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THE PRACTICAL USES OF RISK THEORY

Moderator: ALASTAIR G. LONGLEY-COOK. Panelists: NATHAN H. EPSTEIN, JAMES C. HICKMAN, HARRY H. PANJER

1. Basic concepts in non-technical language.

2. Practical applications - there are some good ones.

3. Making the best use of actuarial students who know risk theory.

MR. ALASTAIR G. LONGLEY-COOK: For the past year or so, I have been involved in the consideration of a new actuarial mathematics textbook for the Society syllabus. This textbook approaches the subject from a risk theory oriented nature - it treats the parameters actuaries deal with (mortality, interest, etc.) as random variables, not deterministic known quantities. Therefore, risk theory is caught up in the whole nature of that book. Five chapters of it were implemented as a study note on the Society syllabus last year.

While working on this project, I came to realize that there existed among many actuaries what I shall label 'risk theory anxiety' - the actuarial equivalent of high school level math anxiety. Many actuaries, including myself, do not feel very comfortable among stochastic processes. Several actuaries asked me if we could have a panel discussion to explain some of these concepts on a very practical level. Lyndon Cole, our Director of Education, also suggested I put something together. So here we are.

We have three experts to speak on the subject. The first is Harry Panjer. Harry is Professor of Actuarial Science at Waterloo. He teaches risk theory on an almost daily basis. He is also a consultant to insurance companies on risk theory matters. He is chairman of the Society's Committee on Research, a member of the Committee on Theory of Risk and the author of many publications on risk theory topics in the Transactions and the Astin Bulletin. His is a name that is well known to anybody who's delved into risk theory.

Second is Nate Epstein, Chief Actuary for Monumental Life. He is also a member of the Committee on the Theory of Risk and co-editor of a soon-to-be-published monograph on mergers and acquisitions in which he says there is absolutely no risk theory at all. Despite that, he is someone who, through his own interest and his work at various insurance companies, has used the theory, if not on a daily basis, then fairly regularly. He will bring perhaps a slightly less "academic" point of view to the discussion.

The third speaker - Jim Hickman - is Professor of Business and Statistics at the University of Wisconsin. He is one of the authors of the new actuarial textbook. He is therefore a co-author of the risk theory study note. He is also an author of numerous publications on risk theory. I'm sure you have seen his name in the Transactions on a regular basis.

Our recorder this morning is Glen Hazlett, good friend and colleague at Aetna. He'll be in charge of making sense of our ramblings later on.

Despite the fact that I am not an expert, and I am aware of the fact that a little knowledge is a dangerous thing, I would like to introduce the topic. I shall then get out of the way.

What is risk theory? Our speakers this morning will answer that question in their own ways. I am visually oriented and so I tend to speak through pictures. This is one that caught my eye. It was in Fortune magazine about a year ago. (Illustration 1) When I first saw it, it occurred to me that this is what risk theory is all about.

Actuaries tend to drive the train on the parallel tracks. We make certain assumptions as to mortality, interest, persistency, expenses. Once those are set, we plug them into our formulas or run our asset shares and we're all set. Now, as long as those assumptions are correct, then the train will run true, everything is fine.

We are kidding ourselves, of course, when we do that in today's environment. We are not on the parallel tracks. We are, in fact, ever emerging into an increasing variance from those assumptions. The further we go away from where we are standing, the further we are to the time horizon, the wider are the possible variations from our assumptions.

So this is a good picture to ponder for somebody who has not been applying risk theory as well as they might. They may be in the position of the puzzled engineer standing next to the train trying to figure out what to do now.

It is rather like that time-worn joke about the actuary who drowned trying to cross the river that averaged 2 ft. deep. The reason why that joke still gets a smile is because of the element of truth that underlies it. We will indeed drown if we assume that the average is what we shall experience.

Why is it becoming increasingly important to recognize this? One reason is interest rate volatility. Illustration 2 shows long-term interest rates measured every 10 business days during a very stable period, 1975-77. (Illustration 2)

