THE 1994 UNINSURED PENSIONER MORTALITY TABLE

SOCIETY OF ACTUARIES UP-94 TASK FORCE[1]

1. EXPERIENCE DATA

The Uninsured Pensioner Mortality Subcommittee[2] was established under the Retirement Plans Experience Committee to develop a recommendation for an uninsured pensioner mortality table, which would thus serve as an update to the UP-1984 Mortality Table (the UP-84 Table).[3]

The Retirement Plans Experience Committee issued a report entitled "Mortality among Members of Uninsured Pension Systems," which appears in the *Transactions of the Society of Actuaries, 1991-92 Reports*. This report compares recent mortality experience for 29 retirement systems with the UP-84 and the 1983 Group Annuity Mortality Table (GAM-83).[4] These systems included the following: Medicare participants in Social Security, the U.S. Federal Civil Service Retirement System (CSRS), the U.S. Military Retirement System, Public Service of Canada, and a combination of 24 private sector systems together with one state system, which is referred to as the private sector uninsured pension plan mortality experience. This report shows that current uninsured pensioner mortality, based on experience from 1985 through 1989, was generally in the range of 82% to 86% of the mortality expected under the UP-84 Table.

When the Uninsured Pensioner Mortality Subcommittee was established, the Group Annuity Valuation Table Task Force was developing a new Group Annuity Mortality Valuation Standard. This standard is based primarily on group annuity mortality experience at ages 66 and older and on CSRS mortality experience for ages under 66. Age is defined as age nearest birthday at the beginning of the year.

The Subcommittee compared the recent experience for uninsured pensioner mortality that had been collected by the Retirement Plans Experience

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[2]This subcommittee was set up as a Task Force in the fall of 1994.

[3]The UP-1984 Mortality Table was promulgated in the *Proceedings of the Conference of Actuaries in Public Practice* XXV (1975): 456–507.

[4]The 1983 Group Annuity Mortality Table was promulgated in the *Transactions of the Society of Actuaries* XXXV (1983): 859–900.

Committee with this group annuity mortality experience and concluded that the uninsured experience was sufficiently close to the insured experience, so that it would be reasonable to use the same underlying data as a basis for both tables.

A comparison of the mortality experience for group annuities, CSRS, uninsured pension plans, and the Railroad Retirement System is displayed in Figures 1 (male) and 2 (female). Experience rates for the Railroad Retirement System are included as an example of how the experience for a particular group, which is predominantly blue collar, can differ from the experience used for the proposed table. As presented in Figures 1 and 2, the mortality rates under Railroad Retirement are considerably higher than the other rates shown, at all ages. The mortality rates cover ages 66 through 95 because the group annuity experience was limited to these ages. All these experience rates were first graduated using a Whittaker-Henderson type B formula to simplify the presentation.

The Group Annuity Mortality Experience (GAM-88) covers years 1986 through 1990, as does the CSRS experience (CSRS-88). The private sector uninsured pension plan experience (UPP-88), which was collected by the Retirement Plans Experience Committee, covers years 1985 through 1989, but was adjusted to central year 1988 based on the ratio of CSRS experience for 1986–1990 to the CSRS experience for 1985–1989. The mortality experience for nondisabled retired lives under the Railroad Retirement System (RRR-88) covers policy anniversaries in years 1988–1991 and was adjusted to a central year 1988 based on the ratio of CSRS experience for the respective years.

Table 1 lists these graduated mortality experience rates, and Table 2 shows the ratios of the rates for CSRS, private sector uninsured private pension plans, and Railroad Retirement to the group annuity rates.

2. PROPOSED UP-94 TABLE

The Subcommittee worked closely with the GAM Task Force to develop a common set of mortality improvement trend factors to be used to project the group annuity mortality experience rates from 1988 to 1994 and to

FIGURE 1

GRADUATED MORTALITY EXPERIENCE FOR MALES FOR GROUP ANNUITIES,
CIVIL SERVICE RETIREMENT, UNINSURED PENSION PLANS, AND RAILROAD RETIREMENT
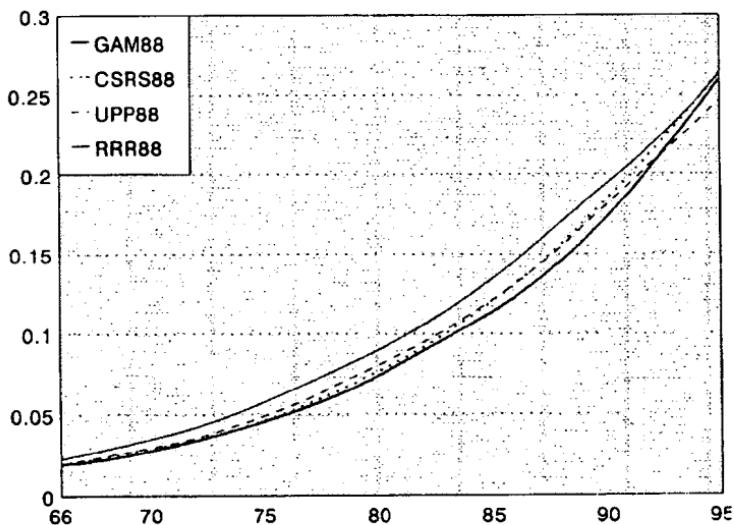

FIGURE 2

GRADUATED MORTALITY EXPERIENCE FOR FEMALES FOR GROUP ANNUITIES,
CIVIL SERVICE RETIREMENT, UNINSURED PENSION PLANS, AND RAILROAD RETIREMENT
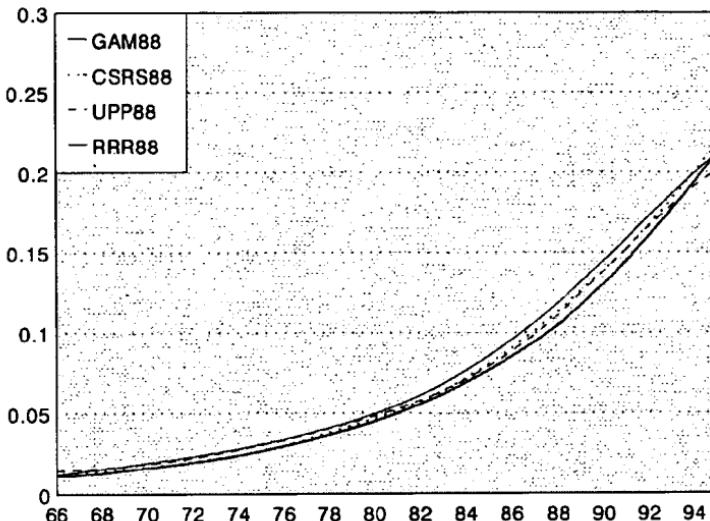
TABLE 1

Graduated Mortality Experience Rates

Age	Male				Female			
	GAM-88	CSRS-88	UPP-88	RRR-88	GAM-88	CSRS-88	UPP-88	RRR-88
66	0.019291	0.019830	0.019690	0.022714	0.011357	0.011772	0.014495	0.012028
67	0.020619	0.021725	0.021895	0.025513	0.011919	0.012814	0.015220	0.013784
68	0.022490	0.023950	0.024532	0.028435	0.013031	0.013965	0.016185	0.015641
69	0.024923	0.026466	0.027113	0.031532	0.014571	0.015303	0.017362	0.017536
70	0.027709	0.029092	0.029575	0.034835	0.016312	0.016824	0.018781	0.019450
71	0.030760	0.031790	0.032285	0.038430	0.018092	0.018469	0.020500	0.021404
72	0.034110	0.034641	0.035674	0.042407	0.019892	0.020205	0.022586	0.023462
73	0.037710	0.038010	0.039812	0.046932	0.021834	0.022075	0.025009	0.025683
74	0.041568	0.041987	0.044323	0.052094	0.024079	0.024230	0.027626	0.028134
75	0.045732	0.046488	0.049067	0.057733	0.026783	0.026869	0.030418	0.030865
76	0.050289	0.051509	0.054302	0.063757	0.029864	0.030113	0.033486	0.033906
77	0.055296	0.057105	0.060363	0.070022	0.033131	0.033928	0.036883	0.037251
78	0.060818	0.063258	0.067107	0.076435	0.036630	0.038073	0.040519	0.040921
79	0.067025	0.069873	0.073933	0.083080	0.040560	0.042379	0.044304	0.044981
80	0.074043	0.076735	0.080648	0.090060	0.045010	0.046843	0.048306	0.049572
81	0.081723	0.084014	0.087512	0.097550	0.049983	0.051736	0.052741	0.054864
82	0.089667	0.092009	0.094918	0.105694	0.055564	0.057343	0.057879	0.061014
83	0.097668	0.100840	0.103074	0.114527	0.061849	0.063855	0.063903	0.068123
84	0.105614	0.110353	0.111850	0.124107	0.068821	0.071462	0.070865	0.076207
85	0.113911	0.120469	0.121186	0.134522	0.076497	0.080246	0.078883	0.085281
86	0.123225	0.131355	0.131299	0.145810	0.084879	0.090212	0.088129	0.095286
87	0.133824	0.143214	0.142430	0.157769	0.094169	0.101211	0.098740	0.106188
88	0.145520	0.156060	0.154600	0.169944	0.104594	0.112951	0.110726	0.117985
89	0.158288	0.169753	0.167461	0.181926	0.116356	0.125226	0.123786	0.130652
90	0.172483	0.184011	0.180597	0.193738	0.129563	0.138110	0.137544	0.144032
91	0.188231	0.198749	0.193776	0.205721	0.144091	0.151794	0.151638	0.157910
92	0.205217	0.214056	0.206887	0.218363	0.159660	0.166536	0.165602	0.171988
93	0.223081	0.230100	0.219987	0.232065	0.175982	0.182425	0.178864	0.185799
94	0.241380	0.246814	0.233184	0.247075	0.192948	0.199250	0.190740	0.198816
95	0.259812	0.263772	0.246742	0.263421	0.210728	0.216547	0.200693	0.210670

TABLE 2

RATIO OF GRADUATED MORTALITY EXPERIENCE RATES
TO GAM-88 GRADUATED MORTALITY RATES

Age	Male			Female		
	CSRS-88/ GAM-88	UPP-88/ GAM-88	RRR-88/ GAM-88	CSRS-88/ GAM-88	UPP-88/ GAM-88	RRR-88/ GAM-88
66	1.028	1.021	1.177	1.037	1.276	1.059
67	1.054	1.062	1.237	1.075	1.277	1.156
68	1.065	1.091	1.264	1.072	1.242	1.200
69	1.062	1.088	1.265	1.050	1.192	1.204
70	1.050	1.067	1.257	1.031	1.151	1.192
71	1.034	1.050	1.249	1.021	1.133	1.183
72	1.016	1.046	1.243	1.016	1.135	1.179
73	1.008	1.056	1.245	1.011	1.145	1.176
74	1.010	1.066	1.253	1.006	1.147	1.168
75	1.017	1.073	1.262	1.003	1.136	1.152
76	1.024	1.080	1.268	1.008	1.121	1.135
77	1.033	1.092	1.266	1.024	1.113	1.124
78	1.040	1.103	1.257	1.039	1.106	1.117
79	1.043	1.103	1.240	1.045	1.092	1.109
80	1.036	1.089	1.216	1.041	1.073	1.101
81	1.028	1.071	1.194	1.035	1.055	1.098
82	1.026	1.059	1.179	1.032	1.042	1.098
83	1.032	1.055	1.173	1.032	1.033	1.101
84	1.045	1.059	1.175	1.038	1.030	1.107
85	1.058	1.064	1.181	1.049	1.031	1.115
86	1.066	1.066	1.183	1.063	1.038	1.123
87	1.070	1.064	1.179	1.075	1.049	1.128
88	1.072	1.062	1.168	1.080	1.059	1.128
89	1.072	1.058	1.149	1.076	1.064	1.123
90	1.067	1.047	1.123	1.066	1.062	1.112
91	1.056	1.029	1.093	1.053	1.052	1.096
92	1.043	1.008	1.064	1.043	1.037	1.077
93	1.031	0.986	1.040	1.037	1.016	1.056
94	1.023	0.966	1.024	1.033	0.989	1.030
95	1.015	0.950	1.014	1.028	0.952	1.000

develop another set of factors to project mortality improvements for years beyond 1994. The factors for projecting mortality improvement from 1988 to 1994 are based on the CSRS mortality experience from 1987 to 1993. The factors for projecting improvement beyond 1994 are based on the average of the CSRS and Social Security mortality improvement trends from 1977 to 1993, with a minimum of 0.5% for ages under 85, and are referred to as Scale AA.

