TRANSACTIONS OF SOCIETY OF ACTUARIES
1992 VOL. 44

FINANCING DEFINED-BENEFIT PENSION PLANS WITH INDEXED BENEFITS

JOHN C. MAYNARD

ABSTRACT

In response to public inquiries into the indexing of pension plan benefits, the financing of defined-benefit plans with fully indexed benefits is explored. A model career-average pension plan is set up with all accrued benefits fully indexed. For different funding conditions, the changes in pension plan surplus are examined under the economic conditions of 1946–1987. The results show that the fluctuations in surplus can be more easily absorbed if the pension plan balance sheet uses market value of assets and a funding objective which is flexibly related to assets and set in terms of a fixed conventional liability and an asset fluctuation reserve.

I. INTRODUCTION

In the past, much attention has been given to ways in which private pensions and annuities can be effective in the presence of inflation. A high point in this attention was Duncan's paper in 1952 [3], in which he described the original variable annuity of the College Retirement Equities Fund. It had the objective of producing income payments correlated to the cost of living by investing a large part of the fund in common stocks. Another high point was the presentation to the Society of three papers in 1982 [1], [4], [8].

In the U.S. recent public opinion has not been favorable to private pension plans [6], one of the issues being the absence of cost-of-living increases in defined-benefit plans [6].

In Canada the effectiveness of private pension plans has been under continuous scrutiny; there have been thirteen reports by private bodies and committees appointed by governments. Four of these were appointed by the Federal Government and four by the Ontario Government. Protection from inflation in defined-benefit plans has not been the only problem addressed in these reports, but it has proved to be the most serious and intractable problem.

In 1986 the Ontario Government introduced a bill which revised the Ontario Pension Benefits Act. The bill did not deal with inflation problems. However, the government affirmed its commitment to mandatory inflation protection and appointed a Task Force on Inflation Protection for Employment Pension Plans. Its terms of reference were to determine the most appropriate formula and phase-in procedures for inflation protection. At the same time, the government announced a moratorium on withdrawals of pension plan surplus. The Task Force issued its lengthy report of analysis, recommendations, and research studies in January 1988 [7]. It is the latest and most comprehensive of the reports on the problem.

The central recommendation of the Task Force was that benefits earned in the future should be increased annually by at least 75 percent of the increase in the Consumer Price Index (CPI), less 1 percent. The Ontario Government has prepared a bill, but as of April 1, 1991, it had not been enacted. Uncertainty continues.

A major concern about defined-benefit plans with indexed benefits is that they are subject to severe fluctuations in surplus.

The author is convinced that the discussions to date have been conducted under a handicap: the lack of demonstration of changes in the pension plan balance sheets of a plan with indexed benefits while the plan is subject to the economic conditions of the recent past.

The objective of this paper is to fill at least part of the gap in knowledge about the funding of these plans by developing a model defined-benefit pension plan with fully indexed benefits, and then by demonstrating how the surplus in the balance sheet changes under the economic conditions of the period 1946–1987. A change in the traditional method of financing is introduced so that fluctuations can be absorbed, and the demonstration shows how this method would have fared under a variety of funding conditions.

The remainder of the paper comprises four sections:

II. Analysis
III. The Model Pension Plan and Its Valuation
IV. Financing
V. Summary and Conclusion

The objectives of the paper are developed in Section IV, Financing.

II. ANALYSIS

In this section, data, bases, and procedures are assembled under the headings:

A. Economic Statistics
B. Valuation of Assets
C. Valuation of Liability
D. Simplifying Assumptions
E. Notation
F. Adjustments for Indexing
G. Formulas.

A. Economic Statistics

The Canadian Institute of Actuaries publishes annually its Report on Canadian Economic Statistics [2]. Building on the work of Ibbotson and Sinquefield [5] and others, the report shows various annual rates going back to 1924. The principal rates are total rates of return on different classes of investment. The total rates include income received and the change in market value from one year-end to the next. Highlights from the 1988 report are shown in Table 1 (the rates in the report are Canadian).

The report and Table 1 include one set of rates submitted by SEI Financial Services Limited, Toronto. These are median total rates of return by calendar year for 2,100 tax-exempt Canadian pension funds. An estimate has been made of the median percentage of assets held in classes of investment by these Canadian pension funds in a recent decade, and this is shown in Table 2.

