Tables 1 & 18 were originally published with errors; corrections are noted on the relevant pages.

TRANSACTIONS OF SOCIETY OF ACTUARIES
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1994 GROUP ANNUITY MORTALITY TABLE AND 1994 GROUP ANNUITY RESERVING TABLE

SOCIETY OF ACTUARIES GROUP ANNUITY VALUATION TABLE TASK FORCE*

EXECUTIVE SUMMARY

The Society of Actuaries Group Annuity Valuation Table Task Force has completed its research and has developed a table that it recommends as suitable for a new Group Annuity Reserve Valuation Standard.

The proposed new table, recommended as suitable for a new Group Annuity Reserve Valuation Standard, if accepted and adopted by regulators, would incorporate the use of generational mortality into statutory reserving requirements for group annuities for the first time. Generational mortality allows for the recognition of explicit assumptions for future mortality improvement in the calculation of reserve values.

The Task Force strongly believes that the use of generational mortality in group annuity reserving is appropriate given the trends in mortality improvement that have been observed in the past and the continued improvement expected to occur in the foreseeable future. Modern systems capabilities are sufficient to allow for the increased refinement and computation intensity that generational mortality requires.

The 1994 Group Annuity Reserving Table

The 1994 Group Annuity Reserving Table appears in Table 1. This table includes q_x values on an age nearest birthday basis for each age in 1994 and projection factors to be used in generating q_x values in years beyond 1994.

Use of the Values in the Table To Produce Projected Mortality Rates

The values in the 1994 Group Annuity Reserving Table are as follows:

q_x^{1994} = the mortality rate for a person age x in 1994.

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TABLE 1

1994 GROUP ANNUITY RESERVING TABLE

	Male		Female	
Age (x)	q_x^{1994}	AA_x	q_x^{1994}	AA_x
1	0.000592	0.020	0.000531	0.020
2	0.000400	0.020	0.000346	0.020
3	0.000332	0.020	0.000258	0.020
4	0.000259	0.020	0.000194	0.020
5	0.000237	0.020	0.000175	0.020
6	0.000227	0.020	0.000163	0.020
7	0.000217	0.020	0.000153	0.020
8	0.000201	0.020	0.000137	0.020
9	0.000194	0.020	0.000130	0.020
10	0.000197	0.020	0.000131	0.020
11	0.000208	0.020	0.000138	0.020
12	0.000226	0.020	0.000148	0.020
13	0.000255	0.020	0.000164	0.020
14	0.000297	0.019	0.000189	0.018
15	0.000345	0.019	0.000216	0.016
16	0.000391	0.019	0.000242	0.015
17	0.000430	0.019	0.000262	0.014
18	0.000460	0.019	0.000273	0.014
19	0.000484	0.019	0.000280	0.015
20	0.000507	0.019	0.000284	0.016
21	0.000530	0.018	0.000286	0.017
22	0.000556	0.017	0.000289	0.017
23	0.000589	0.015	0.000292	0.016
24	0.000624	0.013	0.000291	0.015
25	0.000661	0.010	0.000291	0.014
26	0.000696	0.006	0.000294	0.012
27	0.000727	0.005	0.000302	0.012
28	0.000754	0.005	0.000314	0.012
29	0.000779	0.005	0.000331	0.012
30	0.000801	0.005	0.000351	0.010

	Male		Female	
Age (x)	q_x^{1994}	AA_x	q_x^{1994}	AA_x
31	0.000821	0.005	0.000373	0.008
32	0.000839	0.005	0.000397	0.008
33	0.000848	0.005	0.000422	0.009
34	0.000849	0.005	0.000449	0.010
35	0.000851	0.005	0.000478	0.011
36	0.000862	0.005	0.000512	0.012
37	0.000891	0.005	0.000551	0.013
38	0.000939	0.006	0.000598	0.014
39	0.000999	0.007	0.000652	0.015
40	0.001072	0.008	0.000709	0.015
41	0.001156	0.009	0.000768	0.015
42	0.001252	0.010	0.000825	0.015
43	0.001352	0.011	0.000877	0.015
44	0.001458	0.012	0.000923	0.015
45	0.001578	0.013	0.000973	0.016
46	0.001722	0.014	0.001033	0.017
47	0.001899	0.015	0.001112	0.018
48	0.002102	0.016	0.001206	0.018
49	0.002326	0.017	0.001310	0.018
50	0.002579	0.018	0.001428	0.017
51	0.002872	0.019	0.001568	0.016
52	0.003213	0.020	0.001734	0.014
53	0.003584	0.020	0.001907	0.012
54	0.003979	0.020	0.002084	0.010
55	~~0.04425~~	0.019	0.002294	0.008
56	0.004949	0.018	0.002563	0.006
57	0.005581	0.017	0.002919	0.005
58	0.006300	0.016	0.003359	0.005
59	0.007090	0.016	0.003863	0.005
60	0.007976	0.016	0.004439	0.005

Male q(55) should be 0.004425

Male q(104) should be 0.387855

TABLE 1—*Continued*

Age (x)	Male q_x^{1994}	Male AA_x	Female q_x^{1994}	Female AA_x	Age (x)	Male q_x^{1994}	Male AA_x	Female q_x^{1994}	Female AA_x
61	0.008986	0.015	0.005093	0.005	91	0.167260	0.004	0.128751	0.003
62	0.010147	0.015	0.005832	0.005	92	0.182281	0.003	0.141973	0.003
63	0.011471	0.014	0.006677	0.005	93	0.198392	0.003	0.155931	0.002
64	0.012940	0.014	0.007621	0.005	94	0.215700	0.003	0.170677	0.002
65	0.014535	0.014	0.008636	0.005	95	0.233606	0.002	0.186213	0.002
66	0.016239	0.013	0.009694	0.005	96	0.251510	0.002	0.202538	0.002
67	0.018034	0.013	0.010764	0.005	97	0.268815	0.002	0.219655	0.001
68	0.019859	0.014	0.011763	0.005	98	0.285277	0.001	0.237713	0.001
69	0.021729	0.014	0.012709	0.005	99	0.301298	0.001	0.256712	0.001
70	0.023730	0.015	0.013730	0.005	100	0.317238	0.001	0.276427	0.001
71	0.025951	0.015	0.014953	0.006	101	0.333461	0.000	0.296629	0.000
72	0.028481	0.015	0.016506	0.006	102	0.350330	0.000	0.317093	0.000
73	0.031201	0.015	0.018344	0.007	103	0.368542	0.000	0.338505	0.000
74	0.034051	0.015	0.020381	0.007	104	~~0.387885~~	0.000	0.361016	0.000
75	0.037211	0.014	0.022686	0.008	105	0.407224	0.000	0.383597	0.000
76	0.040858	0.014	0.025325	0.008	106	0.425599	0.000	0.405217	0.000
77	0.045171	0.013	0.028366	0.007	107	0.441935	0.000	0.424846	0.000
78	0.050211	0.012	0.031727	0.007	108	0.457553	0.000	0.444368	0.000
79	0.055861	0.011	0.035362	0.007	109	0.473150	0.000	0.464469	0.000
80	0.062027	0.010	0.039396	0.007	110	0.486745	0.000	0.482325	0.000
81	0.068615	0.009	0.043952	0.007	111	0.496356	0.000	0.495110	0.000
82	0.075532	0.008	0.049153	0.007	112	0.500000	0.000	0.500000	0.000
83	0.082510	0.008	0.054857	0.007	113	0.500000	0.000	0.500000	0.000
84	0.089613	0.007	0.060979	0.007	114	0.500000	0.000	0.500000	0.000
85	0.097240	0.007	0.067738	0.006	115	0.500000	0.000	0.500000	0.000
86	0.105792	0.007	0.075347	0.005	116	0.500000	0.000	0.500000	0.000
87	0.115671	0.006	0.084023	0.004	117	0.500000	0.000	0.500000	0.000
88	0.126980	0.005	0.093820	0.004	118	0.500000	0.000	0.500000	0.000
89	0.139452	0.005	0.104594	0.003	119	0.500000	0.000	0.500000	0.000
90	0.152931	0.004	0.116265	0.003	120	1.000000	0.000	1.000000	0.000

AA_x = the annual improvement factor in the mortality rate for age x.

To produce the mortality rate for a person age x in year (1994+n), the following formula would be used:

$$q_x^{1994+n} = q_x^{1994} (1 - AA_x)^n$$

The application of generational mortality techniques to produce reserve values is described in this report.

Standard Table Names

Several tables are presented in this report. To avoid confusion about what each of these tables represents, the following standard table names are used:

1. The 1994 Group Annuity Mortality Basic (or GAM-94 Basic) Table, which is presented as Table 13, is a static mortality table containing unloaded mortality rates for calendar year 1994.
2. The 1994 Group Annuity Mortality Static (or GAM-94 Static) Table, which is presented as Table 18, is a static mortality table containing loaded mortality rates for calendar year 1994.
3. Projection Scale AA (or Scale AA), which is presented as Table 15, represents the annual rates of mortality improvement by age for projecting future mortality rates beyond calendar year 1994.
4. The 1994 Group Annuity Reserving (or GAR-94) Table, which is presented as Table 1, is a combination of the GAM-94 Static Table and Projection Scale AA. Whenever reference is made to the use of this table, it implies that generational mortality derived from static mortality rates and projection scale factors has been used.

I. INTRODUCTION

A. Charge of The Task Force

The Group Annuity Valuation Table Task Force has been charged by the Society of Actuaries Board of Governors with developing a new Group Annuity Mortality Valuation Standard that would be suitable as a replacement for the current standard, which is based upon the 1983 Group Annuity Mortality Table (GAM-83).

B. New Standard To Replace GAM-83

The Society of Actuaries committee that published the GAM-83 Table recommended that a new mortality table be developed when credible annuitant experience became available, since the GAM-83 was only an update of prior data. The Task Force examined the annuitant experience from 1986 through 1990 and found that this was a sufficient basis for a new mortality table.

Further, that experience shows that mortality improvement has resulted in male actual-to-expected mortality ratios near 1.00, as shown in Table 2. Therefore, the margin included in the male rates in the GAM-83 no longer exists, and a new table with a sufficient margin is warranted.

TABLE 2
RETIRED EXPERIENCE BY ANNUITY INCOME
ACTUAL-TO-EXPECTED MORTALITY RATIOS BY EXPERIENCE YEAR

	Experience Year				
	1986	1987	1988	1989	1990
Males	1.05	1.08	1.06	1.03	1.01
Females	1.21	1.26	1.22	1.18	1.14

Finally, because the data, especially for female annuitants, are much more extensive than those used in the development of previous tables, the results produced in this report are more representative of current mortality.

For these reasons, the Task Force recommends that the new standard, as described in this report, be adopted as a replacement for the GAM-83.

C. Intended Form of the New Standard

The Task Force strongly believes that the new standard should accomplish the goals of:

1. Recognizing mortality improvement
2. Serving for at least 15 years.

As shown in this report, while analyzing the data collected through 1990 and comparing them to GAM-83, the Task Force recognized that the trend in mortality improvement had not abated. Consequently, the Task Force decided that the observed mortality improvement trend should be explicitly recognized in this recommended new standard.

This decision to explicitly recognize mortality improvement was discussed in a 1992 position paper [1]. This position paper generated several very worthwhile suggestions and comments. Many of these suggestions were considered in the development of this recommended new standard.

The Task Force further believes that the new standard should be appropriately designed so that it will be useful for a reasonable time and not need as frequent an update as some of the more traditional standards.

To achieve these two results, the Task Force decided that a generational mortality approach, which is more fully discussed later in this report, would be appropriate. Note also that the great majority of input received by the Task Force in response to the position paper supported such a decision. Thus, the Task Force has proceeded with a recommendation that incorporates a generational approach as part of the new standard. This is the first time that projection scales are being recommended as suitable for a new standard for statutory reserving purposes.

The Task Force further recognizes that this approach departs from the traditional one of solely publishing a static table. Prior papers have published projection scales, but these projection scales were not recommended to be part of the statutory reserving standards. While the implementation of this approach is somewhat more complex than that of previous standards, modern systems capabilities facilitate implementation of this new standard. It is also intended that if and when the new standard is adopted for statutory reserving, insurers should be allowed sufficient time to incorporate this generational approach.

The various sections of this report discuss the development and application of this new standard. Note that additional report(s) will discuss how an adaptation of the new standard also serves as an update to the UP–84 Mortality Table and other related issues.

II. DEVELOPMENT OF 1988 BASE YEAR GROUP ANNUITY MORTALITY TABLE

A. 1988 Base Year Core Experience

Our objective was to develop a 1994 base year mortality table for males and females on an age-nearest-birthday basis based on credible group annuity mortality experience. The core mortality information for ages 66–95 was derived from group annuity mortality experience for retired lives for the 1986–1990 experience years. These data were obtained from the Society of Actuaries Group Annuity Experience Committee. In turn, their data were based upon the collective experience of annuitants in payment status for insured contracts from 11 large insurance companies. Data from contributors that were excluded in reports published by this committee were also excluded from our data. The experience we used was examined for data integrity, and where clearly appropriate, data were excluded when determined to be erroneous.

All experience from the 1986–1990 group annuity mortality studies for the younger ages and for the very old ages were excluded from the experience committee data because of a lack of sufficient exposure at these ages. Mortality rates for these young and old ages were derived using the processes discussed later in this report.

Table 3 presents the crude mortality rates resulting from the income-based experience initially gathered by the committee for these ages. Table 3 forms the core of the initial 1988 base table prior to extensions for younger and older ages.