The proposed UP-94 Mortality Table, along with Scale AA, is shown in Table 3.

TABLE 3

UP-94 MORTALITY RATES AND SCALE AA

Age	UP-94 Rates		Scale AA	
	Male	Female	Male	Female
1	0.000637	0.000571	0.020	0.020
2	0.000430	0.000372	0.020	0.020
3	0.000357	0.000278	0.020	0.020
4	0.000278	0.000208	0.020	0.020
5	0.000255	0.000188	0.020	0.020
6	0.000244	0.000176	0.020	0.020
7	0.000234	0.000165	0.020	0.020
8	0.000216	0.000147	0.020	0.020
9	0.000209	0.000140	0.020	0.020
10	0.000212	0.000141	0.020	0.020
11	0.000223	0.000148	0.020	0.020
12	0.000243	0.000159	0.020	0.020
13	0.000275	0.000177	0.020	0.020
14	0.000320	0.000203	0.019	0.018
15	0.000371	0.000233	0.019	0.016
16	0.000421	0.000261	0.019	0.015
17	0.000463	0.000281	0.019	0.014
18	0.000495	0.000293	0.019	0.014
19	0.000521	0.000301	0.019	0.015
20	0.000545	0.000305	0.019	0.016
21	0.000570	0.000308	0.018	0.017
22	0.000598	0.000311	0.017	0.017
23	0.000633	0.000313	0.015	0.016
24	0.000671	0.000313	0.013	0.015
25	0.000711	0.000313	0.010	0.014
26	0.000749	0.000316	0.006	0.012
27	0.000782	0.000324	0.005	0.012
28	0.000811	0.000338	0.005	0.012
29	0.000838	0.000356	0.005	0.012
30	0.000862	0.000377	0.005	0.010
31	0.000883	0.000401	0.005	0.008
32	0.000902	0.000427	0.005	0.008
33	0.000912	0.000454	0.005	0.009
34	0.000913	0.000482	0.005	0.010
35	0.000915	0.000514	0.005	0.011
36	0.000927	0.000550	0.005	0.012
37	0.000958	0.000593	0.005	0.013
38	0.001010	0.000643	0.006	0.014
39	0.001075	0.000701	0.007	0.015
40	0.001153	0.000763	0.008	0.015
41	0.001243	0.000826	0.009	0.015
42	0.001346	0.000888	0.010	0.015
43	0.001454	0.000943	0.011	0.015
44	0.001568	0.000992	0.012	0.015
45	0.001697	0.001046	0.013	0.016
46	0.001852	0.001111	0.014	0.017
47	0.002042	0.001196	0.015	0.018
48	0.002260	0.001297	0.016	0.018
49	0.002501	0.001408	0.017	0.018
50	0.002773	0.001536	0.018	0.017

TABLE 3—*Continued*

Age	UP-94 Rates		Scale AA	
	Male	Female	Male	Female
51	0.003088	0.001686	0.019	0.016
52	0.003455	0.001864	0.020	0.014
53	0.003854	0.002051	0.020	0.012
54	0.004278	0.002241	0.020	0.010
55	0.004758	0.002466	0.019	0.008
56	0.005322	0.002755	0.018	0.006
57	0.006001	0.003139	0.017	0.005
58	0.006774	0.003612	0.016	0.005
59	0.007623	0.004154	0.016	0.005
60	0.008576	0.004773	0.016	0.005
61	0.009663	0.005476	0.015	0.005
62	0.010911	0.006271	0.015	0.005
63	0.012335	0.007179	0.014	0.005
64	0.013914	0.008194	0.014	0.005
65	0.015629	0.009286	0.014	0.005
66	0.017462	0.010423	0.013	0.005
67	0.019391	0.011574	0.013	0.005
68	0.021354	0.012648	0.014	0.005
69	0.023364	0.013665	0.014	0.005
70	0.025516	0.014763	0.015	0.005
71	0.027905	0.016079	0.015	0.006
72	0.030625	0.017748	0.015	0.006
73	0.033549	0.019724	0.015	0.007
74	0.036614	0.021915	0.015	0.007
75	0.040012	0.024393	0.014	0.008
76	0.043933	0.027231	0.014	0.008
77	0.048570	0.030501	0.013	0.007
78	0.053991	0.034115	0.012	0.007
79	0.060066	0.038024	0.011	0.007
80	0.066696	0.042361	0.010	0.007
81	0.073780	0.047260	0.009	0.007
82	0.081217	0.052853	0.008	0.007
83	0.088721	0.058986	0.008	0.007
84	0.096358	0.065569	0.007	0.007
85	0.104559	0.072836	0.007	0.006
86	0.113755	0.081018	0.007	0.005
87	0.124377	0.090348	0.006	0.004
88	0.136537	0.100882	0.005	0.004
89	0.149949	0.112467	0.005	0.003
90	0.164442	0.125016	0.004	0.003
91	0.179849	0.138442	0.004	0.003
92	0.196001	0.152660	0.003	0.003
93	0.213325	0.167668	0.003	0.002
94	0.231936	0.183524	0.003	0.002
95	0.251189	0.200229	0.002	0.002
96	0.270441	0.217783	0.002	0.002
97	0.289048	0.236188	0.002	0.001
98	0.306750	0.255605	0.001	0.001
99	0.323976	0.276035	0.001	0.001

TABLE 3—*Continued*

Age	UP-94 Rates		Scale AA	
	Male	Female	Male	Female
100	0.341116	0.297233	0.001	0.001
101	0.358560	0.318956	0.000	0.000
102	0.376699	0.340960	0.000	0.000
103	0.396884	0.364586	0.000	0.000
104	0.418855	0.389996	0.000	0.000
105	0.440585	0.415180	0.000	0.000
106	0.460043	0.438126	0.000	0.000
107	0.475200	0.456824	0.000	0.000
108	0.485670	0.471493	0.000	0.000
109	0.492807	0.483473	0.000	0.000
110	0.497189	0.492436	0.000	0.000
111	0.499394	0.498054	0.000	0.000
112	0.500000	0.500000	0.000	0.000
113	0.500000	0.500000	0.000	0.000
114	0.500000	0.500000	0.000	0.000
115	0.500000	0.500000	0.000	0.000
116	0.500000	0.500000	0.000	0.000
117	0.500000	0.500000	0.000	0.000
118	0.500000	0.500000	0.000	0.000
119	0.500000	0.500000	0.000	0.000
120	1.000000	1.000000	0.000	0.000

3. COMPARISON WITH OTHER TABLES

The table and scale are intended to provide actuaries with a standard for measuring and projecting the underlying mortality of pension plans, subject to the actuary's judgment on the mortality for the particular group being valued. The UP-94 Mortality Table is the same as the 1994 Group Annuity Mortality Basic Table. It does not include the 7% margin that was included in the 1994 Group Annuity Mortality Static Table (GAM-94 Static). This 7% margin comprises a 5% margin for random variation in mortality rates and a 2% margin for other contingencies.

Although the UP-84 table was primarily designed to be a unisex table, the Subcommittee believes that sex-distinct tables are more appropriate for actuarial valuations, and since sufficient experience is now available to accurately determine female mortality rates, separate tables should be used whenever feasible. In cases in which unisex factors are required, the actuary can combine the results for male and female in a way that would be most appropriate in that particular situation.

Figures 3 and 4 show a comparison of the mortality rates for the UP-94, GAM-94 Static, GAM-83, and UP-84 Tables. For the UP-84 Table, the rates by sex are based on the recommended four-year age setback for males and the one-year age set-forward for females. The actual mortality rates are listed in Appendix A. The ratios of the UP-94 rates to the GAM-83 rates, and the ratios of the UP-94 rates to the UP-84 rates, are shown in Table 4.

4. USE OF PROJECTION SCALE AA

The use of Scale AA to project mortality improvement on a generational basis was not directly incorporated in the UP-94 Mortality Table as it was for the 1994 Group Annuity Reserving Table (GAR-94), which is a combination of the GAM-94 Static Table and Scale AA. Use of the 1994 Group Annuity Reserving Table implies that the GAM-94 Static mortality has been improved on a generational basis using Scale AA.

However, the Subcommittee believes that projection of future mortality trends is an issue that should be considered in setting up a best estimate of future experience for uninsured pension plans. A considerable body of evidence has accumulated showing that continuous mortality improvements have occurred throughout most of this century. We think that the continuing pace of medical discovery presents a strong argument that provision should be made for mortality improvement in setting a best estimate. In other words, the actuary would have to demonstrate significant factors that would justify not using an improvement trend for current and future retirees under a particular pension plan.

The need to consider the mortality improvement trend in setting assumptions should not be taken to imply that the only appropriate model is one in which mortality improvement trends explicitly appear. In determining liabilities, actuaries must be concerned with a variety of issues. These include the actual population expected to retire under the plan, the interaction of assumptions, the relevance of various assumptions given alternate plan designs, and the significance of a particular assumption given the overall level of precision in the liability model.