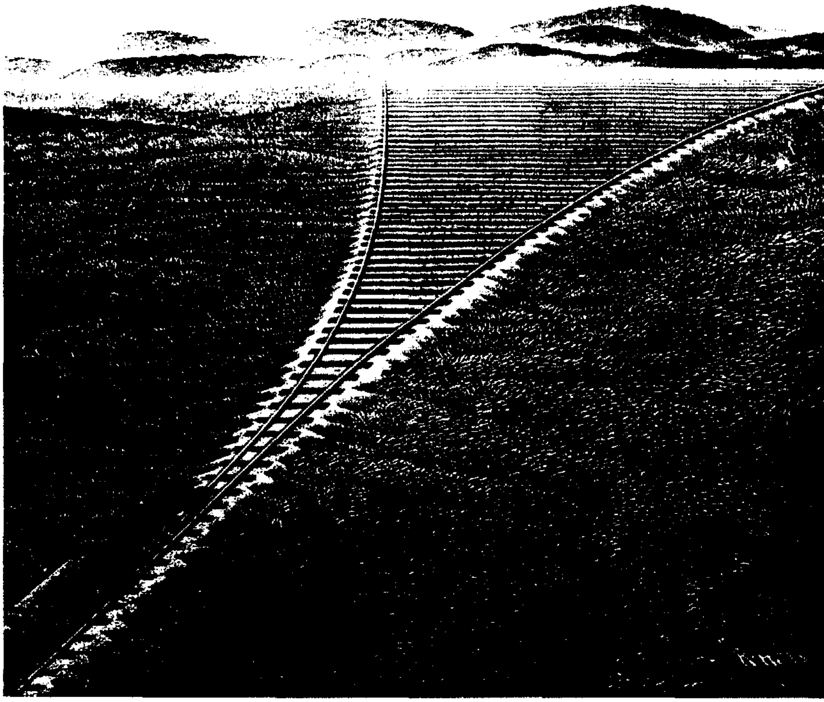

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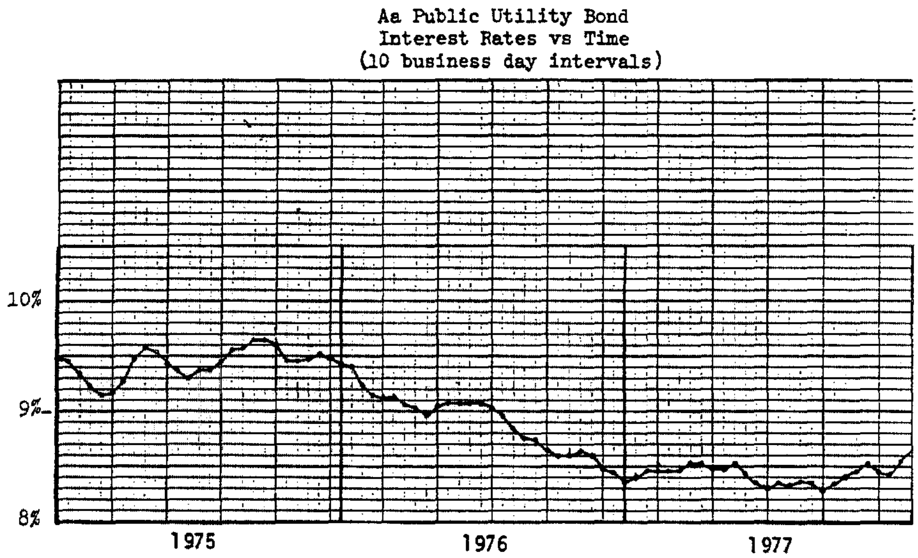
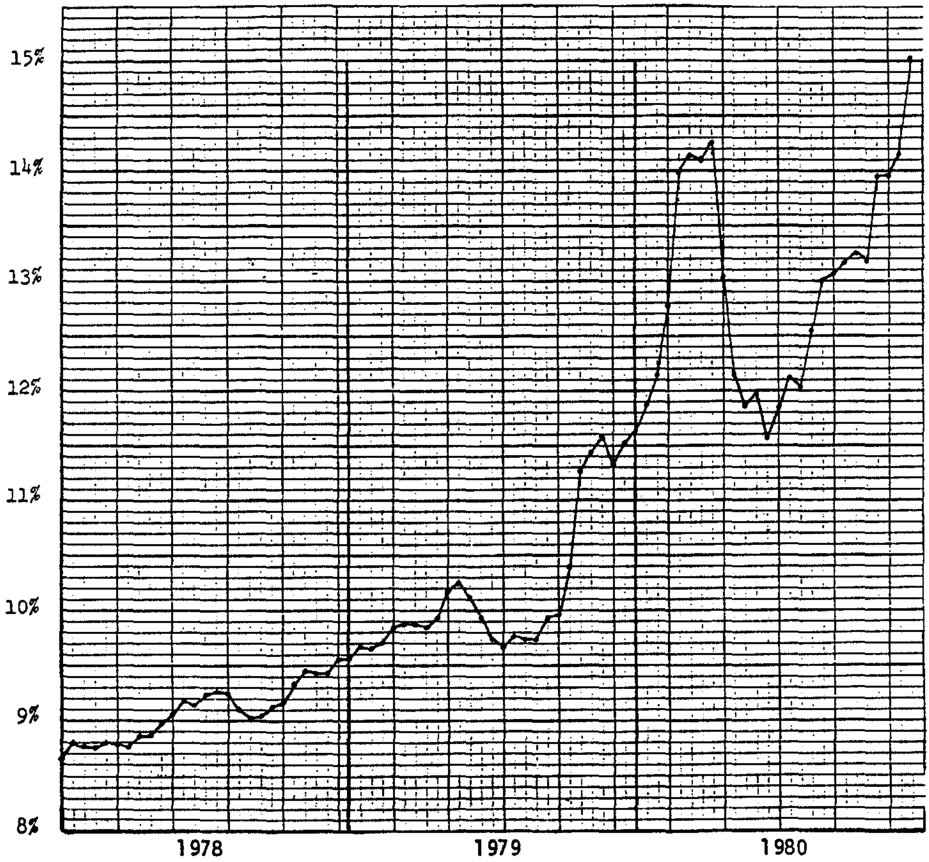
ILLUSTRATION 2



I modeled it as a log-normal stochastic process - which fit very well. I then looked at the next three years. (Illustration 3)

ILLUSTRATION 3



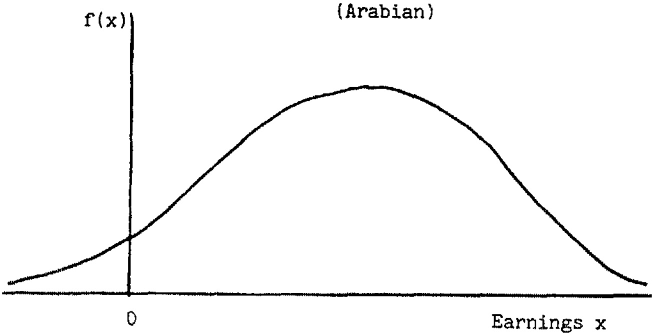
Fitting the same log-normal curve to those interest rates, I found the same good fit but the standard deviation had quadrupled.

We shall talk about distributions this morning. We are not going to spend a lot of time talking about the intricacies of negative binomials and how to derive them, but we are going to talk about distributions. If you are dealing with a manager who does not have a lot of mathematical background, he will not understand the mathematical underpinnings of frequency distributions, but he will understand curves, at least visually. You can say to him 'This is what your earnings could look like', and concentrate on that lower tail. (Illustration 4)

ILLUSTRATION 4

A Normal Distribution of Earnings

(Arabian)

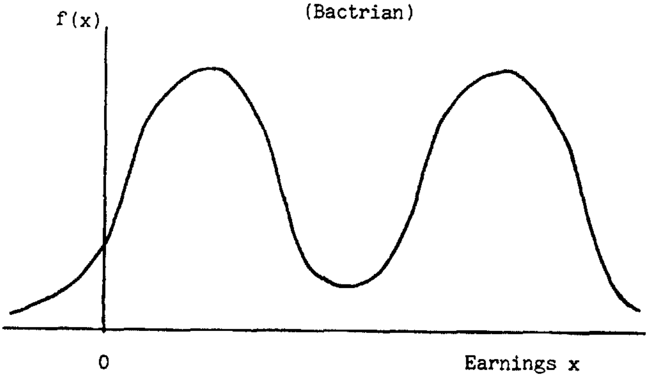


This I would call a 'normal' distribution of earnings. The pun is intended. I would also call it 'Arabian' after the one-humped camel, which is known as the Arabian camel. You might also, when talking to your manager, refer to what I would call an 'abnormal' distribution of earnings (or 'Bactrian' after the two-humped camel). (Illustration 5)

ILLUSTRATION 5

An Abnormal Distribution of Earnings

(Bactrian)



There are many situations where this might occur. One has to do with tax risks. If you're pricing a line of business that relies on a particular federal income tax treatment, the tax decision which the IRS hands down would put you either here or there without a more likely spot in between. So while the panelists will be talking about more rational types of distributions, keep in mind that we have irrational ones, too.

MR. H. H. PANJER: Thank you very much, Alastair. We've split our presentation into three parts. I guess I'm going to give you the good news, Nate Epstein is going to give you the bad news, and our wise sage, Jim Hickman, will probably give you the truth.

I. Introduction

Risk theory is a topic that has managed to terrify most actuarial students who wrote the Society's Part 5 examination since 1970. I wasn't one of those. I mean 'I wasn't one of the group who wrote Part 5 since 1970', not 'I wasn't one of the group who were terrified'. I wrote Part 5 in 1969. Risk theory was moved to Part 5 in 1970, so I managed to dodge it completely. There are my qualifications for speaking to you today.

I plan to discuss risk theory from several points of view. First, the role of mathematical model building. Second, some basic concepts of risk theory. Third, the aspects of the insurance business to which risk theory can be applied successfully. Fourth, the aspects of the insurance business to which risk theory has not been applied very successfully in the past. And lastly, how to get the most out of your young actuaries who know some risk theory.

