TRANSACTIONS OF SOCIETY OF ACTUARIES
1992 VOL. 44

INTERNAL RATE OF RETURN AS AN EVALUATOR OF TAX-PLANNING STRATEGIES

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ABSTRACT

This paper develops an analytical method for evaluating the utility of tax-planning activities that cause initial statutory surplus strain. Many companies would find certain tax-planning activities desirable were it not for the effect on statutory surplus. The application of the concept of Internal Rate of Return (IRR) to invested statutory surplus is well-known to pricing actuaries. This paper expands the horizon beyond normal pricing applications to the evaluation of other activities requiring the initial investment of statutory surplus.

The 1984 Tax Act has given some impetus to this type of activity because, beginning with tax year 1984, tax reserves are computed on a basis different from that for statutory reserves, opening up potential planning situations not previously available.

Also briefly discussed is a determination of the rate of return that should be imposed as a quantitative standard for any use of capital. This hurdle rate concept is then applied to tax-planning strategies.

I. INTRODUCTION

Many tax-reserve-planning strategies involve initial increases in statutory liabilities to obtain beneficial tax-timing differences. Such excess statutory liabilities and tax reserves will reverse over some future time horizon. This paper attempts to provide insight into the measurement and evaluation of those strategies.

Most actuaries are familiar with the concept of IRR; it is typically used in the pricing process and in appraisal work. Concepts of IRR have been used extensively for many years in product development, assumption reinsurance, and company purchase activities and often have been addressed in professional publications, perhaps most notably in the paper by Anderson [1]. The entire financial management process of a business enterprise arguably involves IRR concepts at every material decision point, and any large expense or any activity that necessitates a large initial statutory surplus strain should undergo an IRR consideration.

For those unfamiliar with the concept and because it may mean different things to different people, let us first define IRR generically. For simplicity, we are speaking of an environment in which, as a result of a planned activity, there is an initial statutory surplus strain (an investment of statutory surplus) followed by subsequent contributions to statutory surplus.

Let us simplistically describe such an environment. A company has one line of business (LOB) and a corporate account, containing free statutory capital and surplus. At each year-end, any assets in excess of statutory liabilities and required surplus are swept into the corporate account, and any LOB deficiency is met by a transfer from the corporate account. Assuming the company engages in a given activity, the IRR is simply a rate of return that equates the present value of activity-driven infusions [from the corporate account into the LOB] with the present value of subsequent activity-driven sweepings back into the corporate account. However, in our simplified environment, we are assuming that such an infusion from the corporate account only occurs initially, and that all subsequent year-end computations result in contributions to the corporate account.

As an aside, consider the perspective of an outside investor who is offered an opportunity to invest $100 in a company to support a particular activity. Assume that the $100 investment is immediately utilized by the company's activity, so that its capital and surplus are immediately back where they would have been without that activity; in future years, as a result of that activity, additional profits are made and additional contributions to free surplus (dividendable surplus) take place. Assume also that the outside investor has contracted to reap the reward of those additional contributions to free surplus as they occur. The investor can thus calculate the rate of return on the $100 originally invested, such that the present value of rewards the investor receives will exactly equal the investment of $100, at the IRR. To summarize, the infusion from that outside investor to the corporate account to develop and sell a product, exactly offset by a first-year statutory loss due to development and sales of that product, will [hopefully] result in subsequent years in eventual marginal "sweepings" into the corporate account that go to the outside investor. Measurement of the initial infusion against those subsequent "sweepings" results in that unique IRR that causes the net present value of the "sweepings" to equal that infusion. [The "sweepings" concept is used to clarify that interest on the surplus thus generated is not part of the IRR equation.]

When measuring the rate of return on such investments, it makes no conceptual difference whether monies to finance an activity emanate from an initial *external* infusion, as in the above scenario, or from an initial *internal* contribution of surplus from the corporate account. The internal contribution is the typical IRR activity undertaken in today's environment.