TABLE 3

GROUP ANNUITY MORTALITY EXPERIENCE
UNADJUSTED, UNGRADUATED, BEFORE MARGINS
YEARS 1986–1990
1988 BASE YEAR

	Values of q_x			Values of q_x	
Age	Male	Female	Age	Male	Female
66	0.019269	0.011659	81	0.083702	0.050633
67	0.020827	0.011558	82	0.087230	0.053618
68	0.021989	0.012648	83	0.100734	0.062886
69	0.025223	0.014816	84	0.108259	0.067163
70	0.027970	0.016470	85	0.109440	0.079880
71	0.030305	0.018468	86	0.118562	0.083499
72	0.034400	0.019646	87	0.137411	0.093969
73	0.037566	0.022562	88	0.151901	0.106342
74	0.041715	0.022690	89	0.156454	0.112547
75	0.045670	0.026181	90	0.161550	0.127477
76	0.049899	0.031442	91	0.199729	0.144480
77	0.055961	0.033878	92	0.194778	0.161609
78	0.060834	0.035267	93	0.234746	0.193206
79	0.066465	0.040115	94	0.232451	0.178502
80	0.072808	0.045878	95	0.267373	0.199738

B. Ages 25–65

The first extension of Table 3 was for ages 25 through 65. These mortality rates were derived from Civil Service Retirement System (CSRS) mortality experience by lives for the years 1985–1989 for retired annuitants and 1983–1986 (trending to 1985–1989) for active annuitants. Specifically, experience for active annuitants was used to derive mortality rates for ages 25–50. A blend of experience for active and retired annuitants was used for ages 51–65, based on active/retired distributions of civil service annuitants as shown in Table 4.

TABLE 4

ASSUMED ACTIVE/RETIRED SPLIT OF CIVIL SERVICE ANNUITANTS USED TO DERIVE EXPERIENCE MORTALITY FOR AGES 51–65

	Male Annuitants		Female Annuitants	
Age	Active	Retired	Active	Retired
51	0.96	0.04	0.98	0.02
52	0.95	0.05	0.97	0.03
53	0.93	0.07	0.96	0.04
54	0.92	0.08	0.95	0.05
55	0.84	0.16	0.93	0.07
56	0.68	0.32	0.85	0.15
57	0.63	0.37	0.82	0.18
58	0.57	0.43	0.78	0.22
59	0.53	0.47	0.74	0.26
60	0.45	0.55	0.66	0.34
61	0.37	0.63	0.53	0.47
62	0.29	0.71	0.42	0.58
63	0.21	0.79	0.29	0.71
64	0.17	0.83	0.22	0.78
65	0.13	0.87	0.17	0.83

Because the rates from CSRS based upon number of lives closely matched the rates from 1986–1990 group annuity mortality experience based upon annual income as shown in Table 5, the Task Force concluded that the CSRS experience was a reasonable basis for extension of the initial 1988 base table for ages below 66.

TABLE 5

COMPARISON OF MORTALITY RATES FOR BLENDED CSRS AND 1986–1990 GROUP ANNUITY EXPERIENCE

	Blended CSRS Experience		Group Annuity Experience		Ratios	
Age	Male	Female	Male	Female	Male	Female
65	0.017188	0.009975	0.016831	0.009770	0.97923	0.97945
66	0.019160	0.010456	0.019269	0.011659	1.00569	1.11505
67	0.021456	0.012152	0.020827	0.011558	0.97068	0.95112
68	0.023483	0.012638	0.021989	0.012648	0.93638	1.00079
69	0.026761	0.014862	0.025223	0.014816	0.94253	0.99690
70	0.029621	0.017459	0.027970	0.016470	0.94426	0.94335

The blended mortality rates from the CSRS experience for ages 25–65 were then combined with the Table 3 group annuity experience for ages 66–95. An adjustment of the blended CSRS experience for ages 25–65 to reflect group annuity experience at age 65 was *not* needed, because the mortality rates for the blended CSRS experience were quite similar to the mortality rates for the group annuity experience at ages following age 64, as shown in Table 5.

Table 6 shows the crude mortality rates derived for ages 25 through 65 from the blended CSRS experience.

TABLE 6
BLENDED CSRS EXPERIENCE
UNADJUSTED, UNGRADUATED, BEFORE MARGINS
YEARS 1985–1989

	Values of q_x			Values of q_x	
Age	Male	Female	Age	Male	Female
25	0.000684	0.000365	46	0.002060	0.001202
26	0.000804	0.000280	47	0.002124	0.001232
27	0.000665	0.000369	48	0.002596	0.001387
28	0.000848	0.000324	49	0.002754	0.001763
29	0.000867	0.000375	50	0.003070	0.001540
30	0.000863	0.000414	51	0.003447	0.001766
31	0.000850	0.000411	52	0.003698	0.002068
32	0.000821	0.000381	53	0.004081	0.002153
33	0.000813	0.000438	54	0.004963	0.002313
34	0.000939	0.000555	55	0.004763	0.002522
35	0.001009	0.000539	56	0.005751	0.002669
36	0.000880	0.000585	57	0.007180	0.003222
37	0.000976	0.000620	58	0.007569	0.003703
38	0.000987	0.000568	59	0.008356	0.004186
39	0.001149	0.000810	60	0.009165	0.004759
40	0.001219	0.000701	61	0.010456	0.004990
41	0.001202	0.000991	62	0.011893	0.005865
42	0.001491	0.000861	63	0.013728	0.007110
43	0.001683	0.001265	64	0.015347	0.008633
44	0.001925	0.000993	65	0.017188	0.009975
45	0.001792	0.001065			

C. Extreme Ages (Ages 1–24 and 96–120)

Mortality rates for ages 1–24 and ages 96–120 were developed based on mortality rates from the Life Tables for calendar year 1990 and published in *Actuarial Study No. 107* (*SSA 107*) [2]. U.S. Census statistics, information compiled by the National Center for Health Statistics and published in the volumes of *Vital Statistics of the United States*, and Medicare data are the underlying data sources for *SSA 107*.

The Life Tables were combined with the group annuity experience and the blended CSRS experience as follows:

1. For ages 1–24, mortality rates from the *SSA 107* Life Tables were used with modifications to the rates above age 12. The mortality rates for ages 12–24 were obtained by adjusting the *SSA 107* rates by a formula designed to replicate the *SSA 107* age 12 rate and the age 25 rate from the blended CSRS experience. These values are shown in Table 7.

TABLE 7

SSA 107 LIFE TABLES FOR 1990
MORTALITY RATES BEFORE AND AFTER ADJUSTMENT
TO GROUP ANNUITY EXPERIENCE LEVELS
AGES 1–25

	Before Adjustment		After Adjustment	
Age	Male	Female	Male	Female
1	0.000736	0.000647	0.000736	0.000647
2	0.000497	0.000422	0.000497	0.000422
3	0.000413	0.000315	0.000413	0.000315
4	0.000322	0.000236	0.000322	0.000236
5	0.000295	0.000213	0.000295	0.000213
6	0.000282	0.000199	0.000282	0.000199
7	0.000270	0.000187	0.000270	0.000187
8	0.000249	0.000173	0.000249	0.000173
9	0.000222	0.000159	0.000222	0.000159
10	0.000200	0.000148	0.000200	0.000148
11	0.000209	0.000149	0.000209	0.000149
12	0.000276	0.000172	0.000276	0.000172
13	0.000416	0.000221	0.000314	0.000194
14	0.000608	0.000289	0.000367	0.000226
15	0.000823	0.000368	0.000426	0.000262
16	0.001026	0.000441	0.000481	0.000295
17	0.001203	0.000495	0.000530	0.000320
18	0.001336	0.000520	0.000566	0.000331
19	0.001435	0.000524	0.000593	0.000333
20	0.001533	0.000524	0.000620	0.000333
21	0.001634	0.000530	0.000648	0.000336
22	0.001708	0.000539	0.000668	0.000340
23	0.001747	0.000554	0.000679	0.000347
24	0.001764	0.000574	0.000683	0.000356
25	0.001767	0.000594	0.000684	0.000365

2. For ages 96–119, mortality rates from the *SSA 107* Life Tables were appended to the experience table. The resulting mortality rates were then set at a maximum rate of 0.5. No adjustment was required because the age 95 mortality rates in the experience table and the Life Tables were similar. These values are shown in Table 8.

TABLE 8

SSA LIFE TABLES FOR 1990 MORTALITY RATES
(MODIFIED ABOVE AGE 107)
AGES 96–119

Age	Male	Female	Age	Male	Female
96	0.278505	0.237204	111	0.500000	0.500000
97	0.294423	0.254388	112	0.500000	0.500000
98	0.310198	0.271234	113	0.500000	0.500000
99	0.325708	0.287508	114	0.500000	0.500000
100	0.341993	0.304758	115	0.500000	0.500000
101	0.359093	0.323044	116	0.500000	0.500000
102	0.377047	0.342426	117	0.500000	0.500000
103	0.395900	0.362972	118	0.500000	0.500000
104	0.415695	0.384750	119	0.500000	0.500000
105	0.436479	0.407835			
106	0.458303	0.432305			
107	0.481218	0.458243			
108	0.500000	0.485738			
109	0.500000	0.500000			
110	0.500000	0.500000			

Strong consideration was given to setting an ultimate value equal to 0.5. Setting the highest mortality rate at a value of 0.5 instead of 1.0 would mean that there is no theoretical end to the mortality table. Such a proposed table would depart from past practice by not setting the mortality rate to 1.0 at some ultimate age. This change from tradition could be proposed for two reasons:

1. A number of studies have shown that the ultimate mortality rate peaks at a rate of less than 500 per 1,000, so that a rate of 1.0 is not supported by the facts.
2. Current methods of constructing annuity tables do not require an ultimate value of 1.0.

The mortality curve has long been known to bend upwards during the middle ages, and that is a feature of the proposed new standard table as well as all past tables. Studies of mortality at the very old ages have shown that the mortality rate has a second bendpoint in the 80s or 90s, which reflects a deceleration in the rate of increase. The rate then proceeds to an approximately level ultimate rate after age 100. For example, Bayo and Faber [3] conducted a detailed study of the first OASDI beneficiaries who have now all died. They concluded that the mortality rates began to decelerate at about age 85. Lew and Garfinckel [4] found that the mortality rate first exceeded 0.33 in the late 90s and fluctuated between 0.28 and 0.44 after that point.

The ungraduated group annuity experience is sparse after age 95, but the data show the second bendpoint and the peaking of the rate of mortality. The male rates rise to about 0.25 in the mid-90s and then fluctuate around that point. The female rates also seem to peak at about 0.25 at those ages.

The use of such a mortality table without a final value could be implemented as follows:

1. Add an ultimate value to the annuity
2. Stop the table with a value of 1.0 at a certain age
3. Stop the table at a certain age but use 0.5 as the ultimate rate.

While the Task Force strongly believes an ultimate value of 0.5 is appropriate and could be properly programmed, there are some inconsistencies that could result without an assumed actual "end to the table." To avoid these inconsistent practical applications, the ultimate value is set equal to 1.0 at age 120.

Combining Tables 3, 6, 7, 8 and the ultimate rate of 1.0 at age 120 produces Table 9. This represents ungraduated mortality rates (adjusted for CSRS mortality for ages 25–65 and *SSA 107* Life Tables for ages 1–24 and 96–119), as limited to a maximum rate of 0.5, at all ages except the ultimate age of 120, assuming a base year of 1988. Note that Table 9 does not include any margins.

III. PROJECTION SCALES DECISION-MAKING

The central calendar year of the modified mortality experience shown in Table 9 is 1988. The development of the new standard requires two projections of this 1988 base year mortality experience:

1. To project the mortality experience from the central experience year of 1988 to central year 1994, to produce a 1994 Basic Table
2. To develop the mortality projection scale used to project mortality into the future, after calendar year 1994, for the generational mortality table process.

TABLE 9

MORTALITY EXPERIENCE UNGRADUATED BEFORE MARGINS
1988 BASE YEAR

Age	Values of q_x		Age	Values of q_x		Age	Values of q_x	
	Male	Female		Male	Female		Male	Female
1	0.000736	0.000647	21	0.000648	0.000336	41	0.001202	0.000991
2	0.000497	0.000422	22	0.000668	0.000340	42	0.001491	0.000861
3	0.000413	0.000315	23	0.000679	0.000347	43	0.001683	0.001265
4	0.000322	0.000236	24	0.000683	0.000356	44	0.001925	0.000993
5	0.000295	0.000213	25	0.000684	0.000365	45	0.001792	0.001065
6	0.000282	0.000199	26	0.000804	0.000280	46	0.002060	0.001202
7	0.000270	0.000187	27	0.000665	0.000369	47	0.002124	0.001232
8	0.000249	0.000173	28	0.000848	0.000324	48	0.002596	0.001387
9	0.000222	0.000159	29	0.000867	0.000375	49	0.002754	0.001763
10	0.000200	0.000148	30	0.000863	0.000414	50	0.003070	0.001540
11	0.000209	0.000149	31	0.000850	0.000411	51	0.003447	0.001766
12	0.000276	0.000172	32	0.000821	0.000381	52	0.003698	0.002068
13	0.000314	0.000194	33	0.000813	0.000438	53	0.004081	0.002153
14	0.000367	0.000226	34	0.000939	0.000555	54	0.004963	0.002313
15	0.000426	0.000262	35	0.001009	0.000539	55	0.004763	0.002522
16	0.000481	0.000295	36	0.000880	0.000585	56	0.005751	0.002669
17	0.000530	0.000320	37	0.000976	0.000620	57	0.007180	0.003222
18	0.000566	0.000331	38	0.000987	0.000568	58	0.007569	0.003703
19	0.000593	0.000333	39	0.001149	0.000810	59	0.008356	0.004186
20	0.000620	0.000333	40	0.001219	0.000701	60	0.009165	0.004759

TABLE 9—*Continued*

Age	Values of q_x		Age	Values of q_x		Age	Values of q_x	
	Male	Female		Male	Female		Male	Female
61	0.010456	0.004990	81	0.083702	0.050633	101	0.359093	0.323044
62	0.011893	0.005865	82	0.087230	0.053618	102	0.377047	0.342426
63	0.013728	0.007110	83	0.100734	0.062886	103	0.395900	0.362972
64	0.015347	0.008633	84	0.108259	0.067163	104	0.415695	0.384750
65	0.017188	0.009975	85	0.109440	0.079880	105	0.436479	0.407835
66	0.019269	0.011659	86	0.118562	0.083499	106	0.458303	0.432305
67	0.020827	0.011558	87	0.137411	0.093969	107	0.481218	0.458243
68	0.021989	0.012648	88	0.151901	0.106342	108	0.500000	0.485738
69	0.025223	0.014816	89	0.156454	0.112547	109	0.500000	0.500000
70	0.027970	0.016470	90	0.161550	0.127477	110	0.500000	0.500000
71	0.030305	0.018468	91	0.199729	0.144480	111	0.500000	0.500000
72	0.034400	0.019646	92	0.194778	0.161609	112	0.500000	0.500000
73	0.037566	0.022562	93	0.234746	0.193206	113	0.500000	0.500000
74	0.041715	0.022690	94	0.232451	0.178502	114	0.500000	0.500000
75	0.045670	0.026181	95	0.267373	0.199738	115	0.500000	0.500000
76	0.049899	0.031442	96	0.278505	0.237204	116	0.500000	0.500000
77	0.055961	0.033878	97	0.294423	0.254388	117	0.500000	0.500000
78	0.060834	0.035267	98	0.310198	0.271234	118	0.500000	0.500000
79	0.066465	0.040115	99	0.325708	0.287508	119	0.500000	0.500000
80	0.072808	0.045878	100	0.341993	0.304758	120	1.000000	1.000000

A. *Projection of Mortality Rates to 1994*

For the 1988–1994 projection of mortality reduction, the Task Force considered mortality improvements from the following sources:

1. Projections of mortality improvement in the general population presented in *SSA 107*, with further detail covering the periods 1988–1994 and 1986–1992, from the *1992 Trustees Report* Intermediate Alternative II Assumptions, which are consistent with *SSA 107*
2. CSRS mortality improvement experience
3. Scale H, which was presented with the development of the GAM-83.
4. The Society of Actuaries Group Annuity Mortality Study covering the period 1985–1990.