II. The Role of Mathematical Model Building.

A mathematical model is a representation of a set of mathematical relationships that attempts to represent some phenomenon in the real world. Many people build models. We've all heard about dozens of enormous models of the United States economy and we've also observed how well they have worked for predicting even the short-term, let alone the long-term. This brings us to the major point of why we would, in fact, even be interested in building models. Models are used in some way to predict the future course of events. The models are typically based on the past - that is, a model is first constructed to fit past behavior of the phenomenon under study. The model is then assumed to apply to the future. Models may also simply be based on a set of axioms or premises about certain components or elements of the phenomenon under study. However, even in this case, the past is normally relied upon to estimate the value of any parameters in the model.

For example, reliability engineers or quality assurance engineers model lifetimes of components or devices such as computers or Boeing aircraft. They model the lifetime of the various components independently by either conducting some kind of accelerated experiments on the component or observing past failure times of the component, and they then assemble these models into a large model. They are typically interested in the time to breakdown of the computer or the airplane, and they make certain probability statements about the survivorship of the device.

Obviously, actuaries have been in this game much longer than reliability engineers. Statements about probabilities of survivorship of humans have been made since John Graunt's and Halley's studies of several centuries ago.

So model building is not new to actuaries. Life contingencies embraces a basic model. However, traditional life contingencies is based on a deterministic rather than a stochastic model. That is, the number of deaths or disabilities in a particular year is assumed to be given exactly by the mortality or morbidity table.

Risk theory deals with stochastic models. It attempts to model the stochastic nature of some phenomenon. It attempts to provide better answers to traditional questions as well as to provide answers to new questions that are not answerable through traditional life contingencies. For example, suppose you ask the question, 'Out of a group of l_{30} lives, how many will die before age 31?'. The actuarial student will likely say d_{30}. Clearly this answer is incomplete. A better answer would be, 'The number is a random variable with the binomial distribution with expected value d_{30} and variance $d_{30} \times p_{30}$'. As soon as we introduce random variables, we're using risk theoretic concepts.

Stochastic models describe behavior through systems of equations so that analysis of the process under study can be done analytically. Also, simulation is used to assist in the evaluation of these models. The mathematician's version of a well-known saying might be, 'An equation is worth a thousand numbers'.

III. The Basic Concepts of Risk Theory.

The basic model of risk theory has not really changed very much since it was first enunciated by Filip Lundberg in the first decade of this century. Now I'm talking about what we call collective risk theory. Parallel work was done by Erlange and Einstein in connection with modeling of other phenomena at about the same time.

This basic model is based on the Poisson distribution of the number of claims arising in a fixed time period for a portfolio of risks. That is, if you consider any period of time, the number of claims that arise in that period of time is assumed to have a distribution called the Poisson distribution.

Much work in the 1920's and 1930's was devoted to methods of calculating corresponding distributions of aggregate claims approximately. This was, of course, long before the advent of computers.

The concept of ruin theory was also developed early in the century. It attempted to measure the amount of surplus and/or premium loading necessary for a line of business to never be in a deficit position over a finite or infinite time horizon.

Development in the last few decades, and particularly in the last 10 years, has followed these two thrusts. Much work has been done to generalize the Poisson claim frequency model, as well as to generalize the model used to obtain ruin probability estimates. Indeed, each year since 1970 has seen the publication of dozens of papers on risk theory almost exclusively in the European literature. However, the basic concepts that were enunciated by Filip Lundberg have not really changed very much. Virtually all of the theoretical work in risk theory was carried out by European actuaries centered primarily in Scandinavian countries in the first half of the century. Risk theory did not really capture the imagination of the North American actuary until the 1960's, following the work of Dr. Paul Kahn, one of our own members. Currently, research work is being carried out around the world. There are active researchers in such diverse countries as Australia, Finland, Sweden, Denmark, Norway, Switzerland, the Netherlands, Belgium, Italy, Germany, Japan, the United Kingdom, and of course, the United States and Canada.

IV. Where Can Risk Theory Be Applied?

First of all, it should be said that risk theory should be viewed as only one of the tools in the decision-making process of the actuary. Although risk theory attempts to represent reality, it has not embodied many aspects of the real world, such as market conditions.

Risk theory can be used to model the occurrence through time of events such as the death of insured lives in a particular class, the onset of disability in an individual, admission to hospital of an individual, the occurrence of automobile accidents, fires in insured buildings, or of earthquakes.

In each of the above, some kind of claim-causing event occurs. The Poisson distribution, which is usually used to represent the frequency of such events, can be replaced or generalized in order to reflect observed frequencies, to reflect some uncertainty about the parameter values, or to reflect some contagion or non-independence that may exist. The binomial and the negative binomial distributions are natural alternatives for theoretical reasons, although many others exist.

The second element that must be modeled in order to model the risk process, is a distribution of the size of claims that occur. Currently more and more work is being done to develop and analyze various models of claim size distributions, or as casualty actuaries refer to it, claim severity. A forthcoming book sponsored by the Actuarial and Education Research Fund will deal with some of these models.

The two elements described above, claim frequency and the claim size, are then combined to study the distribution of the aggregated or total claims in a fixed time period or to calculate ruin probabilities or related quantities, such as net stop-loss insurance premiums.

Now to applications.

The current life insurance market is changing rapidly in North America. The prominence of term life insurance in both the individual and group markets, and the unbundling of the investment and mortality risks in products such as universal life, have characterized the current market. Risk theory is a tool that may be very useful in establishing how much surplus is necessary to support blocks of this high mortality risk business which does not have offsetting interest yield gains.

In Canada, valuation standards were modified in 1978. The new law allows actuaries to use realistic assumptions and a great deal of flexibility in valuing life insurance products. This means that there is very little margin in the reserves for any fluctuations in various factors. Risk theory is a useful tool in establishing contingency reserves or, as we call it in Canada, appropriated surplus, for any block of business, at least with respect to the mortality and morbidity risks. In fact, in some Scandinavian countries, solvency standards for insurance business are set out directly in terms of risk theoretic concepts.

Risk theory is also invaluable in making almost any decision in the area of group insurance, particularly when experience rating and stop-loss or excess-of-loss pooling arrangements are involved. Most calculations involving group insurance should become a matter of routine for insurers of all sizes, and without the use of Monte Carlo methods.

Risk theory is a useful tool in establishing an insurer's retention limit. In retention studies using risk theory, the actuary may be able to provide answers to questions such as, 'What are the chances that claims will be more than $5-million in excess of what we expect if we raise our retention limit to $2-million per life, or $1-million per life, or $500,000 per life?'.

Lastly, risk theory has recently been used to model the gains and losses of pension plans, although this application has not yet reached this continent.

As you can imagine from the list of possible items that I've given you, almost any decision in an insurance company regarding the risk being insured can involve some application of risk theoretic concepts. Except for direct applications in group insurance, risk theory is used in making broad management decisions necessary to operate effectively as an insurer. Using risk theory and its specialists may not increase sales or profitability immediately for a particular company, but it may assist in the overall management of risks.