The building blocks for measuring the IRR resulting from an activity are the book profit [$BP(t)$] components, with $BP(t)$ defined as follows:

$$BP(t) = Req(t-1) \times (1+i) + CF(t) - Req(t),$$

where

$Req(t)$ = Required cumulative contribution to the LOB (in terms of statutory liabilities plus any surplus required to be maintained in the LOB)

i = A suitable after-tax interest rate

$CF(t)$ = After-tax cash flows, excluding investment income on $Req(t-1)$.

For a time horizon of n years, solve for the value of v, and consequently the IRR, that resolves the following equation:

$$\sum_{t=1}^{n} (v^t)(BP_t) = 0.$$

Then IRR $= \frac{1}{v} - 1$.

II. GENERAL APPLICATION TO PLANNING STRATEGIES FOR TAX RESERVES AND RELATED ITEMS

Now let us make our definitions more specific to tax-reserve-planning strategies. Suppose that a company is designing a new product and can include a feature which is not expected to add measurable value to the product, but which will increase both statutory and tax reserves. For example, the inclusion in a single-premium deferred annuity of a conservative "bail-out" provision that would waive the contractual heavy initial surrender charge should the crediting interest rate—currently 7.5%—fall below 6%; this provision would cause a material increase in both statutory and tax reserves, as NAIC Guideline XIII [2] makes clear, and therefore has tax-planning significance. Ignoring the marketing considerations of this product design feature, a determination of the IRR associated with the higher reserves should be an element in the company's decision whether to include it.

We define the marginal book profit stream, in a stock life insurance company, resulting from a tax-revenue-planning strategy, as follows:

$$DBP(t) = DRS(t-1) \times (1+i) - DRS(t) + TR \times [DRT(t) - DRT(t-1)] \quad (1)$$

where

$DBP(t)$ = Difference in $BP(t)$ resulting from the strategy
$DRS(t)$ = Difference in statutory reserve resulting from the strategy
$DRT(t)$ = Difference in tax reserve resulting from the strategy
TR = Tax rate.

Note that the following simplifying assumptions have been made:

- The company is fully invested in fixed-income, fully taxable instruments.
- The company is a stock life insurance company.
- The investment income rate is level and wholly classified as ordinary income.
- The income tax rate is level and taxes are paid at end of year.

Arguably, a company can find itself in any of three different economic situations when confronted with a potential tax reserve strategy that requires an investment of statutory surplus:

1. The company has an abundance of surplus and capital.
2. The company is thinly capitalized and critically needs to protect current surplus, relegating long-term tax consequences to a lower priority.
3. The company is in a middle ground: it wishes to invest surplus in tax-reserve-planning strategies that yield a sufficiently high IRR and will discard those where the yield is below such IRR benchmark.

This paper speaks primarily to the third category of companies, only secondarily to the first category, and not at all to the second category. The reasons should be obvious: a company in the first category is in a good position to undertake virtually any tax-reserve-planning strategy that satisfies other criteria, and the temporary surplus strain should not be important; a company in the second category cannot afford any initial investment of surplus at all; a company in the third category, where most companies find themselves, should test its after-tax IRR from a tax-planning strategy against its own "hurdle rates" used in pricing and in other activities.

Now let us explore the IRR resulting from certain specified types of tax reserve strategies.

III. THE CASE IN WHICH THE TAX RESERVE DIFFERENCE EQUALS THE STATUTORY RESERVE DIFFERENCE

Let us assume the simple case in which a planning strategy increases tax and statutory reserves on a dollar-for-dollar basis and takes n years to reverse. Assume further that in each year through the n-th year, the strategy changes both tax and statutory reserves in like amounts. In such case the *post-tax* IRR can be shown to be equal to the company's *pre-tax* investment earnings rate, as follows:

$$\begin{aligned} DBP(t) &= DRS(t-1) \times (1+i) - DRS(t) \\ &\quad + TR \times [DRS(t) - DRS(t-1)] \end{aligned} \tag{2}$$

