REVIEW OF CALCULATOR FUNCTIONS
FOR THE TEXAS INSTRUMENTS BA II PLUS©
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This note presents a review of calculator financial functions for the Texas Instruments BA II PLUS calculator. This note, including a number of the examples used as illustrations, is reprinted with permission from the 3rd edition of the book *Mathematics of Investment and Credit*, by S. Broverman. Also, several examples from SOA/CAS math of finance exams (old Course 2) will be presented illustrating the use of the calculator.

A detailed guidebook for the operation of and functions available on the BA II PLUS can be found at the following internet site: http://education.ti.com/us/global/guides.html#finance. It will be assumed that you have available and have reviewed the appropriate guide book for the calculator that you are using.

Financial functions will be reviewed in the order that the related concepts are covered in Chapters 1 to 8 of *Mathematics of Investment and Credit*. Some numerical values will be rounded off to fewer decimals than are actually displayed in the calculator display.

It will be assumed that unless indicated otherwise, each new keystroke sequence starts with clear registers. Calculator registers are cleared with the keystroke sequences

$\boxed{\text{2nd}}$ $\boxed{\text{CLR WORK}}$ $\boxed{\text{CE/C}}$ and

$\boxed{\text{2nd}}$ $\boxed{\text{CLR TVM}}$ $\boxed{\text{CE/C}}$.

It will also be assumed that the calculator is operating in US date format and US commas and decimals format, with the display showing 9 decimals. These are the default settings for the calculator, but they can be changed in the "FORMAT" work sheet, which is accessed with the keystroke sequence $\boxed{\text{2nd}}$ $\boxed{\text{FORMAT}}$. Although the number of decimals to display is set to 9, in the examples below it will often be the case that dollar amounts are written as rounded to the nearest .01.

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CHAIN (CHN) AND ALGEBRAIC OPERATING (AOS) SYSTEM MODES

When the calculator is operating in chain calculation mode, the usual algebraic order of operations is not respected. For instance, the keystroke sequence $1+2\times3$ $\boxed{=}$ results in an answer of 9. This is true because the calculation of $1+2$ is performed first, resulting in 3, which is then multiplied by 3, resulting in 9. When the calculator is in AOS mode, the result of the keystroke sequence above will be 7. This is true because in the hierarchy of algebraic operations, multiplication is done before addition, so 2×3 is calculated first, resulting in 6, and then the addition operation is applied resulting in $1+6$, which is 7. The order of operations mode can be selected in the "FORMAT" worksheet.

ACCUMULATED AND PRESENT VALUES OF A SINGLE PAYMENT USING A COMPOUND INTEREST RATE

Accumulated values and present values of single payments using annual (or more general periodic) effective interest rates can be determined using the calculator functions as described below.

ACCUMULATED VALUE:

We use Example 1.1 to illustrate this function.
A deposit of 1000 made at time 0 grows at effective annual interest rate 9%.
The accumulated value at the end of 3 years is $1000(1.09)^3 = 1,295.03$.
This can be found using the calculator in two ways.

1. We use standard arithmetic operators in standard calculator mode with the following keystrokes.

   1.09 $\boxed{y^x}$ 3 $\boxed{=}$ $\boxed{\times}$ 1000 $\boxed{=}$

   The screen should display 1,295.029. In this function, $y=1.09$ and $x=3$.

2. We use time value of money functions (TVM).

   $\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ $\boxed{\downarrow}$ 1 (this sets 1 compounding period per year).

   $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ (this returns calculator to standard-calculator mode)

1000 $\boxed{PV}$ (this sets PV to 1000),

9 $\boxed{I/Y}$ (this sets the annual interest rate at 9%)

3 $\boxed{N}$ (this sets the number of years to 3),

$\boxed{CPT}$ $\boxed{FV}$ (this computes the accumulated value, also called future value).

The screen should display $-1,295.029$.

The calculator interprets the PV of 1000 as an amount received (a cash inflow) and the FV as the amount that must be paid back (a cash outflow), so the FV is a "negative" cashflow. If the PV had been entered as -1000, then FV would have been positive. This is part of the "sign convention" used by the BA II PLUS.

PRESENT VALUE:

We use Example 1.5(a) to illustrate this function.
The present value of 1,000,000 due in 25 years at effective annual rate .195 is $1,000,000v^{25} = 1,000,000(1.195)^{-25} = 11,635.96$.
This can be found using the calculator in two ways:

1. 1.195 $\boxed{y^x}$ 25 $\boxed{+/-}$ $\boxed{=}$ $\boxed{\times}$ 1000000 $\boxed{=}$

   The screen should display 11,635.96. This keystroke sequence can be replaced by:

   1.195 $\boxed{1/x}$ $\boxed{y^x}$ $\boxed{=}$ $\boxed{\times}$ 1000000 $\boxed{=}$

2. Using time value of money functions, we have

   $\boxed{2nd}$ $\boxed{P/Y}$ $\boxed{\downarrow}$ 1 $\boxed{ENTER}$ $\boxed{2nd}$ $\boxed{QUIT}$

   1000000 $\boxed{FV}$ 19.5 $\boxed{I/Y}$

   25 $\boxed{N}$ $\boxed{CPT}$ $\boxed{PV}$.

The screen should display $-11,635.96$. (the earlier comment about the negative value applies here).

As a more general procedure, in the equation $(PV)(1+i)^N = FV$, if any 3 of the 4 variables PV, i, N, FV are entered, then the 4th can be found using the $\boxed{CPT}$ function.

UNKNOWN INTEREST RATE:

As an example of solving for the interest rate, we consider Example 1.5(c).

An initial investment of 25,000 at effective annual rate of interest i grows to 1,000,000 in 25 years. Then $25,000(1+i)^{25} = 1,000,000$, from which we get $i = (40)^{1/25} - 1 = .1590$ (15.90%). This can be found using the calculator power function with the following keystrokes:

40 $\boxed{y^x}$ .04 $\boxed{=}$ $\boxed{-}$ 1 $\boxed{=}$, the screen should display 0.158997234.

Using financial functions, the keystroke sequence solving for i is

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ $\boxed{\downarrow}$ 1 $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$
25000 $\boxed{\text{PV}}$ 1000000 $\boxed{+/-}$ $\boxed{\text{FV}}$
25 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{I/Y}}$

The screen should display 15.89972344 (this is the % measure).

UNKNOWN TIME PERIOD:

As an example of solving for an unknown time period, suppose that an initial investment of 100 at monthly compound rate of interest i grows to 300 in n months at monthly interest rate i=.75%. Then $100(1.0075)^n = 300$, from which we get $n = \frac{\ln 3}{\ln 1.0075} = 147.03$ months. This can be found using the calculator $\boxed{\text{LN}}$ function.

