t}$. In the modern day of universal life and adjustable life coverages, one would think that a better fit could rather easily be attained. Under one of these flexible arrangements, with the premium r and the initial face set high, x could plan to reduce the face amount at intervals to better fit the value of the reversionary annuity.

Other Reasons for the Mismatch

To this point there has been an implicit assumption that the actuarial assumptions in the calculation of the reductions, and in the establishment of premiums and settlement options, are identical. Mr. Shigley reminds us that this assumption is untrue, for each of several reasons, among them the necessity for the insurance company to load the theoretical net premium for expenses, contingencies, and/or profit.

I have little to add, except to point out one matter that Mr. Shigley has not directly noted. The mortality assumptions underlying the reduction factors in the pension document will likely be those used in the calculation of annuities and hence conservative from an *annuity* point of view, but it would be an unusual insurer indeed who would price an *insurance* coverage on an annuity table.

The Effect of Federal Income Tax

I am not sure that I follow all that Mr. Shigley has to say about income tax effects, but I agree that pension max is tax-wise inefficient. During the *xy* period, taxable income seems to be increased by *r*; after a CA has commenced, it is presumably fully taxable under the usual form of CA, but may be only partially so under pension max. Whether these two effects are actuarial offsets or not, everybody I know, if given a choice, will opt for less tax early even if it means more tax later. The time value of money, in this case interest on the tax itself, is not to be ignored.

Conclusions

The main advantage of pension max, according to Mr. Shigley, is that the employee retains control of the retirement annuity asset, and the arrangement is more flexible. He tells us that the design of the funding pattern as well as the benefit pattern can be fully controlled by the employee. By funding pattern he seems to mean the use or disuse of the pop-up. In this respect I respectfully disagree. Under the usual arrangements included within a pension plan, there is no pop-up, but some plans provide a pop-up option, and others would if they saw a demand. But under pension max, the pop-up is inevitable, and its disadvantages, as well as its advantages, are built in. The most important disadvantage, not really discernible from the Shigley description, is that the CAs are necessarily lower, because the situation while *x* is alive is somewhat better.

Against the claimed advantages, there are (1) the higher "friction" loss under pension max, (2) the tax inefficiency, (3) the complicated mechanics, and (4) the lack of generality. Under (3) I note that pensions are normally payable monthly, so for consistency the insurance should be monthly as well; thus the offset of pension and insurance premium must happen twelve times a year. Under (4) I include (a) the fact that pension max is not applicable to what I consider the preferred form of retirment income, the J&S option of less than 100%, and (b) that when *x* is in poor health and needs the CA election most, pension max is not available because of insurer underwriting requirements.

After careful consideration of Mr. Shigley's presentation, I have one other concern. Will readers be led to the fallacious conclusion that pension max avoids the characteristic of traditional CA that Mr. Shigley finds offensive (which it does), but at the same time delivers the $(1-r)$ level of CA (which it does not)?

L. TIMOTHY GILES:

This is a fine paper dealing with an exciting topic.

I first encountered this challenge from a marketing department about 10 years ago. My solution was a reversionary annuity. Jordan's *Life Contingencies** devotes an entire chapter to it. I think it was approved in one state. I do not remember whether any were sold. The president of that company, not an actuary but quite astute nonetheless, rejected the possibility of a solution on the grounds that the pension plan factors were on a net basis, whereas any premiums in the company would most certainly be loaded for expenses. At that time I did not have access to the actual amount of typical reductions, an area that Mr. Shigley covers very convincingly. Clearly, any opinion of pension max ought to respond to a specific reduction.

One neat feature of a reversionary annuity is that it is exempt from nonforfeiture benefits. An unhealthy annuitant would choose cash surrender before the policy expires valueless at death. A possible disadvantage is that the annuity payments might be taxed with the investment in the contract being only the premiums paid, instead of the death benefit.

The particular aspects of a couple's situation are also very important. If you are not completely sure that this is the last spouse you will ever have, you will be more receptive to the purchase of pension max. You can always lapse it. Maybe the spouse has a pension and a partial replacement would suffice.

I do not know why reversionary annuities are not available today. The author's solution of decreasing term or level term with an annual partial lapse is more tailored than whole life, but requires researching the current market for annuities that fluctuate with interest rates. The designer of a reversionary annuity has to cast it into the form of annual renewable term with a life annuity as the death benefit. At what interest rates should these annuities be calculated?

I congratulate Mr. Shigley for rekindling an old flame and for providing an insightful analysis.

ELIAS S.W. SHIU:

This paper presents interesting applications of multiple life theory. I hope its results will be incorporated in the syllabus of the Life Contingencies Examination. I have one question. The paper seems to calculate insurance

*JORDAN, C.W. *Life Contingencies.* 2nd ed. Chicago, Ill.: Society of Actuaries, 1982.

values and annuity values with the same mortality table. When an annuitant elects the "pension max" option in the real world, is the *insurance* premium determined by an *annuity* mortality table?

The series expression in Equation (10) of the paper is incorrect because ${}_tp_x\, q_{x+t}$ is the probability that (x) will die in policy year $t+1$, while $\ddot{a}_{y+t}$ is the value of a life annuity starting at time t (which is the beginning of policy year $t+1$ and before the death occurs). It may be useful to point out that the reversionary annuity-due, $\ddot{a}_{x|y}$, is the same as the reversionary annuity-immediate, $a_{x|y}$. One way to see this relationship is as follows:

$$\begin{aligned}\ddot{a}_y - \ddot{a}_{xy} &= (1 + a_y) - (1 + a_{xy})\\ &= a_y - a_{xy}.\end{aligned} \tag{D.1}$$

(Jordan [3], [4] seems to avoid using the reversionary annuity-due symbol.) If we can assume that (x) and (y) are independent lives (even though they are husband and wife), then

$$a_{x|y} = \sum_{j=0}^{\infty} {}_{j|}a_y\ {}_{j|}q_x.$$

Hence

$$\begin{aligned}\ddot{a}_{x|y} &= a_{x|y}\\ &= \sum_{j=0}^{\infty} {}_jE_y\ a_{y+j}\ {}_{j|}q_x\\ &= \sum_{j=0}^{\infty} {}_{j+1}E_y\ \ddot{a}_{y+j+1}\ {}_{j|}q_x\\ &= \sum_{j=0}^{\infty} v^{j+1}\ {}_jp_{xy}\ p_{y+j}\ q_{x+j}\ \ddot{a}_{y+j+1},\end{aligned} \tag{D.2}$$

which is the correct series expression for Equation (10). Unfortunately, the numerical values in the paper calculated according to Equation (10) need to be redone. Formula (D.2) is the same as Exercise 1 on page 231 of Jordan [3] and Exercise 1 on page 265 of Jordan [4].

Suppose that the pension max option is elected. The face amount of the whole life insurance on (x) purchased by an annual premium R is, of course, R/P_x. If (x) predeceases (y) *and* (y) survives to the end of the year of death of (x), then an annuity for (y) is purchased by the death benefit R/P_x paid at the end of the year of death of (x). Let T be the integer such that

$$(N - R)\ddot{a}_{y+T+1} > R/P_x \geq (N - R)\ddot{a}_{y+T+2}.$$

(The integer T here is not exactly the same as the one in the paper.) If (x) predeceases (y) before time $T+1$ *and* (y) survives to the end of the year of death of (x), then the annual payment of the annuity is less than $N-R$. The probability of this event of "losing the bet" is

$$\sum_{j=0}^{T} {}_jp_{xy}\, p_{y+j}\, q_{x+j}. \tag{D.3}$$

The sum (D.3) is not the same as the sum in Equation (9) in the paper; there is the extra term p_{y+j}, which gives the probability that the spouse survives to the end of the year of death of the employee.

Perhaps it is of pedagogical value to reformulate some of the results in the paper. Because the 100% CA option and the 100% J&S option are the same, Equation (1) in the paper can be written as

$$N\, \ddot{a}_x = (N - R)\ddot{a}_{\overline{xy}}. \tag{D.4}$$

Dividing (D.4) by N and rearranging yields

$$\begin{aligned} \frac{R}{N}\ddot{a}_{\overline{xy}} &= \ddot{a}_{\overline{xy}} - \ddot{a}_x \\ &= \ddot{a}_y - \ddot{a}_{xy} \\ &= \ddot{a}_{x|y}. \end{aligned} \tag{D.5}$$

Hence

$$\frac{R}{N} = \frac{\ddot{a}_{x|y}}{\ddot{a}_{\overline{xy}}}, \tag{D.6}$$

numerical values of which can be found in Table 1 of the paper.

