The Distribution of Discounted Compound Renewal Sums

Ya Fang Wang

Department of Mathematics and Statistics
Concordia University, Montreal, Canada

43rd ARC
Regina, August 14–16, 2008

Jointly with Ghislain Léveillé, Université Laval, Québec, Canada
and José Garrido, Concordia University, Montreal, Canada

Research funded by the Natural Sciences and Engineering Research Council of Canada (NSERC)

Outline

Outline

Outline

Outline

Outline

The Sparre Andersen Model (1957) for an aggregate claim is given by:

$$S(t) = \sum_{k=1}^{N(t)} X_k \,, \qquad t \geq 0 \,, \tag{1}$$

where $N(t)$ is a renewal process.

The Sparre Andersen Model (1957) for an aggregate claim is given by:

$$S(t) = \sum_{k=1}^{N(t)} X_k \,, \qquad t \geq 0 \,, \tag{1}$$

where $N(t)$ is a renewal process.

If we consider the effect of the interest and inflation on the claims, Léveillé and Garrido (2001a) propose the following model.

Model

The compound present value sum is denoted by:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta\, T_k}\, X_k\,, \qquad t \geq 0\,, \tag{2}$$

with $Z(t) = 0$ if $N(t) = 0$.

Model

The compound present value sum is denoted by:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta \, T_k} \, X_k \, , \qquad t \geq 0 \, , \tag{2}$$

with $Z(t) = 0$ if $N(t) = 0$.

- The net interest rate $\delta =$ interest rate -inflation rate ≥ 0.

Model

The compound present value sum is denoted by:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta T_k} X_k \,, \qquad t \geq 0 \,, \tag{2}$$

with $Z(t) = 0$ if $N(t) = 0$.

- The net interest rate $\delta =$ interest rate -inflation rate ≥ 0.
- The claim arrival times $\{T_k \,;\, k \in \mathbb{N}^+\}$ form a renewal process.

Model

The compound present value sum is denoted by:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta\, T_k}\, X_k\,, \qquad t \geq 0\,, \tag{2}$$

with $Z(t) = 0$ if $N(t) = 0$.

- The net interest rate $\delta =$ interest rate -inflation rate ≥ 0.
- The claim arrival times $\{T_k\,;\, k \in \mathbb{N}^+\}$ form a renewal process.
- The deflated claim severities $\{X_k\,;\, k \in \mathbb{N}^+\}$ are iid, independent from the times T_k.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

- Taylor (1979), Delbaen and Haezendonck (1987) and Willmot (1989): the moments of the discounted compound Poisson process .

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

- Taylor (1979), Delbaen and Haezendonck (1987) and Willmot (1989): the moments of the discounted compound Poisson process .
- Léveillé and Garrido (2001a, 2001b): the first two moments and recursive formulas for all the moments of $Z(t)$.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

- Taylor (1979), Delbaen and Haezendonck (1987) and Willmot (1989): the moments of the discounted compound Poisson process .
- Léveillé and Garrido (2001a, 2001b): the first two moments and recursive formulas for all the moments of $Z(t)$.
- Kim and Kim (2006): the moments of discounted aggregated claims in a Markovian environment.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

- Taylor (1979), Delbaen and Haezendonck (1987) and Willmot (1989): the moments of the discounted compound Poisson process .
- Léveillé and Garrido (2001a, 2001b): the first two moments and recursive formulas for all the moments of $Z(t)$.
- Kim and Kim (2006): the moments of discounted aggregated claims in a Markovian environment.
- Ren (2008): explicit formula for the first two moments of discounted compound renewal sums.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$

- Taylor (1979), Delbaen and Haezendonck (1987) and Willmot (1989): the moments of the discounted compound Poisson process .
- Léveillé and Garrido (2001a, 2001b): the first two moments and recursive formulas for all the moments of $Z(t)$.
- Kim and Kim (2006): the moments of discounted aggregated claims in a Markovian environment.
- Ren (2008): explicit formula for the first two moments of discounted compound renewal sums.
- Jang (2004): Laplace transform of the discounted compound Poisson process if the claim severities are exponential and mixture of exponential.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$ (...continued)

This talk will present:

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$ (. . . continued)

This talk will present:

- the moment generating function of the discounted compound Poisson aggregate sums when the deflated claims are PH distributed.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$ (...continued)

This talk will present:

- the moment generating function of the discounted compound Poisson aggregate sums when the deflated claims are PH distributed.
- moment generating function of the discounted compound renewal sums.