from Equation (1), substituting the statutory reserve difference in place of the tax reserve difference. Thus,

$$DBP(t) = DRS(t-1) \times (1+i-TR) - DRS(t) \times (1-TR).$$

Multiplying by the expression $[(1-TR)/(1+i-TR)]^t$ and summing from years 1 to n, we obtain the following (assuming for the moment that such expression is a discounting factor):

$$\begin{aligned} PV(DBP) = \sum_{t=1}^{n} \Bigg\{ & \frac{DRS(t-1) \times (1-TR)^t}{(1+i-TR)^{t-1}} \\ & - \frac{DRS(t) \times (1-TR)^{t+1}}{(1+i-TR)^t} \Bigg\}. \end{aligned}$$

This is the sum of a difference, and since $DRS(0) = DRS(n) = 0$, the expression equals zero. But $(1-TR)/(1+i-TR) = 1/[1+i/(1-TR)]$. Therefore,

$$\mathrm{IRR} = i/(1-TR). \qquad \square \tag{3}$$

IV. FURTHER GENERALIZATION OF THE CONCEPT

The above result can be easily expanded to the case in which the tax reserve difference is a constant ratio to the statutory reserve difference.

Assume that the tax-reserve-planning strategy causes the ratio of $DRT(t)$ to $DRS(t)$ to be equal to K; that is,

$$DRT(t) = K \times DRS(t).$$

In such case,

$$DBP(t) = DRS(t-1) \times (1+i) - DRS(t) + TR \times K \times [DRS(t) - DRS(t-1)] \quad (4)$$

from Equation (1), substituting [K times the statutory reserve difference] in place of the tax reserve difference.

Now, by simple analogy with the formulas in the scenario in Section III and the resulting Formula (3), that is, replacing TR with $TR \times K$ in Formula (2), the IRR becomes:

$$\text{IRR} = i/(1 - TR \times K). \quad (5)$$

Thus, for example, when a strategy causes a tax reserve difference to be twice the statutory reserve difference, with a marginal tax rate of 34 percent and an after-tax interest rate of 6 percent, the IRR becomes 18.75 percent, a very desirable situation indeed.

Note a few characteristics of this rather elegant result, for the scenarios given:

- The IRR does not depend on the number of years for the differences to reverse.
- Even if K is less than 1 (that is, where the tax reserve difference is only a fraction of the statutory reserve difference), the after-tax IRR still exceeds the after-tax interest rate on free surplus, still a desirable alternative strategy for a surplus-rich company.

Table 1 illustrates the concept using the deferred annuity referred to in Section II.

Let us generalize once more, to the situation in which the after-tax interest rate varies by year. We now see that there is only a slight conceptual change, into an IRR for a particular year (IRR_t) and a composite IRR for all years in which, suitably generalizing Formula (4), above:

$$\begin{aligned} DBP(t) &= \{DSR_{t-1}(1+i_t) - DSR_t + TR(K)[DSR_t - DSR_{t-1}]\} \\ &= DSR_{t-1}(1 + i_t - K \times TR) - DSR_t(1 - K \times TR). \quad (6) \end{aligned}$$

Multiplying by

$$(1 - K \times TR)^{t-1} \prod_{j=1}^{t} \left(\frac{1}{1 + i_j - K \times TR} \right)$$

TABLE 1

Specifics:	Deferred Annuity, Issue Date	January 1, 1991
	Statutory Valuation Interest Rate	6.50%
	Applicable Federal Interest Rate	8.42%
	Interest Guarantee, 1st four years	5.50%
	Thereafter	4.00%
	Current Interest Rate	7.50%