The formula corresponding to (D.4) for the true pop-up benefit option is

$$N\, \ddot{a}_x = (N - R_p)\ddot{a}_{\overline{xy}} + R_p\, \ddot{a}_{y|x}. \tag{D.7}$$

I think it is clearer to write R_p instead of R as in the paper, because we should distinguish it from the R in (D.4). Equation (D.7) is the same as (7) in the paper. Dividing (D.7) by N and rearranging yields

$$\frac{R_p}{N}(\ddot{a}_{\overline{xy}} - \ddot{a}_{y|x}) = \ddot{a}_{\overline{xy}} - \ddot{a}_x. \tag{D.8}$$

Because

$$\ddot{a}_{\overline{xy}} - \ddot{a}_{y|x} = \ddot{a}_y$$

and

$$\ddot{a}_{\overline{xy}} - \ddot{a}_x = \ddot{a}_{x|y},$$

we have

$$\frac{R_p}{N} = \frac{\ddot{a}_{x|y}}{\ddot{a}_y}, \tag{D.9}$$

numerical values of which can also be found in Table 1 of the paper.

On the other hand, if we formulate the equation for the true pop-up benefit option as

$$N\,\ddot{a}_x = (N - R_p)\ddot{a}_y + N\,\ddot{a}_{y|x}. \tag{D.10}$$

Then (D.9) follows from the identity

$$\ddot{a}_y - \ddot{a}_x + \ddot{a}_{y|x} = \ddot{a}_{x|y}. \tag{D.11}$$

My next remark is motivated by the expression

$$A_x = (1 + r)A^{1}_{xy} \tag{D.12}$$

in the paper. In the very special situation where (x) and (y) are independent lives and each has a constant force of mortality,

$$\mu_{x+t} = \mu_x \tag{D.13}$$

and

$$\mu_{y+t} = \mu_y \tag{D.14}$$

for all $t \geq 0$, one can show that in the continuous case

$$r = \frac{\mu_y}{\mu_x + \delta}. \tag{D.15}$$

Furthermore, suppose that we weaken conditions (D.13) and (D.14) as

$$\frac{\mu_{y+t}}{\mu_{x+t}} = \frac{\mu_y}{\mu_x} \tag{D.16}$$

for all $t \geq 0$. This condition is satisfied if the mortality for both lives follow Gompertz's law with the same parameters B and c. Under (D.16) and the independence assumption we have

$$A_{xy} = \left(1 + \frac{\mu_y}{\mu_x}\right) A^{1}_{xy}. \tag{D.17}$$

To verify (D.17), observe that (D.16) implies

$$\mu_{x+t} = \frac{\mu_x}{\mu_x + \mu_y} (\mu_{x+t} + \mu_{y+t}).$$

Hence

$$\begin{aligned} {}_tp_{xy}\ \mu_{x+t} &= \frac{\mu_x}{\mu_x + \mu_y}\ {}_tp_{xy}(\mu_{x+t} + \mu_{y+t}) \\ &= \frac{\mu_x}{\mu_x + \mu_y}\ {}_tp_{xy}\ \mu_{x+t:y+t} \end{aligned}$$

because of the independence assumption. Let $\lceil t \rceil$ denote the least integer greater than or equal to t. Then

$$\begin{aligned} A^{1}_{xy} &= \int_0^\infty v^{\lceil t \rceil}\ {}_tp_{xy}\ \mu_{x+t}\, dt \\ &= \frac{\mu_x}{\mu_x + \mu_y} \int_0^\infty v^{\lceil t \rceil}\ {}_tp_{xy}\ \mu_{x+t:y+t}\, dt \\ &= \frac{\mu_x}{\mu_x + \mu_y} A_{xy}, \end{aligned}$$

which is (D.17). Furthermore, replacing $\lceil t \rceil$ by $\lceil mt \rceil / m$ in the derivation above yields the more general formula

$$A^{1(m)}_{xy} = \frac{\mu_x}{\mu_x + \mu_y} A^{(m)}_{xy}$$

if (x) and (y) are independent lives and (D.16) holds.

I would like to point out a related paper by Jacka [2], who considers the problem of a trustee faced with investing a sum of money, the interest from which will be received by one party (the life-tenant) during his lifetime, while the capital will go to another party (the survivor) on the death of the life-tenant. Jacka assumes that there are $n+1$ assets in which the trustee may invest—n risky assets of geometric Brownian motion type and one nonrisky asset. Under assumptions about the utility functions of the two parties, he finds the collection of Pareto optimal investment strategies for the trustee together with the corresponding payoffs.

Let me conclude this discussion with an annuity story, as told by the late Dr. Bill Greenough, who was chairman and CEO of TIAA-CREF from 1963 to 1979. Below is from the section entitled "Actuarial Adversities" on pages 31 and 32 of his book [1].*

> A serious problem for the new company was that actuaries in 1918 did not know how long people were going to live in the 1920s, 1930s, and 1980s. Yet the fledgling association issued guarantees reaching that far ahead. It is hard to believe annuities were in their infancy in 1918 when TIAA started. President Pritchett selected, with the help of staff, the McClintock Annuity Mortality Table, 4 percent interest, and no provision for expenses, as the long-term actuarial assumption to calculate annuity rates on which to base guaranteed lifetime income payments.
>
> McClintock's table was the only major annuity mortality table available. Published in 1899, it was based on the annuity experience of 15 American companies before 1892. This table was retrospective in that it was not adjusted to reflect improved mortality rates for each succeeding generation. Actuaries had not yet begun to adjust mortality tables to project how long people were going to live decades in the future, as longevity increased because of such things as medical advances and improving sanitary conditions. And TIAA used the McClintock table from 1918 to 1928, 30 to 40 years after the experience on which it was based.
>
> The second factor of major financial importance was the guaranteed interest rate. TIAA's founders chose 4 percent because prevailing interest rates had always been above that level. And nothing had to be added to the rates for expenses; the Carnegie Foundation would pay all operating expenses.
>
> Pritchett then asked both existing actuarial societies to give advice as to the appropriateness of the new rates. The American Institute of Actuaries said the rates provided "ample financial security." The Actuarial

Society of America suggested the "mortality among college professors may be lower than the McClintock table, thereby creating a loss," but investment interest earnings above 4 percent would easily take care of any deficiency in mortality rates.

Were they ever wrong! Each of the actuarial factors chosen caused trouble, not immediately but within 20 to 30 years. Errors in annuities show up very slowly. The annuitants lived a good deal longer than the table said they would; interest rates fell to below 3 percent; and TIAA grew so rapidly the Carnegie Foundation could not forever pay all of its expenses.

The final result of the choice of inadequate annuity rates for the start of TIAA was unintended but not all that bad. What transpired was a gradual transition from wholly free to wholly financed pensions for the colleges, instead of the intended rapid change. As mentioned, Carnegie Corporation provided the initial capital of $1 million. In 1938, when TIAA and Carnegie Corporation separated, the corporation made additional grants of $6.7 million. And finally, from 1948 to 1958, Carnegie Corporation provided an additional $8.75 million to strengthen the longevity, interest, and expense provisions underlying the original contracts. Exit the free, enter the funded, but oh so slowly!

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ROBERT B. LIKINS:

I thank Mr. Shigley for this timely paper on an important subject. I have also been considering how a prospective buyer could make the choice between taking the higher single-life pension and buying life insurance for face amount L costing premium P on the pensioner's life (pension max) versus taking a lesser contingent spouse annuity (CA) to provide a continuing benefit for the pensioner's spouse. This discussion provides my own thoughts and suggests an analytical tool for evaluating these choices.

The analytical tool is useful for two reasons. First, the prospect here would be giving up the right for a CA, and this type of give-up is not a normal part of a life insurance purchase decision. Second, the pension max and the CA option are more complex to compare than they seem to be.

I simplify my discussion by considering only what the paper calls the $X\%$ CA. By doing this, R is the amount of the actuarial reduction that takes place at retirement for the life of the pensioner. I also assume that $X\%$ is 100%, so the pensioner's spouse gets just what the pensioner was getting before the pensioner died, that is, $N-R$.

This discussion covers the following:

- The comparison between the CA and pension max
- Pension max analysis
- Adequate pension max life insurance
- Margin in the analysis tool
- Additional intricacies of pension max
- Assumptions for use in pension max analysis
- Matrix analytical tool to select likely prospects for pension max
- Conclusion.