The Distribution of the Compound Sum $Z(t)$ if the Net Interest $\delta > 0$ (...continued)

This talk will present:

- the moment generating function of the discounted compound Poisson aggregate sums when the deflated claims are PH distributed.
- moment generating function of the discounted compound renewal sums.
- the comparison Poisson and Erlang(n) models are made.

Outline

Outline

Analytic Results for the M.G.F

Recall our model:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta\, T_k}\, X_k\,, \qquad t \geq 0\,.$$

Recall our model:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta\, T_k} X_k\,, \qquad t \geq 0\,.$$

Léveillé and Garrido (2001b) give the moment generating function (m.g.f.) of $Z(t)$:

$$M_{Z(t)}(s) = \bar{F}_\tau(t) + \int_0^t M_X(se^{-\delta v}) M_{Z(t-v)}(se^{-\delta v}) dF_\tau(v)\,, \quad (3)$$

Recall our model:

$$Z(t) = \sum_{k=1}^{N(t)} e^{-\delta T_k} X_k , \qquad t \geq 0 .$$

Léveillé and Garrido (2001b) give the moment generating function (m.g.f.) of $Z(t)$:

$$M_{Z(t)}(s) = \bar{F}_\tau(t) + \int_0^t M_X(se^{-\delta v}) M_{Z(t-v)}(se^{-\delta v}) dF_\tau(v) , \quad (3)$$

or in term of the renewal function (Léveillé, Garrido and Wang, 2008) we have:

$$M_{Z(t)}(s) = 1 + \int_0^t \left[M_X(se^{-\delta v}) - 1 \right] M_{Z(t-v)}(se^{-\delta v}) dm(v) . \quad (4)$$

Analytic Results for the M.G.F

Continuously substituting $M_{Z(t-v)}(se^{-\delta v})$ in (3) into itself gives an analytical expression of the m.g.f. in terms of F_τ:

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} H_k(t,s)\,, \tag{5}$$

Analytic Results for the M.G.F

Continuously substituting $M_{Z(t-v)}(se^{-\delta v})$ in (3) into itself gives an analytical expression of the m.g.f. in terms of F_τ:

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} H_k(t,s)\,, \tag{5}$$

where $H_k(t,s) = \int_0^t M_X(se^{-\delta v})H_{k-1}(t-v, se^{-\delta v})dF_\tau(v)$, and $H_0(t,s) = \overline{F}_\tau(t)$, for all s.

Analytic Results for the M.G.F

Continuously substituting $M_{Z(t-v)}(se^{-\delta v})$ in (3) into itself gives an analytical expression of the m.g.f. in terms of F_τ:

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} H_k(t,s)\,, \tag{5}$$

where $H_k(t,s) = \int_0^t M_X(se^{-\delta v})H_{k-1}(t-v, se^{-\delta v})dF_\tau(v)$, and $H_0(t,s) = \overline{F}_\tau(t)$, for all s.

Or in term of renewal function

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} I_k(t,s)\,, \tag{6}$$

Analytic Results for the M.G.F

Continuously substituting $M_{Z(t-v)}(se^{-\delta v})$ in (3) into itself gives an analytical expression of the m.g.f. in terms of F_τ:

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} H_k(t,s)\,, \tag{5}$$

where $H_k(t,s) = \int_0^t M_X(se^{-\delta v})H_{k-1}(t-v, se^{-\delta v})dF_\tau(v)$, and $H_0(t,s) = \overline{F}_\tau(t)$, for all s.

Or in term of renewal function

$$M_{Z(t)}(s) = \sum_{k=0}^{\infty} I_k(t,s)\,, \tag{6}$$

where $I_k(t,s) = \int_0^t \left[M_X(se^{-\delta v}) - 1\right] I_{k-1}(t-v, se^{-\delta v})dm(v)$, and $I_0(t,s) = 1$, for all s