Using financial functions, the keystroke sequence solving for n is

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ $\boxed{\downarrow}$ 1 $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$
100 $\boxed{\text{PV}}$ 300 $\boxed{+/-}$ $\boxed{\text{FV}}$
.75 $\boxed{\text{I/Y}}$ $\boxed{\text{CPT}}$ $\boxed{\text{N}}$.

The screen should display 147.03026. Slightly more than 147 months of compounding will be required. The calculator returns a value of n based on compounding including fractional periods, so that the value of 147.03026 means that $100(1.0075)^{147.03026} = 300$.

ACCUMULATED AND PRESENT VALUES OF A SINGLE PAYMENT
USING A COMPOUND DISCOUNT RATE

Present and accumulated values of single payments using an effective rate of discount can be made in the following way. Clear calculator registers before starting the keystroke sequence.

Present Value Using a Compound Discount Rate:

The present value of 500 due in 8 years at effective annual rate of discount 8% is $500(1-.08)^8 = 500(.92)^8 = 256.61$.

This can be found using the calculator in a few ways:

1. We use standard arithmetic operators in standard calculator mode with the following keystrokes.

    $.92\ \boxed{y^x}\ 8\ \boxed{=}\ \boxed{\times}\ 500\ \boxed{=}$

    The screen should display 256.61.

2. $500\ \boxed{FV}\ 8\ \boxed{+/-}\ \boxed{I/Y}$
    $8\ \boxed{+/-}\ \boxed{N}\ \boxed{ENTER}\ \boxed{CPT}\ \boxed{PV}$

The screen should display −256.61. The calculator has calculated

$$PV = -FV(1+I)^{-N} = -500(1-.08)^{-(-8)} = -500(.92)^8 = -256.61.$$

(Remember the sign convention for payments in and payments out.)

The following keystroke sequence could also be used.

3. $500\ \boxed{PV}\ 8\ \boxed{+/-}\ \boxed{I/Y}$
    $8\ \boxed{N}\ \boxed{CPT}\ \boxed{FV}$

We have calculated $-500(1-.08)^8 = -256.61$.

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Accumulated Value Using a Compound Discount Rate:

A deposit of 25 made at time 0 grows at effective annual discount rate 6%. The accumulated value at the end of 5 years is

$$25(1-.06)^{-5} = 25(.94)^{-5} = 34.06 \text{ (nearest .01)}.$$

This can be found using the calculator in two ways.

1. We use standard arithmetic operators in standard calculator mode with the following keystrokes.

.94 $\boxed{y^x}$ 5 $\boxed{+/-}$ $\boxed{=}$ $\boxed{\times}$ 25 $\boxed{=}$

The screen should display 34.06 (rounded to nearest .01).

2. 25 $\boxed{PV}$ 6 $\boxed{+/-}$ $\boxed{I/Y}$

5 $\boxed{+/-}$ $\boxed{N}$ $\boxed{CPT}$ $\boxed{FV}$

The screen should display -34.06 (negative sign indicating outflow). As a third approach, we could also find the effective annual interest rate and accumulate.

CONVERSION BETWEEN EFFECTIVE ANNUAL AND NOMINAL RATES

The nominal annual interest rate compounded m times per year can be found from the effective annual rate of interest and vice-versa using calculator functions as illustrated below.

Nominal Interest Rates

A nominal annual interest rate of .24 (24%) compounded monthly is equivalent to an effective annual rate of interest of $i = .2682\,(26.82\%)$.

The relationship $i = \left(1+\frac{i^{(12)}}{12}\right)^{12} -1$ can be used, or the equivalent rates can be found in the following way using the $\boxed{ICONV}$ function.

Conversion from nominal annual to effective annual interest rate:

We apply the following sequence of keystrokes.

|2nd| |ICONV| (NOM= appears),

24 |ENTER| (the nominal rate) |↓| |↓| (C/Y= appears)

12 |ENTER| (number of compounding periods),

|↓| |↓| (EFF= appears) |CPT| .

The screen should display 26.82. We have converted the nominal annual interest rate of 24% (keyed in as 24) compounded monthly (keyed in as 12) to the equivalent effective annual interest rate of 26.82%.

Conversion from effective annual to nominal annual interest rate:

|2nd| |ICONV| |↓| (EFF= appears)

26.82 |ENTER| |↓| (C/Y= appears)

12 |ENTER| |↓| (NOM= appears)

|CPT|

The screen should display 23.9966 (round to 24). We have converted the effective annual interest rate of 26.82% (key in 26.82) to the equivalent nominal annual interest rate compounded monthly (key in 12) of 24%.

Nominal Discount Rates

The nominal annual discount rate compounded m times per year can be found from the effective annual rate of interest and vice-versa using calculator functions as illustrated below.

A nominal annual discount rate of .09 (9%) compounded quarterly is equivalent to an effective annual rate of interest of $i = .0953$ (9.53%). The relationship $i = \left(1 - \frac{d^{(4)}}{4}\right)^{-4} - 1$ can be used, or the equivalent rate can be found in the following ways.

Use the keystroke sequence $\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 4 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$. This sets the number of compounding periods per year to 4 and returns to standard calculation functions. It is important to do this step first, entering the number of compounding periods in the year.

1. We first find the equivalent effective annual rate of interest from the given nominal annual rate of discount.

4 $\boxed{+/-}$ $\boxed{\text{N}}$ 9 $\boxed{+/-}$ $\boxed{\text{I/Y}}$ 1 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

The display should read -1.0953; we interpret this as indicating that the effective annual rate of interest is 9.53%. We have calculated

$$\left(1+\frac{I}{-N}\right)^{P} = \left(1+\frac{-.09}{4}\right)^{-4} = FV = -1.0953\,,$$

where $P = 4$ was entered with the P/Y function, $N = -4$ was entered with 4 $\boxed{+/-}$ $\boxed{\text{N}}$, and $I = -.09$ (or $d = 9\%$) was entered with 9 $\boxed{+/-}$ $\boxed{\text{I/Y}}$.

2. We now find the equivalent nominal annual rate of discount from the given effective annual rate of interest.

4 $\boxed{+/-}$ $\boxed{\text{N}}$, Key in 1.09531 $\boxed{+/-}$ $\boxed{\text{FV}}$,
Key in 1 $\boxed{\text{PV}}$, Key in $\boxed{\text{CPT}}$ $\boxed{\text{I/Y}}$.
The display should read -9.00; this is the negative of the equivalent nominal annual rate of discount compounded 4 times per year.

Note that when we enter FV, we enter $-(1+\text{EFF})$.

LEVEL PAYMENT ANNUITY VALUATION

The accumulated value and present value of a level payment annuity-immediate can be found using calculator functions. Clear calculator registers before starting the keystroke sequence. The calculator should be in standard calculator mode.

