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Structural Credit Risk Modeling: Merton and Beyond

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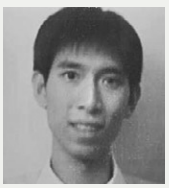
The past two years have seen global financial markets experiencing an unprecedented crisis. Although the causes of this crisis are complex, it is a unanimous consensus that credit risk has played a key role. We will not attempt to examine economic impacts of the credit crunch here; rather, this article provides an overview of the commonly used structural credit risk modeling approach that is less familiar to the actuarial community.



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INTRODUCTION

Although credit risk has historically not been a primary area of focus for the actuarial profession, actuaries have nevertheless made important contributions in the development of modern credit risk modeling techniques. In fact, a number of well-known credit risk models are direct applications of frequency-severity or hazard rate models commonly found in actuarial/insurance literature. As credit risk became an increasing concern in recent years, various advanced methods have been employed extensively to measure credit risk exposures. It is necessary for actuaries to become familiar with these popular methods and their strengths and shortcomings, in order to stay competitive in this dynamic and rapidly evolving area.

Nowadays, structural and reduced form models represent the two primary classes of credit risk modeling approaches. The structural approach aims to provide an explicit relationship between default risk and capital structure, while the reduced form approach models credit defaults as exogenous events driven by a stochastic process (such as a Poisson jump process). In this sense, most actuarial models used for credit risk measurement lie within the reduced form class.

Structural models, pioneered by Black, Scholes and Merton, ingeniously employ modern option pricing theory in corporate debt valuation. Merton model was the first structural model and has served as the cornerstone for all other structural models. To illustrate key concepts behind structural approach, we will review Merton model in detail, and briefly introduce some important extensions to this model. Major advantages and disadvantages of both structural and reduced form models will also be summarized, followed by a quick discussion involving the latest financial crisis.

THE MERTON MODEL

The real beauty of Merton model lies in the intuition of treating a company's equity as a call option on its assets, thus allowing for applications of Black-Scholes option pricing methods. To start reviewing this influential model, we consider the following scenario.

Suppose at time t a given company has asset A_t financed by equity E_t and zero-coupon debt D_t of face amount K maturing at time $T > t$, with a capital structure given by the balance sheet relationship:

$$A_t = E_t + D_t. \tag{1}$$

In practice a debt maturity T is chosen such that all debts are mapped into a zero-coupon bond. In the case $A_T > K$ the company's debtholders can be paid the full amount K, and shareholders' equity still has value $A_T - K$. On the other hand, the company defaults on its debt at T if $A_T < K$, in which case debtholders have the first claim on residual asset A_T and shareholders are left with nothing. Therefore, equity value at time T can be written as:

$$E_T = \max(A_T - K, 0). \tag{2}$$

This is exactly the payoff of a European call option written on underlying asset A_t with strike price K maturing at T. It follows that the well-known Black-Scholes option pricing formulas can be applied if corresponding modeling assumptions are made. Let us assume the asset value follows a geometric Brownian motion (GBM) process, with risk-neutral dynamics given by the stochastic differential equation:

$$\frac{dA_t}{A_t} = r dt + \sigma_A dW_t \tag{3}$$

where W_t is a standard Brownian motion under risk-neutral measure, r denotes the continuously compounded risk-free interest rate, and σ_A is the asset's return volatility. Note that A_t grows at risk-free rate under the risk-neutral measure and thus has drift r in (3), implicitly assuming the continuous tradability of corporate assets. Now applying the Black-Scholes formula for European call option would give:

$$E_t = A_t\Phi(d_+) - Ke^{-r(T-t)}\Phi(d_-) \quad (4)$$

where $\Phi(\cdot)$ denotes the $N(0,1)$ cumulative distribution function, with the quantities d_+ and d_- given by:

$$d_+ = \frac{\ln(\frac{A_t}{K}) + (r + \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}, \quad (5)$$

$$d_- = \frac{\ln(\frac{A_t}{K}) + (r - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}. \quad (6)$$

Under this framework, a credit default at time T is triggered by the event that shareholders' call option matures out-of-money, with a risk-neutral probability:

$$P(A_T < K) = \Phi(-d_-), \quad (7)$$

which is sometimes converted into a real-world probability by extracting the underlying market price of risk.