V. Where Risk Theory Has Not Been Successfully Applied

Currently the single most important risk of an insurer is a risk of yield rates or invested assets falling and the risk of depreciation of asset values; in other words, the risks associated with the asset side of the balance sheet. Only recently have papers been written that deal with such modeling using risk theory. Even such papers make use of quite restrictive assumptions and it is difficult to accept the basic tenets of the models and the resultant conclusions obtained from the use of the models. However, the models are a significant step in the right direction. One of the basic differences of the asset side from the liability side is that risks on the asset side are not nearly as diversifiable as they are on the liability side. Several insurers can get together and pool or share risks to the mutual benefit of all concerned - that is, risks associated with mortality or morbidity. This is because it is assumed that the risks are independent or that a low degree of dependency exists among risks.

This is not true for the stock market. A particular stock's performance over time is correlated with the appropriate market index and with the performance of other insurers' stocks. If a share of value falls for one company, share values tend to fall for all companies. Risks are not reduced by pooling arrangements to nearly the same extent that they are for accidents or other risks. Whether there is a future in this kind of pursuit, I'm not sure. Perhaps persons in the audience would have some ideas on this topic.

VI. <u>How to Best Use Students Who Know Some Risk Theory</u>

In the future, actuarial students will know much more about probability, statistics in general, and about risk theory in particular, than his or her superiors. This is due in part to the upgrading of statistics, operations research, life contingencies, and risk theory currently under way in the associateship syllabus of the Society.

Having been grounded in probability concepts, most students should have a good grasp of risk theory to the extent that it's covered in the new syllabus of Part 5. This syllabus provides a theoretical framework for the subject but does not specifically address a lot of applications. This is appropriate since the associateship syllabus is meant to cover theory rather than practice. Any new associate with no perspective on corporate or clients' problems will not likely be particularly creative in applying risk theory to provide meaningful solutions to broad problems.

It should be stressed that risk theory is only one of the tools that a company may use in addressing a problem. Hence, any manager should not expect that a student well-grounded in probability, statistics, and risk theory will be able to provide complete answers. The key to useful application of risk theory lies in a team oriented problem solving approach. Team members with different perspectives on the problem under study must be able to agree on a common formulation of the problem in order to begin to apply any available theory. The young actuarial student with good technical skills will then be able to assist in solving the well-formulated problem. The student must also, of course, be able to communicate the concepts of risk theory and the details of his work in a non-technical way. This, of course, requires that the student understand the concepts and perspectives of his superiors and this, in turn, requires that he or she be closely involved in the formulation of the problem. This kind of learning environment will provide excellent management training for the student since he or she will gain insight into the nature of large-scale corporate problems and approaches to the solution of such problems.

As our keynote speaker, Dr. Labovitz, said on Monday morning, many of you are going to have to accept the fact that you will be managing students who are in some senses smarter than you. They will be the high tech kids who read the latest technical articles and computer literature and use jargon you have heard but do not necessarily understand. As Dr. Labovitz pointed out, participative management seems to be the key to good management and effective use of these kinds of high tech resources.

The world of insurance is becoming increasingly complex. More and more complicated models will be required to deal with future problems. The actuary is the professional with the most skills to address these problems. We must take up the challenge of the future by using the talents of today and tomorrow effectively.

MR. LONGLEY-COOK: Thank you, Harry. I hope that those with high tech students will use them as a very valuable resource. Nate Epstein will now give us a slightly different perspective.

MR. NATHAN H. EPSTEIN: "The subject of risk theory has been so bedeviled by elaborate mathematics that the ordinary practicing actuary tends to ignore the whole subject and rely on his instinct." This statement was made in 1947 by the British actuary, F. M. Redington, in a discussion of Irving Rosenthal's classic paper, "Limits of Retention for Ordinary Life Insurance." What Mr. Redington said in 1947 has much application today, 35 years later; however, there are some very important differences which I will discuss in due course.

In preparing for this panel, I did an informal survey of practicing actuaries to determine the current state of affairs with respect to practical applications of risk theory. I had expected to bring this audience a catalog of various applications and, indeed, I do have such a catalog. But as risk theory is the study of deviation from the expected, I think it's more important to first report on the current mood - the pulse and the attitudes of practicing actuaries on the subject of risk theory.

I started my survey by calling on the telephone. Without exception the conversation had three stages. The dialogue was something like this: EPSTEIN: Hi! I'm on a panel to discuss risk theory at the Washington meeting, and I'd like your help by getting what you're doing in your shop on risk theory." I was met first of all with humor. SURVEYEE: "Gee, sorry to hear that, Nate. How did that happen?" EPSTEIN: "Well, I got 'Kahned' into it by Paul Kahn, our Chairman of the Committee on Theory of Risk." SURVEYEE: "Who else was on the panel?" EPSTEIN: "The moderator is Alastair Longley-Cook." SURVEYEE: "Not 'the' Longley-Cook!" EPSTEIN: "No, but his son." "Jim Hickman and Harry Panjer." SURVEYEE: "Boy, you're going to need a lot of help!"

However, after the humor, came the apologetic stage - "well, I'm no real expert." "It's kind of theoretical" - "it's elaborate." Note the term "elaborate," the same term that Mr. Redington used. "We should be doing more." "I can recommend you to some people." I'm not an egghead as you know." "I am not really that good at high-powered math." "I have trouble figuring out the difference between random and non-random, but we have this actuarial student - very bright guy - no personality, but I think he can help you on it."

The third stage was, "well, we don't have anything really sophisticated, but we do have some applications." Indeed, I received a lot of material from these practicing actuaries. It's interesting to note that internal company memos on risk theory applications, written for a nontechnical audience, were very clear.

The mood around the country of practicing actuaries was one of recognizing the usefulness of risk theory, apology that they didn't know enough and the feeling that more needed to be done!

Nonetheless, the list of applications was large. So, getting back to Mr. Redington's comment on the Rosenthal paper, risk theory is not being ignored in 1982; it is alive, well, and getting stronger.

I think we have to ask ourselves why there are applications today. Why isn't risk theory being ignored today? In order to answer that question we must define a practical application. A practical application is the solution of a problem.. There are four criteria for the problem. First, you must recognize that you have a problem. The recognition that a problem exists is most fundamental. Secondly, you must be able to formulate that problem - to state it precisely. Thirdly, you have to determine whose problem it is? Is it the insurance industry's problem? Company management's problem? The government's problem? Then you must get involved with the appropriate bodies. If it's an industry-wide problem, the Society's Committee on Theory of Risk prepares a response. If it's company management, you have to get senior management involved. The fourth criterion is that resources exist that can be allocated to the solution of the problem.

Today the four criteria are being met. Today conditions exist that make solutions to the problems much more practical. First of all, what didn't exist in 1947 and does exist today is computer hardware. Anyone involved with computers knows how powerful they are. Secondly, we have some very powerful languages - the APL language, which was introduced to actuaries by the late David Halmstad, makes it very easy to program complex equations. Thirdly, the multi-line company has become more common today than it was in the past, and, with it, is the influence of the property/casualty actuaries who are trained in risk theory. In addition, other disciplines, such as investment, market research, operations research and systems design have all aided in making technical applications much more respectable. We also have the trained human resources within the Society to whom Harry referred.