Accumulated Value of Annuity-Immediate:

A deposit of 1000 is made at the end of each year for 20 years. The deposits earn interest at an effective annual rate of interest of 4%. The accumulated value of the deposits at the time of (and including) the 20th deposit is $1000s_{\overline{20}|.04} = 1000\left[\frac{(1.04)^{20}-1}{.04}\right] = 29,778$. This can be found using the calculator.

Use the keystroke sequence $\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ (this sets the number of compounding periods per year to 1 and returns to standard calculation functions). It is important to do this step first, so that the compounding period corresponds to the payment period.

20 $\boxed{\text{N}}$ 4 $\boxed{\text{I/Y}}$ 0 $\boxed{\text{PV}}$ 1000 $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

The display should read $-29,778.08$, the negative of the accumulated annuity value.

Present Value of Annuity-Immediate:

Payments of 50 will be made at the end of each month for 10 years. The monthly compound interest rate is ¾ %. The present value of the annuity one month before the first payment is made is

$$50a_{\overline{120}|.0075} = 50\left[\frac{1-v_{.0075}^{120}}{.0075}\right] = 3,947.08$$

(10 years, 12 months per year).

The calculator should still have $\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ set to 1 (if not, use the keystroke sequence outlined above to set P/Y to 1). This means that the rate entered as I/Y will be a rate per period. In this example the period is one month. We use the following sequence of keystrokes.

120 $\boxed{\text{N}}$ .75 $\boxed{\text{I/Y}}$ $\boxed{\text{ENTER}}$ 50 $\boxed{\text{PMT}}$ $\boxed{\text{ENTER}}$
0 $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display should read $-3,947.08$, the negative of the present value of the annuity.

In the general equation for an annuity-immediate $PV = PMT \cdot a_{\overline{N}|i}$ if any 3 of the 4 variables PV, PMT, N, i (in %) are given, then the calculator functions can be used to solve for the 4th variable. The same is true for the equation $PMT \cdot s_{\overline{N}|i} = FV$ (keep in mind that when PMT is entered as positive, FV or PV are returned as negative, and vice-versa).

Finding the Payment Amount:

A loan of 1000 is to be repaid with monthly payments for 3 years at a compound monthly interest rate of .5%. The monthly payment is K where $1000 = Ka_{\overline{36}|.005}$, so that $K = \dfrac{1000}{a_{\overline{36}|.005}} = 30.42$.

This can be found using the following sequence of keystrokes:

36 $\boxed{\text{N}}$ .5 $\boxed{\text{I/Y}}$ $\boxed{\text{ENTER}}$ 1000 $\boxed{\text{PV}}$ 0 $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$

The display should read −30.42.

Finding the Interest Rate:

Suppose that the loan payment is 35 for a 36 payment loan of amount 1000 and the interest rate is to be found. Then $1000 = 35a_{\overline{36}|i}$. There is no algebraic solution for i. The following keystrokes give us i.

36 $\boxed{\text{N}}$ 1000 $\boxed{+/-}$ $\boxed{\text{PV}}$ 35 $\boxed{\text{PMT}}$ 0 $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{I/Y}}$

The display should read 1.31(%). That is the effective rate of interest per month.

Finding the Number of Payments:

We will use Example 2.13 to illustrate the calculator function for finding the unknown number of payments. In Example 2.13, Smith wishes to accumulate 1000 by means of semiannual deposits earning interest at nominal annual rate $i^{(2)} = .08$, with interest credited semiannually.

In part (a) of Example 2.13, Smith makes deposits of 50 every six months. We wish to solve for n in the equation $1000 = 50 \cdot s_{\overline{n}|.04}$.

The following keystrokes give us n.

1000 $\boxed{\text{FV}}$ 50 $\boxed{+/-}$ $\boxed{\text{PMT}}$ 4 $\boxed{\text{I/Y}}$ 0 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{N}}$

The display should read 14.9866. 14 deposits are not sufficient and 15 full deposits are more than sufficient. The accumulated value of the deposits just after the 14th deposit is $50 \cdot s_{\overline{14}|.04} = 914.60$, so an additional deposit of 85.40 is needed at the time of the 14th deposit to bring the total accumulated value to 1000. The accumulated value 6 months after the 14th deposit is

$$50 \cdot \ddot{s}_{\overline{14}|.04} = 50(1.04) \cdot s_{\overline{14}|.04} = 951.18,$$

so a deposit of 48.18 is needed at time 15 to bring the accumulated value to 1000.

The next functions reviewed relate to finding the value of an annuity-due.

Annuity-Due:

The accumulated value and present value of a level payment annuity-due can be found using calculator functions. The same methods apply that were used for annuities-immediate, with the additional requirement that keystrokes $\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$ $\boxed{\text{2nd}}$ $\boxed{\text{SET}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ must be entered to make the calculator view payments as being made at the beginning of each period (there is a screen indication of the BGN mode when it is invoked; when in BGN mode, in order to return to END mode, use the keystrokes $\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$ $\boxed{\text{2nd}}$ $\boxed{\text{SET}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$, and BGN should disappear from the screen display, and the calculator is in END mode). In the equation $PV = PMT \cdot \ddot{a}_{\overline{N}|i}$, if any 3 of PV, PMT, N, i are entered, we can find the 4th. In the equation $FV = PMT \cdot \ddot{s}_{\overline{N}|i}$ any 3 of FV, PMT, N, i are entered, we can find the 4th (as before, PMT and PV or FV are opposite signs).

Finding the Time and Amount of a Balloon Payment:

We can use the calculator functions to find the balloon payment required to repay a loan which has level payments for as long as necessary with a final balloon payment. In Example 2.15(a) of Chapter 2, a loan of 5000 is being repaid by monthly payments of 100 each, starting one month after the loan is made, for as long as necessary plus an additional fractional

payment at the time of the final regular payment. At interest rate $i^{(12)} = .09$, we are to find the number of full payments that are required to repay the loan, and the amount of the additional fractional payment required if the additional fractional payment is made at the time of the final regular payment. We find the number of payments needed with the following keystrokes (the calculator should be in END mode).

5000 $\boxed{\text{PV}}$ 100 $\boxed{+/-}$ $\boxed{\text{PMT}}$ .75 $\boxed{\text{I/Y}}$ $\boxed{\text{CPT}}$ $\boxed{\text{N}}$

The display should read 62.9. This indicates that the 62nd payment is not quite enough to repay the loan. The additional payment needed, say X, at the time of the 62nd regular payment of 100 is found from the relationship $X = 5000(1.0075)^{62} - 100 \cdot s_{\overline{62}|.0075} = 89.55$. The keystrokes that will produce the value of X are

5000 $\boxed{\text{PV}}$ 100 $\boxed{+/-}$ $\boxed{\text{PMT}}$ .75 $\boxed{\text{I/Y}}$ 62 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

The display should read -89.55.