The rates in the report allow us to have a great deal of hindsight about investment performance in the past 60 years. The greatest return has consistently been in common stocks and the lowest return in long bonds.

B. Valuation of Assets

The advantages in valuing the assets in a pension plan balance sheet at market value are as follows:

(1) Common stocks are a prominent component of assets, and market value is the most realistic value for them.
(2) Market value brings comparability of value and the corresponding rate of return. This comparability is used widely in analysis of investment performance.
(3) Market value brings understandability.
(4) Market value brings simplicity. It is not necessary to account for profits and losses on sale of investments.

The disadvantage of market value is the characteristically large variation in value and rate of return.

Market value is used in this paper for another reason: Market value permits assets and liabilities to be valued consistently in a way which is described later.

C. Valuation of Liability

Calculations of liability are made at an annual implicit net rate of interest, and it is desirable to define this term.

TABLE 1

ANNUAL PERCENTAGE RATES OF CHANGE/RETURN

Year	Consumer Price Index	Common Stock Index	Canada Long Bonds	Conventional Mortgage Index	91-Day T-bills	Pension Fund Median
1934........	1.39	19.99	19.66	—	—	—
1935........	2.05	30.30	0.83	—	—	—
1936........	1.34	25.77	11.12	—	—	—
1937........	3.97	−15.34	−0.58	—	—	—
1938........	−1.91	7.80	5.63	—	—	—
1939........	2.60	−0.23	−2.98	—	—	—
1940........	4.43	−19.03	8.69	—	—	—
1941........	6.67	2.87	3.80	—	—	—
1942........	3.41	13.75	3.08	—	—	—
1943........	1.10	19.02	3.88	—	—	—
1944........	−1.09	13.25	3.16	—	—	—
1945........	1.10	35.66	5.18	—	—	—
1946........	5.98	−1.51	6.02	—	0.39	—
1947........	14.87	0.79	3.17	—	0.41	—
1948........	8.48	12.24	−2.38	—	0.41	—
1949........	1.23	23.86	4.85	—	0.48	—
1950........	6.10	51.69	−0.12	—	0.54	—
1951........	10.73	25.44	−3.13	—	0.77	—
1952........	−1.73	0.01	1.99	5.18	1.05	—
1953........	0.00	2.56	3.64	2.08	1.66	—
1954........	0.35	39.37	9.99	7.48	1.51	—
1955........	0.35	27.67	−0.34	6.73	1.44	—
1956........	3.15	12.68	−3.63	−2.42	2.86	—
1957........	2.03	−20.58	6.40	3.23	3.86	—
1958........	2.66	31.25	−5.98	8.86	2.28	—
1959........	1.29	4.59	−4.67	1.75	4.69	—
1960........	1.28	1.78	7.10	10.32	3.58	9.5
1961........	0.32	32.75	9.78	7.12	2.96	13.3
1962........	1.57	−7.09	3.05	7.12	4.15	2.0
1963........	1.86	15.60	4.60	7.12	3.67	8.1
1964........	1.82	25.43	6.59	7.12	3.79	11.1
1965........	2.99	6.68	0.96	2.59	3.99	3.5
1966........	3.48	−7.07	1.55	1.58	5.13	−2.3
1967........	4.20	18.09	−2.20	2.21	4.63	7.6
1968........	4.03	22.45	−0.52	2.97	6.43	9.4
1969........	4.65	−0.81	−2.31	−3.15	7.36	−3.2