Scale AA is included to provide actuaries with a tool for projecting the UP-94 Mortality Table, to be applied in accordance with the actuary's

FIGURE 3

COMPARISON OF MORTALITY TABLES FOR MALES
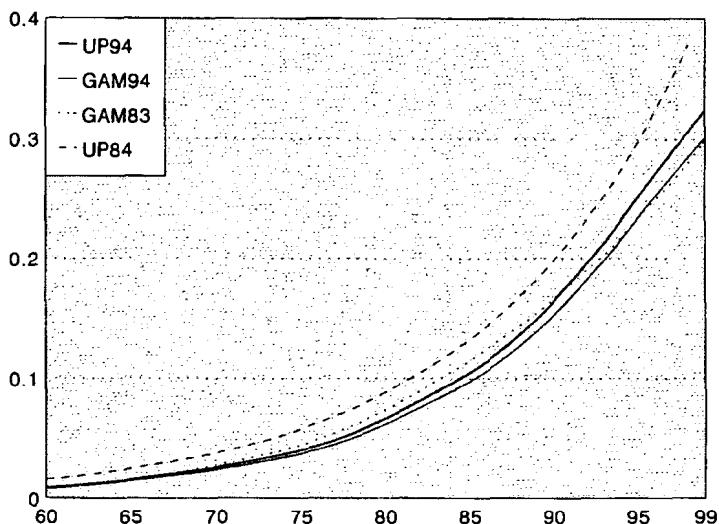
FIGURE 4

COMPARISON OF MORTALITY TABLES FOR FEMALES
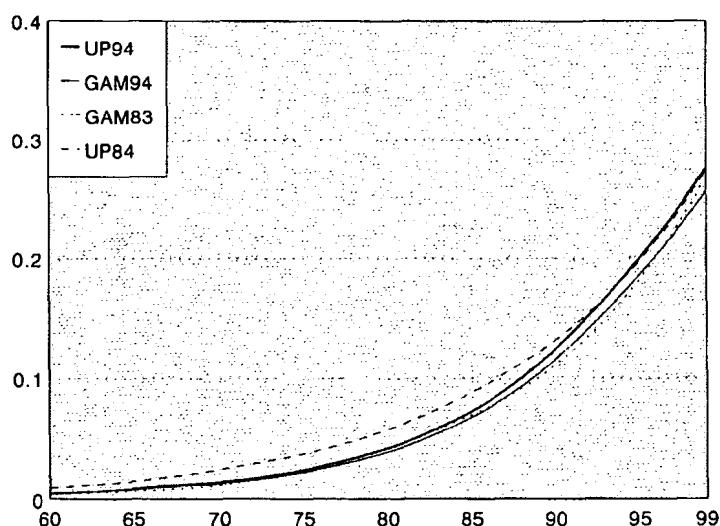
TABLE 4

COMPARISON OF MORTALITY TABLES: RATIOS OF THE UP-94 RATES
TO THE GAM-83 RATES AND TO THE UP-84 RATES

	Male		Female	
Age	UP-94/GAM-83	UP-94/UP-84	UP-94/GAM-83	UP-94/UP-84
1				
2				
3				
4				
5	0.746		1.099	
6	0.767		1.257	
7	0.775		1.398	
8	0.735		1.413	
9	0.716		1.443	
10	0.724		1.469	
11	0.748		1.423	
12	0.799		1.407	
13	0.887		1.451	
14	1.009	0.220	1.550	
15	1.142	0.258	1.664	
16	1.264	0.298	1.752	
17	1.350	0.334	1.767	
18	1.402	0.366	1.744	
19	1.427	0.397	1.682	0.207
20	1.446	0.430	1.614	0.212
21	1.454	0.468	1.532	0.218
22	1.466	0.512	1.467	0.225
23	1.493	0.551	1.391	0.232
24	1.511	0.594	1.310	0.239
25	1.532	0.642	1.237	0.247
26	1.535	0.692	1.179	0.259
27	1.524	0.739	1.141	0.278
28	1.496	0.749	1.119	0.294
29	1.465	0.754	1.113	0.315
30	1.420	0.755	1.102	0.341
31	1.369	0.753	1.102	0.370
32	1.313	0.747	1.101	0.404
33	1.243	0.703	1.097	0.419
34	1.163	0.653	1.088	0.434
35	1.064	0.605	1.080	0.450
36	1.022	0.564	1.096	0.469
37	0.992	0.535	1.106	0.491
38	0.972	0.518	1.122	0.496
39	0.953	0.506	1.136	0.501
40	0.931	0.495	1.147	0.504
41	0.907	0.486	1.154	0.503
42	0.881	0.478	1.146	0.496
43	0.848	0.470	1.120	0.484
44	0.812	0.460	1.079	0.467
45	0.777	0.450	1.036	0.450
46	0.749	0.443	0.995	0.435
47	0.732	0.441	0.967	0.424
48	0.720	0.443	0.949	0.419
49	0.712	0.445	0.936	0.413
50	0.709	0.448	0.933	0.408

TABLE 4—*Continued*

Age	Male		Female	
	UP-94/GAM-83	UP-94/UP-84	UP-94/GAM-83	UP-94/UP-84
51	0.714	0.451	0.940	0.403
52	0.727	0.458	0.956	0.402
53	0.741	0.466	0.967	0.402
54	0.756	0.474	0.968	0.399
55	0.776	0.482	0.970	0.398
56	0.804	0.492	0.983	0.402
57	0.841	0.506	1.012	0.416
58	0.878	0.523	1.049	0.436
59	0.909	0.538	1.087	0.460
60	0.936	0.553	1.125	0.483
61	0.960	0.568	1.164	0.506
62	0.980	0.584	1.204	0.529
63	0.995	0.601	1.244	0.554
64	1.003	0.617	1.283	0.579
65	1.002	0.629	1.315	0.599
66	0.993	0.641	1.333	0.613
67	0.979	0.654	1.333	0.619
68	0.961	0.666	1.304	0.616
69	0.941	0.672	1.251	0.606
70	0.927	0.677	1.192	0.594
71	0.919	0.683	1.138	0.590
72	0.918	0.688	1.098	0.599
73	0.915	0.692	1.067	0.615
74	0.907	0.692	1.039	0.631
75	0.897	0.693	1.017	0.648
76	0.890	0.696	1.002	0.666
77	0.887	0.708	0.994	0.685
78	0.890	0.723	0.990	0.703
79	0.895	0.739	0.986	0.719
80	0.900	0.753	0.986	0.733
81	0.905	0.767	0.992	0.748
82	0.909	0.779	1.003	0.770
83	0.910	0.786	1.016	0.790
84	0.909	0.789	1.028	0.807
85	0.911	0.791	1.042	0.823
86	0.916	0.794	1.058	0.842
87	0.929	0.802	1.077	0.866
88	0.948	0.812	1.097	0.894
89	0.968	0.822	1.110	0.921
90	0.989	0.830	1.119	0.946
91	1.009	0.836	1.125	0.967
92	1.029	0.841	1.126	0.984
93	1.051	0.845	1.121	0.997
94	1.064	0.847	1.112	1.006
95	1.073	0.845	1.098	1.011

TABLE 4—*Continued*

Age	Male		Female	
	UP-94/GAM-83	UP-94/UP-84	UP-94/GAM-83	UP-94/UP-84
96	1.089	0.838	1.079	1.013
97	1.095	0.827	1.064	1.014
98	1.092	0.810	1.048	1.012
99	1.083	0.789	1.029	1.008
100	1.069	0.765	1.007	1.000
101	1.051	0.741	0.981	0.989
102	1.032	0.718	0.950	0.976
103	1.010	0.698	0.921	0.962
104	0.980	0.680	0.890	0.949
105	0.938	0.659	0.851	0.931
106	0.881	0.634	0.803	0.906
107	0.810	0.604	0.744	0.871
108	0.730	0.570	0.679	0.830
109	0.648	0.533	0.612	0.784
110	0.497		0.492	0.736
111				0.686
112				0.636
113				0.586
114				0.541

judgment on future general mortality trends and the nature and demographic attributes of particular retirement plans.

Some actuaries may wish to project the table for a fixed number of years on a static basis, rather than apply mortality improvement on a generational basis. If the mortality rates are projected on a static basis to a specific year, for example, 2004, the projected mortality rate at each age would be given by the following formula:

$$q_x^{2004} = q_x^{1994} \times (1 - AA_x)^{2004-1994}$$

When mortality improvement is applied on a generational basis, the mortality rate for an individual who is age y in the year the decrement is assumed to apply (denoted CYD or calendar year of death) and who was age x in the year of the valuation (denoted CYV or calendar year of valuation) is defined as follows:

$$q_y^{CYV+y-x} = q_y^{1994} \times (1 - AA_y)^{(CYV-1994)+(y-x)}$$

In this formula, the mortality is projected from 1994 to the calendar year of valuation and is also projected from the start of the valuation to the year in which the death is assumed to occur, which is given by the expression

$CYV+y-x$ (and which can also be denoted CYD). Thus, with generational mortality improvement, mortality is projected from 1994 to the calendar year of death, and the formula above can be expressed as follows:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{CYD-1994}$$

In some cases, such as a long-term, open-group projection, an actuary may wish to phase Scale AA into a postulated ultimate long-term mortality improvement trend. A phase-in such as this is used in the projections of Social Security benefits that are done by the Office of the Actuary at the Social Security Administration. See Appendix B for further details.

When the UP-94 Mortality Table is projected on a static basis for a fixed number of years, say, to 2004, it can be referred to as the UP-94 @ 2004. (When the UP-94 Mortality Table is not projected, it is not necessary to append @ 1994 to the title.) The mortality rates that are obtained by projecting the UP-94 Mortality Table on a static basis to the year 1999, and to every fifth year thereafter, through the year 2024, are listed in Appendixes E and F.

The UP-94 Mortality Table with full generational mortality improvement, that is, with mortality projected from 1994 to the calendar year of death, can be referred to as the UP-94G. For a valuation year of, say, 1997, the projection of mortality improvement can be thought of as a two-step process: first, the mortality table is projected three years on a static basis from 1994 to the valuation year of 1997, and then it is projected from 1997 to the calendar year of death. With this interpretation in mind, the mortality table for a valuation year 1997 could also be referred to as the UP-94G @ 1997. However, with full generational mortality improvement, it is understood that mortality is first projected to the valuation year, and the designation @ 1997 is not necessary.[5]

In some cases, an actuary may wish to approximate the effect of using full generational mortality improvement by using the UP-94 Mortality Table projected on a static basis for a fixed number of years. The Subcommittee has found an empirical rule of thumb that can be helpful in this respect. The

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[5]In some cases, an actuary may wish to use a modified version of generational mortality improvement in which the mortality rates would, for example, be the same as for the UP-94G @1997, with the mortality improvement trends applied in exactly the same way, but would be used for a valuation year of, say, 2002. This version of the table could still be referred to as the UP-94G @1997, but in this case the designation @1997 is necessary.

valuation results using the UP-94 Mortality Table with mortality improvement applied on a generational basis turn out to be very close to the results that are obtained by using the UP-94 Mortality Table projected on a static basis for n years, where n is equal to the duration of the liabilities being valued. The "duration" of a liability is the negative of the first derivative of the liability with respect to the change in the valuation interest rate, divided by the liability. For this discussion, the duration is approximated by the following formula:

$$\text{Duration} = \frac{pvb(i) - pvb(i + 0.001)}{pvb(i) \times 0.001}$$

where $pvb(i)$ is the present value of benefits at the valuation interest rate i, and $pvb(i+0.001)$ is the present value of benefits determined with the interest rate increased by one-tenth of one percentage point, that is, by ten basis points.

When the valuation is being performed as of a year that is after 1994, the number of years the static table should be projected should be equal to the duration of the liabilities plus the number of years elapsed between the calendar year of valuation (denoted CYV) and 1994. Thus the mortality rate for each age x is given by the following formula:

$$q_x^{CYV+\text{Duration}} = q_x^{1994} \times (1 - AA_x)^{CYV+\text{Duration}-1994}$$

This formula was tested for deferred and immediate annuities at various issue ages, and for hypothetical populations of active and retired employees, using valuation interest rates of 3%, 7%, and 11%. It was also tested for valuations of the Civil Service Retirement System. The error in using this rule of thumb is generally less than one-half of one percent. See Appendix G for further details.

5. EFFECT OF PROJECTION SCALE AA

To compare the UP-94 Mortality Table with other mortality tables, and to compare the effect of different ways of projecting mortality improvement for the UP-94 Mortality Table, it is helpful to compare the annuity values based on these different tables. Such comparisons can be seen most easily by examining the ratios of the annuity values based on each of the different mortality tables to the values based on the UP-94 Mortality Table.