Part A

Policy Year	Surrender Charge	Fund Beginning of Year	Net Surrender Value	Projected Fund from Issue Date	Tentative CARVM Reserve at Issue Date		Final CARVM Reserve at Beginning of Year	
					Statutory*	Tax	Statutory	Tax
1	6.00%	1000.00	940.00	1000.00	940.00	940.00	972.09	940.00
2	4.00	1075.00	1032.00	1055.00	950.99	934.14	1054.91	1032.00
3	2.00	1155.63	1132.51	1113.03	961.68	927.92	1144.77	1132.51
4	0.00	1242.30	1242.30	1174.24	972.09	921.36	1242.30	1242.30
5	0.00	1335.47	1335.47	1238.82	962.97	896.55	1335.47	1335.47

*Using surrender value at beginning of subsequent year.

CARVM Reserve at Issue Date	Statutory	Tax
Greatest of Present Values Assuming Traditional CARVM Formula	972.09	940.00
With Bailout Provision	1000.00	1000.00
Increase Due to Activity	27.91	60.00
Ratio of Tax Reserve Increase to Statutory Reserve Increase		2.150

Part B Illustrative IRR from Part A

Investment Income Rate	6.00%
Ratio: $\dfrac{\Delta(\text{Tax Reserve})}{\Delta(\text{Statutory Reserve})}$	2.15
Marginal Tax Rate	34.00%
Effective IRR	22.305%†

Year	Activity-Based Increase		T/S Increase Ratio	Book Profit	Present Value of Book Profits
	Statutory	Tax			
1 ...	27.91	60.00	2.150	−7.51	−7.51
2 ...	20.09	43.00	2.140	3.78	3.09
3 ...	10.85	23.11	2.130	3.69	2.47
4 ...	0.00	0.00	0.000	3.57	1.95
5 ...	0.00	0.00	0.000		
					0.00

$$†\text{IRR} = \frac{0.06}{1-(2.15)(0.34)} = 22.305\%.$$

and summing, we obtain:

$$PV(DBP) = \sum_{t=1}^{n} (DSR_{t-1}) \frac{(1 - K \times TR)^{t-1}}{\prod_{j=1}^{t-1} (1 + i_j - K \times TR)}$$

$$- DSR_t \frac{(1 - K \times TR)^t}{\prod_{j=1}^{t} (1 + i_j - K \times TR)} = 0.$$

Thus the composite IRR could be expressed in terms of a composite discount factor for $DBP(t)$ as follows:

$$\left(\frac{1 - K \times TR}{1 + i_1 - K \times TR}\right)\left(\frac{1 - K \times TR}{1 + i_2 - K \times TR}\right) \cdots \left(\frac{1 - K \times TR}{1 + i_t - K \times TR}\right)$$

$$= \left(\frac{1}{1 + \frac{i_1}{1 - K \times TR}}\right)\left(\frac{1}{1 + \frac{i_2}{1 - K \times TR}}\right)$$

$$\cdots \left(\frac{1}{1 + \frac{i_t}{1 - (K \times TR)}}\right).$$

Thus, the IRR_t, though varying by year, is a constant multiple of the after-tax interest rate for year t, that is,

$$IRR_t = \frac{i_t}{1 - TR \times K}. \qquad (7)$$

Unfortunately, in general, these elegant results do not necessarily reflect the real world, inasmuch as the ratio of DTR_t to DSR_t is not level over the typical reversal period. However, we can make some useful statements. If, instead of K, we have a K_t that varies by year (t) and if K_t exceeds K_1 for all $t > 1$, then the IRR is greater than $i/(1 - K_1 \times TR)$. Conversely, it is less than $i/(1 - K_1 \times TR)$ if $K_t < K_1$ for all $t > 1$.

This can be explained intuitively. Any given ratio K_t in year t in excess of K_1 represents an advance tax benefit, to be paid back in a subsequent year. It represents an interest-free loan, which increases the net present value of cash flows (and thus book profits) at any positive IRR. Obviously, the reverse is true for a given ratio less than K_1.