Although debtholders are exposed to default risk, they can hedge their position completely by purchasing a European put option written on the same underlying asset A_t with strike price K. Such a put option will be worth $K - A_T$ if $A_T < K$, and worth nothing if $A_T > K$. Combining these two positions (debt and put option) would guarantee a payoff of K for debtholders at time T, thus forming a risk-free position:

$$D_t + P_t = Ke^{-r(T-t)} \quad (8)$$

where P_t denotes the put option price at time t, which can be determined by applying the Black-Scholes formula for European put option:

$$P_t = Ke^{-r(T-t)}\Phi(-d_-) - A_t\Phi(-d_+). \quad (9)$$

The corporate debt is a risky bond, and thus should be valued at a credit spread (risk premium). Let s denote the continuously compounded credit spread, then bond price D_t can be written as:

$$D_t = Ke^{-(r+s)(T-t)}. \quad (10)$$

Putting (8), (9) and (10) together gives a closed-form formula for s:

$$s = -\frac{1}{T-t}\ln[\Phi(d_-) - \frac{A_t}{K}e^{r(T-t)}\Phi(-d_+)] \quad (11)$$

which allows us to solve for credit spread when asset level and return volatility (A_t and σ_A) are available for given t, T, K, and r . One common way of extracting A_t and σ_A involves assuming another geometric Brownian motion model for equity price E_t and applying Ito's Lemma to show that instantaneous volatilities satisfy:

$$A_t\sigma_A\frac{\partial E_t}{\partial A_t} = E_t\sigma_E. \quad (12)$$

Black-Scholes call option delta can then be substituted into (12) to obtain:

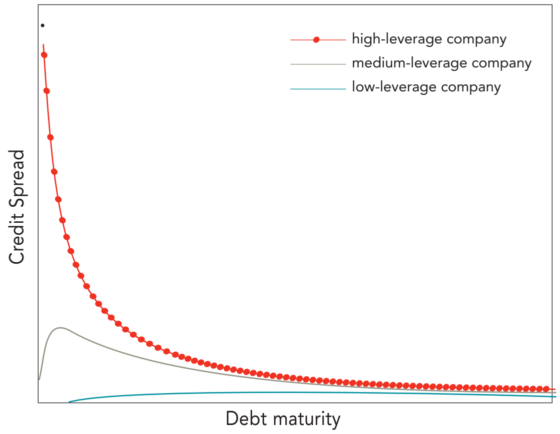
$$A_t\sigma_A\Phi(d_+) = E_t\sigma_E \quad (13)$$

where equity price E_t and its return volatility σ_E are observed from equity market. Finally, (4) and (13) can be solved simultaneously for A_t and σ_A, which are used in (11) to determine credit spread s.

TERM STRUCTURE OF CREDIT SPREADS UNDER MERTON MODEL

Credit spread compensates for exposure to credit risk, and such risk is linked to structural variables (assets, liabilities, etc.) under Merton model. A good risk indicator in Merton's framework is leverage ratio (such as the debt-to-asset ratio), and in (11) the spread is indeed an increasing function of leverage. To better understand implications of this model, we examine term structure of credit spreads determined by (11) and plotted against different debt maturities:

Term Structure of Credit Spreads under the Merton Model



As shown above, the implied credit spread term structure from Merton model appears realistic, with the following

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key observations and facts:

- A low-leverage company has a flatter credit spread term structure with initial spreads close to zero since it has sufficient assets to cover short-term liabilities. Spread slowly increases with debt maturity (reflecting future uncertainties), before it starts to decrease at the long end.
- A medium-leverage company has a humped-shape credit spread term structure. The very short-term spreads are low as the company currently has just enough assets to cover debts. Spread then rises quickly since asset value fluctuations could easily result in insufficient assets, before it gradually drops for longer maturities.
- A high-leverage company has a downward-sloping credit spread term structure which starts very high and decreases for longer maturities as more time is allowed for the company's assets to grow higher and cover liabilities.
- Empirical studies have shown that Merton model tends to underestimate credit spreads, particularly short-term spreads for high-quality debts (recall the very low initial spreads for the low-leverage company mentioned above). This drawback has been tackled by several extended models developed more recently, which are to be discussed next.