What are some of the applications that are being used today? First of all, let me differentiate between what I call product line applications and management applications. The senior management in stock life companies is vitally concerned with GAAP earnings. The Society's Committee on Theory of Risk produced a monograph on adverse deviation. The SOFASIM model was developed for the committee and gives any actuary who wants to test his GAAP assumptions a powerful tool. Unfortunately, it isn't widely used as yet. But I believe as time goes by, and perhaps with a little more exposition, it will begin to be used more widely. I can't think of any more practical application or more useful application for management. Secondly, utility theory is being used in market research. This is an area where trained market researchers get together with actuaries to reach the marketplace. As Peter Drucker has said - business' primary goal is to "create a customer" - and uses of utility theory in market research are very fruitful. A third area is the whole area of surplus management - solvency, surplus allocation, and minimal surplus requirements. The fourth area is investment management of company portfolios and pension plan portfolios.

Product line applications are broken into three categories: 1) pricing, 2) claims distribution and 3) solvency of the line. An especially active area is the group health line. The group annuity line needs attention; and as Harry said, that is beginning to be done. In reinsurance there are programs to do stop-loss and catastrophic risk pricing. In life insurance, more sophisticated pricing models are being developed. The Society of Actuaries C2 risk committee is studying the risk of inadequate pricing in this very competitive environment. Concerning health insurance, the risk of becoming disabled today for a person between ages 25 and 65 is three times as great as dying. That risk, therefore, is becoming an important one in personal financial planning. As a result, the companies that are specializing in disability income are using risk theoretical models to come out with some very fine, sharp pricing. Of course, in the property/casualty line of business, much has been done. One need only read the Casualty Society's publications to see that they really are ahead of us. One area that hasn't used risk theory is the social insurance area. With the Social Security situation, I think that's one area that is ripe for the application of risk theory.

While much work has been in the field of insurance products, a major impediment to future progress lies in the fact that the mathematics is still "elaborate" as Mr. Redington said. The practicing actuary is uncomfortable with it. The root of the problem lies in the dichotomy of nature of actuarial work and the nature of those who write mathematics texts.

In their classic text on Monte Carlo methods, Hammersly and Handscomb point out a new way to classify mathematicians. The old way classified them as "pure" and "applied". Hammersly and Handscomb classify them as "the theoretical" and "the experimental." "Theoretical" and "experimental" are independent of whether the objective is pure or applied. Theoreticians deduce conclusions from postulates whereas experimentalists infer conclusions from observation. It is the difference between deduction and induction. Actuaries are basically experimentalists and induction-type mathematicians. Much of the current literature on risk theory has, however, been written by the theoretician in formal deductive language. What is needed to overcome "this bedevilment of elaborate mathematics," of which Redington spoke, is good elaboration, good elucidation, and good exposition by induction-type mathematician a la Polya and my colleagues on the panel.

At the student level, I think the Society has already made great strides. The new study note is a real step forward in terms of going towards better elaboration. I read the text and I didn't have as much trouble as I thought I would with it. It's a very good text. Chapter 1 deals with utility theory - the theoretical basis for the proposition that insurance systems will increase the total welfare. The chapter on individual risk models is written in clear language. The two chapters on the collective risk model are well motivated and replete with examples. The applications chapter which, by the way, deals with two casualty applications - fire insurance and auto physical damage, and two applications that the life insurance actuaries would be more familiar with - short-term disability and hospital insurance - were very well done and point out that the core of actuarial mathematics is risk theory. I think we've made great progress at the student level.

At the post-fellowship level, more must be done. I think we need some reading lists, annotated bibliographies, more expository writing on an introductory level. There's a very fine text - The Huebner Foundation Monograph 8 - "An Introduction to the Mathematical Theory of Risk" by Hans U. Gerber. It's very good in the sense that he boils down the mathematics to three chapters - Distribution Theory, Stochastic Processes and Martingales. It's a little heavy - and it's formal - and it's theoretical in that it's deductive, but at least you have three chapters that really say it all with the balance of the book being applications. If somebody would write those chapters from an inductive point of view, with a lot of motivation, I think he would greatly aid the practicing actuary.

Professionally, the Society's Committee on the Theory of Risk has published a very fine monograph on adverse deviation. It has on its 1983 agenda: 1) pension modeling, 2) a theory of risk bearing that will develop a taxonomy of risk, and 3) the Society's C2 risk. I look forward to the publication of well written monographs on the results of these researches.

In summary, the practicing actuary has not ignored risk theory in the thirty-five years since Mr. Redington made his remark. This has been due to the recognition that problems needed to be solved, commitments for resources were made, and computer technology and knowledgeable people were available for finding solutions. Yet the mathematics is still elaborate so that good exposition is now necessary more than ever before. The Society has made progress for pre-fellowship students with the new study note. A clear, lucid text for the practicing actuary still needs to be written. It should fill the gap between the study note and Hans Gerber's very fine text. I leave that challenge to my very able colleagues on the panel.

MR. LONGLEY-COOK: I would like to second Nate's comments on the risk theory study note which I find an excellent introduction to those who are a little rusty. As he mentions, utility theory leads it off, then you quickly get into the kinds of applications that Nate enumerated. Your students will be familiar with this material (or they will at least be tested on it). If you can spend some time with the subject material yourself and look into some of the other source books Nate mentioned, I think you'll find it well worth your time. As I mentioned, one of the co-authors of that study note and the actuarial mathematics textbook is with us this morning and that is Jim Hickman.

MR. JAMES C. HICKMAN:

1. The Singularity of Risk Theory

Risk theory is the singular point of actuarial science. This proposition has a corollary. That is, risk theory is what prevents actuarial practice from being a subset of accounting, life office management or some other business occupation. Of course, I do not mean that risk theory is for all practicing actuaries the most important of their intellectual tools. That clearly is not the case. However, when one attempts to state with precision the core of actuarial science, one comes very close to a definition of risk theory. We might have, risk theory (actuarial science) is the construction of models by which deviations from expected financial results can be studied. These models may also serve as guides to organizing and managing insurance systems to reduce the adverse consequences of these deviations.

2. History

To some, risk theory is the study of the compound Poisson process. This is a narrow view and is not in conformity with the history of the development of actuarial science. A mathematics of life contingencies in which premiums and reserves emerge naturally from the requirement that the expected present value (actuarial present value) of future losses (claims less premiums) plus reserves equal zero, was developed in Europe in the nineteenth century. Seal's book (4) provides references by which one can trace this development.

Although I cannot prove my conjecture, I have been intrigued by the fact that the methods and vocabulary of individual risk theory, as developed in Europe in the last century, reappear in statistical decision theory which has developed during the past forty years. That is, both statistical decision theory and individual risk theory start with the formulation of a loss function which is called a risk function in decision theory and the value of a squared loss function which is called mean squared risk in classical individual risk theory. Within both theories the goal is to control, and perhaps minimize, the risk (expected loss). Abraham Wald (5) was one of the founders of statistical decision theory. He was a Hungarian who came to the United States in the late 1930's. I have suspected for a long time that he picked up some of the ideas for statistical decision theory from the earlier individual risk theory, which was well known in central European universities.