Outline

Definition

Let $\boldsymbol{A}$ *be an arbitrary non-singular square matrix of order n such as* $\lim_{x\to\infty} e^{\boldsymbol{A}x} = \boldsymbol{0}$, $\underline{\alpha}$ *be a n-dimensional column vector such that* $\underline{\alpha}'\,\underline{1} = 1$, *where* $\underline{1}$ *is a n-dimensional column vector of* $1'$*s, that is:*

$$\underline{\alpha} = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{pmatrix}', \sum_{i=1}^{n} \alpha_i = 1\,, \alpha_i \geq 0\,,$$

and $$\underline{1} = \begin{pmatrix} 1 & 1 & \cdots & 1 \end{pmatrix}'.$$

Definition

Let $\mathbf{A}$ *be an arbitrary non-singular square matrix of order n such as* $\lim_{x\to\infty} e^{\mathbf{A}x} = \mathbf{0}$, $\underline{\alpha}$ *be a n-dimensional column vector such that* $\underline{\alpha}'\,\underline{1} = 1$, *where* $\underline{1}$ *is a n-dimensional column vector of* $1's$, *that is:*

$$\underline{\alpha} = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{pmatrix}' , \sum_{i=1}^{n} \alpha_i = 1 \, , \alpha_i \geq 0 \, ,$$

and $\underline{1} = \begin{pmatrix} 1 & 1 & \cdots & 1 \end{pmatrix}' .$

If the distribution function F_X *can be written as:*

$$F_X(x) = 1 - \underline{\alpha}' e^{\mathbf{A}x} \underline{1} \, , \qquad x \geq 0, \tag{7}$$

then we say that F_X *is (or* X *has) a phase-type (PH) distribution with parameters* $(\underline{\alpha}\, , \mathbf{A})$.

PH Inter–arrival Times

The mean $\mathbb{E}[N(t)]$ is defined as renewal function and renewal density is given by:

$$m'(t) = \lim_{\Delta t \to 0} \frac{\mathbb{E}\left[N(t+\Delta t)\right] - \mathbb{E}\left[N(t)\right]}{\Delta t} = -\underline{\alpha}' e^{\mathbf{A}[\mathbf{I} - \underline{1}\,\underline{\alpha}']t} \mathbf{A} \underline{1} .$$

Asmussen (2003).

The mean $\mathbb{E}[N(t)]$ is defined as renewal function and renewal density is given by:

$$m'(t) = \lim_{\Delta t \to 0} \frac{\mathbb{E}\left[N(t+\Delta t)\right] - \mathbb{E}\left[N(t)\right]}{\Delta t} = -\underline{\alpha}' e^{\mathbf{A}[\mathbf{I}-\underline{1}\,\underline{\alpha}']t}\mathbf{A}\underline{1}\,.$$

Asmussen (2003).
Substituting it into the m.g.f. equation (6) yields ((Léveillé, Garrido and Wang, 2008):

$$M_{Z(t)}(s) = 1 + \sum_{k=0}^{\infty} \int_0^t \int_0^{t-x_1} \cdots \int_0^{t-\sum_{i=1}^{k} x_i} \prod_{i=1}^{k+1} \left\{ \left[M_X(se^{-\delta \sum_{j=1}^{i} x_j}) - 1 \right] \underline{\alpha}' e^{\mathbf{B}x_i}(-\mathbf{A})\underline{1} \right\} dx_{k+1} \ldots dx_2 dx_1\,, \tag{8}$$

The mean $\mathbb{E}[N(t)]$ is defined as renewal function and renewal density is given by:

$$m'(t) = \lim_{\Delta t \to 0} \frac{\mathbb{E}\left[N(t+\Delta t)\right] - \mathbb{E}\left[N(t)\right]}{\Delta t} = -\underline{\alpha}' e^{\mathbf{A}[\mathbf{I}-\underline{1}\,\underline{\alpha}']t}\mathbf{A}\underline{1}\,.$$

Asmussen (2003).
Substituting it into the m.g.f. equation (6) yields ((Léveillé, Garrido and Wang, 2008):

$$\begin{aligned} M_{Z(t)}(s) &= 1 + \sum_{k=0}^{\infty} \int_0^t \int_0^{t-x_1} \cdots \int_0^{t-\sum_{i=1}^{k} x_i} \prod_{i=1}^{k+1} \\ &\quad \left\{ \left[M_X(se^{-\delta \sum_{j=1}^{i} x_j}) - 1 \right] \alpha' e^{\mathbf{B}x_i}(-\mathbf{A})\underline{1} \right\} dx_{k+1} \ldots dx_2 dx_{\underline{1}}\,, \end{aligned} \tag{8}$$

where $\mathbf{B} = \mathbf{A}(\mathbf{I} - \underline{1}\,\underline{\alpha}')$.