V. IRR AND RISK

So far the discussion has provided the means for computing the IRR implied by a particular tax-reserving strategy. It may be worthwhile to consider whether the computed IRR provides management with a sufficient return for use of its capital. Here principles of risk and reward apply. The riskier the investment, the higher the expected return. We can draw upon capital asset pricing theory [3], which defines the expected return for a portfolio of investments in terms of the market return (usually the S&P 500). Note that the portfolio of investments can contain as few as one stock.

Let us define the following terms:

i_p = Expected portfolio rate of return
i_m = Expected market rate of return (assume S&P 500)
R_f = The risk-free rate of return
β = Beta, a measure of volatility to be defined.

Then from the capital asset pricing model, we have

$$i_p = R_f + (i_m - R_f) \times \beta. \tag{8}$$

Note that a β of 1 reproduces the market rate of return. β's for publicly traded stocks are provided by a number of investment services. The importance of Equation (8) is that the expected return of any investment can be measured in terms of the risk-free rate and a measure of its volatility (riskiness), β, with respect to the market.

Some insights can be gained by pursuing an algebraic path with statistical definitions:

$$\beta = \frac{\sigma_{pm}}{\sigma_m^2}; \tag{9}$$

that is, β is the ratio of covariance between the portfolio and the market to the total market variance. It is a measure of correlation. (Note that this definition of β can also be expressed as the the slope of i_p with respect to i_m in the classical linear regression "least squares" formula.)

From statistics we know that the correlation coefficient is defined in terms of covariance and variance as follows:

$$\rho_{pm} = \frac{\sigma_{pm}}{\sigma_p \sigma_m}. \tag{10}$$

Rewriting β, we have

$$\beta = \frac{\sigma_{pm}}{\sigma_m \sigma_m} \times \frac{\sigma_p}{\sigma_p} = \frac{\sigma_{pm}}{\sigma_m \times \sigma_p} \times \frac{\sigma_p}{\sigma_m} = \rho_{pm} \times \frac{\sigma_p}{\sigma_m}. \qquad (11)$$

Substituting this definition of β into Equation (8), we have

$$i_p = R_f + \frac{(i_m - R_f)}{\sigma_m} \times \rho_{pm} \times \sigma_p. \qquad (12)$$

This leads to an interesting verbal interpretation. Given that the market return is $(i_m - R_f)/\sigma_m$ standard deviations above the risk-free rate of return, we expect our portfolio rate to exceed the risk-free rate by the same number of *its* standard deviations, σ_p, multiplied by the correlation coefficient. Note that the portfolio standard deviation relative to the market standard deviation is a measure of its relative risk.

Actuaries performing valuations of companies quite often define the risk rate of return intuitively. Sometimes profits are discounted at three rates, such as: 12 percent, 15 percent, and 18 percent, and the seller (or buyer) makes the choice based on his/her own assessment of risk. Capital asset pricing theory provides a more objective framework for choosing a risk rate of return. The β for a company (or line of business) could be estimated and the expected risk rate of return derived based on the then-current market conditions with respect to the risk-free rate of return and market yield.

To close the loop, we must select the risk-free rate of return. The next section addresses this problem.

VI. IRR AND DURATION

Whether the IRR derived by the methods of Section IV provides a sufficient rate of return depends on the returns available on alternative investments of similar risk and duration. Risk has been addressed in the prior section. This section addresses the concept of duration.

Suppose for a given strategy we have computed an IRR to be 12 percent. To what should this be compared to determine whether the IRR is sufficient? Ignoring risk temporarily, should this IRR be compared to a 90-day T-bill or a 30-year treasury bond? The term structure of interest rates at times shows a marked difference between yields on bonds of varying maturities. We prefer to compare the IRR to that of a bond of similar duration; that is, if the IRR pertains to a 1-year period, a 1-year bond should be used as the measuring stick. However, an IRR covering a 5- or 10-year period normally does not have a constant amount of investor capital outstanding during the period with a total repayment at the end of the period. Hence, we need a tool to measure the effective period that the investment capital is outstanding. The concept of duration serves as such a tool.