The story of the almost simultaneous development of the compound Poisson process as a model for an insurance company by Lundberg and as a model for a telephone exchange by Erlange is well known. The parallel and almost independent development of risk theory and queueing theory for many years and their later mutually beneficial exchange of ideas has been a favorite topic of Seal (4).

3. Non-use

If risk theory is the singular aspect of actuarial theory and if it has a long and rich intellectual history, why has its impact on the daily practices of actuarial science been so small? This valid question does not have a simple answer. My response will be organized as three separate points.

(a) Technological. Computing second moments (variance) in addition to first moments (expected values) of actuarial loss functions involves more computation. Until recently the practice of actuarial science was restricted by the cost of computation. Many of the computing formulas and approximations have been such an important part of actuarial practice that it is easy to believe that these computational methods are the essence of the science. Today we are largely free of the constraint of computational cost. We can return to first principles and begin to model the distribution of losses.

(b) Educational. Despite the fact that the rudiments of risk theory, including the most useful aspects, are accessible to those with basic training in mathematical statistics, it has only been in recent years that risk theory has been included in the prescribed study program for actuaries in North America.

(c) Theoretical. Almost from the beginning of risk theory, insurance executives have asserted that building models to study random deviations of claims experience from that which is expected does not provide answers to their most pressing problems. In a word, random deviations are almost never the cause of the insolvency of an insurance organization. Of course, this criticism is potentially devastating. It is acknowledged that elements of the criticism remain valid. However, there are several counter points which need to be made. First, because a satisfactory theory for a global problem does not exist, one should not use this as an excuse for not employing the best available model for an important component of the problem. Second, recent technological (competing) and theoretical advances have considerably broadened the scope of risk theory. For example, the assumption that periodic insurance gains are mutually independent random variables, which was required in earlier developments of bounds on the probability of ruin at some future time, has now been relaxed. For a development of a bound where it is assumed that gains follow an autoregressive moving average process, see Gerber (2). Another example of recent advances is the combination of stochastic models for interest and claim processes as shown by Panjer and Bellhouse (3).

4. Example

I would like to present a case study on the application of risk theory in the management of a small insurance system. The case is remarkably simple. It bears almost no resemblance to a company engaged in several lines of business. However, the ideas are applicable to managing particular groups or lines of business that are part of a larger business.

The basic facts are as follows:

a. Identification
University Faculty Association, University of Wisconsin System, administered by National Guardian Life Insurance Company.

b. Members
Approximately 12,800 faculty and academic staff of the University of Wisconsin System. Active faculty and academic staff are members as a condition of employment. Former faculty and staff may remain as members by paying premiums.

c. Actuarial basis
Each member pays $124 per year. The individual loss function for a member age x is

$$L_x = 0 - 24, \text{ if life survives,}$$
$$= B_x - 24, \text{ if life dies.}$$

The equivalence principle yields

$$B_x = 24/q_x$$

and

$$\text{Var}(L_x) = (B_x)^2 p_x q_x = (24)^2 p_x/q_x.$$

d. Current information (approximate)

Scheduled amount of insurance in force	$290,000,000
One-year term dividend	116,000,000
	$406,000,000
Assets	$ 11,900,000

e. Management goal
To provide the maximum amount of term insurance protection for the fixed premium subject to the continuance of the association. Therefore, the basic benefit schedule, or the one-year term insurance dividend, is changed when it can be done consistent with the goal of survival.

f. Probability of ruin
(i) One year ahead

$$\text{Var}(\text{sum}L_x) = (24)x(\text{sum}(24/q_x)p_x)$$
$$\leq (24)x(\text{amount of insurance})$$
$$(\text{Var}(\text{sum}L_x))^{.5} \leq 98{,}700$$

If it is assumed that total loss ($\text{sum}L_x$) has an approximate normal distribution, the probability that annual losses, in excess of annual premium, will exceed
$u = (2.58)x(98{,}700)/(255{,}000)$
is less than .005.

(ii) Infinite time horizon
It will be assumed that annual gains are independently and identically distributed. The probability of ruin at some time in the future is equal to or less than exp(-uR) where u is the current risk reserve and R is the positive square root of

$$M_G(-R) = 1.$$

In this expression, $M_G(t)$ is the moment generating function of annual gains. In the case of a model with a negative binomial distribution of number of claims and individual claim amounts with a gamma distribution, an approximation for life insurance has been obtained by Ammeter (1). The u is set so that the probability of ultimate ruin is less than .005. The resulting formula is

$$u = (.025/a)p^t + (8/a)S_1,$$

where p^t is expected losses (net premium) with the safety loading, a is the fraction of expected claims devoted to the risk reserve, and S_1 is the average claim amount. In this case, p^t = 300,000, S_1 is approximately 16,800 and

$$u = (141,900)/a.$$

The following table can be constructed.

a	u
.05	2,838,000
.10	1,419,000
.20	567,600
.50	283,800

Since most of the interest income on the assets, of approximately $1,900,000, is available for addition to the risk reserve, it is clear that the surplus is adequate to sustain the present benefit scale.

REFERENCE

1. Ammeter, Hans "The Solving Problem in Risk Life Insurance", reprinted in ARCH 1972.3. The same ideas are discussed in RSA Vol. 1, no. 1 pages 153-155 and RSA Vol. 1.1 no. 2, pages 375-402.

2. Gerber, H. U. "Ruin Theory in the Linear Model", Insurance: Mathematics and Economics, Vol. 1, (1982), pages 177-184.

3. Panjer, H. H. and Bellhouse, D. R., "Stochastic Modelling of Interest Rates with Applications to Life Contingencies", JRI, Vol. 47, (1980), pages 991-1110, and "Stochastic Modelling of Interest Rates with Applications to Life Contingencies Part X", JRI, Vol. 48, (1981), pages 628-637.

4. Seal, H. L., Stochastic Theory of a Risk Business, (1969), John Wiley and Sons, Inc., New York.

5. Wald, A., Statistical Decision Functions, (1950), John Wiley and Sons, Inc., New York.

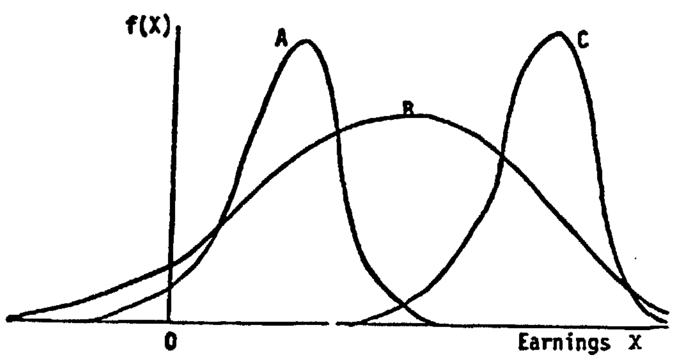
MR. LONGLEY-COOK: Thank you, Jim. That case study I find intriguing. As Jim says - it's a microcosm. If you're dealing in a large corporate structure, that same problem in slightly different forms is becoming increasingly more important. As surplus is strained further and further, and as managers are being asked to drive their surplus harder and harder, I think the type of work that Jim has outlined for us will become more and more part of our daily lives.