ANNUITIES WHOSE INTEREST AND PAYMENT PERIODS DIFFER

A function is available to calculate annuity values when the interest period and payment period do not coincide. The following examples illustrate the use of this calculator function.

1. Annuity-immediate of 10 annual payments of 1 each, with interest at a nominal annual rate of 8% compounded quarterly. The present value one year before the first payment is $a_{\overline{10}|j}$, where

$$j = (1.02)^4 - 1 = .082432$$

    is the effective annual rate of interest that is equivalent to the nominal annual. The present value will be $a_{\overline{10}|.082432} = 6.6367$. The annuity value can be found using financial functions with the following sequence of keystrokes:

    $\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 1 $\boxed{\text{ENTER}}$ (this sets 1 payment per year),

    $\boxed{\downarrow}$ 4 $\boxed{\text{ENTER}}$ (this sets C/Y=4 interest conversion periods per year), $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$.

10 $\boxed{\text{N}}$ (10 annual payments),

8 $\boxed{\text{I/Y}}$ (nominal annual interest rate of 8%),

1 $\boxed{+/-}$ $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 6.637, the present value of the annuity. To find the accumulated value of the annuity, we continue with the following keystrokes: 0 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

The display should read 14.655, the accumulated value of the annuity.

Note that if we find the equivalent effective annual rate of interest first, $i = .082432$, we could have found the annuity value as follows without setting C/Y to 4. This is done in the following way (C/Y and P/Y are both set to 1).

10 $\boxed{\text{N}}$ (10 annual payments),

8.2432 $\boxed{\text{I/Y}}$ (effective annual interest rate),

1 $\boxed{+/-}$ $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display should read 6.637, the present value of the annuity.

2. Annuity-immediate of 1 per month for 5 years at effective annual rate of interest 6%. The equivalent one-month interest rate is $j = (1.06)^{1/12} - 1 = .004868$ and the present value of the annuity one month before the first payment is $a_{\overline{60}|j} = 51.924$. The annuity value can be found using financial functions with the following sequence of keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 12 $\boxed{\text{ENTER}}$ (this sets 12 payments per year),

$\boxed{\downarrow}$ 1 $\boxed{\text{ENTER}}$ (this sets C/Y=1 interest conversion period per year),

then $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$.

5 $\boxed{\text{2nd}}$ $\boxed{\text{xP/Y}}$ $\boxed{\text{N}}$ ($5 \times 12 = 60$ monthly payments),

6 $\boxed{\text{I/Y}}$ (effective annual interest rate),

1 $\boxed{+/-}$ $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display should read 51.924, the present value of the annuity.

As with Case 1 above, we could have used an alternative approach based on the one-month effective interest rate of .004868. P/Y and C/Y should be set to 1 before the following keystrokes.

60 $\boxed{\text{N}}$ .4868 $\boxed{\text{I/Y}}$ (monthly effective interest rate),

1 $\boxed{\text{+/--}}$ $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display should read 51.923, the present value of the annuity.

VALUATION OF INCREASING AND DECREASING ANNUITIES

The values of $(Ia)_{\overline{n}|i}$ (present value) and $(Ds)_{\overline{n}|i}$ (accumulated value) can be found using calculator financial functions. From those values we can then find $(Is)_{\overline{n}|i} = (Ia)_{\overline{n}|i} \cdot (1+i)^n$, and $(Da)_{\overline{n}|i} = (Ds)_{\overline{n}|i} \cdot v^n$.
The following two examples illustrate the method.

Finding $(Ia)_{\overline{n}|i}$:

Suppose that we wish to find $(Ia)_{\overline{20}|.08} = \dfrac{\ddot{a}_{\overline{20}|.08} - 20v^{20}}{.08}$. We first find the numerator with the following keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$ $\boxed{\text{2nd}}$ $\boxed{\text{SET}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$

20 $\boxed{\text{N}}$ 8 $\boxed{\text{I/Y}}$ 1 $\boxed{\text{+/--}}$ $\boxed{\text{PMT}}$ 20 $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display should read 6.3126, which is $\ddot{a}_{\overline{20}|.08} - 20v^{20}$.

In this sequence of keystrokes, we have created a series of 20 payments received of 1 each at the start of each year ($\boxed{\text{BGN}}$), combined with a payment of 20 paid out at the end of 20 years ($\boxed{\text{FV}}$). The net present value is $\ddot{a}_{\overline{20}|.08} - 20v^{20} = 6.3126$. Then, $(Ia)_{\overline{20}|.08} = \dfrac{6.3126}{.08} = 78.908$.

Note that instead of entering 1 $\boxed{\text{+/--}}$ $\boxed{\text{PMT}}$ we could enter 1 $\boxed{\div}$ .08 $\boxed{=}$ $\boxed{\text{+/--}}$ $\boxed{\text{PMT}}$, and instead of entering 20 $\boxed{\text{FV}}$ we could enter 20 $\boxed{\div}$ .08 $\boxed{=}$ $\boxed{\text{FV}}$. Then $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$ includes division by .08. Note that

we cannot use these calculator functions to find $(Is)_{\overline{n}|i}$, but since the numerator of $(Is)_{\overline{n}|i}$ is $\ddot{s}_{\overline{n}|i} - n$, we can find $\ddot{s}_{\overline{n}|i}$ first, then subtract n, and then divide by i.

Finding $(Ds)_{\overline{n}|i}$:

Suppose that we wish to find $(Ds)_{\overline{35}|.04} = \dfrac{35(1.04)^{35} - s_{\overline{35}|.04}}{.04}$. We first find the numerator with the following keystrokes. The calculator payment mode should be set to END.

$$35 \;\boxed{\text{N}}\; 4 \;\boxed{\text{I/Y}}\; \boxed{\text{ENTER}}\; 35 \;\boxed{+/-}\; \boxed{\text{PV}}$$
$$1 \;\boxed{\text{PMT}}\; \boxed{\text{CPT}}\; \boxed{\text{FV}}$$

The display should read 64.4609, which is $35(1.04)^{35} - s_{\overline{35}|.04}$.

In this sequence of keystrokes we have created an initial payment <u>received</u> of 35 at time 0 ($\boxed{\text{PV}}$), and a series of 35 payments of 1 each <u>paid out</u> at the <u>end</u> of each year. The net accumulated value at the end of 35 years is

$$35(1.04)^{35} - s_{\overline{35}|.04} = 64.4609 \;\left(\boxed{\text{FV}}\right).$$

Then,

$$(Ds)_{\overline{35}|.04} = \frac{64.4609}{.04} = 1{,}611.52.$$

Note that we could have incorporated division by .04 into the keystroke sequence by keying in $35 \;\boxed{\div}\; .04 \;\boxed{=}\; \boxed{+/-}\; \boxed{\text{PV}}$ instead of $35 \;\boxed{+/-}\; \boxed{\text{PV}}$, and by keying in $1 \;\boxed{\div}\; .04 \;\boxed{=}\; \boxed{\text{PMT}}$ instead of $1 \;\boxed{\text{PMT}}$.