Comparisons of mortality improvement rates at quinquennial age groups from these studies appear in Table 10.

After much discussion, including interaction with the UP-94 Table Task Force, the Group Annuity Valuation Table Task Force concluded that the CSRS data would provide the most meaningful projection, because they were produced from a large database and also used directly to extend the mortality table for active lives.

This conclusion was arrived at after examining the *SSA 107* experience and the age-by-age trends of the CSRS experience without modification. The *SSA 107* experience did not include actual experience past calendar year 1988, whereas the CSRS data included experience through 1993. The CSRS data would therefore provide the better projection for all ages, even though some slight modification and smoothing were required.

The scale of mortality improvement factors for projecting the mortality rates shown in Table 9 from 1988 to 1994 was based on the average trends for CSRS over the period 1987–1993. A mortality table based on CSRS experience was constructed for each year over this period and graduated by using a Whittaker-Henderson type B method. Then a mortality improvement rate for each age was determined based on a least-squares best fit trend line through logarithms of the death rates for that age. The resulting scale of mortality improvement trends for each age was then itself graduated using the same method and rounded to the nearest one-tenth of one percentage point. However, the trends for females at ages 60–65 were changed from negative to zero, because the group annuity trend experience for these ages was slightly positive. A trend of 2% was used at the younger ages.

Table 11 shows the final mortality improvement factors compared to the actual CSRS 1987–1993 trends.

TABLE 10

ANNUAL MORTALITY IMPROVEMENT RATES FROM VARIOUS STUDIES
RATES IN PERCENTAGE PER YEAR

Ages	SSA 88–94	SSA 86–92	CSRS 86–92	Scale H	SOA 85–90	CSRS Nondisability
Male Lives						
25–29	−2.13	−1.67	0.97	0.10		
30–34	−3.34	−3.04	−1.24	0.75		
35–39	−2.98	−3.51	−0.69	2.00		
40–44	−2.21	−1.39	−0.68	2.00		
45–49	0.64	0.62	1.35	1.75		
50–54	0.91	1.25	2.19	1.75		
55–59	1.41	1.40	2.53	1.50	3.90	2.70
60–64	1.60	1.57	1.78	1.50	2.30	1.77
65–69	1.52	1.08	1.29	1.50	3.00	1.23
70–74	1.03	1.24	1.90	1.25	3.40	1.88
75–79	0.79	0.89	2.18	1.25	2.90	2.18
80–84	0.68	0.57	1.70	1.25	1.00	1.69
85–89	0.73	0.22	1.04	0.75	0.70	1.08
90–94	1.00	−0.23	0.16	0.75	0.30	0.17
Female Lives						
25–29	−0.05	−0.73	3.40	0.75		
30–34	−1.68	−1.47	0.21	1.25		
35–39	0.00	−0.68	2.13	2.25		
40–44	1.29	1.78	3.00	2.25		
45–49	1.81	1.77	0.97	2.00		
50–54	1.39	1.50	1.66	2.00		
55–59	1.20	0.65	−0.26	1.75	0.60	3.06
60–64	0.69	0.74	−0.07	1.75	1.80	−0.44
65–69	0.45	0.35	0.50	1.75	2.60	0.54
70–74	0.54	0.64	1.20	1.75	3.90	1.22
75–79	0.87	0.74	1.20	1.50	2.50	1.19
80–84	1.22	0.86	1.16	1.50	0.70	1.17
85–89	1.19	0.66	1.15	1.00	2.00	1.14
90–94	0.93	0.18	0.85	0.50	2.10	0.81

Table 12 shows the ungraduated base year 1994 mortality table rates before margins. The rates in Table 12 were obtained by taking the 1988 base year mortality rates from Table 9 and projecting them to 1994 using the GAM 88–94 mortality improvement factors in Table 11. The following formula was used to project the mortality rates:

$$q_x^{1994} = q_x^{1988} \times (1 - scale_x)^{(1994-1988)} \qquad \text{(A)}$$

TABLE 11

ANNUAL MORTALITY IMPROVEMENT FACTORS FOR USE IN PROJECTING MORTALITY RATES FROM 1988 TO 1994 (GAM 88–94 COLUMN)

RATES IN PERCENTAGE PER YEAR

	Male		Female			Male		Female	
Age	CSRS 87–93	GAM 88–94	CSRS 87–93	GAM 88–94	Age	CSRS 87–93	GAM 88–94	CSRS 87–93	GAM 88–94
1	2.0	2.0	2.0	2.0	31	−1.5	−1.2	0.8	0.5
2	2.0	2.0	2.0	2.0	32	−1.3	−1.0	0.9	0.5
3	2.0	2.0	2.0	2.0	33	−0.8	−0.7	0.6	0.5
4	2.0	2.0	2.0	2.0	34	−0.1	−0.2	0.3	0.5
5	2.0	2.0	2.0	2.0	35	0.6	0.1	0.1	0.6
6	2.0	2.0	2.0	2.0	36	1.0	0.5	0.4	0.7
7	2.0	2.0	2.0	2.0	37	0.9	0.7	0.9	0.8
8	2.0	2.0	2.0	2.0	38	0.7	1.0	1.3	1.0
9	2.0	2.0	2.0	2.0	39	0.7	1.2	1.4	1.1
10	2.0	2.0	2.0	2.0	40	1.0	1.4	1.2	1.1
11	2.0	2.0	2.0	2.0	41	1.6	1.7	1.0	1.2
12	2.0	2.0	2.0	1.9	42	2.1	1.9	1.0	1.3
13	2.0	1.9	2.0	1.8	43	2.3	2.0	1.2	1.4
14	1.9	1.9	1.8	1.7	44	2.4	2.0	1.5	1.6
15	1.9	1.9	1.6	1.6	45	2.2	2.0	1.8	1.8
16	1.9	1.9	1.5	1.5	46	1.8	1.8	2.2	1.9
17	1.9	1.9	1.4	1.5	47	1.4	1.7	2.3	1.9
18	1.9	1.9	1.4	1.5	48	1.2	1.6	2.1	1.8
19	1.9	1.9	1.5	1.5	49	1.3	1.6	1.6	1.5
20	1.9	1.9	1.6	1.6	50	1.5	1.6	1.0	1.2
21	1.8	1.8	1.7	1.6	51	1.9	1.7	0.7	1.0
22	1.7	1.7	1.7	1.6	52	2.2	1.8	0.6	0.8
23	1.5	1.5	1.6	1.6	53	2.2	1.9	0.6	0.8
24	1.3	1.2	1.5	1.5	54	2.0	1.9	0.8	0.8
25	1.0	0.8	1.4	1.3	55	1.7	1.9	0.9	0.9
26	0.6	0.3	1.2	1.1	56	1.6	1.9	1.0	0.9
27	−0.4	−0.1	0.9	0.9	57	1.7	1.8	1.0	0.8
28	−0.6	−0.6	0.5	0.7	58	1.8	1.9	0.9	0.6
29	−0.9	−1.0	0.3	0.6	59	1.9	1.9	0.4	0.2
30	−1.3	−1.2	0.4	0.5	60	2.0	1.9	−0.1	0.0

TABLE 11—*Continued*

Age	Male CSRS 87–93	Male GAM 88–94	Female CSRS 87–93	Female GAM 88–94	Age	Male CSRS 87–93	Male GAM 88–94	Female CSRS 87–93	Female GAM 88–94
61	1.9	1.9	−0.6	0.0	91	0.9	0.9	0.8	0.8
62	1.9	1.8	−0.8	0.0	92	0.7	0.7	0.8	0.9
63	1.8	1.7	−0.8	0.0	93	0.6	0.6	0.9	0.9
64	1.6	1.5	−0.6	0.0	94	0.5	0.5	1.0	1.0
65	1.3	1.3	−0.2	0.0	95	0.4	0.5	1.1	1.1
66	1.1	1.2	0.2	0.2	96	0.4	0.4	1.1	1.1
67	1.0	1.2	0.8	0.7	97	0.4	0.4	1.1	1.0
68	1.0	1.2	1.3	1.2	98	0.4	0.3	1.1	0.9
69	1.3	1.3	1.7	1.5	99	0.5	0.2	1.0	0.7
70	1.6	1.5	1.9	1.8	100	0.0	0.2	0.0	0.4
71	1.9	1.7	2.0	2.0	101	0.0	0.1	0.0	0.2
72	2.0	1.9	2.0	2.0	102	0.0	0.0	0.0	0.1
73	2.0	2.1	1.9	2.0	103	0.0	0.0	0.0	0.0
74	2.1	2.2	1.8	1.9	104	0.0	0.0	0.0	0.0
75	2.2	2.3	1.8	1.8	105	0.0	0.0	0.0	0.0
76	2.3	2.3	1.7	1.7	106	0.0	0.0	0.0	0.0
77	2.4	2.3	1.6	1.5	107	0.0	0.0	0.0	0.0
78	2.4	2.3	1.4	1.4	108	0.0	0.0	0.0	0.0
79	2.3	2.2	1.2	1.2	109	0.0	0.0	0.0	0.0
80	2.2	2.1	1.1	1.1	110	0.0	0.0	0.0	0.0
81	1.9	1.9	1.0	1.0	111	0.0	0.0	0.0	0.0
82	1.7	1.8	0.9	1.0	112	0.0	0.0	0.0	0.0
83	1.6	1.7	0.9	0.9	113	0.0	0.0	0.0	0.0
84	1.5	1.6	1.0	0.9	114	0.0	0.0	0.0	0.0
85	1.4	1.5	1.0	0.9	115	0.0	0.0	0.0	0.0
86	1.4	1.4	1.0	0.9	116	0.0	0.0	0.0	0.0
87	1.4	1.3	1.0	0.9	117	0.0	0.0	0.0	0.0
88	1.3	1.3	0.9	0.9	118	0.0	0.0	0.0	0.0
89	1.2	1.2	0.8	0.8	119	0.0	0.0	0.0	0.0
90	1.1	1.0	0.8	0.8	120	0.0	0.0	0.0	0.0

TABLE 12

Group Annuity Mortality Rates Ungraduated—No Margin
1994 Base Year

Age	Male	Female	Age	Male	Female	Age	Male	Female
1	0.000652	0.000573	21	0.000581	0.000305	41	0.001084	0.000922
2	0.000440	0.000374	22	0.000603	0.000309	42	0.001329	0.000796
3	0.000366	0.000279	23	0.000620	0.000315	43	0.001491	0.001162
4	0.000285	0.000209	24	0.000635	0.000325	44	0.001705	0.000901
5	0.000261	0.000189	25	0.000652	0.000337	45	0.001587	0.000955
6	0.000250	0.000176	26	0.000790	0.000262	46	0.001847	0.001071
7	0.000239	0.000166	27	0.000669	0.000350	47	0.001916	0.001098
8	0.000221	0.000153	28	0.000879	0.000311	48	0.002357	0.001244
9	0.000197	0.000141	29	0.000920	0.000362	49	0.002500	0.001610
10	0.000177	0.000131	30	0.000927	0.000402	50	0.002787	0.001432
11	0.000185	0.000132	31	0.000913	0.000399	51	0.003110	0.001663
12	0.000244	0.000153	32	0.000872	0.000370	52	0.003316	0.001971
13	0.000280	0.000174	33	0.000848	0.000425	53	0.003637	0.002052
14	0.000327	0.000204	34	0.000950	0.000539	54	0.004423	0.002204
15	0.000380	0.000238	35	0.001003	0.000520	55	0.004245	0.002389
16	0.000429	0.000269	36	0.000854	0.000561	56	0.005126	0.002528
17	0.000472	0.000292	37	0.000936	0.000591	57	0.006439	0.003070
18	0.000504	0.000302	38	0.000929	0.000535	58	0.006746	0.003572
19	0.000529	0.000304	39	0.001069	0.000758	59	0.007448	0.004136
20	0.000553	0.000302	40	0.001120	0.000656	60	0.008169	0.004759

TABLE 12—*Continued*

Age	Male	Female	Age	Male	Female	Age	Male	Female
61	0.009319	0.004900	81	0.074602	0.047670	101	0.356944	0.319187
62	0.010665	0.005865	82	0.078223	0.050480	102	0.377047	0.340376
63	0.012386	0.007110	83	0.090886	0.059566	103	0.395900	0.362972
64	0.014017	0.008633	84	0.098273	0.063617	104	0.415695	0.384750
65	0.015890	0.009975	85	0.099952	0.075662	105	0.436479	0.407835
66	0.017923	0.011520	86	0.108945	0.079090	106	0.458303	0.432305
67	0.019372	0.011081	87	0.127035	0.089007	107	0.481218	0.458243
68	0.020453	0.011764	88	0.140431	0.100727	108	0.500000	0.485738
69	0.023318	0.013532	89	0.145522	0.107252	109	0.500000	0.500000
70	0.025545	0.014769	90	0.152096	0.121479	110	0.500000	0.500000
71	0.027342	0.016360	91	0.189183	0.137682	111	0.500000	0.500000
72	0.030660	0.017403	92	0.186739	0.153076	112	0.500000	0.500000
73	0.033074	0.019986	93	0.226421	0.183005	113	0.500000	0.500000
74	0.036503	0.020223	94	0.225564	0.168056	114	0.500000	0.500000
75	0.039719	0.023478	95	0.259451	0.186913	115	0.500000	0.500000
76	0.043397	0.028368	96	0.271887	0.221973	116	0.500000	0.500000
77	0.048669	0.030941	97	0.287427	0.239501	117	0.500000	0.500000
78	0.052907	0.032406	98	0.304656	0.256913	118	0.500000	0.500000
79	0.058160	0.037312	99	0.321819	0.275642	119	0.500000	0.500000
80	0.064102	0.042932	100	0.337910	0.297517	120	1.000000	1.000000

where

q_x^y = mortality rate in calendar year y at attained age x

$scale_x$ = mortality improvement factor for attained age x.