A differential equation in t is obtained for $M_{Z(t)}$:

$$\frac{\partial}{\partial t} M_{Z(t)}(s) = \left[M_X(se^{-\delta t}) - 1\right]\left[\alpha' e^{\mathbf{B}t}(-\mathbf{A})\underline{1} + f(t,s)\right], \qquad (9)$$

A differential equation in t is obtained for $M_{Z(t)}$:

$$\frac{\partial}{\partial t} M_{Z(t)}(s) = \left[M_X(se^{-\delta t}) - 1\right]\left[\underline{\alpha}' e^{\mathbf{B}t}(-\mathbf{A})\underline{1} + f(t,s)\right], \qquad (9)$$

where

$$f(t,s) = \sum_{k=1}^{\infty} \int_0^t \int_0^{y_k} \cdots \int_0^{y_2} \underline{\alpha}' e^{\mathbf{B}(t-y_k)}(-\mathbf{A})\underline{1} \prod_{i=1}^{k} \left\{\left[M_X(se^{-\delta y_i}) - 1\right]\underline{\alpha}' e^{\mathbf{B}(y_i - y_{i-1})}(-\mathbf{A})\underline{1}\right\} dy_1 \cdots dy_{k-1}\, dy_k ,$$

with $y_0 = 0$.

Outline

Discounted Compound Poisson Sums

If the inter-arrival times are exponential distributed, that is $F_\tau(t) = 1 - e^{-\lambda t} \Rightarrow m(t) = \lambda t$, the m.g.f. can be simplified as (Léveillé, 2002):

$$M_{Z(t)}(s) = e^{\int_0^t [M_X(se^{-\delta v}) - 1] dv} . \tag{10}$$

Discounted Compound Poisson with PH Claim Severities

Considering PH claim severities, the result (10) can be simplified as.

If the deflated claims $\{X_k\}_{k\geq 1}$ have a PH $(\underline{\alpha}, \mathbf{A})$ distribution with $sprad\{s\mathbf{A}^{-1}\} < 1$ and $N = \{N(t), t \geq 0\}$ forms a Poisson process, then for $\delta > 0$

Discounted Compound Poisson with PH Claim Severities

Considering PH claim severities, the result (10) can be simplified as.

If the deflated claims $\{X_k\}_{k\geq 1}$ have a PH $(\underline{\alpha}\,,\mathbf{A})$ distribution with $sprad\{s\mathbf{A}^{-1}\} < 1$ and $N = \{N(t)\,, t \geq 0\}$ forms a Poisson process, then for $\delta > 0$

$$M_{Z(t)}(s) = \exp\left\{\tfrac{\lambda}{\delta}\underline{\alpha}' \ln\left[(\mathbf{I} + se^{-\delta t}\mathbf{A}^{-1})(\mathbf{I} + s\mathbf{A}^{-1})^{-1}\right]\underline{1}\right\}, \quad s \in \mathbb{R}, \tag{11}$$

Discounted Compound Poisson with PH Claim Severities

Considering PH claim severities, the result (10) can be simplified as.

If the deflated claims $\{X_k\}_{k\geq 1}$ have a PH $(\underline{\alpha}, \mathbf{A})$ distribution with $sprad\{s\mathbf{A}^{-1}\} < 1$ and $N = \{N(t), t \geq 0\}$ forms a Poisson process, then for $\delta > 0$

$$M_{Z(t)}(s) = \exp\left\{\frac{\lambda}{\delta}\underline{\alpha}' \ln\left[(\mathbf{I} + se^{-\delta t}\mathbf{A}^{-1})(\mathbf{I} + s\mathbf{A}^{-1})^{-1}\right]\underline{1}\right\}, \quad s \in \mathbb{R}, \tag{11}$$

which is a generalization of Jang (2004).

Corollary

For $\delta > 0$ *we have:*

$$\mathbb{E}[Z(t)] = -\frac{\lambda}{\delta}\left(1 - e^{-\delta t}\right)\underline{\alpha}'\boldsymbol{A}^{-1}\underline{1}\,, \qquad t > 0\,, \tag{12}$$

Corollary

For $\delta > 0$ *we have:*

$$\mathbb{E}[Z(t)] = -\frac{\lambda}{\delta}(1 - e^{-\delta t})\,\underline{\alpha}'\boldsymbol{A}^{-1}\underline{1}\,, \qquad t > 0\,, \tag{12}$$

and

$$\mathbb{V}[Z(t)] = \frac{\lambda}{\delta}(1 - e^{-2\delta t})\,\underline{\alpha}'\boldsymbol{A}^{-2}\underline{1}\,, \qquad t > 0\,. \tag{13}$$