In analytical terms, duration is a measure of price sensitivity to incremental changes in interest rates. Relying on some simple calculus, let us compute the change in the price of a bond, given a change in the interest rate. For simplicity, annual coupons and annual effective rate of interest are assumed.

$$\text{Price} = \sum_{t=0}^{n} (\text{Cash Flow})_t \times (1 + \text{i})^{-t} \tag{13}$$

where the cash flow at time t is a coupon, and at time n, the coupon plus the maturity value.

$$\frac{d\,\text{Price}}{di} = -\sum_{t=0}^{n} t \times (\text{Cash Flow})_t \times (1 + i)^{-(t+1)} \tag{14}$$

or

$$-\frac{1}{(1 + i)} \times \sum_{t=0}^{n} t \times (\text{Cash Flow})_t \times (1 + i)^{-t} \tag{15}$$

Multiplying and dividing by *Price*;

$$\frac{d\,\text{Price}}{di} = -\frac{1}{(1 + i)} \left[\sum_{t=0}^{n} \frac{t \times (\text{Cash Flow})_t \times (1 + i)^{-t}}{\text{Price}} \right] \times \text{Price}. \tag{16}$$

Let:

$$\text{Duration} = \frac{\sum_{t=0}^{n} t \times (\text{Cash Flow})_t \times (1 + i)^{-t}}{\text{Price}}. \tag{17}$$

Then we have

$$\frac{d\,\text{Price}}{di} = -\text{Price} \times \frac{\text{Duration}}{(1 + i)}. \tag{18}$$

Duration as defined above is a simple weighted average in which the weights are the present values of the cash flows at each duration. The denominator is the sum of the weights, or the price of the bond at the market rate of interest, i. This definition is the Macaulay duration and is not without its limitations.

Making use of finite differences and solving Formula (18) for duration, we have

$$\text{Duration} = \frac{-\dfrac{\Delta\text{Price}}{\text{Price}}}{\dfrac{\Delta i}{(1 + i)}} \tag{19}$$

As i becomes convertible more frequently than annually, $\Delta i/(1+i)$ degenerates to Δi.

The advantage of this form for duration over the Macaulay type is that duration can be computed by recomputing the price of a bond at a slightly higher interest rate, for example, $i+0.0001$. Calculus and derivatives could be replaced with computer calculations. Without making the computation unduly sophisticated, the IRR computed in the previous section can be incremented slightly (for example, by 0.0001) and the present value of contributions to the corporate surplus account recomputed. The duration can then be approximated as follows:

$$\text{Duration} = -\frac{\dfrac{PV(@\text{IRR} + 0.0001) - PV(@\text{IRR})}{PV(@IRR)}}{\dfrac{0.0001}{1 + \text{IRR}}}. \tag{20}$$

This computation provides a good ballpark estimate of duration, as long as the increment is small enough to substantially eliminate the effects of convexity (changes in duration with respect to changes in the interest rate, including the resulting changes in the cash flows themselves). A zero-coupon treasury bond (usually referred to as the spot rate) to the same duration (or time to maturity) as computed by Formula (20) can then be used for comparison (the treasury bond yield note being a representation of the risk-free rate of return over the period in consideration). The difference between the IRR and the treasury bond yield rate is regarded as the risk premium. The concept of risk as it related to the IRR was discussed in Section V.

Thus caution should be exercised when choosing between two alternative actions solely on the basis of IRR. Duration analysis helps to put the IRR standard in perspective, inasmuch as the standard varies with the rate of return of an invested asset of similar duration and comparable risk. However, IRR duration analysis should also be viewed with caution because of well-known limitations (such as the convexity mentioned above). In addition, IRR duration analysis depends, for example, on the assumption that the marginal earnings from the activity can be reinvested at the same IRR due to the continued availability of such activities. If that is not so, then it may clearly be better, for example, to generate a 20 percent IRR for a 10-year duration than a 25 percent return for a 1-month duration, since the 20 percent IRR will then be locked in.