In Table 5 and Appendix C, the annuity values for the UP-94 Mortality Table are compared with the annuity values for the following mortality

TABLE 5

Ratio of the Net Single Premium for a Life Annuity to the Premium Based on the UP-94 Table, Assuming 7% Interest (Deferred to Age 62 for Ages under 62, Immediate for Ages over 62)

Age	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G @1994	GAM-94 Static	GAR-94	GAM-83	UP-84
Male								
20	1.041	1.078	1.113	1.162	1.023	1.179	0.972	0.828
25	1.040	1.077	1.112	1.147	1.023	1.165	0.971	0.830
30	1.040	1.077	1.111	1.131	1.022	1.149	0.970	0.831
35	1.040	1.077	1.110	1.115	1.022	1.133	0.969	0.833
40	1.040	1.076	1.109	1.098	1.022	1.117	0.969	0.836
45	1.039	1.075	1.107	1.081	1.021	1.100	0.970	0.842
50	1.037	1.072	1.103	1.063	1.020	1.082	0.974	0.853
55	1.034	1.066	1.095	1.046	1.019	1.064	0.980	0.871
60	1.029	1.056	1.081	1.031	1.017	1.047	0.985	0.897
65	1.030	1.058	1.084	1.025	1.018	1.043	0.982	0.904
70	1.036	1.069	1.100	1.022	1.023	1.045	0.971	0.894
75	1.037	1.073	1.106	1.017	1.029	1.046	0.963	0.885
80	1.034	1.067	1.100	1.012	1.035	1.048	0.964	0.885
85	1.030	1.060	1.089	1.008	1.043	1.051	0.977	0.880
90	1.022	1.044	1.066	1.004	1.052	1.056	1.022	0.878
95	1.014	1.027	1.041	1.002	1.062	1.063	1.064	0.848

TABLE 5—*Continued*

Age	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G @1994	GAM-94 Static	GAR-94	GAM-83	UP-84
				Female				
20	1.014	1.028	1.041	1.061	1.016	1.075	1.020	0.859
25	1.014	1.028	1.041	1.055	1.016	1.069	1.019	0.863
30	1.014	1.027	1.040	1.049	1.015	1.063	1.019	0.867
35	1.014	1.027	1.039	1.042	1.015	1.056	1.019	0.870
40	1.013	1.026	1.038	1.036	1.015	1.050	1.019	0.873
45	1.013	1.025	1.037	1.030	1.015	1.044	1.018	0.877
50	1.012	1.023	1.034	1.024	1.014	1.038	1.018	0.884
55	1.011	1.021	1.031	1.019	1.014	1.032	1.019	0.896
60	1.010	1.019	1.028	1.014	1.012	1.026	1.018	0.916
65	1.011	1.022	1.032	1.012	1.014	1.026	1.018	0.920
70	1.015	1.029	1.042	1.012	1.018	1.030	1.012	0.911
75	1.019	1.037	1.054	1.011	1.023	1.034	1.009	0.914
80	1.021	1.041	1.060	1.009	1.029	1.038	1.018	0.926
85	1.018	1.036	1.054	1.006	1.037	1.042	1.041	0.951
90	1.015	1.029	1.044	1.003	1.046	1.049	1.060	0.989
95	1.010	1.021	1.031	1.001	1.055	1.057	1.039	1.005

tables: UP-94 @ 2004, UP-94 @ 2014, UP-94 @ 2024, UP-94G @ 1994, GAM-94 Static, GAR-94, GAM-83, and UP-84. The annuity values based on the GAM-94 Static Table are based on the mortality table for 1994 and do not reflect any mortality improvement for years beyond 1994. For ages under 62, the annuity being valued is a deferred annuity payable at the beginning of each month, commencing at age 62 and issued at five-year age intervals, from age 20 through 60. For ages over 62, it is an immediate annuity payable at the beginning of each month, issued at five-year age intervals, from age 65 through 95. These annuity values are based on 7% interest. The ratios of the annuity values for each of these tables to the annuity values for the UP-94 Mortality Table are shown in Table 5, while the actual annuity values are listed in Appendix C.

6. OTHER COMPARISONS

It may also be helpful to compare the effect on the annuity values of using different interest rates with the effect of using different mortality tables or different mortality improvement trends. These comparisons can be facilitated by examining the ratios of the annuity values based on certain alternative interest rates to the annuity values based on the 7% interest rate. Table 6

TABLE 6

COMPARISON OF NET SINGLE PREMIUMS FOR LIFE ANUITIES FOR THE UP-94 TABLE
(DEFERRED TO AGE 62 FOR AGES UNDER 62; IMMEDIATE FOR AGES OVER 62)

| | Ratio of the Premiums Assuming 8% and 9% Interest to the Premium Assuming 7% Interest | | | |
| | Male | | Female | |
Age	8%	9%	8%	9%
20	0.629	0.399	0.624	0.393
25	0.659	0.437	0.654	0.431
30	0.690	0.480	0.685	0.473
35	0.723	0.526	0.718	0.519
40	0.757	0.577	0.752	0.569
45	0.793	0.633	0.788	0.625
50	0.831	0.695	0.825	0.685
55	0.871	0.762	0.865	0.752
60	0.912	0.836	0.906	0.825
65	0.934	0.876	0.927	0.864
70	0.942	0.891	0.936	0.879
75	0.951	0.906	0.945	0.895
80	0.959	0.922	0.954	0.911
85	0.967	0.937	0.963	0.928
90	0.975	0.951	0.971	0.943
95	0.981	0.962	0.978	0.957

shows these ratios for alternative interest assumptions of 8% and 9%. The actual annuity values are shown in Appendix D.

Table 7 shows a comparison of the expected age at death for each of these mortality tables. It is equal to the current age plus the life expectancy at that age.

ACKNOWLEDGMENT

TABLE 7

COMPARISON OF EXPECTED AGE AT DEATH

(CURRENT AGE PLUS LIFE EXPECTANCY)

Age	UP-94	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G @1994	GAM-94 Static	GAR-94	GAM-83	UP-84
					Male				
20	78.6	79.9	81.0	82.1	83.9	79.4	84.6	77.9	73.8
25	78.8	80.0	81.1	82.2	83.6	79.6	84.3	78.0	74.1
30	79.0	80.2	81.3	82.4	83.3	79.7	84.0	78.1	74.3
35	79.2	80.4	81.5	82.6	83.0	79.9	83.7	78.3	74.6
40	79.4	80.6	81.7	82.7	82.7	80.1	83.4	78.5	74.9
45	79.7	80.8	81.9	82.9	82.4	80.4	83.1	78.7	75.4
50	80.0	81.1	82.2	83.2	82.2	80.7	82.9	79.2	76.0
55	80.5	81.5	82.5	83.5	82.2	81.1	82.9	79.8	76.9
60	81.2	82.2	83.1	83.9	82.5	81.8	83.1	80.6	78.1
65	82.3	83.1	83.9	84.7	83.1	82.8	83.7	81.7	79.7
70	83.8	84.5	85.2	85.9	84.3	84.3	84.9	83.2	81.7
75	85.7	86.2	86.7	87.2	86.0	86.1	86.4	85.2	84.0
80	88.0	88.3	88.7	89.0	88.1	88.4	88.5	87.6	86.8
85	90.9	91.1	91.3	91.5	90.9	91.2	91.3	90.7	90.0
90	94.2	94.3	94.4	94.5	94.2	94.4	94.4	94.3	93.5
95	97.9	98.0	98.0	98.1	98.0	98.2	98.2	98.2	97.4

TABLE 7—*Continued*

Age	UP-94	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G @1994	GAM-94 Static	GAR-94	GAM-83	UP-84
				Female					
20	83.4	84.0	84.6	85.1	86.2	84.1	86.9	84.1	78.4
25	83.5	84.1	84.7	85.2	86.0	84.2	86.7	84.2	78.8
30	83.6	84.2	84.7	85.3	85.9	84.3	86.5	84.3	79.1
35	83.7	84.3	84.8	85.4	85.7	84.4	86.4	84.4	79.3
40	83.8	84.4	84.9	85.5	85.6	84.5	86.2	84.5	79.6
45	84.0	84.6	85.1	85.6	85.5	84.7	86.2	84.7	79.9
50	84.2	84.7	85.2	85.7	85.5	84.9	86.1	84.9	80.4
55	84.5	85.0	85.5	85.9	85.5	85.2	86.2	85.2	81.0
60	85.0	85.4	85.9	86.3	85.8	85.6	86.4	85.7	81.9
65	85.7	86.1	86.5	87.0	86.3	86.3	86.9	86.3	83.1
70	86.8	87.2	87.6	87.9	87.2	87.3	87.7	87.1	84.7
75	88.1	88.5	88.8	89.1	88.4	88.6	88.9	88.4	86.7
80	89.9	90.2	90.4	90.7	90.0	90.3	90.5	90.2	89.0
85	92.2	92.3	92.5	92.7	92.2	92.5	92.6	92.6	91.8
90	95.0	95.1	95.2	95.3	95.1	95.3	95.4	95.4	95.0
95	98.5	98.6	98.6	98.6	98.5	98.8	98.8	98.7	98.5

APPENDIX A

COMPARISON OF MORTALITY RATES FOR VARIOUS MORTALITY TABLES

Age	Male				Female			
	UP-94	GAM-94 Static	GAM-83	UP-84	UP-94	GAM-94 Static	GAM-83	UP-84
1	0.000637	0.000592			0.000571	0.000531		
2	0.000430	0.000400			0.000372	0.000346		
3	0.000357	0.000332			0.000278	0.000258		
4	0.000278	0.000259			0.000208	0.000194		
5	0.000255	0.000237	0.000342		0.000188	0.000175	0.000171	
6	0.000244	0.000227	0.000318		0.000176	0.000163	0.000140	
7	0.000234	0.000217	0.000302		0.000165	0.000153	0.000118	
8	0.000216	0.000201	0.000294		0.000147	0.000137	0.000104	
9	0.000209	0.000194	0.000292		0.000140	0.000130	0.000097	
10	0.000212	0.000197	0.000293		0.000141	0.000131	0.000096	
11	0.000223	0.000208	0.000298		0.000148	0.000138	0.000104	
12	0.000243	0.000226	0.000304		0.000159	0.000148	0.000113	
13	0.000275	0.000255	0.000310		0.000177	0.000164	0.000122	
14	0.000320	0.000297	0.000317	0.001453	0.000203	0.000189	0.000131	
15	0.000371	0.000345	0.000325	0.001437	0.000233	0.000216	0.000140	
16	0.000421	0.000391	0.000333	0.001414	0.000261	0.000242	0.000149	
17	0.000463	0.000430	0.000343	0.001385	0.000281	0.000262	0.000159	
18	0.000495	0.000460	0.000353	0.001351	0.000293	0.000273	0.000168	
19	0.000521	0.000484	0.000365	0.001311	0.000301	0.000280	0.000179	0.001453
20	0.000545	0.000507	0.000377	0.001267	0.000305	0.000284	0.000189	0.001437