Corollary

For $\delta > 0$ *we have:*

$$\mathbb{E}[Z(t)] = -\frac{\lambda}{\delta}\left(1 - e^{-\delta t}\right)\underline{\alpha}' A^{-1}\underline{1}\,, \qquad t > 0\,, \tag{12}$$

and

$$\mathbb{V}\left[Z(t)\right] = \frac{\lambda}{\delta}\left(1 - e^{-2\delta t}\right)\underline{\alpha}' A^{-2}\underline{1}\,, \qquad t > 0\,. \tag{13}$$

which is consistent with Léveillé and Garrido (2001a).

Erlang(2) Inter–arrival Times

Considering Erlang(2) inter–arrival times, then the m.g.f. of $Z(t)$ satisfies:

$$\frac{\partial^2}{\partial t^2} M_{Z(t)}(s) = a_1(t) \frac{\partial}{\partial t} M_{Z(t)}(s) + a_0(t) M_{Z(t)}(s) , \qquad t \geq 0 , s \in \mathbb{R} , \tag{14}$$

Erlang(2) Inter–arrival Times

Considering Erlang(2) inter–arrival times, then the m.g.f. of $Z(t)$ satisfies:

$$\frac{\partial^2}{\partial t^2} M_{Z(t)}(s) = a_1(t) \frac{\partial}{\partial t} M_{Z(t)}(s) + a_0(t) M_{Z(t)}(s) \,, \qquad t \geq 0 \,, s \in \mathbb{R} \,, \tag{14}$$

with initial values $M_{Z(0)}(s) = 1$ and $\frac{\partial}{\partial t} M_{Z(t)}(s)|_{t=0} = 0$,

Erlang(2) Inter–arrival Times

Considering Erlang(2) inter–arrival times, then the m.g.f. of $Z(t)$ satisfies:

$$\frac{\partial^2}{\partial t^2} M_{Z(t)}(s) = a_1(t) \frac{\partial}{\partial t} M_{Z(t)}(s) + a_0(t) M_{Z(t)}(s) \,, \qquad t \geq 0 \,, s \in \mathbb{R} \,, \tag{14}$$

with initial values $M_{Z(0)}(s) = 1$ and $\frac{\partial}{\partial t} M_{Z(t)}(s)\big|_{t=0} = 0$, where $a_1(t) = \frac{\frac{\partial}{\partial t}\left[M_X(se^{-\delta t})-1\right]}{\left[M_X(se^{-\delta t})-1\right]} - 2\lambda$, $a_0(t) = \lambda^2 \left[M_X(se^{-\delta t}) - 1\right]$ and M_X is the m.g.f. of the deflated claim severity X.

Outline

Example

Let inter-arrival time be Erlang(2) ($\underline{\alpha} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,
$\mathbf{A} = \begin{pmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{pmatrix}$, deflated claim X be exponential distribution (θ) and $\delta = 0.01$, $\lambda = 0.01$, $\theta = 1$. We have homogeneous differential equation:

Example

Let inter-arrival time be Erlang(2) ($\underline{\alpha} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,
$\mathbf{A} = \begin{pmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{pmatrix}$, deflated claim X be exponential distribution (θ) and $\delta = 0.01$, $\lambda = 0.01$, $\theta = 1$. We have homogeneous differential equation:

$$\frac{\partial^2}{\partial t^2} M_{Z(t)}(s) = a_1(t) \frac{\partial}{\partial t} M_{Z(t)}(s) + a_0(t) M_{Z(t)}(s) \, ,$$

Example

Let inter-arrival time be Erlang(2) ($\underline{\alpha} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,
$\mathbf{A} = \begin{pmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{pmatrix}$, deflated claim X be exponential distribution (θ) and $\delta = 0.01$, $\lambda = 0.01$, $\theta = 1$. We have homogeneous differential equation:

$$\frac{\partial^2}{\partial t^2} M_{Z(t)}(s) = a_1(t) \frac{\partial}{\partial t} M_{Z(t)}(s) + a_0(t) M_{Z(t)}(s) \,,$$

where

$$\begin{aligned} a_1(t) &= \frac{\frac{\partial}{\partial t}\left[M(t,s)\right]}{M(t,s)} - 2\lambda = \frac{0.01(2se^{-0.01t} - 3)}{1 - se^{-0.01t}} \,, \\ a_0(t) &= \lambda^2 M(t,s) = \frac{0.0001se^{-0.01t}}{1 - se^{-0.01t}} \,, \quad M(t,s) = \frac{\theta}{\theta - se^{-\delta t}} - 1 \,. \end{aligned}$$