The resulting rates are an ungraduated set of mortality rates for ages 1–120, by sex, with a base experience year of 1994.

B. Graduated Mortality Rates

The resulting set of mortality rates in Table 12 was then graduated by using the Karup-King four point graduation formula, as follows. Mortality rates were averaged by quinquennial age groups q_n, q_n, q_n, q_n, ... Graduated mortality rates $q_{n\,t}$ were derived based on the following formula:

$$q_{n+t} = A_1 \times q_{n-5} + A_2 \times q_n + A_3 \times q_{n+5} + A_4 \times q_{n+10} \qquad \text{(B)}$$

where

$A_1 = -0.5 \times S_1 \times (1 - S_1)^2$

$A_2 = 1.5 \times S_1^3 - 2.5 \times S_1^2 + 1$

$A_3 = -1.5 \times S_1^3 + 2 \times S_1^2 + 0.5 \times S_1$

$A_4 = 0.5 \times S_1^2 \times (S_1 - 1)$

$S_1 = t/5$

At the extreme ages (under age 7 and over age 102), minor adjustments were made.

The adjusted mortality rates with a base year of 1994 of Table 12 *after graduation* are shown in Table 13. Table 13 is the 1994 Group Annuity Mortality Basic (GAM–94 Basic) Table.

C. Projection of Mortality Rates beyond 1994

For the projection of mortality reduction beyond 1994, the Task Force decided to use a blend of the CSRS and *SSA 107* mortality reduction trends based upon experience between years 1977 through 1993, with adjustments. A mortality improvement scale based entirely on CSRS data over the period 1977–1993 was constructed. The starting point was a mortality table for each year 1977 through 1993, graduated by Whittaker-Henderson type B. Then a mortality improvement trend for each age was determined based on

TABLE 13

1994 GROUP ANNUITY MORTALITY TABLE
GRADUATED—NO MARGIN
1994 BASE YEAR

Age	Male	Female	Age	Male	Female	Age	Male	Female
1	0.000637	0.000571	21	0.000570	0.000308	41	0.001243	0.000826
2	0.000430	0.000372	22	0.000598	0.000311	42	0.001346	0.000888
3	0.000357	0.000278	23	0.000633	0.000313	43	0.001454	0.000943
4	0.000278	0.000208	24	0.000671	0.000313	44	0.001568	0.000992
5	0.000255	0.000188	25	0.000711	0.000313	45	0.001697	0.001046
6	0.000244	0.000176	26	0.000749	0.000316	46	0.001852	0.001111
7	0.000234	0.000165	27	0.000782	0.000324	47	0.002042	0.001196
8	0.000216	0.000147	28	0.000811	0.000338	48	0.002260	0.001297
9	0.000209	0.000140	29	0.000838	0.000356	49	0.002501	0.001408
10	0.000212	0.000141	30	0.000862	0.000377	50	0.002773	0.001536
11	0.000223	0.000148	31	0.000883	0.000401	51	0.003088	0.001686
12	0.000243	0.000159	32	0.000902	0.000427	52	0.003455	0.001864
13	0.000275	0.000177	33	0.000912	0.000454	53	0.003854	0.002051
14	0.000320	0.000203	34	0.000913	0.000482	54	0.004278	0.002241
15	0.000371	0.000233	35	0.000915	0.000514	55	0.004758	0.002466
16	0.000421	0.000261	36	0.000927	0.000550	56	0.005322	0.002755
17	0.000463	0.000281	37	0.000958	0.000593	57	0.006001	0.003139
18	0.000495	0.000293	38	0.001010	0.000643	58	0.006774	0.003612
19	0.000521	0.000301	39	0.001075	0.000701	59	0.007623	0.004154
20	0.000545	0.000305	40	0.001153	0.000763	60	0.008576	0.004773

TABLE 13—*Continued*

Age	Male	Female	Age	Male	Female	Age	Male	Female
61	0.009663	0.005476	81	0.073780	0.047260	101	0.358560	0.318956
62	0.010911	0.006271	82	0.081217	0.052853	102	0.376699	0.340960
63	0.012335	0.007179	83	0.088721	0.058986	103	0.396884	0.364586
64	0.013914	0.008194	84	0.096358	0.065569	104	0.418855	0.389996
65	0.015629	0.009286	85	0.104559	0.072836	105	0.440585	0.415180
66	0.017462	0.010423	86	0.113755	0.081018	106	0.460043	0.438126
67	0.019391	0.011574	87	0.124377	0.090348	107	0.475200	0.456824
68	0.021354	0.012648	88	0.136537	0.100882	108	0.485670	0.471493
69	0.023364	0.013665	89	0.149949	0.112467	109	0.492807	0.483473
70	0.025516	0.014763	90	0.164442	0.125016	110	0.497189	0.492436
71	0.027905	0.016079	91	0.179849	0.138442	111	0.499394	0.498054
72	0.030625	0.017748	92	0.196001	0.152660	112	0.500000	0.500000
73	0.033549	0.019724	93	0.213325	0.167668	113	0.500000	0.500000
74	0.036614	0.021915	94	0.231936	0.183524	114	0.500000	0.500000
75	0.040012	0.024393	95	0.251189	0.200229	115	0.500000	0.500000
76	0.043933	0.027231	96	0.270441	0.217783	116	0.500000	0.500000
77	0.048570	0.030501	97	0.289048	0.236188	117	0.500000	0.500000
78	0.053991	0.034115	98	0.306750	0.255605	118	0.500000	0.500000
79	0.060066	0.038024	99	0.323976	0.276035	119	0.500000	0.500000
80	0.066696	0.042361	100	0.341116	0.297233	120	1.000000	1.000000

a least-squares best-fit trend line through the logarithms of the rates for that age. The opening year of 1977 was chosen because it provided a reasonable representation of anticipated trends in the future and, properly, did not reflect more rapid mortality improvement rates found in the experience of prior periods.

The trends for Social Security are based on data from *SSA 107* along with additional data used in this study, which were provided by the Office of the Actuary at the Social Security Administration. These additional data included central death rates for five-year age groups for each calendar year over the period 1960–1988. Before the Social Security trends for 1977–1988 could be blended with the CSRS trends for 1977–1993, it was necessary to extend the Social Security trends up through 1993. This extension was based on mortality improvement trends for the CSRS from 1988 through 1993. The *SSA 107* extended central death rates for each year 1989 through 1993 were obtained by multiplying the SSA central death rate for 1988 by the ratio of the CSRS central death rate for the corresponding year to the CSRS central death rate for 1988. Then the average trend for each central age over the entire 1977–1993 period was determined based on a least-squares best-fit trend line through the logarithms of these central death rates. The Social Security data did not cover central ages beyond age 92, and the CSRS data at these older ages were limited. The mortality improvement trends for individual ages were interpolated from the trends for the central ages by using the Karup-King four-point interpolation formula.

The trends at ages 1–25 were based on Social Security data and on the Social Security assumptions for future trends listed in *SSA 107* and start out at a rate of improvement of 2% per year. Then the CSRS mortality improvement trend for each age was averaged with the corresponding trend for Social Security. These average trends were then rounded to the nearest one-tenth of one percentage point. The resulting mortality improvement factors are shown in Table 14.

To obtain the mortality improvement factors for projecting mortality beyond 1994, the following modifications were made in this scale:

1. Any mortality improvement factors that were less than 0.5% for ages under 85 were changed to 0.5%, because the Task Force thought that the use of lower factors would result in excessive mortality rates in the future.
2. A maximum mortality improvement rate of 2.0% was set for ages under 60. This reduced the highest rate of 2.3% to 2.0% at male ages 52–54 and provided a smoother progression of rates around these ages.

TABLE 14

ANNUAL MORTALITY IMPROVEMENT FACTORS FROM THE *SSA* 107 AND CSRS STUDIES BASED UPON 1977–1993 EXPERIENCE RATES IN PERCENTAGE PER YEAR

	Male			Female		
Age	SS 77–93	CSRS 77–93	Average	SS 77–93	CSRS 77–93	Average
1	2.0	2.0	2.0	2.0	2.0	2.0
2	2.0	2.0	2.0	2.0	2.0	2.0
3	2.0	2.0	2.0	2.0	2.0	2.0
4	2.0	2.0	2.0	2.0	2.0	2.0
5	2.0	2.0	2.0	2.0	2.0	2.0
6	2.0	2.0	2.0	2.0	2.0	2.0
7	2.0	2.0	2.0	2.0	2.0	2.0
8	2.0	2.0	2.0	2.0	2.0	2.0
9	2.0	2.0	2.0	2.0	2.0	2.0
10	2.0	2.0	2.0	2.0	2.0	2.0
11	2.0	2.0	2.0	2.0	2.0	2.0
12	2.0	2.0	2.0	2.0	2.0	2.0
13	2.0	2.0	2.0	2.0	2.0	2.0
14	1.9	1.9	1.9	1.8	1.8	1.8
15	1.9	1.9	1.9	1.6	1.6	1.6
16	1.9	1.9	1.9	1.5	1.5	1.5
17	1.9	1.9	1.9	1.4	1.4	1.4
18	1.9	1.9	1.9	1.4	1.4	1.4
19	1.9	1.9	1.9	1.5	1.5	1.5
20	1.9	1.9	1.9	1.6	1.6	1.6
21	1.8	1.8	1.8	1.7	1.7	1.7
22	1.7	1.7	1.7	1.7	1.7	1.7
23	1.5	1.5	1.6	1.6	1.6	1.6
24	1.3	1.3	1.3	1.5	1.5	1.5
25	1.0	1.0	1.0	1.4	1.4	1.4
26	0.6	0.6	0.6	1.2	1.2	1.2
27	0.3	0.4	0.3	1.1	0.9	1.0
28	0.0	−0.4	−0.2	0.9	1.4	1.2
29	−0.5	−1.0	−0.8	0.8	1.6	1.2
30	−1.0	−1.4	−1.2	0.6	1.5	1.0
31	−1.4	−1.6	−1.5	0.5	1.1	0.8
32	−1.7	−1.5	−1.6	0.5	0.6	0.6
33	−1.7	−1.2	−1.4	0.7	0.2	0.4
34	−1.5	−0.9	−1.2	1.0	0.0	0.5
35	−1.2	−0.5	−0.9	1.3	0.1	0.7
36	−0.9	−0.3	−0.6	1.6	0.4	1.0
37	−0.6	−0.1	−0.4	1.8	0.8	1.3
38	−0.3	0.0	−0.2	1.9	1.0	1.5
39	0.0	0.0	0.0	2.0	1.1	1.5
40	0.3	0.2	0.2	2.0	1.0	1.5
41	0.6	0.4	0.5	2.0	0.8	1.4
42	0.9	0.7	0.8	2.0	0.7	1.3
43	1.1	1.0	1.1	2.0	0.7	1.4
44	1.3	1.3	1.3	2.0	0.9	1.4
45	1.5	1.6	1.5	2.0	1.2	1.6

TABLE 14—*Continued*

Age	Male SS 77–93	Male CSRS 77–93	Male Average	Female SS 77–93	Female CSRS 77–93	Female Average
46	1.6	1.8	1.7	1.9	1.4	1.7
47	1.7	2.1	1.9	1.9	1.6	1.8
48	1.8	2.3	2.0	1.9	1.6	1.8
49	1.8	2.5	2.2	1.9	1.6	1.8
50	1.9	2.7	2.3	1.9	1.5	1.7
51	1.9	2.8	2.3	1.9	1.3	1.6
52	1.9	2.7	2.3	1.9	1.0	1.4
53	1.8	2.6	2.2	1.7	0.8	1.2
54	1.8	2.4	2.1	1.5	0.6	1.0
55	1.7	2.2	1.9	1.3	0.4	0.8
56	1.6	2.0	1.8	1.0	0.2	0.6
57	1.6	1.8	1.7	0.8	0.1	0.5
58	1.6	1.7	1.6	0.6	0.0	0.3
59	1.6	1.5	1.6	0.4	−0.2	0.1
60	1.7	1.4	1.6	0.2	−0.4	0.0
61	1.8	1.3	1.5	0.0	−0.5	−0.2
62	1.8	1.2	1.5	0.0	−0.5	−0.2
63	1.7	1.2	1.4	0.0	−0.5	−0.2
64	1.6	1.2	1.4	0.0	−0.4	−0.1
65	1.5	1.2	1.4	0.1	−0.2	0.0
66	1.4	1.3	1.3	0.2	0.0	0.1
67	1.3	1.4	1.3	0.3	0.1	0.2
68	1.3	1.5	1.4	0.4	0.3	0.3
69	1.3	1.5	1.4	0.5	0.3	0.4
70	1.3	1.6	1.5	0.7	0.4	0.5
71	1.4	1.6	1.5	0.8	0.4	0.6
72	1.4	1.6	1.5	0.9	0.4	0.6
73	1.3	1.6	1.5	0.9	0.5	0.7
74	1.3	1.6	1.5	0.9	0.5	0.7
75	1.3	1.6	1.4	1.0	0.6	0.8
76	1.2	1.5	1.4	0.9	0.6	0.8
77	1.2	1.4	1.3	0.9	0.5	0.7
78	1.1	1.3	1.2	0.9	0.5	0.7
79	1.0	1.2	1.1	0.9	0.5	0.7
80	0.9	1.0	1.0	0.9	0.5	0.7
81	0.9	0.9	0.9	0.9	0.5	0.7
82	0.8	0.8	0.8	0.9	0.6	0.7
83	0.8	0.8	0.8	0.9	0.5	0.7
84	0.8	0.7	0.7	0.8	0.5	0.7
85	0.8	0.6	0.7	0.8	0.4	0.6
86	0.8	0.6	0.7	0.8	0.3	0.5
87	0.7	0.5	0.6	0.7	0.2	0.4
88	0.6	0.5	0.5	0.7	0.1	0.4
89	0.5	0.4	0.5	0.7	0.0	0.3
90	0.4	0.3	0.4	0.7	0.0	0.3
91	0.3	0.2	0.3	0.6	0.0	0.3
92	0.2	0.2	0.2	0.6	0.0	0.3

3. At the higher ages the mortality improvement rates were graded to a value of 0.1% at age 100 and set to 0 for all ages greater than 100.
4. Other minor adjustments were made as described below.