The Comparison

As the paper points out, pricing for the pensioner's CA option and the pension max life insurance is not done by using the same assumptions. And the life insurance face amount, L, that is bought does not precisely match the life insurance embodied in the CA option because it is not available as a policy. So the decision about the pension max approach is not as easy as just comparing the pension reduction, R, to the pension max life insurance premium, P.

A significant concern is the potential mismatch in benefits between what the spouse could have gotten with the CA and would get with the pension max life insurance. Mr. Shigley calls this "a bet," and I agree. If a pension max sale is for less than an adequate amount of life insurance (to fully replace the desired CA benefit of $N-R$) at some point after the pensioner starts his or her pension, then the pension max approach includes a bet.

The following is notation I have used:

CA = Spouse payments of $N-R$, for the 100% CA situation, beginning at the pensioner's death.

R = The reduction in the pensioner's annuity when he or she selects a CA for his or her spouse.

L = The face amount of life insurance needed to provide an after-tax spouse annuity equivalent to $(N-R)$ (1 −postretirement tax rate) right after the pensioner retires, at the spouse's age y. This will be a larger face amount that F, which provides a spouse annuity less than this and more than zero if purchased at the spouse's age y. L would be [$(N-R)$ times (the current immediate annuity rate for a spouse age y) times (1 −postretirement tax rate)] plus (the taxes to be paid on the gross annuity payments to the spouse at the postretirement tax rate, but only on the nonexcluded part of the payments where the exclusion ratio is the cost of the payments, L, divided by the IRC expected amount of the payments).*

P = Premium for pension max life insurance of face amount L.

By establishing L in this way we eliminate the bet mentioned in the paper that exists if the pensioner dies too soon. L is conservative because it provides more than the needed funds for the spouse annuity once the pensioner survives for some time after retirement. This is shown graphically in Figure 1, where F is not sufficient to buy the full CA at retirement of [$(N-R)$ times (immediate annuity rate for spouse age y), adjusted for taxes].

Pension Max Analysis

One way to make the pension max decision is to compare the present values of the cash flows in each approach. This would provide a comparison of cash flows, after income taxes and the time value of money have been considered. We calculate and compare to find which has a larger value, (a) the pension plan's contingent spouse annuity (CA) option or (b) the pension max approach.

(a) The CA option contains: the smaller $N-R$ pension benefit plus the residual surviving spouse benefit of $N-R$

*Using actuarial notation and assuming the exclusion ratio applies to all the annuity payments:

$N-R$ = gross CA payments

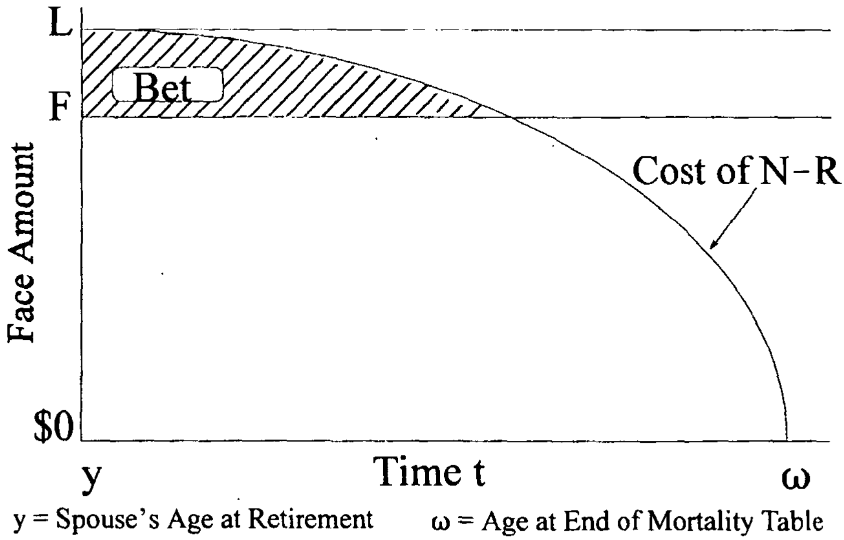
TR = postretirement tax rate

$\ddot{e}_y$ = IRC expected number of annual payments

$\ddot{a}_y$ = Current immediate annuity cost for 1 per year

$$L = \frac{(N-R)(1-TR)}{\dfrac{1}{\ddot{a}_y} - TR\left(\dfrac{1}{\ddot{a}_y} - \dfrac{1}{\ddot{a}_y}\right)}$$

FIGURE 1
THE BET USING FACE AMOUNT *F*



(b) The pension max approach contains: the larger *N* single-life pension benefit plus income from the life insurance death benefit, *L*, less the life insurance premium payments, *P*.

The analysis I describe here assumes that the pensioner lives until life expectancy, determined at his or her retirement age, and that the spouse lives that long and to her or his life expectancy determined at the end of the retiree's life expectancy. The analysis covers the time from now until the end of the spouse's life expectancy. Fixing the order of death dictates the need for a spouse income for financial security.

When the pension max life insurance is purchased before retirement, life insurance is bought for the face amount, *L*, from preretirement up to retirement. This benefit is not explicitly provided by the retiree's pension plan. Therefore, even though it adds value, in this analytical tool it is not included in the calculation comparing the pension max to the CA option, because the pensioner can still make the CA selection at the pensioner's actual retirement some years later, after this part of the death benefit coverage of the life insurance has passed.

In each present value below, n is the number of years from the date of the valuation to the date of the cash flow payment.

(a) For the CA option, the present-value cash-flow stream is:

(1) Zero to the pensioner prior to retirement age
$= 0$

(2) $N-R$ from retirement age until the pensioner's death

$$= (N - R)(1-\text{postretirement tax rate}) \times \left[\frac{1}{(1 + \text{interest discount rate})}\right]^n$$

(3) CA spouse benefit from the pensioner's death until the spouse's death, $N-R$ in the 100% CA situation

$$= (N - R)(1 - \text{postretirement tax rate}) \times \left[\frac{1}{(1 + \text{interest discount rate})}\right]^n$$

(b) For the pension max approach, the present-value cash-flow stream is:

(1) Negative in an amount equal to the out-of-pocket life insurance premium, P, from the point of sale to retirement. This P is the premium that buys the needed amount of life insurance, L, so that the desired spouse annuity $N-R$, adjusted for taxes, can be purchased if the pensioner dies just after retiring. P can be reduced by policy dividends. If a COLI is involved, the L could be largest at a time several years after retirement

$$= -P\left[\frac{1}{(1 + \text{interest discount rate})}\right]^n$$

(2) N, the single-life pension benefit, minus P, the premium for life insurance that must be paid out of pocket (P can be reduced by policy dividends) from retirement age until the pensioner's death.

$$= [N(1 - \text{postretirement tax rate}) - P] \times \left[\frac{1}{(1 + \text{interest discount rate})}\right]^n$$

(3) For consistency with the CA being given up, only an amount equal to that in (a)-(3) above. There is no comparable CA

pension benefit to the pension max additional spouse benefit that results from the excess postretirement death benefit coming from the decreasing cost of the spouse annuity as the surviving spouse ages. Note that getting (a)-(3) size annuity payment requires a smaller gross annuity payment than $N-R$ because tax is paid only on the nonexclusion ratio part of the annuity payment.

$$= (N - R)(1 - \text{postretirement tax rate}) \times \left[\frac{1}{(1 + \text{interest discount rate})}\right]^n$$

These cash flows can be valued for a variety of situations. If the purchase of life insurance is done at retirement rather than before retirement, then (b)-(1) is zero.

If the pension max is purchased before retirement but the policyowner wants to know the comparison at retirement age before making the CA decision, then in place of (b)-(1) the policy's cash surrender value is used as a negative amount (cost) because this value is left in the policy so that we can retain the policy's use in the pension max approach (something like a drop-in premium to get this policy to where it is). If the pension max approach is going to be used along with other, existing in-force policies, then the new policy's face amount is simply the amount needed, L, less the face amount(s) of the existing policy(ies); the cash flows (b)-(1) and (b)-(2) would be as follows.

(a) Negative in an amount equal to (i) the out-of-pocket life insurance premium for the new policy and for the in-force policy(ies) being considered in the pension max analysis, from the point of sale of the new policy to retirement and (ii) the in-force policy(ies)'s cash surrender value(s) at the point of sale of the new policy.