	Male				Female			
Age	UP-94	GAM-94 STATIC	GAM-83	UP-84	UP-94	GAM-94 STATIC	GAM-83	UP-84
21	0.000570	0.000530	0.000392	0.001219	0.000308	0.000286	0.000201	0.001414
22	0.000598	0.000556	0.000408	0.001167	0.000311	0.000289	0.000212	0.001385
23	0.000633	0.000589	0.000424	0.001149	0.000313	0.000292	0.000225	0.001351
24	0.000671	0.000624	0.000444	0.001129	0.000313	0.000291	0.000239	0.001311
25	0.000711	0.000661	0.000464	0.001107	0.000313	0.000291	0.000253	0.001267
26	0.000749	0.000696	0.000488	0.001083	0.000316	0.000294	0.000268	0.001219
27	0.000782	0.000727	0.000513	0.001058	0.000324	0.000302	0.000284	0.001167
28	0.000811	0.000754	0.000542	0.001083	0.000338	0.000314	0.000302	0.001149
29	0.000838	0.000779	0.000572	0.001111	0.000356	0.000331	0.000320	0.001129
30	0.000862	0.000801	0.000607	0.001141	0.000377	0.000351	0.000342	0.001107
31	0.000883	0.000821	0.000645	0.001173	0.000401	0.000373	0.000364	0.001083
32	0.000902	0.000839	0.000687	0.001208	0.000427	0.000397	0.000388	0.001058
33	0.000912	0.000848	0.000734	0.001297	0.000454	0.000422	0.000414	0.001083
34	0.000913	0.000849	0.000785	0.001398	0.000482	0.000449	0.000443	0.001111
35	0.000915	0.000851	0.000860	0.001513	0.000514	0.000478	0.000476	0.001141
36	0.000927	0.000862	0.000907	0.001643	0.000550	0.000512	0.000502	0.001173
37	0.000958	0.000891	0.000966	0.001792	0.000593	0.000551	0.000536	0.001208
38	0.001010	0.000939	0.001039	0.001948	0.000643	0.000598	0.000573	0.001297
39	0.001075	0.000999	0.001128	0.002125	0.000701	0.000652	0.000617	0.001398
40	0.001153	0.001072	0.001238	0.002327	0.000763	0.000709	0.000665	0.001513

APPENDIX A—*Continued*

Age	Male				Female			
	UP-94	GAM-94 Static	GAM-83	UP-84	UP-94	GAM-94 Static	GAM-83	UP-84
41	0.001243	0.001156	0.001370	0.002556	0.000826	0.000768	0.000716	0.001643
42	0.001346	0.001252	0.001527	0.002818	0.000888	0.000825	0.000775	0.001792
43	0.001454	0.001352	0.001715	0.003095	0.000943	0.000877	0.000842	0.001948
44	0.001568	0.001458	0.001932	0.003410	0.000992	0.000923	0.000919	0.002125
45	0.001697	0.001578	0.002183	0.003769	0.001046	0.000973	0.001010	0.002327
46	0.001852	0.001722	0.002471	0.004180	0.001111	0.001033	0.001117	0.002556
47	0.002042	0.001899	0.002790	0.004635	0.001196	0.001112	0.001237	0.002818
48	0.002260	0.002102	0.003138	0.005103	0.001297	0.001206	0.001366	0.003095
49	0.002501	0.002326	0.003513	0.005616	0.001408	0.001310	0.001505	0.003410
50	0.002773	0.002579	0.003909	0.006196	0.001536	0.001428	0.001647	0.003769
51	0.003088	0.002872	0.004324	0.006853	0.001686	0.001568	0.001793	0.004180
52	0.003455	0.003213	0.004755	0.007543	0.001864	0.001734	0.001949	0.004635
53	0.003854	0.003584	0.005200	0.008278	0.002051	0.001907	0.002120	0.005103
54	0.004278	0.003979	0.005660	0.009033	0.002241	0.002084	0.002315	0.005616
55	0.004758	0.004425	0.006131	0.009875	0.002466	0.002294	0.002541	0.006196
56	0.005322	0.004949	0.006618	0.010814	0.002755	0.002563	0.002803	0.006853
57	0.006001	0.005581	0.007139	0.011863	0.003139	0.002919	0.003103	0.007543
58	0.006774	0.006300	0.007719	0.012952	0.003612	0.003359	0.003443	0.008278
59	0.007623	0.007090	0.008384	0.014162	0.004154	0.003863	0.003821	0.009033
60	0.008576	0.007976	0.009158	0.015509	0.004773	0.004439	0.004241	0.009875

843

	Male				Female			
Age	UP-94	GAM-94 Static	GAM-83	UP-84	UP-94	GAM-94 Static	GAM-83	UP-84
61	0.009663	0.008986	0.010064	0.017010	0.005476	0.005093	0.004703	0.010814
62	0.010911	0.010147	0.011133	0.018685	0.006271	0.005832	0.005210	0.011863
63	0.012335	0.011471	0.012391	0.020517	0.007179	0.006677	0.005769	0.012952
64	0.013914	0.012940	0.013868	0.022562	0.008194	0.007621	0.006386	0.014162
65	0.015629	0.014535	0.015592	0.024847	0.009286	0.008636	0.007064	0.015509
66	0.017462	0.016239	0.017579	0.027232	0.010423	0.009694	0.007817	0.017010
67	0.019391	0.018034	0.019804	0.029634	0.011574	0.010764	0.008681	0.018685
68	0.021354	0.019859	0.022229	0.032073	0.012648	0.011763	0.009702	0.020517
69	0.023364	0.021729	0.024817	0.034743	0.013665	0.012709	0.010922	0.022562
70	0.025516	0.023730	0.027530	0.037667	0.014763	0.013730	0.012385	0.024847
71	0.027905	0.025951	0.030354	0.040871	0.016079	0.014953	0.014128	0.027232
72	0.030625	0.028481	0.033370	0.044504	0.017748	0.016506	0.016160	0.029634
73	0.033549	0.031201	0.036680	0.048504	0.019724	0.018344	0.018481	0.032073
74	0.036614	0.034051	0.040388	0.052913	0.021915	0.020381	0.021092	0.034743
75	0.040012	0.037211	0.044597	0.057775	0.024393	0.022686	0.023992	0.037667
76	0.043933	0.040858	0.049388	0.063142	0.027231	0.025325	0.027185	0.040871
77	0.048570	0.045171	0.054758	0.068628	0.030501	0.028366	0.030672	0.044504
78	0.053991	0.050211	0.060678	0.074648	0.034115	0.031727	0.034459	0.048504
79	0.060066	0.055861	0.067125	0.081256	0.038024	0.035362	0.038549	0.052913
80	0.066696	0.062027	0.074070	0.088518	0.042361	0.039396	0.042945	0.057775

844

Age	Male				Female			
	UP-94	GAM-94 Static	GAM-83	UP-84	UP-94	GAM-94 Static	GAM-83	UP-84
81	0.073780	0.068615	0.081484	0.096218	0.047260	0.043952	0.047655	0.063142
82	0.081217	0.075532	0.089320	0.104310	0.052853	0.049153	0.052691	0.068628
83	0.088721	0.082510	0.097525	0.112816	0.058986	0.054857	0.058071	0.074648
84	0.096358	0.089613	0.106047	0.122079	0.065569	0.060979	0.063807	0.081256
85	0.104559	0.097240	0.114836	0.132174	0.072836	0.067738	0.069918	0.088518
86	0.113755	0.105792	0.124170	0.143179	0.081018	0.075347	0.076570	0.096218
87	0.124377	0.115671	0.133870	0.155147	0.090348	0.084023	0.083870	0.104310
88	0.136537	0.126980	0.144073	0.168208	0.100882	0.093820	0.091935	0.112816
89	0.149949	0.139452	0.154859	0.182461	0.112467	0.104594	0.101354	0.122079
90	0.164442	0.152931	0.166307	0.198030	0.125016	0.116265	0.111750	0.132174
91	0.179849	0.167260	0.178214	0.215035	0.138442	0.128751	0.123076	0.143179
92	0.196001	0.182281	0.190460	0.232983	0.152660	0.141973	0.135630	0.155147
93	0.213325	0.198392	0.203007	0.252545	0.167668	0.155931	0.149577	0.168208
94	0.231936	0.215700	0.217904	0.273878	0.183524	0.170677	0.165103	0.182461
95	0.251189	0.233606	0.234086	0.297152	0.200229	0.186213	0.182419	0.198030
96	0.270441	0.251510	0.248436	0.322553	0.217783	0.202538	0.201757	0.215035
97	0.289048	0.268815	0.263954	0.349505	0.236188	0.219655	0.222044	0.232983
98	0.306750	0.285277	0.280803	0.378865	0.255605	0.237713	0.243899	0.252545
99	0.323976	0.301298	0.299154	0.410875	0.276035	0.256712	0.268185	0.273878
100	0.341116	0.317238	0.319185	0.445768	0.297233	0.276427	0.295187	0.297152

APPENDIX A—*Continued*

	Male				Female			
Age	UP-94	GAM-94 Static	GAM-83	UP-84	UP-94	GAM-94 Static	GAM-83	UP-84
101	0.358560	0.333461	0.341086	0.483830	0.318956	0.296629	0.325225	0.322553
102	0.376699	0.350330	0.365052	0.524301	0.340960	0.317093	0.358897	0.349505
103	0.396884	0.368542	0.393102	0.568365	0.364586	0.338505	0.395843	0.378865
104	0.418855	0.387855	0.427255	0.616382	0.389996	0.361016	0.438360	0.410875
105	0.440585	0.407224	0.469531	0.668696	0.415180	0.383597	0.487816	0.445768
106	0.460043	0.425599	0.521945	0.725745	0.438126	0.405217	0.545886	0.483830
107	0.475200	0.441935	0.586518	0.786495	0.456824	0.424846	0.614309	0.524301
108	0.485670	0.457553	0.665268	0.852659	0.471493	0.444368	0.694885	0.568365
109	0.492807	0.473150	0.760215	0.924666	0.483473	0.464469	0.789474	0.616382
110	0.497189	0.486745	1.000000		0.492436	0.482325	1.000000	0.668696
111	0.499394	0.496356			0.498054	0.495110		0.725745
112	0.500000	0.500000			0.500000	0.500000		0.786495
113	0.500000	0.500000			0.500000	0.500000		0.852659
114	0.500000	0.500000			0.500000	0.500000		0.924666
115	0.500000	0.500000			0.500000	0.500000		
116	0.500000	0.500000			0.500000	0.500000		
117	0.500000	0.500000			0.500000	0.500000		
118	0.500000	0.500000			0.500000	0.500000		
119	0.500000	0.500000			0.500000	0.500000		
120	1.000000	1.000000			1.000000	1.000000		

APPENDIX B

ISSUES IN CHOOSING A MORTALITY IMPROVEMENT TREND

Social Security Trends

The methods and assumptions used in determining the mortality improvement trends for Social Security are summarized in *Actuarial Study No. 107*. As part of this process, the long-term trends in age-adjusted central death rates were first examined. The average annual rates of improvement, which appear in Table 4 on page 9 of *Actuarial Study No. 107*, are summarized below. The data reveal several distinct periods of mortality improvement since 1900.

AVERAGE ANNUAL REDUCTIONS IN AGE-ADJUSTED CENTRAL DEATH RATES

Age	1900–36	1936–54	1954–68	1968–82	1982–88	1900–88
Male						
25–64	0.87%	1.69%	−0.18%	2.27%	0.79%	1.09%
65+	0.21	1.15	−0.32	1.49	0.23	0.52
Female						
25–64	1.07	3.30	0.62	2.13	0.61	1.59
65+	0.33	1.84	0.79	2.01	−0.17	0.95

Because the reduction in mortality has varied greatly by cause of death, for purposes of determining the Social Security mortality improvement trend assumptions, mortality rates were also calculated and analyzed by age group and sex for ten different groups of causes of death.