Tax strategies could be regarded as risk-free, assuming that no pre-tax economic value is given up by the company in pursuit of those strategies, were it not for several risk factors. Such risk factors include the following:

- There is a risk that the company will not be in the same tax status or "corridor" in subsequent years. First, laws can change. Second, the company can develop net operating loss carryovers or incur a reallocation with non-life entities in a consolidated return. Further, a company can change status between regular tax and alternative minimum tax, or incur a change of small life company status.
- There is a risk that the company will not be able to sustain the planned tax treatment of the strategy. The degree of aggressiveness of the strategy must be assessed as well as the relationship to other current or potential audit issues.
- Administrative and expense risk must be considered. Some strategies (such as a choice of tax basis mortality table on a new block of business) might involve little more than "a stroke of the pen," while others may involve substantial—and currently unknown—administrative support.

Therefore, it is probably appropriate to assess the aggressiveness of the approach, the stability of company tax status under current law, administrative aspects, and the political climate. Further, the longer the reversal period, the greater the exposure to deviation from the expected rate of return.

This exercise may appear fraught with concepts difficult to quantify, but this is not a significant departure from the general difficulties of establishing appropriate hurdle rates.

Thus, instead of the economic risks normally associated with commercial activities, we have a different set of political and economic risks in establishing a desired rate of return.

VII. APPLICATION TO MUTUAL LIFE INSURANCE COMPANIES

For tax strategies for mutual companies, the equations become somewhat more complex, in order to account for the Internal Revenue Code Section 809 tax benefit on the equity base decrease caused by the activity. The equivalent book profit difference formula is approximately as follows:

$$DBP(t) = DRS(t-1) \times (1+j) - DRS(t) + \frac{TR \times \{DRT(t) - DRT(t-1) + 0.5 \times DER(t) \times [DRT(t) + DRT(t-1)]\}}{1 + TR \times [DER(t) + DER(t+1)] \times 0.5} \tag{21}$$

where

j = After-tax mutual company interest rate

$DER(t)$ = Differential earnings rate. [Assume it to be a level rate (DER) in the formula developments below.]

A few words of explanation are in order for Expression (21). First, the denominator provides for the fact that any incremental tax accrual itself forms an offsetting effect on the equity base and therefore provides a minor offset to the tax otherwise incurred. The expression is the result of summing the following power series. For each dollar of incremental tax otherwise incurred, the equity base reduction causes the following offset:

$$1 - DER \times TR + (DER \times TR)^2 - (DER \times TR)^3 + \text{etc.} = \frac{1}{1 + DER \times TR}$$

A minor item being ignored is the fact that the DER is affected half in the current year and half in the next year, and thus deserves a slight interest discount.

Second, the interest rate (j) is subject to various interpretations of what the post-tax interest rate earned on surplus really is in a mutual company. It is dependent to a great extent on the assumption of how quickly such interest is paid out to policyholders. If one assumes that it is paid out at year-end, then it appears that, for i' defined as the before-tax interest rate, that is,

$$i' = i/(1 - TR),$$

then

$$j = i' - TR \times (i' + DER \times i)/(1 + TR \times DER).$$

The calculation of the IRR is somewhat analogous to the stock company scenario. Let us start with Formula (21), simplifying it for a level *DER*:

$$DBP(t) = DRS(t - 1) \times (1 + j) - DRS(t)$$
$$+ \frac{TR \times \{DRT(t) - DRT(t - 1) + 0.5 \times DER \times [DRT(t) + DRT(t - 1)]\}}{1 + TR \times (DER)} \tag{22}$$

Let us now modify Formula (4) for mutual companies, with *K* as defined as in Section IV. Define

$$TR' = K \times TR/(1 + TR \times DER)$$
$$TR'' = K \times TR \times 0.5 \times DER/(1 + TR \times DER).$$