(b) Same as (b)-(2) above but including the out-of-pocket premiums for all new and in-force policies being considered in the pension max analysis.

Practical questions on using in-force policies in a pension max analysis are: Can it be economically built into the analysis tool/illustration system? Can another company's in-force policy be part of the analysis? Has the original need for the in-force policy been satisfied? Would an analysis using in-force policies inappropriately encourage the financing of one policy with the values of another?

The discount rate and other assumptions and techniques are described under "Assumptions."

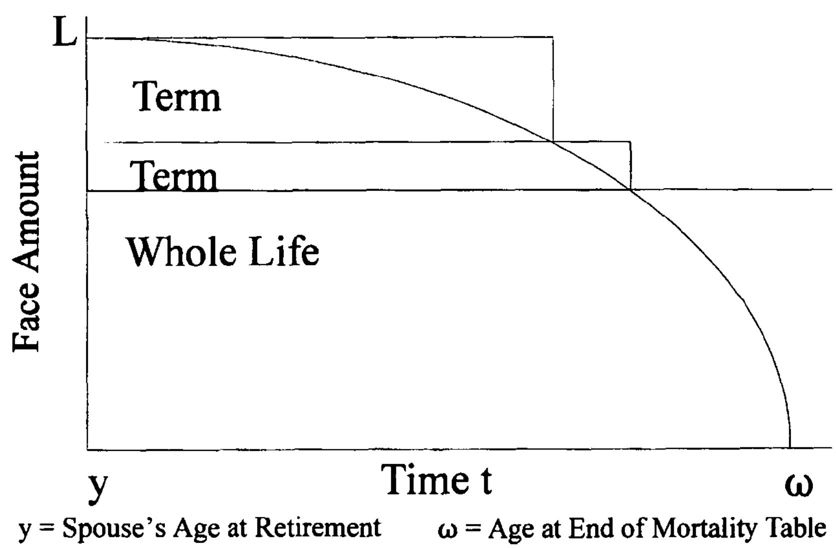
Adequate Pension Max Life Insurance

There are several ways of providing an adequate Pension Max life insurance death benefit when the pensioner retires. First, whole life insurance can be purchased at retirement for premium P in an amount L large enough to fully replace what would have been provided to the spouse in the CA option immediately after the pensioner's retirement, at the spouse's age y. This option is shown in Figure 1.

The second option would be to provide term insurance in addition to a smaller amount of whole life insurance in such a way that their combination provides for an adequate spouse annuity. The term insurance can be reduced at several year intervals to somewhat follow the curve of the amount needed to purchase the contingent spouse annuity while always staying above that amount. This option is seen in Figure 2.

FIGURE 2

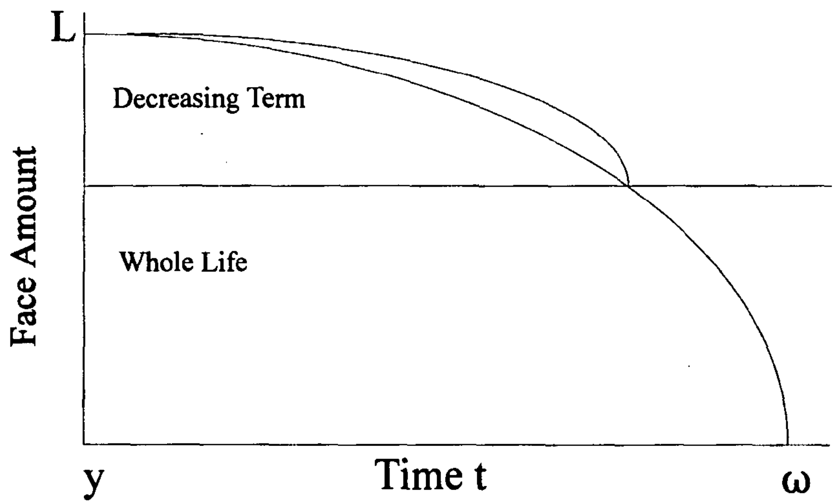
ADEQUATE TERM AND WHOLE LIFE INSURANCE



A third option is to have decreasing term insurance that closely follows the decrease in the contingent annuity purchase rate curve going down to a modest amount of whole life insurance; see Figure 3.

FIGURE 3

ADEQUATE DECREASING TERM AND WHOLE LIFE INSURANCE



y = Spouse's Age at Retirement ω = Age at End of Mortality Table

And finally, term insurance for precisely the face amount needed to buy the contingent annuity would be the most comparable amount to purchase, without excess, as described in the paper. This is simply the decreasing curve shown in the several previous figures.

And as Formula (11) points out, even this is more insurance than would be provided by the CA (assuming the pension plan does not provide a pop-up feature for the pensioner's reduced $N-R$ benefit, back up to N if the spouse predeceases the pensioner), because it provides A^{2}_{xy} when the pensioner dies second, in addition to the CA benefit of A^{1}_{xy}, which pays off when $\ddot{a}_{x|y}$ starts paying and only pays when the pensioner dies first.

Margin in the Analysis Tool

I suggest that a margin be used in the CA versus pension max cash-flow comparison. The prospect might consider the economics of the pension max approach when the present values are close because there are benefits in the overall pension max approach that are not quantified in the cash-flow comparison I have described. The prospect should consider the purchase of the pension max approach if the present value of the pension max cash flow is at least, say, 90% as large as the present value of the CA cash flow. This 10% margin recognizes that:

- When the pension max life insurance is purchased before retirement, this analysis ignores the value of the death benefit before retirement.
- The pension max whole life insurance is used at its initial face amount, L, but will likely have a larger death benefit than that if it is participating.
- The pension max insurance is level or steps down after retirement, but as long as it provides insurance equal or greater than the decreasing CA benefit, it provides insurance greater than the amount needed to buy the annuity for the spouse with a decreasing life expectancy.
- There is value in the pension max life insurance if the spouse predeceases the pensioner and there is no value in the CA benefit in this situation.

Furthermore, it would be reasonable to vary the margin to be, say, 5% below the total CA cash flows for a 50% CA and, say, 10% below for a 100% CA. And when term insurance is used in the pension max approach, the margin could be reduced to recognize that a significant part of the nonrequired death benefit from level face amount whole life insurance will not be present and the term insurance has no cash value if the spouse predeceases the pensioner.

It would be possible to not use a simple margin but instead to value all the extra benefits that are not part of the CA option. Then the present value of the CA cash flow could be compared to (1) the present value of the pension max cash flow without the "extras" and to (2) the present value of the pension max cash flow with the "extras."

Additional Intricacies of Pension Max

Vanishing-Premium Payment Approach

An option in pension max is to buy a participating policy and accumulate the dividends as paid-up additions under the policy with the intent of discontinuing premium payments, P, in the future and letting those payments be made by the accumulated and future dividends. The vanishing-premium concept introduces a nonguaranteed element into the premium payment cash flow because dividends are not guaranteed.

Spouse Contingent Annuity Tied to Health Benefits

Another situation arises when health insurance for the spouse is provided only after the pensioner's death if the spouse is receiving a CA provided by the pension plan. If the pensioner can select the amount of the contingent annuity (100%, 75%, 50%, and so on), then the pensioner can select a CA, to allow the spouse to obtain the postretirement health insurance, but a smaller CA percentage than desired, with the remainder of the desired spouse annuity being provided by the pension max approach if it is more economical. In this case the pension max analysis would be done simply by using the amount of spouse annuity to be provided by life insurance rather than the entire desired CA benefit.

COLI Adjustment

While the paper mentions that it would not be particularly difficult to figure the cost-of-living increases (COLI) into the pension max comparison, this is likely to involve assumptions and risk. The comparison could require assumptions about (1) how often and (2) in what amounts the COLIs will occur. Several decades of COLIs could be provided after retirement to a pensioner and subsequently to a CA. If a sufficient amount of life insurance in the pension max approach is purchased to provide for the COLIs, which will depend upon the time of death, then the spouse will be left with an adequate annuity. But the spouse is unlikely to be able to buy an annuity that contains COLIs, so future increases would also need to be considered after the death of the pensioner if the CA is eligible for post-pensioner's death COLIs.

Possibly most of the COLIs can be covered in the pension max approach by purchasing level face amount insurance for the whole of life. If COLIs are provided only as long as the pensioner is alive, if the CA pension benefit is determined at the pensioner's death, and if the COLIs are 3% per year, then the decrease in the cost of the spouse's annuity is quite close to the 3% compound increases in the COLI pension benefit, so level whole life insurance of L provides good coverage. For larger COLIs or COLIs to the spouse after the pensioner's death, the life insurance face amount would need to be larger than the L described in this discussion.