After age 37, the factors for males start to increase fairly rapidly from one age to the next, going from a factor of 0.2% (before change) at age 38 to 2.3% at age 50.

When there are large mortality improvement factor increases from one age to the next like this, it is possible that, after the mortality improvement scale has been applied for a number of years, the mortality rate for a particular age could become lower than the rate for an age one year younger. To minimize this possibility, it was decided to limit the increase in the factor from one age to the next to one-tenth of one percentage point. As a result, the mortality improvement factors for males were modified so that they increase from 0.5% at age 37 to 2.0% at age 52. The factors for some ages were increased by this process, and factors for other ages were reduced.

There are also significant age-to-age increases for females in the factors from ages 33 through 38. The factors for females for ages 32 through 38 were therefore also modified, as were the factors at female ages 41 to 44.

Mortality improvement factors to be used in the new Group Annuity Mortality Table when projecting mortality rates beyond 1994 are shown in Table 15 and are referred to as the Projection Scale AA. Figure 1 displays a graph of the Projection Scale AA factors for males. Figure 2 displays a graph of the Projection Scale AA factors for females.

IV. MARGINS

Consistent with accepted actuarial practice and precedent set in the development of existing mortality tables used in reserving, the Task Force deemed it necessary and appropriate to add margins to the q_x values of the 1994 Group Annuity Mortality Basic Table. The overall margin comprises two components:

1. Margins for random variation in mortality rates
2. Margins for other contingencies.

A. Margins for Random Variation in Mortality Rates

The unloaded 1994 Group Annuity Mortality Basic Table q_x values shown in Table 13 represent expected values. Considering current reserving theory, the Task Force decided to incorporate margins to produce annuity reserve

TABLE 15

PROJECTION SCALE AA
MORTALITY IMPROVEMENT FACTORS TO BE USED IN THE NEW TABLE WHEN PROJECTING MORTALITY RATES BEYOND 1994
FACTORS ARE SHOWN AS PERCENTAGE PER YEAR

Attained Age	Male Factor	Female Factor	Attained Age	Male Factor	Female Factor	Attained Age	Male Factor	Female Factor
1	2.0	2.0	21	1.8	1.7	41	0.9	1.5
2	2.0	2.0	22	1.7	1.7	42	1.0	1.5
3	2.0	2.0	23	1.5	1.6	43	1.1	1.5
4	2.0	2.0	24	1.3	1.5	44	1.2	1.5
5	2.0	2.0	25	1.0	1.4	45	1.3	1.6
6	2.0	2.0	26	0.6	1.2	46	1.4	1.7
7	2.0	2.0	27	0.5	1.2	47	1.5	1.8
8	2.0	2.0	28	0.5	1.2	48	1.6	1.8
9	2.0	2.0	29	0.5	1.2	49	1.7	1.8
10	2.0	2.0	30	0.5	1.0	50	1.8	1.7
11	2.0	2.0	31	0.5	0.8	51	1.9	1.6
12	2.0	2.0	32	0.5	0.8	52	2.0	1.4
13	2.0	2.0	33	0.5	0.9	53	2.0	1.2
14	1.9	1.8	34	0.5	1.0	54	2.0	1.0
15	1.9	1.6	35	0.5	1.1	55	1.9	0.8
16	1.9	1.5	36	0.5	1.2	56	1.8	0.6
17	1.9	1.4	37	0.5	1.3	57	1.7	0.5
18	1.9	1.4	38	0.6	1.4	58	1.6	0.5
19	1.9	1.5	39	0.7	1.5	59	1.6	0.5
20	1.9	1.6	40	0.8	1.5	60	1.6	0.5

TABLE 15—*Continued*

Attained Age	Male Factor	Female Factor	Attained Age	Male Factor	Female Factor	Attained Age	Male Factor	Female Factor
61	1.5	0.5	81	0.9	0.7	101	0.0	0.0
62	1.5	0.5	82	0.8	0.7	102	0.0	0.0
63	1.4	0.5	83	0.8	0.7	103	0.0	0.0
64	1.4	0.5	84	0.7	0.7	104	0.0	0.0
65	1.4	0.5	85	0.7	0.6	105	0.0	0.0
66	1.3	0.5	86	0.7	0.5	106	0.0	0.0
67	1.3	0.5	87	0.6	0.4	107	0.0	0.0
68	1.4	0.5	88	0.5	0.4	108	0.0	0.0
69	1.4	0.5	89	0.5	0.3	109	0.0	0.0
70	1.5	0.5	90	0.4	0.3	110	0.0	0.0
71	1.5	0.6	91	0.4	0.3	111	0.0	0.0
72	1.5	0.6	92	0.3	0.3	112	0.0	0.0
73	1.5	0.7	93	0.3	0.2	113	0.0	0.0
74	1.5	0.7	94	0.3	0.2	114	0.0	0.0
75	1.4	0.8	95	0.2	0.2	115	0.0	0.0
76	1.4	0.8	96	0.2	0.2	116	0.0	0.0
77	1.3	0.7	97	0.2	0.1	117	0.0	0.0
78	1.2	0.7	98	0.1	0.1	118	0.0	0.0
79	1.1	0.7	99	0.1	0.1	119	0.0	0.0
80	1.0	0.7	100	0.1	0.1	120	0.0	0.0

FIGURE 1

MORTALITY IMPROVEMENT FACTORS—MALE
SCALE AA

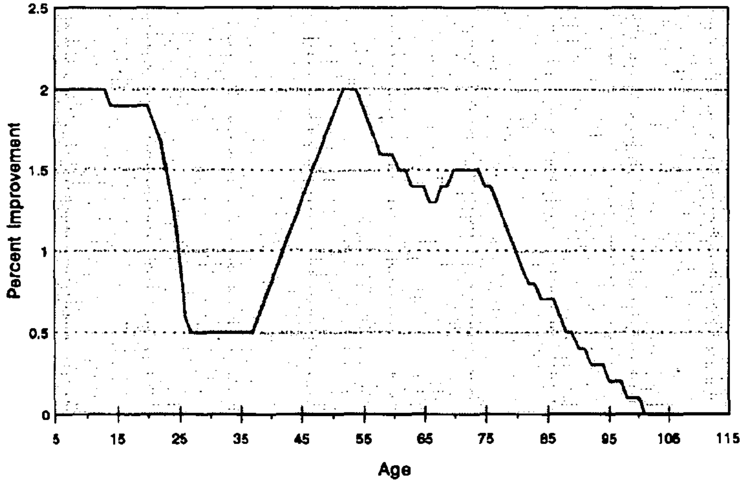
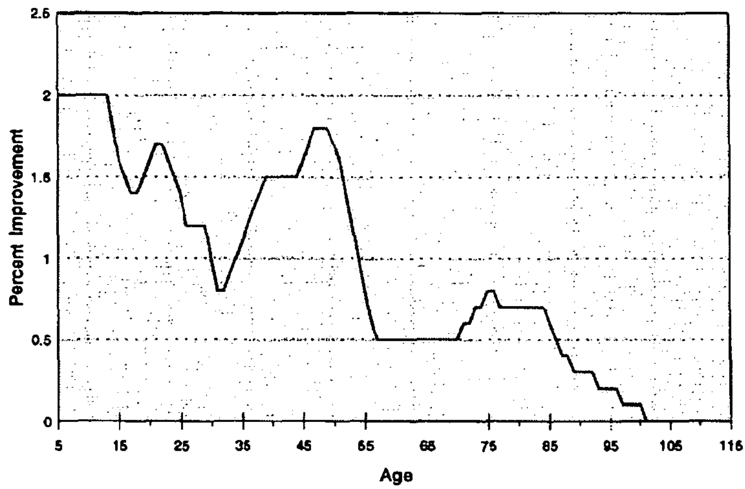


FIGURE 2

MORTALITY IMPROVEMENT FACTORS—FEMALE
SCALE AA



values that would be adequate for random variation of two standard deviations from expected mortality.

Probability theory was used to develop variances of distributions of annuity values as indicated below.

For a single life age x, assume Y is a random variable representing the present value of annuity payments received. Y would have the following probability distribution:

Y	$\Pr(Y = y)$
0	$1 - p$
$a_{\overline{1}\mid}$	$_{1\mid}q_y \; [= p_y(1 - p_{y+1})]$
$a_{\overline{2}\mid}$	$_{2\mid}q_y$
$a_{\overline{3}\mid}$	$_{3\mid}q_y$
.	.
.	.
.	.

The mean, variance, and standard deviation of this distribution would be determined as follows:

$$\mu = E[Y] = \sum_{t=0}^{\infty} a_{\overline{t}\mid} \times \Pr(Y = a_{\overline{t}\mid}) \tag{C}$$

$$E[Y^2] = \sum_{t=0}^{\infty} a^2_{\overline{t}\mid} \times \Pr(Y = a_{\overline{t}\mid}) \tag{D}$$

$$\sigma^2 = E[Y^2] - (E[Y])^2 \tag{E}$$

$$\sigma = \sqrt{\sigma^2} \tag{F}$$

For a distribution of annuity values for N lives age x, assumed to be independent, the mean, variance and standard deviations would be calculated as follows:

$$\mu_N = N \times \mu \tag{G}$$

$$\sigma_N^2 = N \times \sigma^2 \tag{H}$$

$$\sigma_N = \sqrt{N} \times \sigma \tag{I}$$

As the size of a company's group annuity block of business increases, the required margins for random variations decrease. The Task Force reviewed recent statutory annual statement data on group annuity business to determine an appropriate company block of business volume assumption to use in calculating the random variation margin component. To ensure that the new standard would provide at least a two-standard-deviation margin for the vast majority of companies (more than 95%) having insured group annuity business, the Task Force decided that a 3,000-life block of business would be appropriate for computing margins for random variation.

Tables 16 and 17 show the results of applying these concepts and the determination of required margins to be built into the GAM-94 Basic Table q_x values shown in Table 13. Expected values and standard deviations were calculated by using the formulas presented in this section with a value of N 3,000. The interest rate used in the analysis was 6%. Note that use of other interest assumptions and forms of annuity did not significantly change the level of required margins.

TABLE 16

RANDOM VARIATION ANALYSIS OF REQUIRED MARGINS FOR MALES
GAM-94 BASIC TABLE EXPECTED MORTALITY
3,000-LIFE GROUP, INTEREST AT 6%

Annuity Type	Age	Expected Value	Standard Deviation	Required Margins: 1 Standard Deviation	Required Margins: 2 Standard Deviations
Immediate Life Annuities	45	41,400	128	3.0%	6.0%
	50	38,987	148	2.8	5.6
	55	36,043	167	2.6	5.2
	60	32,574	184	2.5	4.9
	65	28,724	194	2.3	4.6
	70	24,697	195	2.2	4.3
	75	20,459	188	2.0	4.0
	80	16,180	175	1.9	3.8
Deferred to Age 65 Life Annuity	30	3,281	33	2.3%	4.6%
	35	4,410	43	2.3	4.6
	40	5,931	58	2.3	4.6
	45	7,990	76	2.3	4.6
	50	10,804	101	2.3	4.7
	55	14,713	132	2.4	4.7
	60	20,301	167	2.4	4.7

Based on these results, the Task Force concluded that a 5% margin would make adequate provision for random variations in mortality for reserving purposes.