After these past trends had been examined, ultimate annual percentage reductions in central death rates were postulated for years after 2016, by age, sex, and cause of death, after considering such factors as: the development and application of new diagnostic, surgical and life sustaining techniques; the presence of environmental pollutants; improvements in exercise and nutrition; the incidence of violence; the isolation and treatment of causes of disease; the emergence of new forms of disease; improvements in prenatal care; the prevalence of cigarette smoking; the misuse of drugs (including alcohol); the extent to which people assume responsibility for their own health; education on health; and changes in our conception of the value of life.

For years after 1990, the reductions in mortality were assumed to gradually change from an initial rate of 100% of the average annual reductions

observed for the 1968–88 period to the postulated ultimate percentage reductions, which were assumed to apply after the year 2016. These ultimate trend rates are generally in the range of 0.5% to 0.6% per year, depending on age.

Use of an Ultimate Trend

Scale AA is based on an average of Social Security and Civil Service overall mortality improvement trends, based on experience for the years 1977 through 1993, and it applies to all future years and does not phase into a different long-term ultimate trend. However, for some applications, such as a long-term open group projection, an actuary may prefer to phase Scale AA into some ultimate trend, which can be denoted "Scale AB." For example, Scale AB may be 0.005 (that is, 0.5%) for all ages, or it might be the lesser of 0.005 and Scale AA. First, we shall assume that the phase-in is over 30 years, from 1994 to 2024.

If a linear interpolation is used, for a person age y, the mortality improvement factor in calendar year CY would be:

$$1 - AA_y \times (2024 - CY)/30 - AB_y \times (CY - 1994)/30$$

However, when using a linear interpolation formula such as this, it is not possible to come up with a simple mathematical expression for the cumulative improvement for each year in the future, and a geometric interpolation formula works better, where the mortality improvement factor in the year CY would be:

$$(1 - AA_y)^{(2024-CY)/30} \times (1 - AB_y)^{(CY-1994)/30}$$

When generational mortality improvement is applied, the mortality rate is projected to the year in which the decrement is assumed to occur, which we denote CYD, or calendar year of death. Using geometric interpolation (and assuming the 30-year phase-in), it can be shown that if the mortality rate in 1994 at age y is q_y^{1994}, then the mortality rate in the year CYD is:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{(CYD-1994)(2053-CYD)/60}$$
$$\times (1 - AB_y)^{(CYD-1994)(CYD-1993)/60}$$

This formula would be true for CYD less than or equal 2024. For years greater than 2024, the formula would be:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{14.5} \times (1 - AB_y)^{CYD-1994-14.5}$$

In the more general case, in which the mortality improvement is being phased from Scale AA to Scale AB over N years (rather than 30 years in the example shown above), the mortality rate in year CYD would be:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{(CYD-1994)(1993+2N-CYD)/2N}$$
$$\times (1 - AB_y)^{(CYD-1994)(CYD-1993)/2N}$$

This formula would be true for a CYD less than or equal $1994+N$. For years greater than $1994+N$, the formula would be:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{(N-1)/2} \times (1 - AB_y)^{CYD-1994-(N-1)/2}$$

Note that if the Scale AB were equal to Scale AA at a particular age, the above formulas would simplify to the following at that age:

$$q_y^{CYD} = q_y^{1994} \times (1 - AA_y)^{(CYD-1994)}.$$

APPENDIX C

COMPARISON OF NET SINGLE PREMIUMS FOR LIFE ANNUITIES ASSUMING 7% INTEREST
(DEFERRED TO AGE 62 FOR AGES UNDER 62, IMMEDIATE FOR AGES OVER 62)

Age	UP-94	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G 1994	GAM-94 Static	GAR-94	GAM-83	UP-84
				Male					
20	0.532	0.554	0.574	0.592	0.618	0.544	0.628	0.517	0.440
25	0.749	0.779	0.807	0.832	0.859	0.766	0.872	0.727	0.621
30	1.054	1.096	1.135	1.171	1.192	1.078	1.212	1.023	0.876
35	1.485	1.544	1.599	1.649	1.656	1.518	1.683	1.439	1.237
40	2.093	2.176	2.252	2.322	2.298	2.138	2.338	2.029	1.750
45	2.956	3.071	3.176	3.273	3.194	3.018	3.250	2.867	2.490
50	4.189	4.345	4.489	4.621	4.452	4.274	4.531	4.079	3.575
55	5.979	6.183	6.371	6.544	6.253	6.093	6.362	5.859	5.209
60	8.646	8.898	9.131	9.347	8.910	8.792	9.052	8.520	7.758
65	9.413	9.695	9.958	10.203	9.645	9.584	9.814	9.242	8.512
70	8.243	8.538	8.814	9.071	8.425	8.432	8.613	8.006	7.368
75	6.984	7.244	7.491	7.726	7.105	7.183	7.306	6.729	6.179
80	5.684	5.878	6.067	6.251	5.755	5.886	5.958	5.480	5.028
85	4.506	4.641	4.775	4.906	4.542	4.698	4.735	4.401	3.968
90	3.418	3.493	3.568	3.643	3.432	3.594	3.609	3.493	3.002
95	2.560	2.595	2.630	2.664	2.564	2.718	2.722	2.723	2.171

APPENDIX C—*Continued*

Age	UP-94	UP-94 @2004	UP-94 @2014	UP-94 @2024	UP-94G @1994	GAM-94 Static	GAR-94	GAM-83	UP-84
				Female					
20	0.610	0.618	0.627	0.635	0.647	0.619	0.655	0.622	0.523
25	0.856	0.868	0.880	0.891	0.903	0.870	0.915	0.873	0.739
30	1.203	1.220	1.236	1.251	1.262	1.221	1.278	1.226	1.043
35	1.691	1.714	1.736	1.757	1.762	1.716	1.786	1.723	1.470
40	2.378	2.410	2.441	2.470	2.464	2.414	2.498	2.423	2.075
45	3.351	3.394	3.435	3.474	3.451	3.400	3.498	3.412	2.937
50	4.728	4.784	4.838	4.889	4.842	4.796	4.906	4.815	4.178
55	6.694	6.765	6.834	6.901	6.818	6.785	6.907	6.820	5.999
60	9.541	9.634	9.724	9.812	9.672	9.660	9.789	9.717	8.740
65	10.439	10.554	10.665	10.773	10.567	10.583	10.710	10.623	9.607
70	9.342	9.478	9.610	9.738	9.458	9.507	9.622	9.451	8.512
75	8.058	8.207	8.352	8.492	8.148	8.241	8.331	8.131	7.368
80	6.672	6.809	6.943	7.073	6.731	6.867	6.926	6.795	6.179
85	5.288	5.384	5.478	5.571	5.317	5.482	5.512	5.505	5.028
90	4.011	4.070	4.128	4.186	4.024	4.195	4.209	4.252	3.968
95	2.987	3.018	3.049	3.080	2.991	3.153	3.157	3.103	3.002

APPENDIX D

COMPARISON OF NET SINGLE PREMIUMS FOR LIFE ANNUITIES FOR THE UP-94 TABLE, ASSUMING INTEREST RATES OF 7%, 8%, AND 9% (DEFERRED TO AGE 62 FOR AGES UNDER 62, IMMEDIATE FOR AGES OVER 62)

Age	Male			Female		
	7%	8%	9%	7%	8%	9%
20	0.532	0.335	0.212	0.610	0.381	0.240
25	0.749	0.493	0.327	0.856	0.560	0.369
30	1.054	0.728	0.506	1.203	0.824	0.569
35	1.485	1.074	0.781	1.691	1.213	0.877
40	2.093	1.585	1.208	2.378	1.788	1.354
45	2.956	2.345	1.872	3.351	2.639	2.093
50	4.189	3.482	2.910	4.728	3.902	3.239
55	5.979	5.206	4.556	6.694	5.787	5.031
60	8.646	7.887	7.227	9.541	8.641	7.867
65	9.413	8.793	8.245	10.439	9.681	9.017
70	8.243	7.769	7.343	9.342	8.741	8.208
75	6.984	6.642	6.331	8.058	7.612	7.209
80	5.684	5.454	5.241	6.672	6.364	6.081
85	4.506	4.359	4.222	5.288	5.090	4.907
90	3.418	3.331	3.249	4.011	3.894	3.784
95	2.560	2.510	2.463	2.987	2.921	2.858

APPENDIX E
UP-94 PROJECTED MALE MORTALITY RATES

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
1	.000576	.000520	.000470	.000425	.000384	.000347
2	.000389	.000351	.000318	.000287	.000259	.000235
3	.000323	.000292	.000264	.000238	.000215	.000195
4	.000251	.000227	.000205	.000186	.000168	.000152
5	.000230	.000208	.000188	.000170	.000154	.000139
6	.000221	.000199	.000180	.000163	.000147	.000133
7	.000212	.000191	.000173	.000156	.000141	.000128
8	.000195	.000176	.000160	.000144	.000130	.000118
9	.000189	.000171	.000154	.000140	.000126	.000114
10	.000192	.000173	.000157	.000142	.000128	.000116
11	.000202	.000182	.000165	.000149	.000135	.000122
12	.000220	.000199	.000179	.000162	.000147	.000133
13	.000249	.000225	.000203	.000184	.000166	.000150
14	.000291	.000264	.000240	.000218	.000198	.000180
15	.000337	.000306	.000278	.000253	.000230	.000209
16	.000382	.000348	.000316	.000287	.000261	.000237
17	.000421	.000382	.000347	.000315	.000287	.000260
18	.000450	.000409	.000373	.000337	.000306	.000278
19	.000473	.000430	.000391	.000355	.000323	.000293
20	.000495	.000450	.000409	.000371	.000337	.000307
21	.000521	.000475	.000434	.000396	.000362	.000331
22	.000549	.000504	.000462	.000424	.000390	.000358
23	.000587	.000544	.000505	.000468	.000434	.000402
24	.000629	.000589	.000551	.000516	.000484	.000453
25	.000676	.000643	.000612	.000582	.000553	.000526
26	.000727	.000705	.000684	.000664	.000644	.000625
27	.000763	.000744	.000725	.000707	.000690	.000673
28	.000791	.000771	.000752	.000734	.000715	.000698
29	.000817	.000797	.000777	.000758	.000739	.000721
30	.000841	.000820	.000800	.000780	.000760	.000742
31	.000861	.000840	.000819	.000799	.000779	.000760
32	.000880	.000858	.000837	.000816	.000796	.000776
33	.000889	.000867	.000846	.000825	.000805	.000785
34	.000890	.000868	.000847	.000826	.000805	.000786
35	.000892	.000870	.000849	.000828	.000807	.000787
36	.000904	.000882	.000860	.000839	.000818	.000798
37	.000934	.000911	.000889	.000867	.000845	.000824
38	.000980	.000951	.000923	.000895	.000869	.000843
39	.001038	.001002	.000967	.000934	.000902	.000871
40	.001108	.001064	.001022	.000982	.000943	.000906