Administrative Systems

It is helpful if the administrative systems are able to:

- Record the basis of sale as pension max.
- Record the basis of sale as vanishing premium, if applicable.
- Record the expected retirement date.
- If there is a change in underwriting class at issue from what was used in the analysis, request that a new analysis be performed.
- Implement communications for policy actions that will affect the policy's ability to fulfill its purpose, for example, for the vanishing premium life cycle (that is, the impact of loans, surrenders, and so on on the vanish date).
- Communicate the effect on pension max of transfer of ownership, beneficiary changes, policy face decreases/increases, other policy changes.
- Produce a "retirement approaching" letter to the agent and client before retirement and suggest that the CA-versus-pension max decision be reevaluated.

Assumptions for Use in Pension Max Analysis

Assumptions such as the following will be needed to prepare the analysis:

(1) *Dividend Scale*—Probably the current dividend scale, not guaranteed.

(2) *Settlement Option Rates*—Probably the current settlement option (immediate annuity) rates, not guaranteed.

(3) *Interest Discount Rate*—A 4%, 5%, or 6% rate could be justified based on historical inflation rates of 4% for the last 10 years, 6% the last 20 years, 5% the last 30 years, and 4% the last 40 years. This rate is not as high as an investment return rate, but, consistent with this, one might not want to reduce it for the effect of taxes.

(4) *Postretirement Tax Rate*—Analysis calculations might assume that the client is in the 25% tax bracket. This is used to compare the analysis calculations on an after-tax basis for all cash flows. A higher rate will disfavor the pension max sale.

(5) *Mortality Table Used for Life Expectancy*—The insurance company's single-premium immediate annuity mortality can be used to calculate life expectancy for the retiree at retirement age and for the spouse at the end of the retiree's life expectancy. *Tax Facts* Appendix A Table V ("Ordinary Life Annuity—One Life Expected Return Multiple") can be used to estimate the spouse's annuity taxation (exclusion ratio)

in determining the life insurance needed for pension max. It is on a unisex basis.

(6) *Pensioner's Death*—The pensioner and spouse are assumed to live to the year of the pensioner's life expectancy. Life expectancy of the pensioner is determined as of his or her retirement. For analysis calculations, the spouse's life expectancy is calculated as of the end of the pensioner's life expectancy. This provides a deterministic, not probabilistic, set of cash flows.

(7) *Preretirement Death Benefit*—This benefit has not been valued in my cash flows. It does help to support the use of a margin.

(8) *Postretirement Death Benefit*—This death benefit has three pieces:

   (a) That which is needed to provide the surviving spouse's income. This is a decreasing amount after retirement and it is used in the analysis calculation.

   (b) That which is more than (a) but not more than the guaranteed death benefit. This is an increasing amount after retirement, and it is not used in the analysis calculation because it is not required to replace the CA spouse's benefit.

   (c) That which is more than the guaranteed death benefit. This is a nonguaranteed amount, and it is not used in the analysis calculation because it is not required to replace the CA spouse's benefit and it is not guaranteed.

(9) *Pension Max Margin*—As described previously.

(10) *Premiums*—For the pension max cash flows, because the income to the pensioner from the pension benefit is stopped after the life expectancy of the pensioner (assumed time of death), we also assume no further premium payments after that time.

(11) *First Year of Retirement Death Benefit*—The analysis calculates a death benefit, L, so that the survivor will have at least as much after-tax income at the pensioner's retirement date, using the guaranteed face amount of the policy L and current settlement option rate, as would be provided to the spouse under the pension's CA option.

(12) *Policy Rating Classes*—Ratings above standard may nonqualify themselves based on their higher cost (greater negative cash flow for the pension max approach). A rated pensioner situation favors the CA option over the pension max approach because the pension plan would not normally change extra for the CA if the pensioner is sick at retirement.

(13) *Youngest Age*—Marketing of the pension max concept might be limited to prospects at some age (for example, 35 or 45) and above, because marketing the CA give-up and determining the pension benefits at younger ages is difficult.

(14) *Pension Pop-up*—This is an increase in the pensioner's benefit if the CA option is used and if the pensioner's spouse dies before the pensioner. It might be available in a pension. Our analysis assumes a scenario in which the pensioner dies before the spouse, so it does not add value to the pension max approach for the contingency in which the spouse dies before the pensioner. If the analysis includes both the pop-up's value in the CA cash flow and the policy's cash value at the spouse's death in the pension max cash flow, as the years pass after retirement the value to the pensioner of the pop-up benefit, in the year of the spouse's death, goes down and the value of the whole life policy's cash value goes up. Because this analysis uses fixed lifetimes for the pensioner and spouse, it would be difficult to bring in the value of this pop-up comparison. Leaving it out leaves the pension max approach with a bet if a pop-up is provided. Pension max results in less than full pop-up coverage when the spouse dies shortly after the pension max life insurance is purchased and the cash value is less than needed to replace the pop-up benefit.

Matrix Analytical Tool to Select Likely Prospects for Pension Max

For agents in the pension max market it can be particularly helpful if they have a tool for analyzing the likelihood of successfully selling the pension max approach to employees covered by a certain pension plan. A matrix display can be helpful; that is, general information on the pension plan is put into it, such as the reduction that will result when pensioners take the CA at various percentages of the pension benefit. The output is a matrix of pension max comparisons showing combinations of spouses' and pensioners' ages at retirement for a couple of spouse CA percentages (for example, 100% and 50%) and at several durations before retirement when the pension max approach is purchased (for example, 0, 5, and 10). By running large pension plans in an agent's territory through such a matrix, the agent can determine the likely candidates for a pension max sale based on where the sale provides the best deal for pensioners and their spouses. This can be seen in Table 1.

TABLE 1

PENSION MAX PLAN ANALYSIS MATRIX FOR XYZ PENSION PLAN*
{(PV PENSION MAX DIVIDED BY PV PENSION CA BENEFIT) PLUS A 10% MARGIN}

Pensioner's Retirement Age	Spouse's Age versus Pensioner's	Purchased N Years before Retirement					
		$N = 0$		$N = 5$		$N = 10$	
		100% CA	50% CA	100% CA	50% CA	100% CA	50% CA
		Male					
65	+5	92	97	91	98	89	97
	+0	95	98	94	99	92	98
	−5	98	100	97	101	95	100
60	+5	96	101	94	100	96	101
	+0	98	103	96	102	98	102
	−5	101	104	99	103	101	104
55	+5	97	103	99	103	99	103
	+0	100	104	101	104	101	104
	−5	102	105	104	105	104	105
		Female					
65	+5	104	106	104	106	103	105
	+0	107	107	107	107	105	106
	−5	111	109	111	109	110	108
60	+5	106	107	104	106	104	106
	+0	108	108	107	108	107	107
	−5	112	110	110	109	110	109
55	+5	105	107	105	107	106	107
	+0	108	108	107	108	108	108
	−5	110	110	110	109	111	109

*Pension max sale looks most promising when ratio is 100 or more.

Some pension plans are potentially good ones to approach for the pension max sale, while others are more generous and a sale is not likely to look good economically. And even when the general plan matrix display described above is available, the specific pension max analysis tool is still helpful when approaching a particular pensioner.

Conclusion

Mr. Shigley's excellent paper points out the complexities of the pension max versus contingent spouse annuity (CA) decision. My discussion presents additional issues. The parties to the sale can benefit from a pension max analysis tool. The tool would use financial and demographic facts about the prospect, the pension and the spouse, combine them with assumptions and calculation techniques, and provide a comparison.

There is another view of the pension max decision. Some pensioners simply want to get the largest single-life pension possible. They may not want an analytical tool to justify their decision, and they may not want to buy all or any of the pension max life insurance called for to provide an annuity for their spouse at their death. That is reality, so the need for an analytical tool depends on how prospects view the pension max decision and the level of support the company and agent are able to provide prospects who are approaching this decision.

ROBERT T. McCRORY:

I enjoyed reading Mr. Shigley's paper on the pension max election. He has done a fine job of describing the election from the point of view of the member. It is also important to look at pension max from the point of view of the pension plan sponsor. In the presence of a significant number of pension max elections, the plan sponsor is confronted with a number of problems.