It is also possible to use the cashflow worksheet to find present and accumulated values of a series of non-level payments. There is a limitation that allows an initial payment, CF_0, and up to 24 more payment amounts.

This would be a less efficient way of finding $(Ia)_{\overline{20}|.08}$, for instance.

DEPRECIATION

An asset has a purchase price of 1200 and a salvage value of 100. According to the sum-of-years-digits method with a 10-year depreciation period, the depreciation amounts and book values will be

$$D_1 = \tfrac{10}{55} \cdot (1200 - 100) = 200,$$

$$D_2 = \tfrac{9}{55} \cdot (1200 - 100) = 180, \dots, D_{10} = 20,$$

$$B_1 = 1000, \; B_2 = 820, \dots, B_9 = 120, \; B_{10} = 100.$$

These values can be found using the calculator depreciation worksheet with the following sequence of keystrokes.

1 [2nd] [DEPR], then [2nd] [SET] until SYD shows in the display.

Then, [↓] gets us to LIF=, and we key in 10 [ENTER] (depreciation over 10 years).

Then [↓] gets us to MOl =, which we skip,

and [↓] again gets us to CST=, and we enter 1200 [ENTER] (initial cost).

Then [↓] gets us to SAL=, and we enter 100 [ENTER] (salvage value).

Then [↓] gets us to YR=, and we can enter any year, say 5 [ENTER] .

Then [↓] gets us DEP=120, which is D_5;

Then [↓] gets us RBV=400, which is B_5.

We can use the same functions for declining balance (a discount rate must be entered), and straight line depreciation.

LOAN AMORTIZATION

For a loan with level payments, there are calculator functions for finding outstanding balances, interest or principal paid in a single payment, and interest or principal paid in a range of payments. Parameters are entered for P/Y, N, I/Y, PV as with the usual annuity valuation. The calculator will give us the payment using the PMT key. The following example illustrates how we get the amortization quantities.

We use Example 3.3 to illustrate these functions. A homebuyer borrows $250,000 to be repaid over a 30-year period with level monthly payments beginning one month after the loan is made. The interest rate on the loan is a nominal annual rate of 9% compounded monthly. The loan payment is $K = \frac{250,000}{a_{\overline{360}|.0075}} = 2,011.56$.

The outstanding balance at the end of the first year (after the 12th monthly payment) is $OB_{12} = 2,011.56a_{\overline{348}|.0075} = 248,292.01$. The principal repaid in the 12th payment is $PR_{12} = Kv^{360-12+1} = 2,011.56v^{349} = 148.25$, and the interest paid in the 12th payment is $I_{12} = K(1-v^{360-12+1}) = 1,863.30$. The principal repaid in the 2nd year (the 13th through 24th payments inclusive) is

$$PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24} = K(v^{348} + v^{347} + \cdots + v^{338} + v^{337})$$
$$= 1,868.21,$$

the interest paid in the 2nd year is

$$I_{13} + I_{14} + \cdots + I_{23} + I_{24} = K(1-v^{348} + 1-v^{347} + \cdots + 1-v^{338} + 1-v^{337})$$

$$= 12K - (PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24}) = 22,270.46.$$

These calculations can be completed using calculator functions. We first clear the calculator registers.

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ (sets payment period to 1)
360 $\boxed{\text{N}}$ .75 $\boxed{\text{I/Y}}$ 250000 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$

The value $-2,011.56$ is returned (we must key in $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$ in order to use the amortization functions).

To find OB_{12} use the following keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{AMORT}}$ (this opens the amortization worksheet).

The display reads P1 =. Key in 12 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$.

The display reads P2 =. Key in 12 $\boxed{\text{ENTER}}$.

Then using $\boxed{\downarrow}$ again, the display should read $BAL = 248,292.01$.
This is OB_{12}.

Using $\boxed{\downarrow}$ again gives the display PRN= -148.25; this is $-PR_{12}$.

Using $\boxed{\downarrow}$ again gives the display INT= $-1,863.30$; this is $-I_{12}$.

To find $I_{13} + I_{14} + \cdots + I_{23} + I_{24}$ and $P_{13} + P_{14} + \cdots + P_{23} + P_{24}$ use the following keystrokes while still in the amortization worksheet (we leave a worksheet using the $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ sequence of keystrokes).

Use $\boxed{\downarrow}$ until P1= appears again and enter 13 (key in 13 $\boxed{\text{ENTER}}$).

Then use $\boxed{\downarrow}$ and P2= appears and we enter 24 (key in 24 $\boxed{\text{ENTER}}$).

The next use of $\boxed{\downarrow}$ given us $BAL = 246,473.79$, the outstanding balance at the end of the period to time 24 months).

$\boxed{\downarrow}$ again gives us PRN = $-1,868,21$; this is $-(P_{13} + P_{14} + \cdots + P_{23} + P_{24})$, the negative total amount of principal paid in payments 13 to 24 (the second year).

$\boxed{\downarrow}$ again gives us INT = $-22,270.46$; this is $-(I_{13} + I_{14} + \cdots + I_{23} + I_{24})$, the negative total amount of interest paid in payments 13 to 24.

Note that $P_{13} + P_{14} + \cdots + P_{23} + P_{24} = OB_{12} - OB_{24}$ could be found from OB_{12} and OB_{24}, and

$$I_{13} + I_{14} + \cdots + I_{23} + I_{24}$$
$$= 12K - (PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24})$$
$$= 12K - (OB_{12} - OB_{24}).$$

BOND VALUATION AND AMORTIZATION

It is possible to calculate the price or yield to maturity of a bond using the calculator's bond worksheet. We use Example 4.1(a) to illustrate the bond worksheet functions.

Finding The Bond Price on a Coupon Date:

A 10% bond with semiannual coupons has a face amount (par value) of 100 and is issued on June 18, 1990. The bond has a maturity date of June 18, 2010. We wish to find the price of the bond on its issue date using a nominal annual yield rate of 5% convertible semi-annually.