TABLE 17

RANDOM VARIATION ANALYSIS OF REQUIRED MARGINS FOR FEMALES
GAM-94 BASIC TABLE EXPECTED MORTALITY
3,000-LIFE GROUP, INTEREST AT 6%

Annuity Type	Age	Expected Value	Standard Deviation	Required Margins: 1 Standard Deviation	Required Margins: 2 Standard Deviations
Immediate Life Annuities	45	43,301	107	3.3%	6.5%
	50	41,342	125	3.0	6.0
	55	38,861	145	2.8	5.6
	60	35,810	165	2.7	5.3
	65	32,307	180	2.5	4.9
	70	28,439	186	2.3	4.6
	75	24,039	187	2.1	4.3
	80	19,406	180	2.0	4.0
Deferred to Age 65 Life Annuity	30	3,907	30	2.6%	5.1%
	35	5,239	40	2.6	5.1
	40	7,032	53	2.6	5.1
	45	9,453	70	2.6	5.1
	50	12,727	92	2.6	5.1
	55	17,192	120	2.6	5.1
	60	23,381	153	2.6	5.1

B. Margins for Other Contingencies

The Task Force thought that the 5% margin was adequate for random variation for most insurance companies. However, blocks of business of less than 3,000 lives would have a greater standard deviation than shown above. Also, variations in the mix of business that companies write may cause the underlying mortality for a given company to differ from the underlying mortality in the valuation standard. Examples of business characteristics that could affect the underlying mortality averages include:

1. The mix of white-collar and blue-collar workers
2. The mix of higher-income and lower-income annuitants
3. Degree of concentration by geographic area.

For these reasons, the Task Force decided to recommend a specific margin to be added to the 5% statistical margin. The conclusion was to add 2% to the 5% statistical margin to produce a total 7% margin. It is anticipated that this margin produces reserves that are adequate to cover various business characteristics and random variations.

The resulting q_x values, including the 7% margin, comprise the 1994 Group Annuity Mortality Static Table and are presented in Table 18. Table 18 is calculated as 93% of the corresponding Table 13 values, with modification after age 102. No margin was applied to the mortality rates of 0.5

TABLE 18

1994 GROUP ANNUITY MORTALITY STATIC TABLE
1994 BASE YEAR

Age	Values of q_x		Age	Values of q_x		Age	Values of q_x	
	Male	Female		Male	Female		Male	Female
1	0.000592	0.000531	21	0.000530	0.000286	41	0.001156	0.000768
2	0.000400	0.000346	22	0.000556	0.000289	42	0.001252	0.000825
3	0.000332	0.000258	23	0.000589	0.000292	43	0.001352	0.000877
4	0.000259	0.000194	24	0.000624	0.000291	44	0.001458	0.000923
5	0.000237	0.000175	25	0.000661	0.000291	45	0.001578	0.000973
6	0.000227	0.000163	26	0.000696	0.000294	46	0.001722	0.001033
7	0.000217	0.000153	27	0.000727	0.000302	47	0.001899	0.001112
8	0.000201	0.000137	28	0.000754	0.000314	48	0.002102	0.001206
9	0.000194	0.000130	29	0.000779	0.000331	49	0.002326	0.001310
10	0.000197	0.000131	30	0.000801	0.000351	50	0.002579	0.001428
11	0.000208	0.000138	31	0.000821	0.000373	51	0.002872	0.001568
12	0.000226	0.000148	32	0.000839	0.000397	52	0.003213	0.001734
13	0.000255	0.000164	33	0.000848	0.000422	53	0.003584	0.001907
14	0.000297	0.000189	34	0.000849	0.000449	54	0.003979	0.002084
15	0.000345	0.000216	35	0.000851	0.000478	55	0.004425	0.002294
16	0.000391	0.000242	36	0.000862	0.000512	56	0.004949	0.002563
17	0.000430	0.000262	37	0.000891	0.000551	57	0.005581	0.002919
18	0.000460	0.000273	38	0.000939	0.000598	58	0.006300	0.003359
19	0.000484	0.000280	39	0.000999	0.000652	59	0.007090	0.003863
20	0.000507	0.000284	40	0.001072	0.000709	60	0.007976	0.004439

Female q(83) should be 0.054857

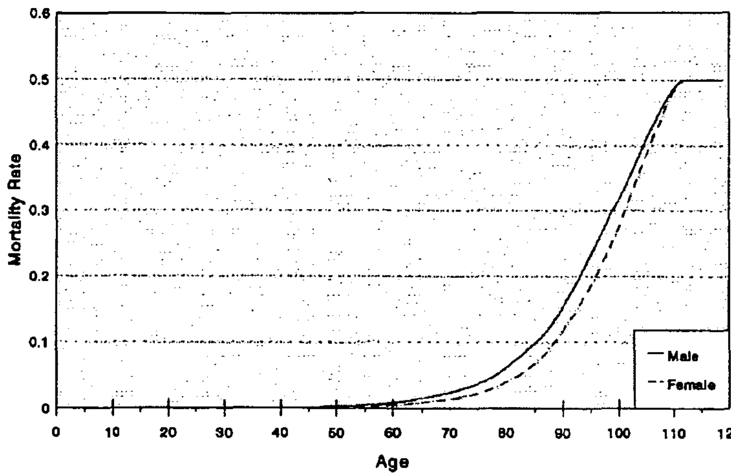
TABLE 18—*Continued*

Age	Values of q_x		Age	Values of q_x		Age	Values of q_x	
	Male	Female		Male	Female		Male	Female
61	0.008986	0.005093	81	0.068615	0.043952	101	0.333461	0.296629
62	0.010147	0.005832	82	0.075532	0.049153	102	0.350330	0.317093
63	0.011471	0.006677	83	0.082510	~~0.054847~~	103	0.368542	0.338505
64	0.012940	0.007621	84	0.089613	0.060979	104	0.387855	0.361016
65	0.014535	0.008636	85	0.097240	0.067738	105	0.407224	0.383597
66	0.016239	0.009694	86	0.105792	0.075347	106	0.425599	0.405217
67	0.018034	0.010764	87	0.115671	0.084023	107	0.441935	0.424846
68	0.019859	0.011763	88	0.126980	0.093820	108	0.457553	0.444368
69	0.021729	0.012709	89	0.139452	0.104594	109	0.473150	0.464469
70	0.023730	0.013730	90	0.152931	0.116265	110	0.486745	0.482325
71	0.025951	0.014953	91	0.167260	0.128751	111	0.496356	0.495110
72	0.028481	0.016506	92	0.182281	0.141973	112	0.500000	0.500000
73	0.031201	0.018344	93	0.198392	0.155931	113	0.500000	0.500000
74	0.034051	0.020381	94	0.215700	0.170677	114	0.500000	0.500000
75	0.037211	0.022686	95	0.233606	0.186213	115	0.500000	0.500000
76	0.040858	0.025325	96	0.251510	0.202538	116	0.500000	0.500000
77	0.045171	0.028366	97	0.268815	0.219655	117	0.500000	0.500000
78	0.050211	0.031727	98	0.285277	0.237713	118	0.500000	0.500000
79	0.055861	0.035362	99	0.301298	0.256712	119	0.500000	0.500000
80	0.062027	0.039396	100	0.317238	0.276427	120	1.000000	1.000000

at ages 112 and older. A modified Karup-King graduation process was used to obtain a smooth transition from the rates under age 103 to the rates at age 112 and above. Figure 3 displays a graph of the mortality rates for male and female ages 1–119 shown in Table 18. Figures 4, 5 and 6 display those rates by the age categories of 1–40, 40–70, and 70–119, respectively.

FIGURE 3

1994 GROUP ANNUITY MORTALITY STATIC TABLE RATES
1994 BASE YEAR
AGES 1–119



V. THE GENERATION MORTALITY TABLE

A. Development of Generation Mortality Tables

The Task Force was now in a position to produce the generation mortality tables for males and females.

Prior mortality table generation methodologies included mortality tables produced from projection scales. Thus, if we have a static mortality table that is appropriate for 1994, together with mortality improvement factors that are assumed to apply in the calendar years 1995 and later, we can produce a static mortality table for each calendar year 1995 and later.

FIGURE 4
1994 GROUP ANNUITY MORTALITY STATIC TABLE RATES
1994 BASE YEAR
AGES 1–40

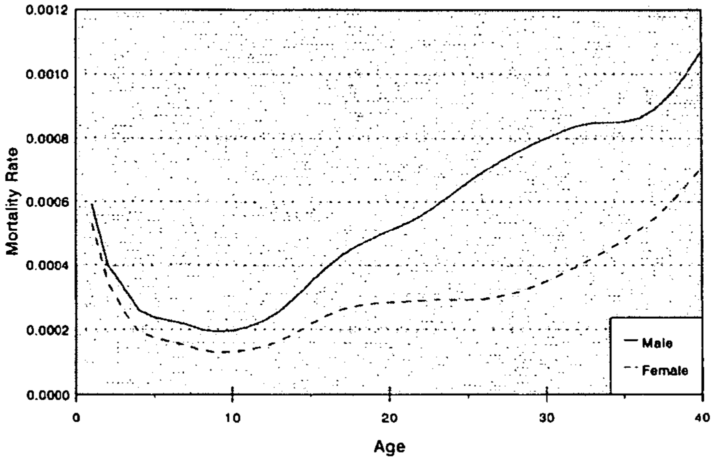


FIGURE 5
1994 GROUP ANNUITY MORTALITY STATIC TABLE RATES
1994 BASE YEAR
AGES 40–70

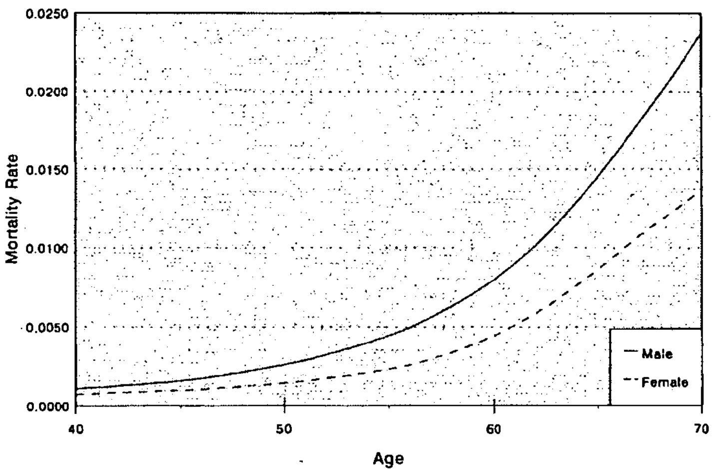
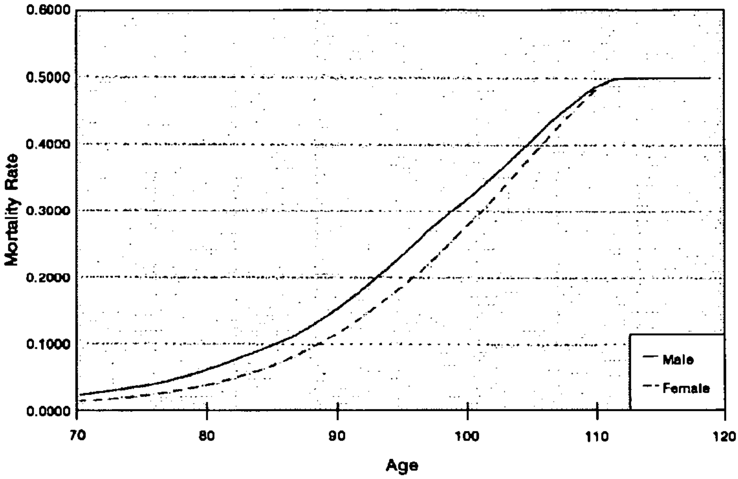


FIGURE 6

1994 GROUP ANNUITY MORTALITY STATIC TABLE RATES
1994 BASE YEAR
AGES 70–119



For example, assume a set of generation mortality rates is required to calculate annuity values for issue age 65 in calendar year 1997. The attained age 65 q_x value would be taken from the 1997 static table. The attained age 66 q_x value would be taken from the 1998 static table. This process would be continued until the ultimate age q_x value is taken from the appropriate final static table. Table 19 illustrates this process if we understand that the columnar rates come from the individual static mortality tables.

$$q_x^y = q_x^{1994} (1 - AA_x)^{y-1994} \tag{J}$$

An abbreviated example illustrates the principles involved in determining the q_x values needed to calculate an annuity value using generational mortality techniques.

As a specific example, assume one wishes to calculate, in 1994, a five-year temporary life annuity for a male age 63, using the GAM-94 Static Table from Table 18 and Projection Scale AA from Table 15. This requires determination of mortality rates for male ages 63–67 that would be applicable in 1994–1998. This example requires only five abbreviated "static" tables. However, a life annuity calculation would require the generation table

TABLE 19

ILLUSTRATION OF THE DEVELOPMENT OF A GENERATION MORTALITY TABLE FROM STATIC MORTALITY TABLES

Age	1994	1995	1996	1997	1998	1999	2000	2001	...	2052
65	q_{65}^{1994}	q_{65}^{1995}	q_{65}^{1996}	q_{65}^{1997}	q_{65}^{1998}	q_{65}^{1999}	q_{65}^{2000}	q_{65}^{2001}	...	q_{65}^{2052}
66	q_{66}^{1994}	q_{66}^{1995}	q_{66}^{1996}	q_{66}^{1997}	q_{66}^{1998}	q_{66}^{1999}	q_{66}^{2000}	q_{66}^{2001}	...	q_{66}^{2052}
67	q_{67}^{1994}	q_{67}^{1995}	q_{67}^{1996}	q_{67}^{1997}	q_{67}^{1998}	q_{67}^{1999}	q_{67}^{2000}	q_{67}^{2001}	...	q_{67}^{2052}
68	q_{68}^{1994}	q_{68}^{1995}	q_{68}^{1996}	q_{68}^{1997}	q_{68}^{1998}	q_{68}^{1999}	q_{68}^{2000}	q_{68}^{2001}	...	q_{68}^{2052}
69	q_{69}^{1994}	q_{69}^{1995}	q_{69}^{1996}	q_{69}^{1997}	q_{69}^{1998}	q_{69}^{1999}	q_{69}^{2000}	q_{69}^{2001}	...	q_{69}^{2052}
.	.	.	.	.	.	.	.	.	...	.
.	.	.	.	.	.	.	.	.	...	.
.	.	.	.	.	.	.	.	.	...	.
120	q_{120}^{1994}	q_{120}^{1995}	q_{120}^{1996}	q_{120}^{1997}	q_{120}^{1998}	q_{120}^{1999}	q_{120}^{2000}	q_{120}^{2001}	...	q_{120}^{2052}

to comprise 57 "static" tables, using age 120 as the last attained age in the calculations.