Since our panelists have kept very diligently to their schedule, I would like to take a couple of minutes to talk about an application that I found to be very helpful to me in my work, and then open it up for questions.

As Nate said, with an inductive approach, you start out with a problem - the problem I was facing was to try to measure the profitability of various different lines of business. Looking ahead, if you take the risk theory approach, you don't have different expected earnings numbers, you have different distributions of possible earnings. Here are three at which we might be looking. (Illustration 6)

ILLUSTRATION 6

Three Distributions of Earnings



If we're choosing between two different products, or two different lines of business, B and C, then the decision would not be very difficult. Obviously, with the higher mean and the smaller standard deviation, it's not too hard to go with C if you were given that choice. The choice between A and C is also fairly routine. And frankly, that's what most people look for - 'Which has the higher expected value of earnings?'. The usual management goal, if you can put it down mathematically, is to maximize the expected value of earnings.

However, if you were given the choice between A and B - the choice becomes a little more difficult. Does the higher mean of B compensate for the greater standard deviation?

The choice that managers face all the time is the choice between A and B. How you determine which one to take is really wedded to your degree of risk aversion. In order to get a handle on your degree of risk aversion, utility theory provides you with a mathematical discipline. Usually you have to make various assumptions to use it, but it is better to build on the sand than the void.

Now, the usual way to measure prospective earnings is by the use of some rate of return, such as ROSHE, return on shareholder's equity. If you're using it prospectively, you should talk in terms of its expected value, because you don't know what it is yet. (Illustration 7)

ILLUSTRATION 7

$$E(ROSHE) = \frac{\mu}{u}$$

where, μ = expected earnings for the line of business,

u = surplus allocated to the line.

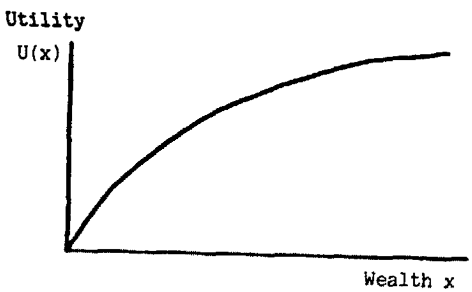
Using that concept, we have expected value of earnings divided by the surplus which is allocated to that line (or if you're talking about an entire company, the entire surplus).

As you can see, we are still on the two parallel tracks. We are still looking at only expected values and trying to maximize them. It is true that if the surplus has been allocated properly, there will be more surplus in the denominator if the variance is expected to be greater than for other lines. That will drag down your E(ROSHE) and adjust for risk. But often the allocation of surplus is driven by other considerations, not necessarily completely in line with the actual risk of that particular line of business.

Bringing utility theory to bear on the problem, we bring the effect of these variations directly into the numerator. If the company is risk averse then each additional dollar is worth less. Conversely, as it gets poorer and poorer, it is more and more concerned about each dollar it loses. (Illustration 8)

ILLUSTRATION 8

Risk Averse Utility Curve



If we now map our earnings distribution through that curve so that instead of taking expected value of earnings, we take the expected value of the utility of earnings, we bring into play the effect of variations. We can now look at a concept I've coined REROSHE for risk equivalent return on shareholder's equity. It calculates the risk-free investment rate to which our rate of return is equivalent on a utility basis. In other words, given a choice between investing in that line of business, or investing in totally risk-free investments, what would be the equivalent risk-free return on a utility basis? (Illustration 9)

ILLUSTRATION 9

$$\text{REROSHE} = \frac{\mu - \frac{1}{2} a \sigma^2}{u}$$

The result, making some simplifying assumptions, turns out to be quite meaningful. You get your expected value of ROSHE back again, but you're subtracting out what I call a risk-adjustment factor, which is dependent upon the variance (that makes sense because the greater the variance, the more it would drag down your equivalent ROSHE) and it's also dependent upon a - your degree of risk aversion (the concavity of your utility curve).

This allows you to set risk margins; it allows you to decide which line of business is indeed the better one in which to be; and make decisions of that nature.

It also allows you to get a handle on what your value of a is. One of the reasons utility theory has not been used very much is that it's very hard to determine just how risk averse a company is. Through use of REROSHE and looking at what the company has been doing in the past, you can get a handle on just how risk averse it has been and, therefore, make some projections based on that value of a.

I have written a paper on REROSHE, and I'll be glad to send it to anyone who sends in their business card. I am hoping that it will be used as a management tool to help us analyze business risks and make decisions.

I am very grateful to the three panelists who have spoken this morning. At this point, I shall entertain questions from the audience.

MR. DONALD D. CODY: I'm a consulting actuary but, more importantly, I'm the Chairman of the Society's Committee on Valuation. The most serious problem facing the industry today is the determination of the contingency surplus needed for the various risks of each insurance company. This problem is important, not only because it deals with the solvency and viability of insurance companies, but also because we are under a very tight time schedule brought upon us by the economic environment. It is very likely that valuation actuaries will be given the responsibility of looking at the extent to which asset and liability durations are matched. If we don't accomplish that, I would anticipate that the regulators will move up the level of reserves, freezing out the flexibility that insurance companies will have by the very existence of surplus where the dollar has many uses, rather than being locked up in reserves. So this Committee has the responsibility of developing the knowledge and seeing that the education of actuaries is developed and guiding the Academy and Technical Advisory Committees of the AIC in the utilization of this knowledge.

The area covered is known as the C1, C2 and C3 risks. C1 risk is the risk of defaults in bonds or the variations in the stock market. C2 risk is the risk of inappropriate pricing assumptions. C3 risk is the risk of changes in the interest environment.

About all you can do is try to keep companies away from the edges. There are ways of doing that. The way we approach this is to run very large computer programs, either for individual products or for whole companies consisting of single products through a complete universe of interest scenarios. There's no way of indicating the distribution of economic conditions. You can look at the past and you can form judgments as to how bad it can get - I think it has more chance of getting worse than it has of getting better.

This all reminds me of the great heyday of operations research during the Second World War, when the timeframe was very short, the problem was very important, and we had the best academic minds in the country working on the problem and we did the best we could. It was pretty good indeed because we didn't have time to do it too elaborately.

The C2 risk can be handled with respect to death benefits. You also get into medical care and disability income and some of the casualty area. The theory may not be very bad because you have wild cards there - catastrophe-type risks like inflation and the tendency of the federal government to shift medical costs to the private sector. We have all the theory we need and the part that can be handled stochastically and the wild cards have to be handled in the best way we can; deterministically, if you will.

The C1 risk can be handled deterministically - this method is credible because your investment officers do it. They determine the amount of money they need for the ruin from irrecoverable losses from bond and mortgage defaults in real estate, common stocks, and so on. You can also develop a stochastic model - it's dominated by the wild card - the one great claim like a depression into which you fall. But you can equate the wild card to what you get deterministically and get some idea of the variation, which is very comforting.