Example

$$M_{Z(t)}(s) = \frac{1}{s^2}\left\{(s-1)\left[se^{-0.01t}-2\right]\ln\left[\frac{1-s}{1-se^{-0.01t}}\right] + se^{-0.01t}(s-2)+2s\right\}.$$

Example

$$M_{Z(t)}(s) = \frac{1}{s^2}\left\{(s-1)\left[se^{-0.01t}-2\right]\ln\left[\frac{1-s}{1-se^{-0.01t}}\right] + se^{-0.01t}(s-2)+2s\right\}.$$

The asymptotic behavior of the $M_{Z(t)}(s)$:

$$M_{Z(\infty)}(s) = \frac{2}{s} + \frac{2(1-s)\ln(1-s)}{s^2}.$$

Outline

Comparison of Erlang(n)

We have the following assumption for the comparison

- Erlang(n) inter-arrival times $n = 1, 2, 3, 4$.

Comparison of Erlang(n)

We have the following assumption for the comparison

- Erlang(n) inter-arrival times $n = 1, 2, 3, 4$.
- Erlang(2) with mean 2 deflated claim severities.

Comparison of Erlang(n)

We have the following assumption for the comparison

- Erlang(n) inter-arrival times $n = 1, 2, 3, 4$.
- Erlang(2) with mean 2 deflated claim severities.
- $\delta = 0.01$, $\lambda = \frac{n}{200}$ for $n = 1, 2, 3, 4$.

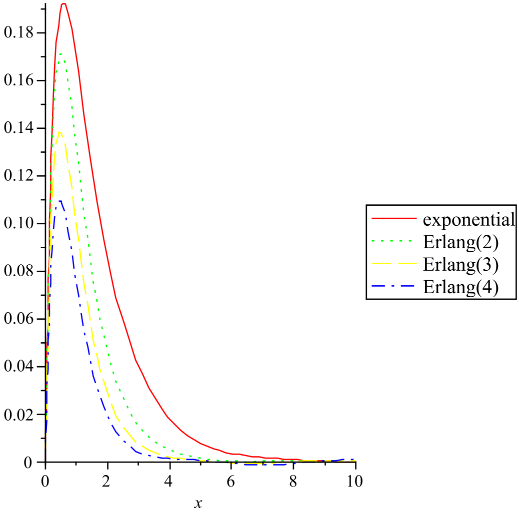

Figure: Density function at time 100

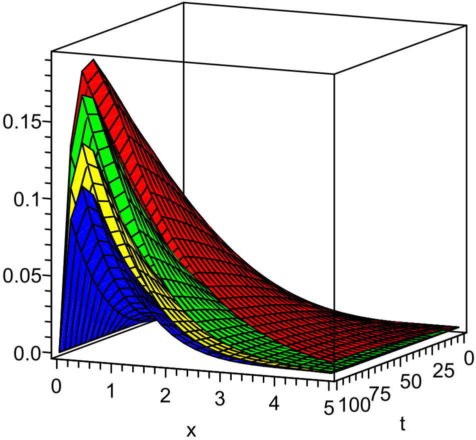

Figure: Density of $Z(t)$ (Exponential, Erlang(2), Erlang(3), Erlang(4))

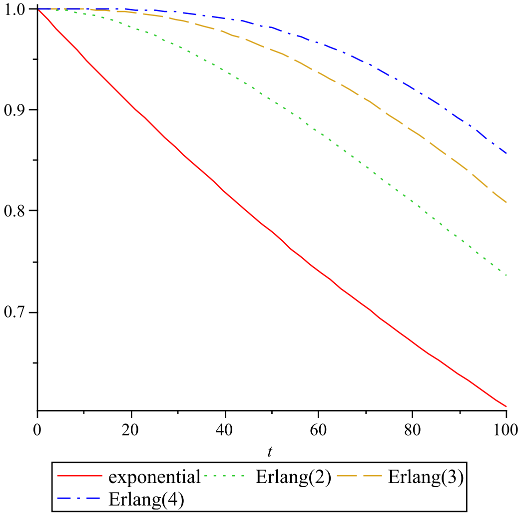

Figure: Masses at zero

The mass at $x = 0$ of the distribution of $Z(t)$ differs for Erlang claim inter–arrival distributions.