The bond price is $100v_{.025}^{40} + 100(.05) \cdot a_{\overline{40}|.025} = 162.76$ (nearest .01). If the bond has maturity value 110. then the price is

$$110v_{.025}^{40} + 100(.05) \cdot a_{\overline{40}|.025} = 166.48.$$

These prices can be found using the following sequence of keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{BOND}}$ (this opens the bond worksheet).
The display shows SDT = (there may be a date here).
Key in 6.1890 $\boxed{\text{ENTER}}$ (valuation date, June 18, 1990)

Using $\boxed{\downarrow}$ gives us CPN=, and we key in 10 $\boxed{\text{ENTER}}$ (coupon amount of 10 per year).

Using $\boxed{\downarrow}$ gives us RDT=, and we key in 6.1810 $\boxed{\text{ENTER}}$ (redemption date 20 years later, June 18, 2010).

Using $\boxed{\downarrow}$ gives us RV=, and we key in 100 $\boxed{\text{ENTER}}$. (redemption amount of 100)

Using $\boxed{\downarrow}$ gives us ACT, which we leave (refers to actual day count for bond valuation).

Using $\boxed{\downarrow}$ gives us 2/Y (we leave this setting since the coupons are paid twice per year; for coupons once per year we would use $\boxed{\text{2nd}}$ $\boxed{\text{SET}}$ to change the display to 1/Y, which means we want "1 coupon per year").

Using $\boxed{\downarrow}$ gives us YLD=, and we key in 5 $\boxed{\text{ENTER}}$ (nominal yield rate per year, compounded 2/Y).

20

Using $\boxed{\downarrow}$ gives us PRI=, and using $\boxed{\text{CPT}}$ results in 162.76 (nearest .01) on the display. This is the bond price based on a face amount of 100. Multiplying by 1,000,000 gives the price for a 100,000,000 face amount bond.

To change the maturity value to 110, we use $\boxed{\downarrow}$ until we reach RV=, and we enter 110. Then use $\boxed{\downarrow}$ until we get to PRI= again, and use $\boxed{\text{CPT}}$. The resulting display should be 166.48.

Finding The Bond Yield on a Coupon Date:

We can use the worksheet to find the yield rate from the price. Suppose that the bond above with face amount 100 has a price of 150. There is no algebraic solution for the yield-to-maturity $i^{(2)} = 2j$, where j, the 6-month yield rate is the solution of the equation $150 = 100v_j^{40} + 5a_{\overline{40}|j}$.

The yield-to-maturity can be found using the calculator in the bond worksheet as follows.

SDT=6-18-1990 (enter 6.1890),
CPN=10, RDT=6-18-2010 (enter 6.1810),
RV=100, ACT, 2/Y, bypass YLD=, PRI=150.

Use $\boxed{\downarrow}$ to get to YLD= and then use $\boxed{\text{CPT}}$ to calculate the yield rate.

The display should read 5.76 (YTM is $i^{(2)} = .0576$, which is $j = 2.88\%$ every 6-months).

The previous calculations can also be done using the calculator annuity functions as follows.

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 2 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$
(ensures yield period and coupon period are the same).

40 $\boxed{\text{N}}$ (40 6-month periods),

5 $\boxed{\text{I/Y}}$ (nominal annual yield rate),

5 $\boxed{+/-}$ $\boxed{\text{PMT}}$ (coupon payment every 6 months),

100 $\boxed{+/-}$ $\boxed{\text{FV}}$ (maturity value),

$\boxed{\text{CPT}}$ $\boxed{\text{PV}}$ should result in 162.76.

For the bond with maturity amount 110 we do the following.

40 $\boxed{\text{N}}$ 5 $\boxed{\text{I/Y}}$ 5 $\boxed{+/-}$ $\boxed{\text{PMT}}$ 110 $\boxed{+/-}$ $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

This should result in 166.48.

To find the yield rate for a price of 150 we do the following.

40 $\boxed{\text{N}}$ 150 $\boxed{\text{PV}}$ 5 $\boxed{+/-}$ $\boxed{\text{PMT}}$ 100 $\boxed{+/-}$ $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{I/Y}}$

This should result in 5.76 (the nominal annual yield).

Bond Amortization:

The bond amortization components can be found using the calculator's amortization worksheet in much the same way they are found for loan amortization.

A bond has face amount 1000, coupon rate 5% per coupon period, maturity value 1000, 20 coupon periods until maturity and yield-to-maturity 6% (per coupon period). The bond's amortized value just after the 5th coupon is

$$BV_5 = 1000v_{.06}^{15} + 1000(.05) \cdot a_{\overline{15}|.06} = 902.88.$$

This can be found using the following keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{P/Y}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$

20 $\boxed{\text{N}}$ 6 $\boxed{\text{I/Y}}$ 50 $\boxed{+/-}$ $\boxed{\text{PMT}}$ 1000 $\boxed{+/-}$ $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The 20-year bond price of 885.30 should appear.

Then $\boxed{\text{2nd}}$ $\boxed{\text{AMORT}}$ should result in P1 =. Key in 5 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$.
This should result in P2 =, and again enter 5 $\boxed{\text{ENTER}}$.

Then $\boxed{\downarrow}$ should result in 902.88, the balance just after the 5th coupon.

Using the $\boxed{\downarrow}$ key again gives PRN=3.94, which is the negative of the principal repaid in the 5th payment (the amount of write-up is 3.94, since the bond was purchased at a discount). Using the $\boxed{\downarrow}$ key again gives INT=-53.94, which is the negative of the amount of interest due in the 5th payment.

Bond Price and Yield Between Coupon Dates:

The bond examples considered so far have had valuation take place on a coupon date. It is also possible to use the bond worksheet functions to find the price (given the yield) or the yield (given the price) of a bond at any time, on or between coupon dates.

We use Example 4.2 to illustrate the valuation of a bond between coupon dates. A bond has face amount 100, with an annual coupon rate of 10% and coupons payable semi-annually. The bond matures on June 18, 2010 and is purchased on August 1, 2000 at a yield rate of 5% (nominal annual yield compounded semi-annually). The quoted purchase price from Example 4.2 is 138.60. This can be found using the following keystrokes in the bond worksheet.

SDT=8-01-2000 (enter 8.0100),
CPN=10, RDT=6-18-2010 (enter 6.1810),
RV=100, ACT, 2/Y, YLD=5 ($\boxed{\text{ENTER}}$ must be used after each entry).

At PRI=, use $\boxed{\text{CPT}}$ to calculate the price.
The display should read 138.60. Note that this is the quoted price which excludes the accrued coupon. The accrued coupon amount is found at AI= .
If a price had been entered instead of a yield rate, we could have computed the yield.

INTERNAL RATE OF RETURN AND NET PRESENT VALUE

The internal rate of return for a series of payments received and payments made can be found in a couple of different ways, depending upon the nature of the series of payments. When we consider a level payment annuity with or without a balloon payment at the time of the last annuity payment, we can enter values into the variables

$\boxed{\text{N}}$, $\boxed{\text{PV}}$, $\boxed{\text{PMT}}$ and $\boxed{\text{FV}}$, and then use $\boxed{\text{CPT}}$ $\boxed{\text{I/Y}}$ to find the interest rate which satisfies the relationship $PV = PMT \cdot a_{\overline{n}|j} + FV \cdot v_j^n$.