TABLE 1—*Continued*

Year	Consumer Price Index	Common Stock Index	Canada Long Bonds	Conventional Mortgage Index	91-Day T-bills	Pension Fund Median
1970........	1.48	−3.57	21.98	11.87	6.68	1.3
1971........	4.87	8.01	11.55	13.90	3.84	12.5
1972........	5.10	27.38	1.11	8.92	3.58	18.4
1973........	9.27	0.27	1.71	6.87	5.34	−2.1
1974........	12.32	−25.93	−1.69	4.50	8.12	−12.7
1975........	9.53	18.48	2.82	12.20	7.53	13.2
1976........	5.91	11.02	19.02	14.21	9.43	12.4
1977........	9.46	10.71	5.97	14.62	7.85	8.7
1978........	8.36	29.72	1.29	6.84	8.77	13.5
1979........	9.80	44.77	−2.62	5.66	12.21	15.0
1980........	11.19	30.13	2.06	8.10	13.80	18.3
1981........	12.10	−10.25	−3.02	9.98	19.96	1.5
1982........	9.26	5.54	42.98	29.15	15.59	21.1
1983........	4.55	35.49	9.60	20.46	9.87	20.0
1984........	3.76	−2.39	15.09	12.36	12.07	8.8
1985........	4.35	25.07	25.26	16.72	9.86	23.5
1986........	4.17	8.95	17.54	13.34	9.47	12.8
1987........	4.15	5.88	0.45	10.26	8.61	4.4
AVERAGE ANNUAL COMPOUND PERCENTAGE RATES OF CHANGE/RETURN						
1938–1947...	3.62	6.34	3.92	—	—	—
1948–1957...	3.00	15.76	1.64	—	1.45	—
1958–1967...	2.14	11.33	1.96	5.54	3.89	—
1968–1977...	6.62	5.71	5.66	8.54	6.60	5.38
1978–1987...	7.12	16.01	10.04	13.09	11.97	13.68

Source: Reference 2.

TABLE 2

APPROXIMATE MEDIAN PERCENTAGE OF ASSETS* HELD IN CLASSES OF INVESTMENT BY CANADIAN PENSION FUNDS

Year	Equities	Bonds	Short-Term Instruments	Mortgages	Other	Total
1978........	36	32	12	15	5	100%
1983........	43	35	10	3	9	100
1987........	45	39	9	0	7	100

*Based on an inquiry to SEI Financial Services Limited, Toronto. The approximation is made by the author.

Earned pension benefits are assumed to be increased annually by the rate of change in an index. If:

r = the appropriate future annual rate of change in the index
h = the appropriate future annual total rate of return on assets
i = the appropriate annual implicit net rate of interest

then:
i is defined in terms of the other two rates by the equation

$$1 + i = (1 + h) \div (1 + r).$$

If A is a single benefit earned today, payable n years hence, and subject to increases by the index without any time lag, then A gives rise to a payment of $A(1+r)^n$ and the liability to be held today for the payment is

$$\frac{A(1 + r)^n \text{ (probability of survival)}}{(1 + h)^n}$$

or

$$\frac{A\text{(probability of survival)}}{(1 + i)^n} .$$

Liability calculations at an implicit net rate of interest provide for future increases in benefits without the increases appearing specifically in the calculation. The use of the net rate is a mathematical convenience.

D. Simplifying Assumptions

The defined-benefit plan provides an earned pension of a percentage of salary for each year of service. A number of assumptions are made which simplify the demonstration:
(1) Retirements take place at age y.
(2) The only postretirement decrement is death, and postretirement mortality is according to the single mortality table used throughout the paper in calculations of current service contributions and liability.
(3) The only pre-retirement decrements are termination and death, and at the time of these decrements, benefits in the form of single payments are paid. There are no decrements for disability or early retirement.

(4) Contributions and liability are according to the unit credit actuarial cost method. Under this method the liability is the present value of accrued benefits.
(5) Calculations of contributions and liability provide for the single decrement of postretirement mortality and use an implicit net rate of interest. For an active member, age z, the liability factor is of the form $v^{y-z}\,\bar{a}_y$.
(6) No provision is made for tax or regulatory controls or for integration with Social Security, so that the effect of economic factors can be seen as clearly as possible.
(7) The pension plan and fund are self-administered, and no expenses are charged to the fund.

Several other assumptions are made which simplify the calculations:
(i) The financial year is the calendar year.
(ii) Age is recorded as age nearest birthday at the beginning of the calendar year.
(iii) Benefit payments are made from the pension fund, and income payments of both contributions and investment income are made to it, only at mid-year.
(iv) Simple interest is applied to income and disbursements occurring during a year.

The cell used in the analysis is the x/t rhombic shaded area in Figure 1. Members of the plan, represented by the rhombic shaded area during year t, may be thought of as moving along the diagonal from age x toward age $x+1$ at the end of the year.