Antiselection

Mr. Shigley notes that pension max is likely to be most attractive to plan members who experience lower-than-average mortality. There are several reasons for this:

- The underlying structure of the pension max election favors members with low mortality rates, as shown in Table 4.
- By law, most pension plans offer unisex factors for converting from the normal form to one providing a death benefit. Such factors are usually based on combined male and female mortality rates, thus charging female pensioners more for the death benefit than they would pay under a female-only mortality table. As Mr. Shigley points out, this makes the pension max election more favorable than the plan death benefit for most females.
- The policies offered as part of the pension max election are individually underwritten. This means, of course, that the issuing insurance company can reject impaired lives. Therefore, pension max elections will be issued primarily to healthy retirees.

As a result, the healthiest members of the retiree population are those most likely to elect pension max and a life-only annuity form. The least healthy segment of the retiree population is left to elect a death benefit from the plan. The impact of such systematic, institutionalized antiselection on the plan's cost could be significant, depending on the aggressiveness with which pension max is marketed.

Incorrect Information

Sales of the pension max election are usually accompanied by a comparison of the benefits expected from the plan's death benefit election with those expected from pension max. I have reviewed a few such comparisons, and while my review is hardly exhaustive, I have yet to find one that I considered correct. A consistent problem is that cost-of-living adjustments (COLA) are seldom handled correctly, especially in the public sector.

The analysis of pension max is not easily extended to COLAs because COLAs may not be easy to compute. For example:

- In some cases, the annual COLA is equal to the increase in the Consumer Price Index (CPI), with no limit.
- Some COLAs are based on the earnings of an active member in the grade occupied by the retiree just before retirement.
- Many COLAs are driven by investment results: When earnings on plan assets exceed a certain level, a 13th monthly check may be issued or an increase in benefits may occur.
- Often, statutory minimum COLAs are accompanied by a program of regular, ad hoc benefit increases approved by the governing body.

In the above cases, projection of future COLAs may be difficult or impossible. Furthermore, the factors used by the pension plan for converting from the normal form to an annuity with a death benefit may totally or partially ignore the COLA. As a result, the sales illustrations accompanying pension max may misstate or omit projected COLAs. This means that the plan's death benefit may be significantly undervalued, and the member may be misinformed during the sales process.

Member Relations

Pension max is sometimes presented as a superior financial alternative to the plan's forms of benefit. In some cases, this may be correct, but probably not as often as portrayed during the sales process.

During the sales process, agents may sell against the plan sponsor, describing pension max as a better deal being offered by the insurance company than that offered by the inefficient pension plan. Often the sales literature is quite negative about the pension plan: One sales letter referred to the pension plan's election as a "no-win situation." This type of approach can be particularly effective against public sector plans, given the current bias against all things governmental.

Clearly, the plan sponsor has an interest in ensuring that the value of the plan is accurately appreciated by all active and retired members.

How should a plan sponsor react when confronted with numerous pension max elections? Here are some ideas:

1. The plan may offer more and better designed optional benefit forms, tailored to match some of the advantages of pension max. Such forms could include:
   - "Pop-up" annuity forms
   - Lump-sum death benefits
   - Preretirement savings for survivor annuities.
2. All benefit option factor tables should be reviewed for currency of interest rates and mortality tables.
3. Communication materials should be drafted to equip retirees with the information necessary to ask the right questions if approached for a pension max sale.
4. Incorrect or misleading sales literature or illustrations should be immediately brought to the attention of the state insurance commissioner.

I am certain that neither Mr. Shigley nor his company would engage in any intentionally deceptive practices. My concerns about the effect of pension max on the plan sponsor should not detract in any way from my regard for the high quality of Mr. Shigley's research and the fine paper that resulted from it.

CONRAD M. SIEGEL:

Mr. Shigley's paper represents a valuable addition to the pension actuarial literature because the subject of pension max has heretofore been discussed primarily in the press, such as Jane Bryant Quinn in *Newsweek* and periodic *Wall Street Journal* articles that have quoted actuaries. These articles have been largely negative to pension max. Mr. Shigley's conclusions in Section 5, on balance, seem positive, although he does attempt to provide suitable caveats.

I practice as a consulting actuary in Harrisburg, Pa., the capital city of Pennsylvania. The two major governmental plans in the state (state employees and public school employees) have a major influence on benefits of residents of our area, because they constitute a very large proportion of the pension dollars being paid to our area's retirees. The plans are generous: a 2% of final 3 years average pay for each year of service (including military service buyback options), not offset by Social Security. Member contributions are 5%–6.25% of pay. There is no lump-sum option, but member contribution accounts may be withdrawn at retirement with a reduction in benefits based on 4% actuarial factors.

The basic benefit is a modified cash refund pension (if member contributions have not been withdrawn, otherwise life only) and there are three principal options:

(1) Full cash refund
(2) 100% contingent annuity
(3) 50% contingent annuity

The attractiveness of this market is such that "free seminars" are available all year long, and especially in the heavy retirement months of June and December, at which insurance agents, banks, financial planners, and others give advice on investing the lump-sum refunds and on pension max.

From public sources I have attempted to gauge the effect of changes in election patterns over a six-year period in the public school system. The data, based on statewide dollars of annuity in force at each valuation date, are as follows:

	Males 1988	Males 1994	Females 1988	Females 1994
No option	49%	57%	81%	79%
Full CR	11	9	10	10
100% CA	11	12	3	4
50% CA	29	22	6	7

A reason suggesting *increased* use of the CA options was the decision by the governing board, after the Norris case, to "top-up" the factors to the best of any combination of sexes of the member and the contingent annuitant. During these six years there were two ad-hoc COLAs and some early retirement incentives, leading to an 88% increase in dollars of annuity in force with only a 23% increase in number of annuitants.

These data seem to indicate that CA options are more popular for male members than for females. The proportion of in-force male CA annuities dropped from 40% to 34%. The drop in male CA elections in 1988–94 *retirements* would be much greater. Pension max may have been responsible for some of this drop. Other reasons for the CA drop could be increased work force activity of spouses earning their own pensions.

I was particularly intrigued by Mr. Shigley's concern about antiselection against the insurance company in his description of a contingent first-to-die life insurance policy, leading him to suggest a change in nonforfeiture laws (Section 3). Of course the pension max proposal has, at its core, antiselection against the pension plan in which the plan's unisex actuarial factors vary from the "correct factors" (his conclusion 4 in Section 5).

This prompted me to revisit a paper written by the late John Hanson in 1961 entitled "What Is the Added Cost to Permit Unrestricted Election of Optional Forms of Retirement Income" [*TSA* XIII, Part I (1961): 169]. At that time the critical issue was whether pension plans would become "actuarially unsound" if the five-year election period requirement for CA options in group annuity plans was relaxed or eliminated! The actuaries employed by insurance companies were very concerned, and the consultants had solutions involving small-employer cost increases or factor changes. In the 1960s I wrote an article for Ralph Edward's newsletter (a homegrown predecessor to *The Actuary*), suggesting actuarial factors based on a linear formula involving the ages of the two persons. My suggestion was criticized by a giant of the profession as "unsound." My how things have changed!

The core idea of Mr. Shigley's paper is an economic justification of pension max to a prospective retiree. Our firm counsels many retirees of the major plans in retirement and divorce situations. Our experience is that the concepts of "economic value" and "expected value" as represented in his formulas are far too difficult for the typical retiree to understand. Further, the use of a single number to justify the purchase or to recommend against it is not sufficient, in my view.

The employee and spouse are interested in benefit levels. Mr. Shigley's Figure 2, while described as involving relative benefit levels, really involves relative *present value* of survivor's benefits. The graph would show a level line for the CA option and an increasing curve for the pension max option if dollars of annual income were graphed. While an insurance professional may look at CA as decreasing-face-amount whole life insurance, the retiree looks at it as income replacement after death.

Why do people select CA options? Very obviously to provide income to the surviving spouse. Since historically females have interrupted periods of employment with broken pension service and lower wages than males, the female dependent has a need for income after the death of the male employee. How much? Something less than the full income paid to both after retiring, since one can live less expensively than two, but not at 50% of the cost of two. Social security can be viewed as a 66.7% J&S annuity, if both persons retired after the early reduction ages.

Health is another reason for the CA election or lack thereof, or for the FCR election.

Since the intent is to enable the retiree to make a decision, I use a practical example of a reasonable comparison based on the state employees' plan. I find the paper's use of broad-brush assumptions for insurance policy costs not sufficiently rigorous.