The mass at $x = 0$ of the distribution of $Z(t)$ differs for Erlang claim inter–arrival distributions.

The comparison should be made for the conditional density functions of $Z(t)$, given that $x > 0$ (see following figures).

The mass at $x = 0$ of the distribution of $Z(t)$ differs for Erlang claim inter–arrival distributions.

The comparison should be made for the conditional density functions of $Z(t)$, given that $x > 0$ (see following figures).

We can see that the exponential claim inter–arrival times have the heaviest tail; i.e. compound Poisson discounted sum is the most dangerous Erlang compound renewal sum.

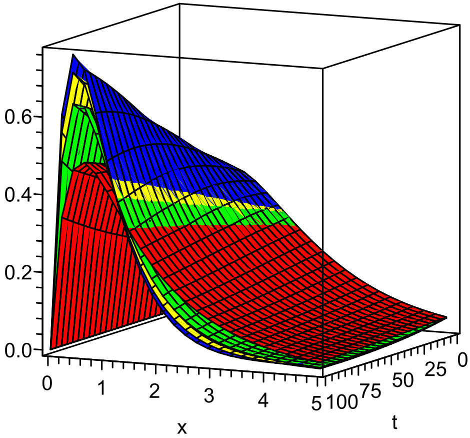

Figure: Conditional density of $Z(t)$ (Exponential, Erlang(2), Erlang(3), Erlang(4))

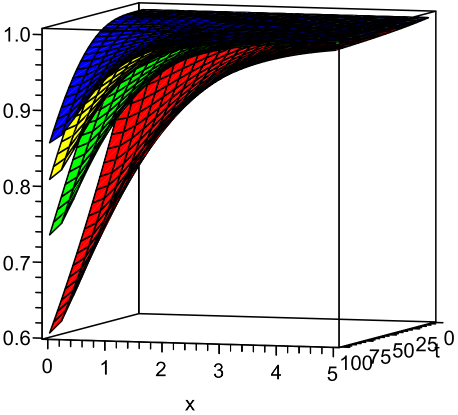

Figure: C.d.f. of $Z(t)$ (Exponential, Erlang(2), Erlang(3), Erlang(4))

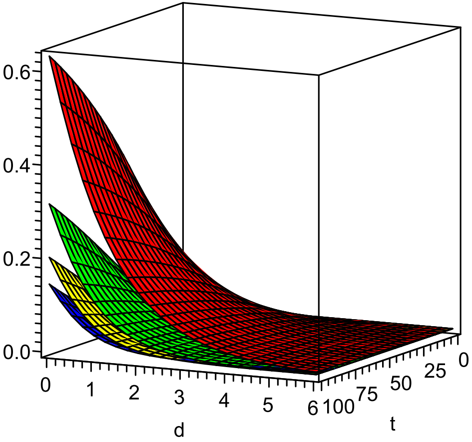

Figure: Stop-loss premium (Exponential, Erlang(2), Erlang(3), Erlang(4))

Outline

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.
- obtaining homogeneous differential equation for m.g.f. if the inter-arrival times are Erlang(2) distributed.

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.
- obtaining homogeneous differential equation for m.g.f. if the inter-arrival times are Erlang(2) distributed.
- asymptotic results as $t \to \infty$ and $\delta \to 0$ are also considered.

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.
- obtaining homogeneous differential equation for m.g.f. if the inter-arrival times are Erlang(2) distributed.
- asymptotic results as $t \to \infty$ and $\delta \to 0$ are also considered.
- some numerical examples are given to illustrate the results.

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.
- obtaining homogeneous differential equation for m.g.f. if the inter-arrival times are Erlang(2) distributed.
- asymptotic results as $t \to \infty$ and $\delta \to 0$ are also considered.
- some numerical examples are given to illustrate the results.
- the results can be applied to delated and stationary renewal processes.

Summary

- developing a close form for m.g.f. if the discounted compound sums is Poisson process with PH claim severities.
- obtaining homogeneous differential equation for m.g.f. if the inter-arrival times are Erlang(2) distributed.
- asymptotic results as $t \to \infty$ and $\delta \to 0$ are also considered.
- some numerical examples are given to illustrate the results.
- the results can be applied to delated and stationary renewal processes.
- the computation of the stop-loss premium.

Thanks