The internal rate of return is j.

We can use the cashflow worksheet ($\boxed{\text{CF}}$) to enter a cashflow at time 0, CF_0, along with up to 24 additional cashflows at the end of 24 successive periods, $C01, C02, \ldots, C24$. Once these cashflow amounts are entered, we can use the $\boxed{\text{IRR}}$ function (internal rate of return) to calculate an internal rate of return. It is a solution j to the relationship

$$CF_0 + C01 \cdot v_j + C02 \cdot v_j^2 + \cdots + C24 \cdot v_j^{24} = 0.$$

The cashflow amounts can each be positive (an amount received) or negative (an amount paid out).

Calculating Internal Rate of Return

We illustrate how the internal rate of return in Example 5.1 can be found in this way. Example 5.1 has the following series of cashflows, where time is measured in 6-month intervals:

$$C_0 = -5100, C_1 = 0, \quad C_2 = -2295, \text{ and } C_3 = 7982.5 .$$

The cashflow worksheet is cleared using $\boxed{\text{CF}}$ $\boxed{\text{2nd}}$ $\boxed{\text{CLR WORK}}$
The following series of keystrokes solves for the internal rate of return j, where j will be the 6-month internal rate of return:

Key in $\boxed{\text{CF}}$, the display should read CFo= ,

Key in 5100 $\boxed{+/-}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$, the display should read C01=,

Key in 0 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ $\boxed{\downarrow}$, the display should read C02=,

Key in 2295 $\boxed{+/-}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ $\boxed{\downarrow}$, the display should read C03=,

Key in 7982.5 $\boxed{\text{ENTER}}$,

Key in $\boxed{\text{IRR}}$ $\boxed{\text{CPT}}$. The display should read 3.246 .
This is the 6-month internal rate of return.

As seen in Example 5.2, a series of cashflows may have a unique internal rate of return, it may have more than one internal rate of return, or it may have no internal rate of return. If we attempt to solve for the internal rate of return with the calculator function for Example 5.2, we get the following results:

5.2(a) $C_0 = -1, C_1 = 2.3,$ and $C_2 = -1.33,$ and the calculator returns an "Error 7" message when $\boxed{\text{IRR}}$ $\boxed{\text{CPT}}$ is keyed in, which generally occurs when there is no solution for the internal rate of return.

5.2(b) $C_0 = -1, C_1 = 2.3,$ and $C_2 = -1.32,$ and the calculator returns an IRR of 10%. This is the smaller of the two solutions, 10% and 20%.

5.2(c) $C_0 = -1, C_1 = 2.3, C_2 = -1.3125,$ and the calculator returns an IRR of 5%, again the smaller of the two solutions, 5% and 25%.

5.2(d) $C_0 = -1, C_1 = 2.3, C_2 = -1.2825,$ and the calculator returns an IRR of -5%, again the smaller of the two solutions, -5% and 35%.

Calculating Net Present Value:

The calculator $\boxed{\text{NPV}}$ function can be used to calculate the present value of a series of cashflows that have been entered into the cashflow worksheet. We use Example 5.2(a) as an illustration. Suppose that we wish to find the present value at time 0 of the three cashflows

$$C_0 = -1, C_1 = 2.3, \text{ and } C_2 = -1.33,$$

using an interest rate of 10%.

This would be $-1 + 2.3v_1 - 1.33v_1^2$. We can find this present value as follows. We enter the cashflows $C_0 = -1, C_1 = 2.3,$ and $C_2 = -1.33,$ into the $\boxed{\text{CF}}$ worksheet as shown above. Immediately after C_2 has been entered, and while still in the cashflow worksheet, press $\boxed{\text{NPV}}$. The display will read I=0. We key in 10 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ $\boxed{\text{CPT}}$.

The display should read NPV = -0.008264. This is the present value of the cashflows valued at an interest rate of 10% per period.

In the cashflow worksheet it is possible to specify an initial cashflow amount CF0, and 24 additional amounts C1 to C24. Each cashflow amount may be made multiple times ("grouped" cashflows of the same amount made at successive points in time).

We use Example 2.5 to illustrate this application. Suppose that 10 monthly payments of 50 each are followed by 14 monthly payments of 75 each. If interest is at a monthly effective rate of 1%, what is the accumulated value of the series at the time of the final payment? The following keystrokes give us the accumulated value.

$\boxed{\text{CF}}$ 0 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$

Key in 50 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 10 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ (this sets a payment amount of C01=50 to be made for F01=10 successive periods),

Key in 75 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 14 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ (this sets a payment amount of C02=75 to be made for F02=14 successive periods after the first 10 periods).

Key in $\boxed{\text{NPV}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ $\boxed{\text{CPT}}$

The screen should display 1,356.47. This is the present value of the 24 payments one month before the first payment.

With NPV = 1,356.47 still displayed, Key in $\boxed{\text{PV}}$.
Key in $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ 24 $\boxed{\text{N}}$ 1 $\boxed{\text{I/Y}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

This should result in the display FV $= -1,722.36$. This is the accumulated value of the series of 24 payments at time of the final payment.

As a variation on Example 2.5, suppose that 10 deposits of 50 per month are made into an account earning monthly interest rate i. Suppose further that one month after the 10th deposit, monthly withdrawals are made from the account of amount 75 per month. The account balance is 0 just after the 14th withdrawal. We wish to find the monthly interest rate on the account. We wish to solve for i in the equation $50s_{\overline{10}|i} = 75a_{\overline{14}|i}$.

This equation is equivalent to $50a_{\overline{10}|i} = 75v^{10}a_{\overline{14}|i}$. We can place this in the context of internal rate of return, where we wish to find the internal rate of return for a sequence of cashflows of 10 payments <u>paid</u> of 50 each (C01=50, F01=10) followed by 14 payments received of 75 each (C02=$-$75, F02=14), and whose total initial present value is CF0=0.

We use the following keystrokes:

CF 0 ENTER ↓ 50 ENTER ↓ 10 ENTER ↓
75 +/− ENTER ↓ 14 ENTER IRR CPT .

The display should read IRR = 6.518.

EXAMPLES FROM SOA/CAS EXAM FM/2
(FORMERLY COURSE 2 COMPOUND INTEREST)

| May 2003, #33 (Annuity Valuation) |

At an effective annual interest rate of i, $i > 0$, , both of the following annuities have a present value of X:

(i) A 20-year annuity-immediate with annual payments of 55
(ii) A 30-year annuity-immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years.

Calculate X.