Table 20 shows our assumptions of mortality improvement factors and $1000q_x$ values in columns (1) and (2), respectively, and the resulting calculated values for future years in columns (3) to (6). Column (1) shows the final male mortality improvement factors from Table 15 by attained age, ages 63–67 in our example. Column (2) shows the GAM-94 Static Table of death rates in 1994 for attained ages 63–67 from Table 18. Columns (3) to (6) show calculated generation table death rates during the calendar years 1995–1998.

TABLE 20

GENERATION MORTALITY TABLE FOR THE YEARS 1994–1998 BASED ON GAM-94 STATIC TABLE FOR MALES WITH FULL GENERATION USING PROJECTION SCALE AA

SPECIMEN $1000q_x$ MORTALITY RATES FOR ISSUE YEAR 1994

Attained Age	(1) Mortality Improvement Factor	Values of $1000q_x$				
		(2) 1994	(3) 1995	(4) 1996	(5) 1997	(6) 1998
63	1.4%	11.471	11.310	11.152	10.996	10.842
64	1.4	12.940	12.759	12.580	12.404	12.230
65	1.4	14.535	14.332	14.131	13.933	13.738
66	1.3	16.239	16.028	15.820	15.614	15.411
67	1.3	18.034	17.800	17.568	17.340	17.114

The values in columns (3) to (6) for age 63 are calculated as 11.471 multiplied successively by (1–0.014). For age 65 values under columns (3) to (6), 14.535 would be multiplied successively by (1–0.014).

Our required mortality rates are therefore found along the diagonal beginning with 11.471, followed by 12.759, 14.131, 15.614, and 17.114.

Generation mortality rates from the GAM-94 Static Table for males and females at *issue age 65* until attained age 120 are shown in Table 21 and Table 22, respectively. These tables compare the rates of mortality for issues of 1994, 1999, 2004, and 2009. A similar set of tabular rates applies to each issue age, for each issue year. Note that the mortality rates by issue year are the same for attained ages 101 and older because no mortality improvement is assumed at these advanced ages.

Note that the generation tables shown for each issue year in Table 21 (male) and Table 22 (female) reflect projected mortality using the general formula on page 909:

TABLE 21

GENERATION MORTALITY RATES PER 1,000 FOR ISSUES OF 1994, 1999, 2004, AND 2009
AT MALE ISSUE AGE 65 IN THE INDICATED YEAR
BASED UPON GAM-94 STATIC TABLE WITH FULL GENERATION AND PROJECTION SCALE AA

	Male Issue Age 65 in the Year			
Age	1994	1999	2004	2009
65	14.535	13.546	12.624	11.764
66	16.028	15.013	14.062	13.171
67	17.568	16.456	15.413	14.437
68	19.037	17.741	16.533	15.408
69	20.537	19.140	17.837	16.623
70	22.003	20.401	18.917	17.540
71	23.701	21.976	20.377	18.894
72	25.622	23.757	22.028	20.425
73	27.648	25.635	23.770	22.040
74	29.720	27.557	25.552	23.692
75	32.318	30.118	28.068	26.157
76	34.988	32.607	30.387	28.319
77	38.607	36.162	33.872	31.727
78	42.918	40.404	38.037	35.809
79	47.847	45.273	42.837	40.532
80	53.347	50.732	48.246	45.881
81	59.374	56.750	54.242	51.844
82	65.892	63.298	60.806	58.412
83	71.403	68.592	65.892	63.298
84	78.416	75.710	73.097	70.574
85	84.495	81.579	78.763	76.045
86	91.282	88.132	85.090	82.153
87	101.327	98.323	95.409	92.581
88	113.153	110.352	107.621	104.957
89	123.646	120.585	117.601	114.690
90	138.350	135.605	132.915	130.277

TABLE 21—*Continued*

	Male Issue Age 65 in the Year			
Age	1994	1999	2004	2009
91	150.708	147.717	144.787	141.914
92	168.078	165.572	163.103	160.671
93	182.385	179.665	176.987	174.348
94	197.701	194.754	191.850	188.989
95	219.989	217.798	215.628	213.481
96	236.375	234.021	231.690	229.382
97	252.134	249.622	247.136	244.675
98	276.012	274.635	273.265	271.901
99	291.221	289.768	288.322	286.883
100	306.321	304.793	303.272	301.759
101	333.461	333.461	333.461	333.461
102	350.330	350.330	350.330	350.330
103	368.542	368.542	368.542	368.542
104	387.855	387.855	387.855	387.855
105	407.224	407.224	407.224	407.224
106	425.599	425.599	425.599	425.599
107	441.935	441.935	441.935	441.935
108	457.553	457.553	457.553	457.553
109	473.150	473.150	473.150	473.150
110	486.745	486.745	486.745	486.745
111	496.356	496.356	496.356	496.356
112	500.000	500.000	500.000	500.000
113	500.000	500.000	500.000	500.000
114	500.000	500.000	500.000	500.000
115	500.000	500.000	500.000	500.000
116	500.000	500.000	500.000	500.000
117	500.000	500.000	500.000	500.000
118	500.000	500.000	500.000	500.000
119	500.000	500.000	500.000	500.000
120	1000.000	1000.000	1000.000	1000.000

TABLE 22

GENERATION MORTALITY RATES PER 1,000 FOR ISSUES OF 1994, 1999, 2004, AND 2009 AT FEMALE ISSUE AGE 65 IN THE INDICATED YEAR
BASED UPON GAM-94 STATIC TABLE WITH FULL GENERATION AND PROJECTION SCALE AA

	Female Issue Age 65 in the Year			
Age	1994	1999	2004	2009
65	8.636	8.422	8.214	8.010
66	9.646	9.407	9.174	8.947
67	10.657	10.393	10.136	9.885
68	11.587	11.301	11.021	10.748
69	12.457	12.148	11.848	11.554
70	13.390	13.059	12.736	12.420
71	14.423	13.995	13.580	13.178
72	15.825	15.356	14.901	14.459
73	17.342	16.743	16.165	15.607
74	19.132	18.472	17.835	17.219
75	20.935	20.111	19.319	18.559
76	23.183	22.271	21.394	20.552
77	26.073	25.173	24.304	23.465
78	28.958	27.959	26.994	26.062
79	32.050	30.944	29.876	28.845
80	35.456	34.232	33.051	31.910
81	39.280	37.924	36.615	35.351
82	43.620	42.115	40.661	39.258
83	48.341	46.673	45.062	43.507
84	53.360	51.518	49.740	48.024
85	60.057	58.276	56.549	54.873
86	67.819	66.140	64.503	62.907
87	76.931	75.405	73.909	72.443
88	85.558	83.860	82.197	80.566
89	97.317	95.866	94.437	93.029
90	107.852	106.244	104.660	103.099

TABLE 22—*Continued*

Age	Female Issue Age 65 in the Year			
	1994	1999	2004	2009
91	119.076	117.301	115.552	113.829
92	130.911	128.959	127.036	125.142
93	147.431	145.962	144.508	143.069
94	161.050	159.446	157.858	156.286
95	175.358	173.612	171.882	170.171
96	190.350	188.454	186.577	184.719
97	212.734	211.672	210.616	209.565
98	229.993	223.845	227.703	226.567
99	248.126	246.888	245.656	244.430
100	266.915	265.583	264.258	262.339
101	296.629	296.629	296.629	296.629
102	317.093	317.093	317.093	317.093
103	338.505	338.505	338.505	338.505
104	361.016	361.016	361.016	361.016
105	383.597	383.597	383.597	383.597
106	405.217	405.217	405.217	405.217
107	424.846	424.846	424.846	424.846
108	444.368	444.368	444.368	444.368
109	464.469	464.469	464.469	464.469
110	482.325	482.325	482.325	482.325
111	495.110	495.110	495.110	495.110
112	500.000	500.000	500.000	500.000
113	500.000	500.000	500.000	500.000
114	500.000	500.000	500.000	500.000
115	500.000	500.000	500.000	500.000
116	500.000	500.000	500.000	500.000
117	500.000	500.000	500.000	500.000
118	500.000	500.000	500.000	500.000
119	500.000	500.000	500.000	500.000
120	1000.000	1000.000	1000.000	1000.000

$$q_{65+n}^{1994+n+t} = q_{65+n}^{1994} \times (1 - AA_{65+n})^{n+t} \tag{K}$$

where
n attained age less 65
t issue year less 1994.

B. The 1994 Group Annuity Reserving Table

As initially indicated, the Task Force was charged with recommending a new Group Annuity Mortality Valuation Standard that would be suitable for calculating group annuity valuation reserves. By definition, this new standard shall be known as the 1994 Group Annuity Reserving (GAR-94) Table.

The GAR-94 Table combines three components:

1. Projection Scale AA, whose mortality improvement factors are shown in Table 15, for projecting mortality beyond the year 1994
2. The GAM-94 Static Table, whose q_x values are shown in Table 18
3. All the generation tables produced by multiplying the Projection Scale AA mortality improvement factors by the respective GAM-94 Static Table q_x values (of which examples for *issue age 65* and certain issue years are shown in Table 21 and Table 22).

The complete GAR-94 Table appears as Table 1 in the Executive Summary.

Note that this approach implies that a different set of mortality rates should be used for each different issue year for a specific issue age. However, it also implies that the same mortality rate should be used when the attained age and issue year offsets are the same. Thus, the mortality rate for issue age 65 in 1994 five years after issue is the same as that for issue age 70 in 1999 (and issue age 67 in 1996 two years after issue).

C. Financial Values Using the GAR-94 Tables

Table 23 shows and compares the life annuity net single premiums for an annuity due of $1 per year, payable monthly, for various issue ages based upon GAM-83 mortality and 7% level interest and for various issue ages and issue years based upon GAR-94 mortality and the same interest rate. In this table, on the GAR-94 basis, the net single premiums are significantly greater (at least 3%) for male issue ages 40–90 in 1994, 1999 and 2004, and 35–90 in 2009. At male issue age 65, for these issue years, the percentages are 6.2%, 7.7%, 9.1%, and 10.4%, respectively. Female issue ages show no significantly greater net single premiums in 1994 and only for issue age 75 in 1999. Because of the improving mortality, issues in 2004 show

TABLE 23

Life Annuity Net Single Premiums Assuming 7% Level Interest Rate and Mortality from GAR-94 Table versus Mortality from GAM-83 Table

	Value of $\ddot{a}_x^{(12)}$								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Issue Year							
		1994		1999		2004		2009	
Issue Age	GAM-83	GAR-94	(2)/(1)	GAR-94	(4)/(1)	GAR-94	(6)/(1)	GAR-94	(8)/(1)
				Male					
20	14.334	14.463	1.009	14.477	1.010	14.490	1.011	14.502	1.012
25	14.169	14.346	1.012	14.363	1.014	14.378	1.015	14.393	1.016
30	13.947	14.190	1.017	14.213	1.019	14.234	1.021	14.255	1.022
35	13.649	13.970	1.024	14.002	1.026	14.032	1.028	14.061	1.030
40	13.252	13.653	1.030	13.699	1.034	13.741	1.037	13.782	1.040
45	12.737	13.215	1.038	13.278	1.042	13.337	1.047	13.393	1.052
50	12.099	12.625	1.043	12.708	1.050	12.786	1.057	12.860	1.063
55	11.329	11.861	1.047	11.963	1.056	12.059	1.064	12.151	1.073
60	10.380	10.915	1.052	11.034	1.063	11.149	1.074	11.259	1.085
65	9.242	9.814	1.062	9.950	1.077	10.080	1.091	10.206	1.104
70	8.006	8.613	1.076	8.757	1.094	8.896	1.111	9.031	1.128
75	6.729	7.306	1.086	7.434	1.105	7.560	1.123	7.682	1.142
80	5.480	5.958	1.087	6.055	1.105	6.150	1.122	6.244	1.139
85	4.401	4.735	1.076	4.803	1.091	4.870	1.107	4.937	1.122
90	3.493	3.609	1.033	3.647	1.044	3.685	1.055	3.723	1.066
95	2.723	2.722	1.000	2.740	1.006	2.757	1.012	2.775	1.019

TABLE 23—*Continued*

Issue Age	Value of $\ddot{a}_x^{(12)}$								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Issue Year							
		1994		1999		2004		2009	
	GAM-83	GAR-94	(2)/(1)	GAR-94	(4)/(1)	GAR-94	(6)/(1)	GAR-94	(8)/(1)
				Female					
20	14.510	14.552	1.003	14.560	1.003	14.567	1.004	14.574	1.004
25	14.400	14.456	1.004	14.466	1.005	14.475	1.005	14.483	1.006
30	14.253	14.320	1.005	14.333	1.006	14.344	1.006	14.355	1.007
35	14.054	14.133	1.006	14.149	1.007	14.165	1.008	14.180	1.009
40	13.786	13.877	1.007	13.898	1.008	13.918	1.010	13.937	1.011
45	13.430	13.532	1.008	13.553	1.009	13.583	1.011	13.607	1.013
50	12.964	13.066	1.008	13.096	1.010	13.125	1.012	13.154	1.015
55	12.359	12.450	1.007	12.484	1.010	12.518	1.013	12.552	1.016
60	11.586	11.657	1.006	11.701	1.010	11.745	1.014	11.787	1.017
65	10.623	10.710	1.008	10.764	1.013	10.818	1.018	10.871	1.023
70	9.451	9.622	1.018	9.688	1.025	9.752	1.032	9.816	1.039
75	8.131	8.331	1.025	8.404	1.034	8.476	1.042	8.547	1.051
80	6.795	6.926	1.019	6.993	1.029	7.060	1.039	7.126	1.049
85	5.505	5.512	1.001	5.560	1.010	5.607	1.019	5.654	1.027
90	4.252	4.209	0.990	4.288	0.997	4.268	1.004	4.297	1.011
95	3.103	3.157	1.017	3.172	1.022	3.188	1.027	3.203	1.032

significantly greater net single premiums for ages 70–80. At female issue age 65, for these issue years, the percentages are 0.8%, 1.3%, 1.8%, and 2.3%, respectively. The progression of ratios of GAR-94 net single premiums for males to those of the GAM-83 is relatively smooth and increasing until issue age 80 and then proceeds to decrease. Such ratios for females begin to materially decrease starting at issue age 80 but then show a sharp increase at issue age 95.