I have set up a strawman comparison of equal after-tax income while both are alive and the use of insurance proceeds to buy an annuity to achieve some rigor in achieving a fair comparison. This precludes proposals of starting the policy before retirement or pouring in the proceeds of sale of another asset including the cash value of another insurance policy. While the use of insurance proceeds to invest in a business or to buy municipal bonds may be of interest, they do not facilitate an apples-to-apples comparison. They do point to added flexibility in dealing with insurance proceeds.

A male employee age 60 with a wife age 57 is entitled to a benefit of \$2,000 per month for life, or \$1,748 per month, for the 100% CA option. If he elects the no-refund annuity, he has \$252 per month before tax, or \$214 per month after 15% federal tax, to buy a life insurance policy to keep the gross pension income the same, \$1,748 per month. I called the local agency of a major mutual company for values, both guaranteed and illustrated. The \$214 premium would purchase a whole life policy in the face amount of \$51,000+ for a male nonsmoker at age 60.

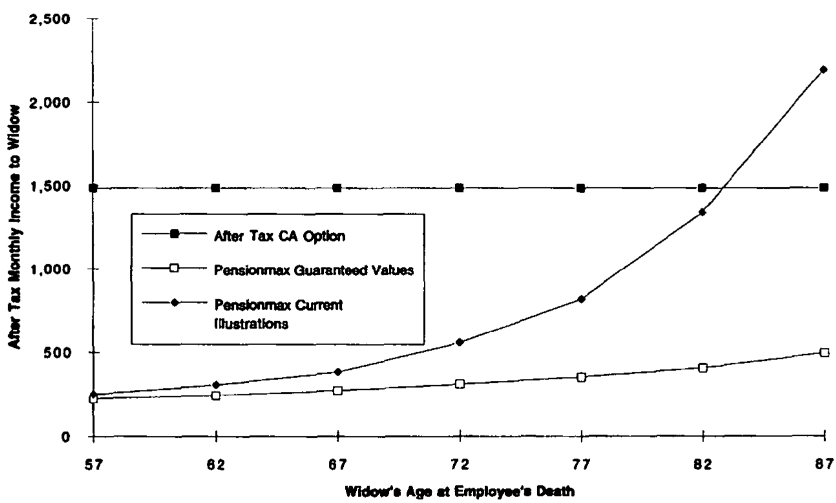
Because the policy is a whole life policy, it is participating in some form. There is no point to receiving dividends in cash each year, since the strawman comparison of equal income while both are alive must be maintained. The interest earned on dividend accumulations would also upset the tax comparison. The remaining possibility is paid-up additions or some form of supplemental term insurance. The result is that the insurance proceeds at death increase with duration since retirement. The resulting annuity from the proceeds will, if annuity rates do not change, increase with duration since annuity rates decrease with age. Since annuity rates change frequently, the actual annuity is not predictable except using settlement option rates in the policy, which are usually very unattractive.

Is an annuity that increases in initial amount with the age at death a desirable result? I do not think so. In the first five or ten years after retirement, the expenditures for travel and vacations are typically higher. When benefits commence at a very old age, the needs are less, yet that is the time when the largest initial annuity is payable to the surviving beneficiary.

The specific policy annuity proceeds are compared with the CA on an after-tax basis. The annuity payments are partially taxed, because some of the payments consist of a return of the basis. Nevertheless, Figure 1 shows the CA payment to be much superior to pension max, on both the guaranteed values and the illustrative dividends. At the time this illustration was done (December 1995), the low interest rates currently available provided unattractive illustrations.

I cannot get very concerned about the case in which the spouse dies first. The pop-up option makes no sense to me. Why would the employee want a larger pension after the death of the spouse (when he only has one person, himself, to support) than while two are alive? The cash value of the insurance policy could produce additional annuity income for the employee, again resulting in the same outcome: more income while one is alive than was received by two.

FIGURE 1
CA VERSUS PENSION MAX



Two additional caveats should accompany the pension max proposal.

(1) The results illustrated are for a participating policy and are not guaranteed. In recent years insurance companies have failed to meet their illustrations at times, as interest rates have fallen. The benefits resulting from minimum guaranteed results are also shown.

(2) Major insurance companies have gotten into financial difficulties in recent years, and the state-operated system of guarantee programs has a very spotty record in terms of speed of reaction and extent of coverage.

An additional caveat applies in the two Pennsylvania plans.

The benefit is increased by an ad hoc COLA every five years, computed as some combination of flat dollars and percentages applied to some or all of current pension, years of service, years since retirement, and so on. There is a state constitutional prohibition against increasing the pension of a beneficiary after the death of an employee. The CA payment can be increased by a COLA after retirement but before the death of the employee. Thus the CA annuity will increase up to the date of death of the employee. Conversely the single-life annuity will be larger during the member's life under the pension max proposal, since some portion of the COLA is based on the current pension.

Mr. Shigley's paper stimulated my thinking, but has not changed my mind. If a friend asks for a one-word answer on pension max, the answer is "careful!" If a client wants a more extensive answer, we will do it based upon a comprehensive analysis specifically tailored to the client's income, assets, dependency, and tax position.

(AUTHOR'S REVIEW OF DISCUSSIONS)

KLAUS O. SHIGLEY:

I very much appreciate the six fine discussions of my paper. I reply to each discussant in turn.

Charles L. Trowbridge

Mr. Trowbridge's discussion adds some insights that I had not previously fully appreciated. Before commenting on these substantive insights, however, I believe it would be constructive to comment on some of the other issues he raises.

To begin, Mr. Trowbridge concludes that the main advantage for pension max, promoted within the paper, derives from an "inherent defect" in the CA approach. While the paper does not make this sufficiently clear, *the main advantage of pension max derives from the potential arbitrage between the qualified plan reduction factors and the true actuarial equivalents.* And it is not clear to me from reading Mr. Trowbridge's discussion that he appreciates this point. Because of the Supreme Court's *Norris* decision, a qualified plan must calculate identical reduction factors for a male employee age 65 with spouse age 60 and a female employee age 65 with spouse age 60. But obviously the same reduction cannot be correct for both scenarios. In the special case in which the plan uses the UP-84 7% Table to calculate these reductions and actual mortality is assumed to be GAM-83 (0, −6) 7%, the male employee is undercharged and the female is overcharged (see Table 3). The overcharge for the female is on the order of 89% of the correct charge.

Second, Mr. Trowbridge points out that insurers do not price life insurance on annuity tables. Page 498 deals with that issue:

> Life insurance mortality assumptions are often based on multiples of the 1975–80 Basic Table. The GAM-83 (0, −6) Table, which is used for most of this discussion, produces mortality rates roughly equivalent to the ultimate rates from the 1975–80 Basic Table at ages 60 and beyond. Policies sold with preferred mortality [*in practice*] are commonly priced at less than 100% of the ultimate rates in the 1975–80 Basic Table. This logic would establish the GAM-83 (0, −6) Table as a conservative standard for judging the relative economics of "pension max" for individuals who qualify for preferred mortality rates.

Mr. Trowbridge also points out, in his discussion of the tax effects, that the "time value of money is not to be ignored." Section 4-C states: "Since

$$R\ddot{a}_x = (N - R)\ddot{a}_{x|y} = FA_x$$

it follows that

$$(1 - T)\ R\ddot{a}_x = (1 - T)(N - R)\ddot{a}_{x|y} = (1 - T)FA_x."$$

I believe this adequately covers the time value of money issue except for the caveat specifically noted thereafter.

Among the disadvantages, Mr. Trowbridge cites lack of generality. Although pension max is a natural fit for CA options, it is not obvious how to replicate J&S options of less than 100%. This is true. However, between the CA and the J&S option, the CA option is by far the more *prevalent option* within qualified plans. Among qualified annuities being paid at John Hancock, 97% of reversionary annuities are of the CA type and 3% are of the J&S type. Be that as it may, for individuals who prefer the J&S option (with continuance percentages less than 100%), a modification of Table 6 could be designed that would at least test the proposition that the reduction factor offered by the plan provides fair value at the "regulated unisex" assumptions. And if it does not, we could attempt to replicate the desired annuity flows by purchasing a sufficient amount of life insurance on the employee to provide the required reversionary annuity to the spouse. All things equal, the outlay for such insurance would need to be higher than the J&S reduction.

No doubt many other parameters could have been considered besides those above. In my view, however, these will have only second-order relevance when the "regulated unisex" plan reduction factors are clearly excessive compared with "free market" actuarial equivalents.