EXTENSIONS AND IMPROVEMENTS TO MERTON MODEL

Ever since the works of Black, Scholes and Merton started the literature of structural credit risk modeling, many researchers have proposed extensions to Merton model, which has been criticized for basing on a number of simplifying assumptions. The extended structural models represent important improvements for Merton's original framework as they are more realistic and able to better align with market data (e.g., CDS spreads). Some of these areas of improvements are introduced below:

- In Merton's framework, a company could only default at its debt maturity date. The model can be modified to allow for early defaults by specifying a threshold level such that a default event occurs when asset value A_t falls below this critical level. The methods for pricing barrier options can be applied in this setting. Such threshold level sometimes results from shareholders' optimal default strategy to maximize equity value. Extensions to Merton model along this direction were pioneered by Black and Cox, and this group of models is often referred to as First Passage Time models.
- The constant interest rate assumption is not reliable, and a stochastic interest rate model can be incorporated into Merton model or its extended versions. In this case, correlation between asset and interest rate processes can also be introduced if needed.
- Mapping all debts into a single zero-coupon bond is not always feasible. It has been shown that multiple debts with different characteristics can also be modeled using a structural approach. The *Geske Compound Option model* developed by Robert Geske was the first structural model of this category.
- Several more sophisticated structural models involving stochastic volatility, jump diffusion and even regime-switching methods have also been proposed. These applications can help explain market observations with higher accuracy, but they often involve a high level of analytical complexity.

ADVANTAGES AND DISADVANTAGES OF CREDIT RISK MODELS

Structural approach, led by Merton model, has the highly appealing feature of connecting credit risk to underlying structural variables. It provides both an intuitive economic interpretation and an endogenous explanation of credit defaults, and allows for applications of option pricing methods. As a result, structural models not only facilitate security valuation, but also address the choice of financial structure.

The main disadvantage of structural models lies in the difficulty of implementation. For example, the continuous tradability assumption for corporate assets is unrealistic, and calibrating stochastic asset processes using publicly available information is sometimes more difficult than anticipated. Furthermore, although improved structural models have addressed several limitations of earlier models, they tend to be analytically complex and computationally intensive.

"To make the best possible use of models and avoid repeating costly mistakes, a sound ERM framework is needed where model outputs alone cannot dominate the decision-making process."

Reduced form models do not consider endogenous cause of defaults; rather, they rely on exogenous specifications for credit default and debt recovery. This feature is both a strength and a weakness—while these models suffer from the lack of economic insights about default occurrence, they offer more degrees of freedom in functional form selection. Such flexibility contributes to analytical tractability and ease of implementation and calibration (compared to structural models). However, reduced form models' dependence on historical data may result in good in-sample fitting properties but limited out-of-sample predictive power.

In general, structural models are particularly useful in areas such as counterparty credit risk analysis, portfolio/security analysis and capital structure monitoring, while the difficulty in calibration limits their presence in front-office environments. Reduced form models, on the other hand, are widely used on credit security trading floors where traders require fast computation tools to help them react to market movements quickly.

REFLECTION ON THE CURRENT FINANCIAL CRISIS: ROLE OF RISK MODELS

The current financial crisis originated in 2007 from the U.S. subprime mortgages and related credit products markets, and quickly imposed severe adverse consequences on financial markets worldwide, leading to a global recession. Today, lack of regulations and failures of well-known risk models are being blamed, especially credit risk models considering the origin of the crisis. To conclude this article we will briefly discuss the role of risk models.

Let us acknowledge the obvious: there has been a rapid growth of financial risk modeling in recent years thanks to technological developments and an increasing supply of human capital. The acceptable performance of various risk models during stable market periods often leads risk managers to overlook these models' inherent limitations, resulting in overreliance on popular modeling approaches and related analyses. This is particularly dangerous during a crisis, when major flaws of risk models are highlighted and cause significant losses. Furthermore, the popularity of certain models may lead many market participants to execute similar strategies, which in turn quickly dries up liquidity and destabilizes prices, thus amplifying the crisis.

At the end of the day, risk models are constructed based on simplifying assumptions and inputs; therefore, they are only as good as these assumptions and inputs, and even risk measures generated by highly regarded models should be treated with caution. In order to make the best possible use of models and avoid repeating costly mistakes, a sound enterprise risk management framework is needed where model outputs alone cannot dominate the decision-making process. As the saying goes: "All models are wrong, but some are useful." ✦

Note: The opinions expressed in this article are those of the author and do not reflect the views of Manulife Financial.