The point I'm making is that there's a risk that you can handle by traditional theory in the life insurance area. Those things can be handled if you know that they can be handled. The others - you have to do the best you can.

Now, the problem that I have talked to a number of you about and which I still present to you, is that you have a combination of stochastic risks, catastrophe risks, and deterministic risks, some of which are determined by a universe of scenarios. In order to put them together to get a total surplus need, you must have some idea of the probability law we are talking about. Despite the fact that the theory doesn't allow for this, we must do it, and we must do it quickly as well as we can.

So this is the kind of problem that we face - risk theory is indeed very valuable - in some things you have to use judgment because they're wild cards and depend upon political influences. At any rate, I thought I'd put that out on the table as the real world today for your consideration. Perhaps you'd like to comment on it.

MR. LONGLEY-COOK: The work of the C1, C2 and C3 Risk Task Forces is bringing up considerations that have not been addressed. Jim's example and Harry's method of determining surplus are each a microcosm. We have too long considered surplus as being free surplus, something that we could take out at any time. However, if you follow through the risk theory methodology or do the stochastic modeling, you find it is very necessary - an inseparable part of the financial structure. Trying to get a handle on how big surplus should be in relation to your risks is what those task forces are all about and indeed what a lot of risk theory is all about.

MR. ALLAN BRENDER: My impression is that one of the main reasons we're here is to discuss the fact that risk theory exists and in fact can be used. In particular, I'm concerned with this whole question of students and of how managers will use them. There's an analogy here. There are still many managers around who are not terribly comfortable with computers, but at least they know what the computers can do for them today. A lot of people have learned that they don't have to know how to use computers to be able to manage people who can produce the answers.

I think that risk theory has reached that stage where managers have to find out what techniques are around - what kinds of problems they can solve. The managers in fact know their own problems. The rest of the job is to find somebody who has some technical ability. I don't think a manager has to understand the mathematics. He just needs confidence that he knows what his problems are and that hopefully there are mathematical techniques around, and his job is to hire people who can find and use those techniques. The students we produce wouldn't know the models, but they ought to know the problems. They also won't have any sense most of the time as to really how good the models are. The whole idea then is to train the student to appreciate the model and train the manager to appreciate the kinds of answers he can get. The changes that are coming up in the syllabus in operations research, statistics, and risk theory tell me that people are going to have all kinds of technical resources around their companies and they should use them. They should just appreciate that the resources are there and have the courage to use them without necessarily understanding the mathematics behind them.

MR. HICKMAN: I wouldn't want to argue with you but I think your computer analysis is liberal. There have been billions of dollars wasted in American business because top management either undersold or underestimated the profound impact computers could have or they underestimated the complexity of creating the computer programs. The next generation of managers has to be much more acquainted with probabilistic methods and probabilistic ways of thought, or they're sunk! They won't be able to effectively use these vibrant people you're talking about and they should not take it completely on trust. They've got to ask some of the right questions. They cannot be experts at programming, nor experts in probability and statistics, but we've got to raise that level of general management understanding. I think your example of computers is a good one. We wasted a lot of money in American business because the top people didn't understand what those computers could do for us.

MR. BRENDER: I agree. I think that the new syllabus will produce the kind of people that you're talking about. I'm just much more concerned with what happens in the next 5-10 years before those people are really out and doing the managing. We need to have people today using what's available.

MR. EPSTEIN: I'd just like to comment that I get turned off when I hear the phrase 'managers don't have to know'. I recently read a study that reports that 15% of the Fortune 500 CEO's are Ph.D's. I think 90% have Masters degrees. You only have to be acquainted with some CEO's, for example, L. E. Goldstone of Eastern Gas & Fuel and Dr. Wang of Wang Computers, to see that managers have to know. I think the real lesson is that those who don't know won't be managers for long. What has happened in this industry is that we must be sharper.

MR. HAROLD N. LUND: One thing that was just briefly touched on today is the contagion element. I'd like either Dr. Hickman or Dr. Panjer to comment on how one accounts for contagion elements.

MR. PANJER: Contagion is probably the most difficult part in the model. There aren't really very well known models or ways of modeling contagion other than the linear model, which leads you to the negative binomial distribution. That's a very difficult problem. The non-independence of claims in catastrophe modeling is also a difficult problem.

MR. HICKMAN: On the claim number side, of course, there's a whole family of contagion models. It's perhaps best developed for actuaries in a single source in the Buelman book. You start with a kind of Poisson model, then you 'muck it up'; the Poisson parameter may be drifting one way or another, depending upon what's happened before. The theory is quite well known and they're not all that hard to use.

The question of independence between claim numbers and claim amounts is somewhat harder. You certainly can formulate it mathematically because the generality of claim numbers does get in kind of formula form, although it is difficult. Another question of contagion I alluded to is that, if you look simply at total gains for a line, most of the old models required independence across years to get either asymptotic or finite ruin. These have been pretty well removed by several developments, perhaps the most accessible one being in Hans Gerber's paper (2). It is true that Hans restricts himself to the autoregressive moving-average family, but that's a pretty rich family and can encompass a lot of dependence across time. Certainly the problem isn't solved, but we're making progress.

MR. JOSEPH J. BUFF: I'd like to share briefly some of my experiences which I think tie into things I've heard some of the panelists say, especially Nate's comments on the question of the psychological self image. I think that a manager in any technical field, whether it be aerospace technology or insurance company work, will continually find new generations of students knowing more technical things than he knows, simply because they're in the educational mainstream. This puts two responsibilities on the manager.

The first is to try to understand the new techniques at least to the point where he has some idea of what his workers are doing. He just pushes buttons, they go into their offices or sit at their desks, come back and give him some numbers. He must have some understanding of the techniques, have some idea of what's going on.

The second concerns the use of jargon. I think that an actuarial manager is in no better position and certainly no worse position than any other manager to enforce on his subordinates an abandonment of unnecessary jargon. We as actuaries have our own pretty terrific jargon, and if we allow our students to start using this against us, it's going to harm our effectiveness as managers. I think we have to be aware of that risk and firstly, not be threatened when we see students throwing around technical phrases that we don't understand or have never heard before, and secondly, do what I think is the naturally right thing to do, which is to ask for an explanation. We should get these students to practice their communication skills because 20 years from now they will be trying to communicate with senior VP's or presidents. They have to start to learn to communicate with their own managers who may be relatively young or middle level fellows themselves, and if you create this environment, these young people coming into the companies will develop the right habits of teamwork with other students and with the managers. Then we won't have a lot of these problems institutionalizing themselves. If the jargon problems become institutionalized there's going to be tremendous resistance to change and all these technical methods are going to be met with resistance. The students will become impatient. There'll be turnover and all kinds of other problems and the company won't take advantage of the resources.

MR. LONGLEY-COOK: I agree. I think jargon is perhaps the biggest barrier to becoming familiar with the topic. Jargon is used for various purposes. One is to simplify things. Another is to make what you are saying sound more important. If we can eliminate the latter, then we'll all benefit.