APPENDIX E—*Continued*

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
41	.001188	.001136	.001085	.001037	.000992	.000948
42	.001280	.001217	.001158	.001101	.001047	.000996
43	.001376	.001302	.001232	.001165	.001103	.001043
44	.001476	.001390	.001308	.001232	.001159	.001092
45	.001590	.001489	.001395	.001306	.001224	.001146
46	.001726	.001608	.001499	.001397	.001302	.001213
47	.001893	.001756	.001628	.001509	.001399	.001298
48	.002085	.001923	.001774	.001637	.001510	.001393
49	.002296	.002107	.001934	.001775	.001629	.001495
50	.002532	.002312	.002112	.001928	.001761	.001608
51	.002806	.002549	.002316	.002104	.001912	.001737
52	.003123	.002823	.002552	.002307	.002085	.001885
53	.003484	.003149	.002846	.002573	.002326	.002102
54	.003867	.003495	.003160	.002856	.002582	.002334
55	.004323	.003927	.003568	.003242	.002945	.002676
56	.004860	.004438	.004053	.003701	.003380	.003086
57	.005508	.005055	.004640	.004259	.003909	.003588
58	.006249	.005765	.005318	.004906	.004526	.004175
59	.007032	.006487	.005985	.005521	.005093	.004699
60	.007912	.007299	.006733	.006211	.005730	.005286
61	.008960	.008308	.007703	.007142	.006622	.006140
62	.010117	.009381	.008698	.008065	.007478	.006933
63	.011495	.010713	.009984	.009304	.008671	.008081
64	.012967	.012084	.011262	.010495	.009781	.009115
65	.014565	.013574	.012650	.011789	.010986	.010239
66	.016356	.015320	.014350	.013441	.012590	.011793
67	.018163	.017013	.015935	.014926	.013981	.013095
68	.019900	.018546	.017284	.016107	.015011	.013989
69	.021774	.020292	.018910	.017623	.016424	.015306
70	.023659	.021937	.020340	.018860	.017487	.016214
71	.025874	.023991	.022245	.020626	.019124	.017732
72	.028396	.026329	.024413	.022636	.020989	.019461
73	.031107	.028843	.026744	.024797	.022992	.021319
74	.033949	.031478	.029187	.027063	.025093	.023267
75	.037288	.034750	.032385	.030181	.028126	.026212
76	.040943	.038156	.035559	.033138	.030883	.028780
77	.045494	.042613	.039914	.037386	.035018	.032801
78	.050828	.047851	.045048	.042409	.039925	.037586
79	.056834	.053776	.050883	.048145	.045555	.043104
80	.063427	.060319	.057362	.054551	.051878	.049335

APPENDIX E—*Continued*

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
81	.070519	.067402	.064423	.061576	.058855	.056253
82	.078020	.074949	.071998	.069164	.066441	.063826
83	.085228	.081873	.078650	.075554	.072580	.069723
84	.093032	.089821	.086721	.083728	.080839	.078049
85	.100950	.097466	.094102	.090854	.087719	.084691
86	.109829	.106038	.102379	.098845	.095434	.092140
87	.120690	.117113	.113641	.110273	.107004	.103832
88	.133157	.129862	.126647	.123513	.120456	.117474
89	.146237	.142618	.139088	.135645	.132288	.129014
90	.161179	.157981	.154847	.151775	.148763	.145812
91	.176281	.172783	.169355	.165995	.162701	.159473
92	.193079	.190200	.187364	.184570	.181818	.179107
93	.210144	.207011	.203924	.200884	.197888	.194938
94	.228478	.225071	.221715	.218409	.215153	.211945
95	.248687	.246210	.243758	.241330	.238926	.236547
96	.267747	.265081	.262440	.259826	.257239	.254677
97	.286169	.283319	.280497	.277703	.274937	.272199
98	.305219	.303696	.302181	.300673	.299172	.297679
99	.322359	.320751	.319150	.317558	.315973	.314396
100	.339414	.337720	.336035	.334358	.332689	.331029
101	.358560	.358560	.358560	.358560	.358560	.358560
102	.376699	.376699	.376699	.376699	.376699	.376699
103	.396884	.396884	.396884	.396884	.396884	.396884
104	.418855	.418855	.418855	.418855	.418855	.418855
105	.440585	.440585	.440585	.440585	.440585	.440585
106	.460043	.460043	.460043	.460043	.460043	.460043
107	.475200	.475200	.475200	.475200	.475200	.475200
108	.485670	.485670	.485670	.485670	.485670	.485670
109	.492807	.492807	.492807	.492807	.492807	.492807
110	.497189	.497189	.497189	.497189	.497189	.497189
111	.499394	.499394	.499394	.499394	.499394	.499394
112	.500000	.500000	.500000	.500000	.500000	.500000
113	.500000	.500000	.500000	.500000	.500000	.500000
114	.500000	.500000	.500000	.500000	.500000	.500000
115	.500000	.500000	.500000	.500000	.500000	.500000
116	.500000	.500000	.500000	.500000	.500000	.500000
117	.500000	.500000	.500000	.500000	.500000	.500000
118	.500000	.500000	.500000	.500000	.500000	.500000
119	.500000	.500000	.500000	.500000	.500000	.500000
120	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

APPENDIX F
UP-94 PROJECTED FEMALE MORTALITY RATES

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
1	.000516	.000467	.000422	.000381	.000345	.000311
2	.000336	.000304	.000275	.000248	.000224	.000203
3	.000251	.000227	.000205	.000186	.000168	.000152
4	.000188	.000170	.000154	.000139	.000126	.000113
5	.000170	.000154	.000139	.000126	.000113	.000103
6	.000159	.000144	.000130	.000117	.000106	.000096
7	.000149	.000135	.000122	.000110	.000100	.000090
8	.000133	.000120	.000109	.000098	.000089	.000080
9	.000127	.000114	.000103	.000093	.000084	.000076
10	.000127	.000115	.000104	.000094	.000085	.000077
11	.000134	.000121	.000109	.000099	.000089	.000081
12	.000144	.000130	.000117	.000106	.000096	.000087
13	.000160	.000145	.000131	.000118	.000107	.000097
14	.000185	.000169	.000155	.000141	.000129	.000118
15	.000215	.000198	.000183	.000169	.000156	.000144
16	.000242	.000224	.000208	.000193	.000179	.000166
17	.000262	.000244	.000227	.000212	.000198	.000184
18	.000273	.000254	.000237	.000221	.000206	.000192
19	.000279	.000259	.000240	.000222	.000206	.000191
20	.000281	.000260	.000239	.000221	.000204	.000188
21	.000283	.000259	.000238	.000219	.000201	.000184
22	.000285	.000262	.000240	.000221	.000203	.000186
23	.000289	.000266	.000246	.000227	.000209	.000193
24	.000290	.000269	.000250	.000231	.000215	.000199
25	.000292	.000272	.000253	.000236	.000220	.000205
26	.000297	.000280	.000264	.000248	.000234	.000220
27	.000305	.000287	.000270	.000254	.000240	.000226
28	.000318	.000300	.000282	.000265	.000250	.000235
29	.000335	.000316	.000297	.000280	.000263	.000248
30	.000359	.000341	.000324	.000308	.000293	.000279
31	.000385	.000370	.000355	.000341	.000328	.000315
32	.000410	.000394	.000379	.000364	.000349	.000336
33	.000434	.000415	.000396	.000379	.000362	.000346
34	.000458	.000436	.000415	.000394	.000375	.000357
35	.000486	.000460	.000435	.000412	.000390	.000369
36	.000518	.000487	.000459	.000432	.000407	.000383
37	.000555	.000520	.000487	.000456	.000428	.000400
38	.000599	.000558	.000520	.000485	.000452	.000421
39	.000650	.000603	.000559	.000518	.000480	.000445
40	.000707	.000656	.000608	.000564	.000523	.000485

APPENDIX F—*Continued*

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
41	.000766	.000710	.000658	.000611	.000566	.000525
42	.000823	.000763	.000708	.000656	.000609	.000564
43	.000874	.000811	.000752	.000697	.000646	.000599
44	.000920	.000853	.000791	.000733	.000680	.000630
45	.000965	.000890	.000821	.000758	.000699	.000645
46	.001020	.000936	.000859	.000788	.000724	.000664
47	.001092	.000997	.000911	.000832	.000759	.000694
48	.001184	.001082	.000988	.000902	.000824	.000752
49	.001286	.001174	.001072	.000979	.000894	.000816
50	.001410	.001294	.001188	.001090	.001001	.000918
51	.001555	.001435	.001324	.001221	.001127	.001039
52	.001737	.001619	.001509	.001406	.001310	.001221
53	.001931	.001818	.001711	.001611	.001517	.001428
54	.002131	.002027	.001927	.001833	.001743	.001658
55	.002369	.002276	.002186	.002100	.002017	.001938
56	.002673	.002594	.002517	.002443	.002370	.002300
57	.003061	.002986	.002912	.002840	.002769	.002701
58	.003523	.003435	.003350	.003267	.003187	.003108
59	.004051	.003951	.003853	.003758	.003665	.003574
60	.004655	.004540	.004427	.004318	.004211	.004107
61	.005340	.005208	.005079	.004954	.004831	.004711
62	.006116	.005964	.005817	.005673	.005532	.005395
63	.007001	.006828	.006659	.006494	.006333	.006177
64	.007991	.007793	.007600	.007412	.007229	.007050
65	.009056	.008832	.008613	.008400	.008192	.007990
66	.010165	.009913	.009668	.009429	.009195	.008968
67	.011288	.011008	.010736	.010470	.010211	.009958
68	.012335	.012030	.011732	.011442	.011158	.010882
69	.013327	.012997	.012675	.012361	.012056	.011757
70	.014398	.014041	.013694	.013355	.013024	.012702
71	.015602	.015140	.014691	.014256	.013833	.013423
72	.017222	.016711	.016216	.015735	.015269	.014816
73	.019043	.018386	.017751	.017139	.016547	.015976
74	.021159	.020428	.019723	.019043	.018385	.017751
75	.023433	.022510	.021624	.020773	.019955	.019170
76	.026159	.025129	.024140	.023190	.022277	.021400
77	.029448	.028432	.027451	.026503	.025589	.024705
78	.032938	.031801	.030703	.029644	.028620	.027633
79	.036712	.035445	.034221	.033040	.031900	.030799
80	.040899	.039487	.038125	.036809	.035538	.034312

APPENDIX F—*Continued*

Age	Projected to Year					
	1999	2004	2009	2014	2019	2024
81	.045629	.044054	.042534	.041066	.039648	.038280
82	.051029	.049268	.047567	.045926	.044341	.042810
83	.056950	.054985	.053087	.051255	.049486	.047778
84	.063306	.061121	.059012	.056975	.055008	.053110
85	.070677	.068582	.066549	.064576	.062662	.060805
86	.079013	.077057	.075150	.073290	.071476	.069706
87	.088555	.086798	.085076	.083388	.081734	.080112
88	.098880	.096919	.094996	.093111	.091264	.089453
89	.110790	.109138	.107511	.105908	.104329	.102773
90	.123152	.121316	.119507	.117725	.115970	.114241
91	.136378	.134344	.132341	.130368	.128424	.126509
92	.150384	.148141	.145933	.143757	.141613	.139502
93	.165998	.164345	.162708	.161087	.159483	.157894
94	.181696	.179886	.178095	.176321	.174565	.172826
95	.198235	.196260	.194305	.192370	.190454	.188557
96	.215614	.213466	.211340	.209235	.207151	.205088
97	.235009	.233837	.232670	.231509	.230353	.229204
98	.254330	.253060	.251798	.250541	.249291	.248047
99	.274657	.273287	.271923	.270566	.269216	.267873
100	.295750	.294274	.292805	.291344	.289890	.288444
101	.318956	.318956	.318956	.318956	.318956	.318956
102	.340960	.340960	.340960	.340960	.340960	.340960
103	.364586	.364586	.364586	.364586	.364586	.364586
104	.389996	.389996	.389996	.389996	.389996	.389996
105	.415180	.415180	.415180	.415180	.415180	.415180
106	.438126	.438126	.438126	.438126	.438126	.438126
107	.456824	.456824	.456824	.456824	.456824	.456824
108	.471493	.471493	.471493	.471493	.471493	.471493
109	.483473	.483473	.483473	.483473	.483473	.483473
110	.492436	.492436	.492436	.492436	.492436	.492436
111	.498054	.498054	.498054	.498054	.498054	.498054
112	.500000	.500000	.500000	.500000	.500000	.500000
113	.500000	.500000	.500000	.500000	.500000	.500000
114	.500000	.500000	.500000	.500000	.500000	.500000
115	.500000	.500000	.500000	.500000	.500000	.500000
116	.500000	.500000	.500000	.500000	.500000	.500000
117	.500000	.500000	.500000	.500000	.500000	.500000
118	.500000	.500000	.500000	.500000	.500000	.500000
119	.500000	.500000	.500000	.500000	.500000	.500000
120	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

APPENDIX G

APPROXIMATING THE EFFECT OF FULL GENERATIONAL MORTALITY IMPROVEMENT BY USING A STATIC TABLE PROJECTED *N* YEARS BEYOND 1994

For uninsured pension plans, actuaries may wish to use the UP-94 Mortality Table with mortality rates at each age projected a fixed number of years using Scale AA, rather than to apply the mortality improvement on a generational basis, as in the Group Annuity Reserving valuation standard.