Moving forward to the heart of Mr. Trowbridge's discussion, I agree with most of what he says.

Fundamentally the paper deals with the following equality:

$$R\ddot{a}_x = (N - R)\ddot{a}_{x|y}$$

The left side represents the available funds; this is what we have to spend. The right side represents the benefit configuration that is offered by the qualified plan.

The basic premise of the paper is that participants are free to choose other benefit structures. The only constraint is that they must all have the same present value, that is, $R\ddot{a}_x$. For the sake of this discussion, let us assume the participant can select from any number of the following benefit structures: for simplicity, it is best to think of each of these structures as being fully paid up with purchase price equal to $R\ddot{a}_x$.

1. $(N-R)\ddot{a}_{x|y}$
2. $(N-R^{(2)})\ddot{a}_{x|y} + R\ddot{a}_{y|x}$
3. $F A_x$
4. $F(A^{1}_{xy} + A^{2}_{xy})$; since $A^{1}_{xy} + A^{2}_{xy} = A_x$
5. $F(A^{1}_{xy}) + R^{(3)}\ \ddot{a}_{y|x}$
6. $(1+r)F A^{1}_{xy}$; where r is defined on p. 488.
7. $(N-R) \sum_{t=0}^{\infty} v^t\ {}_tp_{xy}\ q_{x+t}\ \ddot{a}_{y+t}$

Benefit structure no. 1 is the CA option. Benefit structure no. 3 is the conventional pension max option. Benefit structure no. 7 is an alternative pension max formulation, presented on p. 486, which exactly replicates benefit structure no. 1. Benefit structure no. 6 is the level benefit equivalent of benefit structure no. 7. Mr. Trowbridge points out that for benefit structure no. 7 to replicate the reversionary annuity, the policy cannot be lapsed even if the spouse has died. Stated another way, if benefit structure no. 7 has been fully funded, the cash value remaining at the spouse's death must revert to the insurance company to fund CAs who outlive their spouses. Benefit structure no. 7 does not permit any money to pop up or revert to the participant. In other words, 100% of benefit structure no. 7 is allocated to the spouse. Mr. Trowbridge and I are in complete agreement on this point.

In the pension max alternative, we substitute benefit structure no. 3 for benefit structure no. 1. To the extent that benefit structure no. 3, that is, conventional life insurance in an amount, F, must comply with standard nonforfeiture laws, it must be presumed that the employee will lapse the policy if the spouse dies first. Thus the value of the benefit structure *must be split* between the spouse and the employee. The portion that reverts to the employee is the implicit pop-up. The balance of the benefit structure is all that is available to the spouse. Under pension max, therefore, the expected value of the spouse's portion will be smaller than that under the conventional CA. Thus, even though pension max and the CA option produce identical expected values for the entire family unit, the allocation between employee and spouse will be different. Mr. Trowbridge's discussion thus begs the question of how the present value of the pension max benefit structure is split between the employee and the spouse. This is an area that was not developed in the paper. Mr. Trowbridge's discussion provides the insight to answer this question.

Mr. Trowbridge argues that the relative shares to the spouse and the employee can be calculated with reference to benefit structure no. 2. I agree that if we fund for a reversionary annuity plus an *expectation* of a pop-up of R after the joint life period xy, then only $(N-R^{(2)})$ of reversionary annuity remains available. But this is *not* the benefit structure that is being purchased. Under pension max, we purchase benefit structure no. 3, which buys F amount of conventional insurance. If the spouse dies first, the employee takes the surrender value *and* stops future premiums. In practice, therefore, pension max is really an *implicit purchase of benefit structure no. 5* under which the employee buys contingent first-to-die insurance of F. And since premiums of R are more than sufficient to purchase this benefit, the redundant premium then pops up if y dies first. Thus the spouse's share of the pension max benefit would be calculated as the ratio of $F(A^{1}_{xy})$ to FA_x. Page 488 calculates this fraction as 85% for a male employee age 65/spouse age 60. Note, however, notwithstanding the fact that the spouse's share of FA_x is expected to be less than 100%, the probability that pension max is a winning bet is still correctly given in Table 2.

Mr. Trowbridge's discussion has thus served to define an algorithm to solve for the pension max outlay that preserves the *expected value* of the payout to the spouse at $(N-R)$ of reversionary annuity.

Mr. Trowbridge ends his discussion with a caution that readers should not conclude that with pension max it is possible to get *both* a pop-up *and* (on an expected value basis), to provide the full $(N-R)$ reversionary annuity to the spouse. I concur. Nevertheless, the entire family unit may be better off selecting the pension max approach because plan reduction factors often act more like "fixed exchange rates" than actuarial equivalents. This can and does happen because of the requirement to use unisex factors.

L. Timothy Giles

It had not occurred to me to develop a reversionary annuity to replicate the qualified annuity. This would certainly be practical in those situations in which the actuarial reductions calculated by the qualified plan are too high.

Note, however, all other things being equal, the calculated premium for a reversionary annuity would be higher than the reduction developed by the qualified plan, because the premiums for a commercial policy are uncollectible in the $y|x$ period. The qualified plan, on the other hand, in its calculation of the reversionary annuity reduction, does collect this premium. This is the point brought forth in Mr. Trowbridge's discussion. Thus it might be difficult to convince a potential customer that the commercial annuity is a better buy because the outlays are likely to be higher. In practice, this might restrict the applicability to those situations in which the plan's reduction factors are disproportionately large.

Elias S.W. Shiu

I am grateful to Dr. Shiu for pointing out his technical correction to Formula (10). The exhibits have been recalculated to reflect his correction. The resulting changes do not materially affect any of the conclusions.

Dr. Shiu raises the following question: "When an annuitant elects the "pension max" option in the real world, is the insurance premium determined by an annuity mortality table?" The paper attempts to capture this point in Table 6, in which an attempt is made to create an algorithm that solves for the "better" value between the plan's reversionary annuity and pension max. In Table 6, assumption set d is used as a proxy for the insurer's mortality and interest assumptions. As indicated in the paper, this is a reasonable proxy for individuals who qualify for preferred underwriting.

Robert B. Likins

Mr. Likins has given a great deal of thought to the practical implementation of pension max. My objectives, which were more limited, were to present a comparison between the reversionary annuity and pension max along two dimensions, the funding dimension and the benefit dimension, and then to develop a conceptually simple algorithm to test the relative economics of the two different benefit structures.

Mr. Likins' discussion adds a valuable practical dimension. Of particular significance, he starts from the premise that the insurance amount must be big enough to *eliminate* the financial exposure of the spouse. In addition, he assumes that life insurance will be used to purchase an annuity and he applies the tax mechanics for annuities. He also develops a decision matrix for screening specific applications. I think it would be worthwhile to try to investigate the characteristics of this decision matrix more fully. One way to do that would be to compare the results it produces against the algorithm presented in the paper.

Robert T. McCrory

I am in complete agreement with Mr. McCrory's comments on the importance of avoiding deceptive sales practices. This was certainly one of the objectives that motivated me to write this paper. Having once observed a sales seminar on this subject, I found that the cadence of the presentation soon outran my ability to keep pace with the supporting arguments. Subsequently, as I tried to confirm the representations made during this presentation, I ultimately concluded that the issue is more complex than it first appears. Furthermore, as Mr. McCrory's discussion points out, it seems that I have underestimated the complexity of extending the analysis to plans with COLAs.

Mr. McCrory's discussion also touches on some of the issues that pension max presents for the plan. My own reaction to these issues was to identify those demographic subgroups that are being severely overcharged for the CA election within my own company's plan.

Conrad M. Siegel

Mr. Siegel's observation that employees are more interested in benefit levels than "expected value" is a good one. And I agree that the use of a single "economic value" number may be insufficient to justify or recommend a purchase. But I also believe that a single measure of "economic value" is a valuable screening mechanism for false positives *as well as false negatives*. Moreover, I start from the premise that retirement annuities, like other financial assets, are fungible and they can be exchanged for similar goods with comparable value. If there is sufficient economic justification for selecting pension max, then it ought to be fairly evaluated. And in this connection, as several other discussants have pointed out, the use of universal life in conjunction with target term capabilities would be more appropriate than whole life.

For the record, I do not advocate a change in nonforfeiture laws. I merely state in Section 3 that it would be difficult in practice to develop a *contingent first-to-die policy* without relief from current nonforfeiture laws. This problem can be solved by using a reversionary annuity, which, as Mr. Giles points out, is exempt from a cash value nonforfeiture rule.