Then $DBP(t)$ becomes

$$DRS(t - 1) \times (1 + j) - DRS(t) + TR' \times [DRS(t) - DRS(t - 1)]$$
$$+ TR'' \times [DRS(T) + DRS(t - 1)].$$

Therefore,

$$DBP(t) = DRS(t - 1) \times (1 + j - TR' + TR'')$$
$$- DRS(t) \times (1 - TR' - TR'').$$

We can then define the following discounting factor, w, where

$$w = \frac{1 - TR' - TR''}{1 + j - TR' + TR''}.$$

Analogously to the summations in the above sections, the sum of $DBP(t) \times w^t$ from 1 to n can easily be shown to be zero.

The IRR can then be calculated as $(1/w) - 1$, or

$$\text{IRR} = (j + 2 \times TR'')/(1 - TR' - TR''). \tag{23}$$

Here again, the *IRR* is independent of the time to reversal, n.

Let us examine this formula for its properties relative to the stock company formula, as it compares to the after-tax interest rate. Using the definitions of *TR'* and *TR"* above, we find that Formula (23) becomes

$$\text{IRR} = \frac{j + k \times TR \times DER/(1 + TR \times DER)}{1 - K \times TR \times (1 + 0.5 \times DER)/(1 + TR \times DER)}. \quad (24)$$

The numerator here is affected by an additional positive term, not present in the stock company scenario, and the denominator is less than in the stock company scenario, assuming that the *DER* is positive and that the tax rate is less than 50 percent. That leads us to the obvious conclusion that, with j normally less than or equal to i, given a particular tax-reserve-planning strategy, and other facts being equal, the *IRR* is generally a greater multiple of the after-tax investment income rate for a mutual company than for a stock company. This greater effect is due to the reducing effect of the strategy on the equity base, beginning immediately in the year of implementation of the strategy. See Table 2 for an illustration of the Formula (24) concept.

VIII. CONCLUSION

Because of the rapid evolution of tax law, a host of ambiguities and opportunities have sprung up, particularly with respect to tax reserve issues, an area where large planning numbers are commonly encountered. In addition, since 1988 the industry has confronted a series of both tax laws and statutory requirements that are pushing statutory liabilities and tax basis liabilities ever further apart.

Planning activities that bring tax basis liabilities and statutory liabilities closer are often valuable tools in the tax-planning process. This paper has attempted to provide a means of evaluating certain of those strategies, for both stock and mutual life assurance companies under current law, by means of computing an IRR by which to measure their value, and it has provided guidance in computing a standard against which to measure that rate of return. The IRR is not a perfect tool, but we have attempted to address the most significant pitfalls to the approach.

TABLE 2

ILLUSTRATIVE IRR FOR A MUTUAL LIFE COMPANY

Investment Income Rate	6.00%
Ratio: $\dfrac{\Delta(\text{Tax Reserve})}{\Delta(\text{Statutory Reserve})}$	2.40
Differential Earnings Rate (DER)	5.00%
Marginal Tax Rate	34.00%
Effective IRR	56.379%*

Year	Reserve Increment		Book Profit†	Present Value of Book Profits
	Statutory	Tax		
1....	1000	2400	−177.58	−177.58
2....	850	2040	126.76	81.06
3....	700	1680	111.74	45.69
4....	600	1440	87.84	22.97
5....	500	1200	77.83	13.01
6....	400	960	67.82	7.25
7....	300	720	57.81	3.95
8....	200	480	47.79	2.09
9....	100	240	37.78	1.06
10....	0	0	27.77	0.50
Total				0.00

$$*TR' = \frac{(2.4)(0.34)}{1+0.34(0.05)} = 0.80236$$

$$TR'' = \qquad = 0.02006$$

$$IRR = \qquad = 56.379\%$$

†Refer to Formulas (22) and (23).

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