The mortality rate equivalence under GAR-94 mortality outlined above implies the same equivalence between net single premiums and reserves. Thus the net single premium for issue age 70 in 1999 is the same as the fifth-year reserve for issue age 65 in 1994, while the net single premium for issue age 75 in 2004 is the same as the tenth year reserve for issue age 65 in 1994 (and the fifth-year reserve for issue age 70 in 1999). Thus, for male issue age 65 in 1994, the percentage increases of the initial and fifth-, tenth-, and fifteenth-year reserves relative to the GAM-83 values are 6.2, 9.4, 12.3, and 13.9, respectively. For female issue age 65, the values are 0.8, 2.5, 4.2, and 4.9.

This analysis further confirms the need for a new reserve valuation standard to replace the GAM-83.

7. CONCLUSION

Present-day mortality levels have eroded the margins built into the 1983 Group Annuity Mortality Tables. They are no longer adequate for valuation purposes. Therefore, the Task Force has developed the 1994 Group Annuity Reserving Table presented in this report. The Task Force recommends that this new table replace the 1983 Group Annuity Mortality Table for use as a Valuation Mortality Standard.

A. Potential Uses of the New Standard

This report does not preclude other uses of the new standard, as long as the user clearly understands the development and coinciding limitations (margins, annual mortality improvement, and so on) of this new standard. Other reports that will be released will discuss additional uses of the tables presented in this report.

B. Acknowledgment

The Chair would like to thank each Task Force member as well as their employers for the time and unceasing effort devoted to this endeavor. This report and incorporated recommendation would not be as complete or as well-defined without the effort extended by each of these Task Force members. In addition, the Society of Actuaries staff and especially our assigned actuarial liaison's support have been invaluable throughout the process.

The Task Force thanks the following individuals for their written comments on the Exposure Draft of this report: Robert L. Brown and Shaun Wang, William H. Crosson, Harvey Fishman and Zachary Granovetter, G. Thomas Mitchell, Michael Mudry, Bruce E. Nickerson, Owen A. Reed, Robert Stalzer, David A. Wiener, William S. Wright, and especially Walter J. McLaughlin. Their comments only served to improve the final report.

Special thanks go to Charles F. Brown and Marian Rivera from Bankers Security Life for their tireless assistance in developing the Exposure Draft and presenting it in its final format. To try and name each and every individual who helped the Task Force would be to forget someone who should not be forgotten. However, we want to acknowledge the many helpful suggestions we received during the development of the tables. Thus, our appreciation to all, even those not named, is total.

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DISCUSSION OF PRECEDING PAPER

JACQUES F. CARRIERE:

The purpose of this discussion is to present a parametric model or mathematical formula that will explain the pattern of mortality for the male and female GAM-94 tables. In my opinion, parametric formulas for mortality rates are always preferable to tabular rates, if the formula gives a good fit. Generally, the parametric approach always yields very smooth rates.

Before proceeding, it is instructive to plot the crude and graduated rates and examine the pattern of mortality in the GAM-94 tables. Consider the function

$$y_x = \log_e\{-\log_e(1 - q_x)\}$$

where q_x is the mortality rate for a life aged x. Remember that $\mu_{x+0.5} \approx \exp(y_x)$, where $\mu_{x+0.5}$ is a force of mortality. Let y_x^{crude} be the value based on the crude or ungraduated rates found in Table 12 and let y_x^{grad} be the value based on the graduated rates found in Table 13. Figure 1 presents plots of y_x^{crude}, y_x^{grad}, and $y_x^{crude}-y_x^{grad}$ versus $x=1, 2, \ldots, 108$ for both the female and male rates. The graduated values y_x^{grad} are based on the classical Karup-King method, which did an excellent job.

The parametric models for the male and female rates have nine parameters, denoted as $\theta=(\psi_1, m_1, \sigma_1, m_2, \sigma_2, \psi_3, m_3, \sigma_3)$. The parametric formula for the *male* rates is

$$y_x^{form}(\theta) = \psi_1 \times r_x(m_1, \sigma_1) + \psi_2 + s_x(m_2, \sigma_2) + \psi_3 \times t_x(m_3, \sigma_3),$$

while the formula for the *female* rates is

$$y_x^{form}(\theta) = \psi_1 \times r_x(m_1, \sigma_1) + \psi_2 \times r_x(m_2, \sigma_2) + \psi_3 \times t_x(m_3, \sigma_3),$$

where

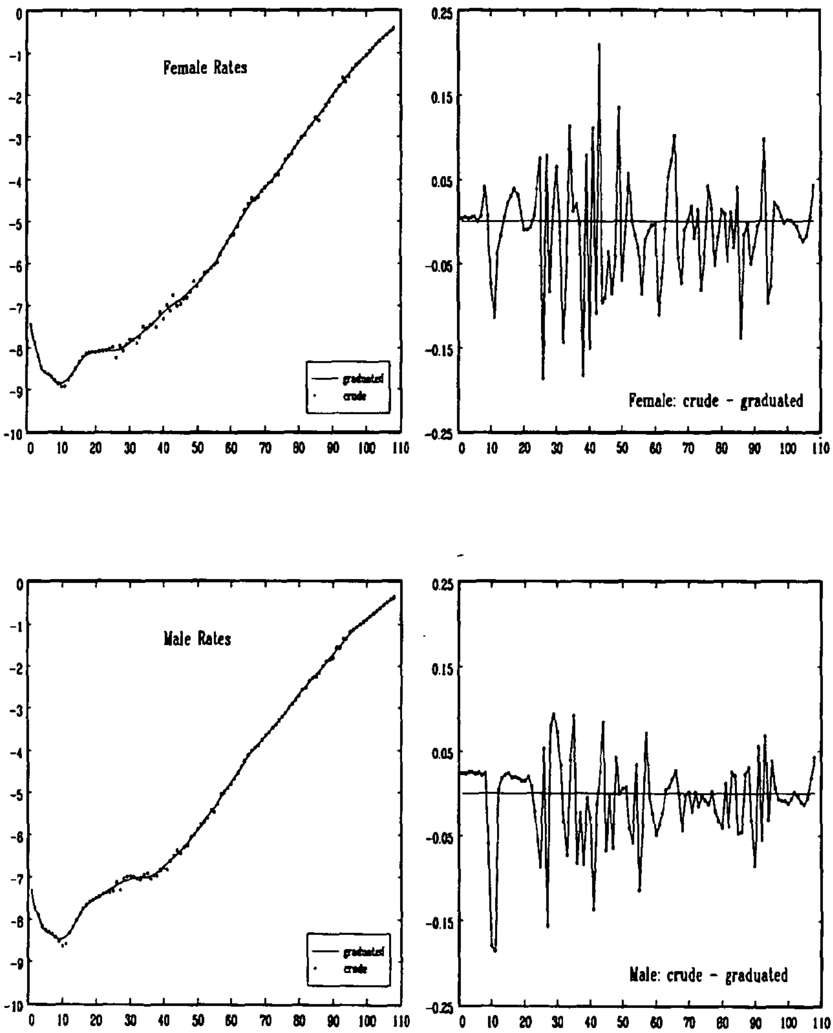
$$r_x(m, \sigma) = \exp\left\{-\left(\frac{x}{m}\right)^{m/\sigma}\right\},$$

$$s_x(m, \sigma) = 1 - \exp\left\{-\left(\frac{x}{m}\right)^{-m/\sigma}\right\},$$

$$t_x(m, \sigma) = \frac{1}{\sigma}\left(\frac{x}{m}\right)^{(m/\sigma)-1} \exp\left\{-\left(\frac{x}{m}\right)^{m/\sigma}\right\}.$$

FIGURE 1

A COMPARISON OF THE CRUDE AND GRADUATED RATES FROM THE GAM-94.
THE HORIZONTAL AXIS ON ALL GRAPHS IS THE ATTAINED AGE x.
THE FIRST COLUMN OF GRAPHS GIVES $y_x = \log_e\{-\log_e(1-q_x)\}$ VERSUS x.
THE SECOND COLUMN SHOWS $y_x^{crude} - y_x^{grad}$, THE DIFFERENCE IN CRUDE AND GRADUATED RATES.



Notice that the male formula is a function of r_x, s_x, and t_x, while the female formula is a function of r_x, r_x, and t_x, so the male and female formulas are different. Both the male and female formulas were specifically created for the GAM-94 tables. Thus, applying these formulas on other tables may or may not yield good results. To estimate the parameters θ, we minimized the loss function

$$L(\theta) = \sum_{x=1}^{108} [y_x^{form}(\theta) - y_x^{crude}]^2.$$

The parameter estimates are shown in Table 1. Figure 2 presents plots of y_x^{crude}, y_x^{form}, and $y_x^{crude} - y_x^{form}$ versus $x=1, 2, \ldots, 108$ for both the female and male rates. The graduated values, y_x^{form}, based on our formulas did an excellent job. Comparison of Figure 1 and Figure 2 shows y_x^{form} is smoother than y_x^{grad}, but the Karup-King values, y_x^{grad}, are slightly better fitting than y_x^{form}.

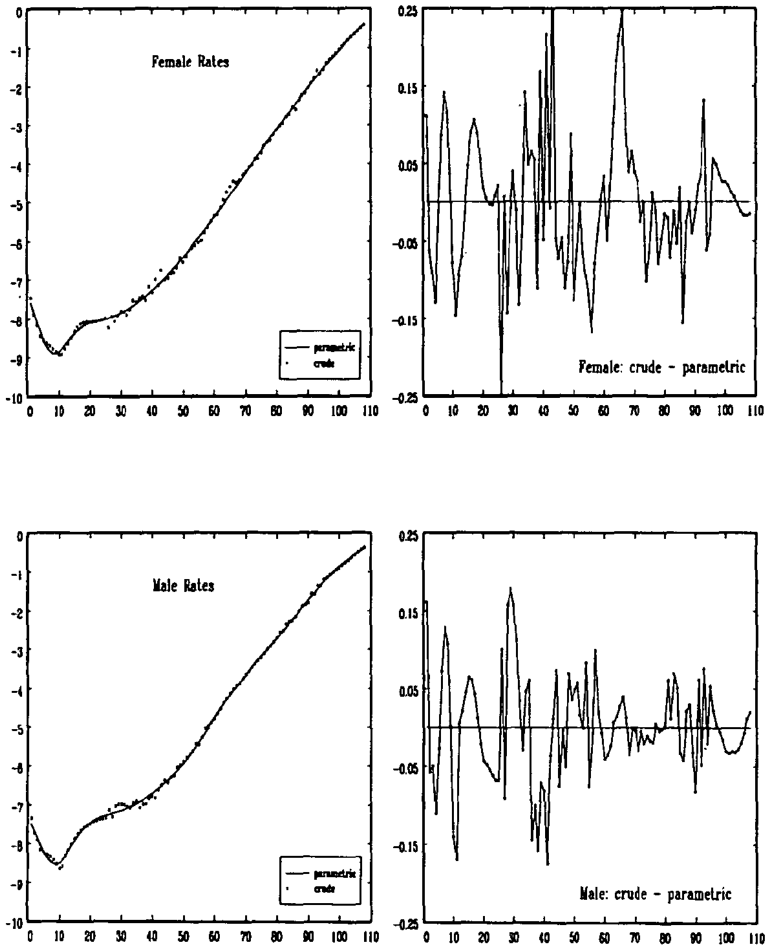
TABLE 1

PARAMETER ESTIMATES FOR THE PARAMETRIC MODELS

Model	ψ_1	m_1	σ_1	ψ_2	m_2	σ_2	ψ_3	m_3	σ_3
Male	9.800	82.09	22.15	−2.428	75.88	11.67	13.82	11.20	4.882
Female	188.3	96.59	59.35	−180.9	97.84	62.85	12.10	9.973	4.459

FIGURE 2

A COMPARISON OF THE CRUDE AND GRADUATED RATES FROM THE FORMULAS.
THE HORIZONTAL AXIS ON ALL GRAPHS IS THE ATTAINED AGE x.
THE FIRST COLUMN OF GRAPHS GIVES $y_x=\log_e\{-\log_e(1-q_x)\}$ VERSUS x.
THE SECOND COLUMN SHOWS $y_x^{crude}-y_x^{form}$, THE DIFFERENCE IN CRUDE AND FORMULA RATES.



(AUTHOR'S REVIEW OF DISCUSSION)

LINDSAY J. MALKIEWICH:

The Task Force thanks Dr. Carriere for his thought-provoking discussion on a possible parametric model for explaining mortality patterns. While it is interesting to note that such a model can help to explain some of the GAR-94 results, it seems that an exploration of the model's computational viability would enhance the discussion. In other words, whereas the solution to a one-parameter model is readily developed, additional parameters will surely increase the difficulty of finding such a solution.

In addition to the increased difficulty of solving a multiple-parameter problem, the issue of parameter sensitivity should be addressed. Small changes in the provided formula's coefficients can cause drastic changes in the estimates of the given parameters. Therefore, an easy solution for a model making use of nine parameters would, logically, be quite difficult to find. It would be an interesting addition if the discussion explored various methods that would be used to discover such solutions.

In a related observation, a range of choices was shown for these parameter estimates in Table 1. Some of the differences between a given male versus female parameter are quite large. Therefore, it follows that small changes in the formula's coefficients may yield very different results, given the same parameters or even slightly different ones. Is this situation intended? Based on the discussion alone, it is unclear.

While the above concerns suggest areas in which additional demonstrations and more research could enhance the model, this discussion does serve as a worthwhile start. As stated, the model as presented does help to explain some of the GAR-94 results. The Task Force is, of course, pleased that such a model could do so. Perhaps, additional research could be that much more enlightening.