(A) 575   (B) 585   (C) 595   (D) 605   (E) 615

| SOLUTION |

The series of cashflows representing the difference between (i) and (ii) is a series of 10 payments of $55 - 30 = 25$ each, followed by a series of 10 payments of $55 - 60 = -5$ each, followed by a series of 10 payments of -90 each. The interest rate that makes the present value of this series equal to 0 is found using the IRR function as follows.

Key in $\boxed{\text{CF}}$ 0 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 25 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 10 $\boxed{\text{ENTER}}$,

$\boxed{\downarrow}$ 5 $\boxed{+/-}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 10 $\boxed{\text{ENTER}}$,

$\boxed{\downarrow}$ 90 $\boxed{+/-}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 10 $\boxed{\text{ENTER}}$,

$\boxed{\text{IRR}}$ $\boxed{\text{CPT}}$. The display should read IRR = 7.177.

This is the interest rate per year. Set P/Y and C/Y to 1. The following keystrokes give us the value of X.

$\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ $\boxed{\text{2nd}}$ $\boxed{\text{CLR TVM}}$

20 $\boxed{\text{N}}$ 7.177 $\boxed{\text{I/Y}}$ 55 $\boxed{+/-}$ $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 574.74. Answer: A

May 2003, #8 (Annuity Valuation) (*This is also Exercise 2.2.2*)

Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an effective annual interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.

(A) 4695   (B) 5070   (C) 5445   (D) 5820   (E) 6195

SOLUTION

We denote the 4-year rate of interest by j. Then the accumulated value at the end of 40 years is $X = 100\ddot{s}_{\overline{10}|j}$. (10 4-year periods, with valuation one full 4-year period after the 10th deposit). The accumulated value at the end of 20 years is $100\ddot{s}_{\overline{5}|j}$.

We are given that $100\ddot{s}_{\overline{10}|j} = 5 \times 100\ddot{s}_{\overline{5}|j}$, which is the same as

$$100s_{\overline{10}|j} - 5 \times 100s_{\overline{5}|j} = 0$$

(after we multiply both sides of the equations by v_j). This can be interpreted as saying that 5 payments of 100 per period, followed by 5 payments of -400 per period has an accumulated value (and present value) of 0. We can use the IRR function to find j as follows.

Key in $\boxed{\text{CF}}$ 0 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 100 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 5 $\boxed{\text{ENTER}}$,

$\boxed{\downarrow}$ 400 $\boxed{+/-}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 5 $\boxed{\text{ENTER}}$ $\boxed{\text{IRR}}$ $\boxed{\text{CPT}}$.

The display should read IRR = 31.95.
The 4-year interest rate is $j = .3195$.
Then we apply the following keystrokes.

$\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$ $\boxed{\text{2nd}}$ $\boxed{\text{CLR TVM}}$

$\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$ $\boxed{\text{2nd}}$ $\boxed{\text{SET}}$ $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$

10 $\boxed{\text{N}}$ 31.95 $\boxed{\text{I/Y}}$

100 $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.

The display should read $-6,194.44$.   Answer: E

May 2003, #26 (Decreasing Annuity) (*Exercise 2.3.13*)

1000 is deposited into Fund X, which earns an effective annual rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an effective annual rate of 9%. Determine the accumulated value of Fund Y at the end of year 10.

(A) 1519　　(B) 1819　　(C) 2085　　(D) 2273　　(E) 2431

SOLUTION

The 10 deposits to Fund Y are $160, 154, 148, ..., 112, 106$.

These can be entered as 10 separate cashflows in the ⎡CF⎤ worksheet. The NPV function at 9% will give a present value (one year before the deposit of 160) of 880.59. Then the FV at the end of 10 years will be 2,085. Answer: C

November 2001, #16 (Increasing Annuity) (*Exercise 2.3.12*)

Olga buys a 5-year increasing annuity for X. Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2. The nominal interest rate is 9% convertible quarterly. Calculate X.

(A) 2680　　(B) 2730　　(C) 2780　　(D) 2830　　(E) 2880

SOLUTION

With monthly rate j, $X = 2(Ia)_{\overline{60}|j}$.

We are given 3-month rate .0225, so that $(1+j)^3 = 1.0225$, and therefore, $j = .007444$. The numerator of $(Ia)_{\overline{60}|j}$ can be found by the following keystrokes.

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

[2nd] [P/Y] 12 [ENTER] [↓] [C/Y] 4 [ENTER] [2nd] [QUIT] ,

60 [N] 1 [+/−] [PMT] 9 [I/Y] 60 [FV] [CPT] [PV]
This results in the display reading PV = 10.1587.

Then [÷] .007444 [×] 2 results in 2,729 on the display. Answer: B

May 2003, #25 (Loan Repayment)

John borrows 1000 for 10 years at an effective annual interest rate of 10%. He can repay this loan using the amortization method with payments of P at the end of each year. Instead, John repays the 1000 using a sinking fund that pays an effective annual rate of 14%. The deposits to the sinking fund are equal to P minus the interest on the loan and are made at the end of each year for 10 years.

Determine the balance in the sinking fund immediately after repayment of the loan.

(A) 213   (B) 218   (C) 223   (D) 230   (E) 237

SOLUTION

P is found as follows. Clear all registers and be sure that I/Y and P/Y are both set to 1 and payment mode is END. Apply the following keystrokes.

10 $\boxed{\text{N}}$ 10 $\boxed{\text{I/Y}}$ 1000 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$

The screen displays −162.75. The deposits to the sinking fund are 62.75 each. The accumulated value of the sinking fund deposits at 14% is found as follows.

10 $\boxed{\text{N}}$ 14 $\boxed{\text{I/Y}}$ 62.75 $\boxed{\text{PMT}}$ 0 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$

The display reads −1,213.42.

After the principal loan amount of 1,000 is paid, 213 is left in the sinking fund.   Answer: A

May 2003, #42 (Bond Amortization)

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an effective annual rate of 6%.

Calculate the interest portion of the 7th coupon.

(A) 632   (B) 642   (C) 651   (D) 660   (E) 667

SOLUTION

The AMORT functions can be used as follows.

10 $\boxed{\text{N}}$ 6 $\boxed{\text{I/Y}}$ 800 $\boxed{+/-}$ 10000 $\boxed{+/-}$ $\boxed{\text{FV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$

The display reads 11,472.02 (the bond price). Then

$\boxed{\text{2nd}}$ $\boxed{\text{AMORT}}$ 7 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ 7 $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$

shows BAL = 10,534.60 (the book value after the 7th coupon).

Then $\boxed{\downarrow}$ shows PRN = -158.42 (the negative of the principal repaid),

and $\boxed{\downarrow}$ shows INT = -641.58 is the negative of the interest paid.

Answer: B