The valuation results that are obtained using the UP-94 Mortality Table projected *N* years are very close to the results using the UP-94 with full generational mortality improvement if *N* is equal to the duration of the liabilities being valued, where the "duration" of a liability is the negative of the first derivative of the liability with respect to the change in the valuation interest rate, divided by the liability. (See "An Introduction to Duration for Pension Actuaries" by Richard Daskais and David LeSueur in *The Pension Forum* for June 1993.) For this discussion, the duration is approximated by the following formula:

$$\text{Duration} = \frac{pvb(i) - pvb(i + 0.001)}{pvb(i) \times 0.001}$$

where $pvb(i)$ is the present value of benefits at the valuation interest rate i, and $pvb(i+0.001)$ is the present value of benefits determined with the interest rate increased by one-tenth of one percentage point, that is, by ten basis points.

When the valuation is being performed as of a year (denoted *CYV*) that is after 1994, the number of years the static table should be projected should be equal to the duration of the liabilities plus the number of years elapsed between the valuation year and 1994 *(CYV*+duration− 1994).

This formula was tested for the following cases: for deferred annuities issued at ages 35 through 60 (at five-year age intervals) and commencing at age 62; for immediate annuities issued at ages 65 through 90 (at five-year age intervals); for a hypothetical population of active employees; for a hypothetical population of retired employees; and for these active and retired employee populations combined. Each of these cases was valued for male and female separately, using valuation interest rates of 3%, 7%, and 11%, for valuation year 1994.

The error from using this rule of thumb was less than one-half of 1% in most of the cases tested. The hypothetical population of retired employees was based on the population of CSRS annuitants who are age 62 and older. The hypothetical population of active employees was assumed to comprise an equal number of employees at each age from age 35 through 61, where the number at each age was set equal to the number of CSRS annuitants at age 61.

Finally, the rule of thumb was tested using the valuation program for civil service retirement for federal employees for valuation year 1994. An average duration was determined for both sexes combined, and both male and female mortality rates were projected using this average duration. There are two tiers of benefits for federal employees: one for pre-1984 hires, which is referred to as the Civil Service Retirement System, or CSRS, and one for post-1983 hires, which is referred to as the Federal Employees Retirement System, or FERS. The valuation assumptions include: 7% interest, 4.5% inflation, and 4.5% general salary increases. CSRS has a COLA equal to inflation, and FERS has a COLA of inflation minus 1%. In determining the duration for CSRS and FERS, the ten-basis-point increase in the interest rates was not assumed to imply a corresponding increase in the inflation assumption.

The results of these tests are shown in Tables G-1 through G-4, where the numbers in the columns are defined as follows:

A *For 35–60.* This is the issue age of a deferred annuity commencing at age 62.

A *For 65–90.* The issue age of an immediate annuity.

A *For Active.* Results are for the hypothetical population of active employees.

A *For Retired.* Results are for the hypothetical population of retired employees.

A *For Combined.* Results are for the hypothetical population of active and retired employees combined.

B Ratio of present value of benefits based on the UP-94 table with full generational mortality improvement, that is, the UP-94G @1994, to the present value of benefits based on UP-94 static table with no projection.

C Exact number of years the UP-94 table must be projected on a static basis to give the same present value of benefits as for the UP-94G @1994.

TABLE G-1

ANALYSIS OF THE ACCURACY OF RULE OF THUMB FOR APPROXIMATING FULL GENERATIONAL
MORTALITY IMPROVEMENT WITH A STATIC TABLE PROJECTED TO CYV+DURATION
ASSUMING AN INTEREST RATE OF 3%

(A) Age	(B) UP-94G @1994/UP-94	(C) Exact N	(D) Duration of Liabilities	(E) UP-94 @1994+Duration/ UP-94G @1994
Male				
35	1.157	34.0	35.3	1.004
40	1.135	28.9	30.6	1.006
45	1.112	24.1	25.8	1.007
50	1.090	19.5	21.1	1.006
55	1.068	15.6	16.3	1.003
60	1.047	12.1	11.6	0.998
65	1.037	9.4	8.8	0.998
70	1.031	6.9	7.5	1.002
75	1.023	5.2	6.2	1.004
80	1.016	4.0	5.0	1.004
85	1.010	2.8	3.9	1.003
90	1.005	2.0	3.0	1.002
Active	1.091	20.5	21.1	1.002
Retired	1.031	7.6	7.7	1.001
Combined	1.061	14.3	14.5	1.001
Female				
35	1.064	36.0	36.4	1.001
40	1.055	31.5	31.7	1.000
45	1.046	27.2	27.0	1.000
50	1.038	23.5	22.3	0.998
55	1.030	20.0	17.5	0.996
60	1.023	16.1	12.7	0.995
65	1.020	12.7	9.9	0.996
70	1.018	9.5	8.5	0.998
75	1.015	6.7	7.1	1.001
80	1.011	4.7	5.7	1.003
85	1.007	3.3	4.5	1.002
90	1.004	2.4	3.4	1.002
Active	1.039	24.2	22.4	0.997
Retired	1.018	9.7	8.7	0.998
Combined	1.028	16.6	15.6	0.998

D Duration of liabilities, using the approximate formula described above.

E Ratio of present value of benefits based on the UP-94 table projected N
years, where N is equal to the duration of the liabilities, that is the UP-
94 @(1994+duration), to the present value of benefits based on the UP-
94G @1994.

TABLE G-2

ANALYSIS OF THE ACCURACY OF RULE OF THUMB FOR APPROXIMATING FULL GENERATIONAL
MORTALITY IMPROVEMENT WITH A STATIC TABLE PROJECTED TO $CYV+$ DURATION
ASSUMING AN INTEREST RATE OF 7%

(A) Age	(B) UP-94G @1994/UP-94	(C) Exact N	(D) Duration of Liabilities	(E) UP-94 @1994+Duration/ UP-94G @1994
Male				
35	1.115	31.5	32.2	1.002
40	1.098	26.5	27.7	1.003
45	1.081	21.8	23.1	1.004
50	1.063	17.4	18.6	1.004
55	1.046	13.6	14.0	1.001
60	1.031	10.5	9.3	0.997
65	1.025	8.2	7.0	0.996
70	1.022	6.1	6.0	1.000
75	1.017	4.6	5.1	1.002
80	1.012	3.6	4.2	1.002
85	1.008	2.6	3.4	1.002
90	1.004	1.9	2.6	1.002
Active	1.057	16.8	16.6	1.000
Retired	1.021	6.5	6.1	0.999
Combined	1.035	10.5	10.2	0.999
Female				
35	1.042	32.5	32.9	1.001
40	1.036	28.1	28.4	1.000
45	1.030	24.1	23.9	1.000
50	1.024	20.7	19.3	0.998
55	1.019	17.7	14.7	0.997
60	1.014	14.1	10.1	0.996
65	1.012	11.2	7.7	0.996
70	1.012	8.5	6.8	0.998
75	1.011	6.0	5.8	1.000
80	1.009	4.2	4.8	1.001
85	1.006	3.0	3.9	1.001
90	1.003	2.2	3.0	1.001
Active	1.022	20.1	17.5	0.997
Retired	1.012	8.3	6.8	0.998
Combined	1.016	12.2	10.9	0.998

TABLE G-3

ANALYSIS OF THE ACCURACY OF RULE OF THUMB FOR APPROXIMATING FULL GENERATIONAL
MORTALITY IMPROVEMENT WITH A STATIC TABLE PROJECTED TO $CYV+$DURATION
ASSUMING AN INTEREST RATE OF 11%

(A) Age	(B) UP-94G @1994 / UP-94	(C) Exact N	(D) Duration of Liabilities	(E) UP-94 @1994+Duration / UP-94G @1994
Male				
35	1.088	29.3	29.8	1.001
40	1.075	24.4	25.4	1.003
45	1.060	19.7	21.0	1.003
50	1.046	15.5	16.6	1.003
55	1.032	11.9	12.2	1.001
60	1.020	9.0	7.7	0.997
65	1.017	7.1	5.6	0.997
70	1.016	5.4	5.0	0.999
75	1.013	4.1	4.3	1.001
80	1.010	3.3	3.6	1.001
85	1.007	2.4	2.9	1.001
90	1.004	1.8	2.3	1.001
Active	1.036	13.7	13.3	0.999
Retired	1.015	5.5	5.0	0.999
Combined	1.021	8.0	7.5	0.999
Female				
35	1.030	29.1	30.3	1.001
40	1.025	24.9	25.9	1.001
45	1.020	21.1	21.5	1.000
50	1.016	18.0	17.1	0.999
55	1.012	15.4	12.7	0.998
60	1.008	12.2	8.2	0.997
65	1.008	9.8	6.1	0.997
70	1.009	7.6	5.5	0.998
75	1.008	5.4	4.8	0.999
80	1.007	3.8	4.1	1.000
85	1.005	2.8	3.3	1.001
90	1.003	2.1	2.6	1.001
Active	1.013	16.6	13.8	0.998
Retired	1.008	7.0	5.5	0.998
Combined	1.009	9.2	8.0	0.999

TABLE G-4

ANALYSIS OF THE ACCURACY OF RULE OF THUMB FOR APPROXIMATING FULL GENERATIONAL
MORTALITY IMPROVEMENT WITH A STATIC TABLE PROJECTED TO CYV+DURATION
FOR VALUATIONS OF CIVIL SERVICE RETIREMENT

	(B) UP-94G @1994/UP-94	(C) Exact N	(D) Duration of Liabilities	(E) UP-94 @1994+Duration/ UP-94G @1994
CSRS active	1.052	23.9	21.8	.996
CSRS retired	1.023	9.8	9.5	.999
CSRS total	1.038	16.7	15.9	.998
FERS active	1.063	33.5	30.4	.995