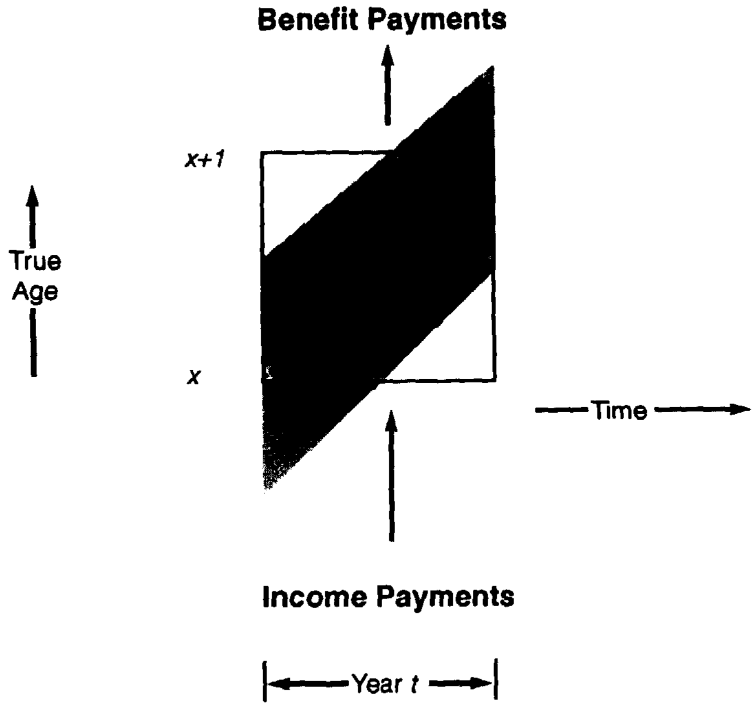
E. Notation

Symbols are defined as follows:

C = annual contribution for current service benefits
${}^{T}P$ = total of annual payments on terminations of active lives
${}^{D}P$ = total of annual payments on deaths of active lives
${}^{R}P$ = total of annual payments to retired lives
P = total payments from the plan = ${}^{T}P + {}^{D}P + {}^{R}P$
${}^{T}L$ = total of liability released on terminations of active lives
${}^{D}L$ = total of liability released on deaths of active lives
L = liability at year-end by the unit credit method
r = annual rate of change in the index, which is applied to increase benefits

FIGURE 1

AGE/TIME DIAGRAM



h = annual total rate of return on assets at market value

i = annual implicit net rate of interest; when used without a subscript, this symbol is the net rate used in the valuation of liability and contributions

${}_tr$ = annual rate of change in the index during year t

${}_th$ = annual total rate of return on assets at market value during year t

${}_ti$ = annual implicit net rate of interest earned on assets at market value during year t

${}_{t|m}r$ = average annual rate of change in the index in the m years following the year t

${}_{t|m}h$ = average annual total rate of return on assets at market value in the m years following year t

${}_{t|m}i$ = average annual implicit net rate of interest earned on assets at market value in the m years following year t
A = market value of assets at year-end
S = surplus at year-end, being the excess of assets over liability
AR = asset fluctuation reserve
n = number of lives at year-end
d = lives leaving the plan during the year
ab = average accrued annual pension benefit per life at year-end
Sal = average salary per life at beginning of year
x = age nearest birthday at beginning of year.

Two subscripts and two superscripts may be applied to symbols. They have these meanings:

Status (T, D, or R)	Rate of Interest (If required to be defined)

$$ {}^{D}_{t}L^{j}_{x} $$

Year	Age

Almost all symbols require the year subscript. If the symbol pertains to age and the age subscript is omitted, the symbol represents the sum for all relevant ages.

F. Adjustments for Indexing

An index is chosen which is suitable for adjusting accrued benefits for inflation.

A method is needed which allows for a necessary time lag between the occurrence of inflation and the adjustment for it.

At the end of year $t-1$, salary scales are set for year t, considering index levels and other matters at that time.

During year t, the current service benefits for that year are earned at the salaries set. Contributions for the current service benefits are also determined at the salaries set, and the payment of contributions together with the payment of investment income are regarded as being made to the fund at mid-year. Pension payments to retired lives and single payments for deaths and terminations among active lives are based on benefits as recorded at the end of year $t-1$, and are regarded as being made from the fund at mid-year.

At the end of year t, for each active member the accrued earned pension at the end of year $t-1$ is increased by the factor ${}_{t-1}r$ and added to the unadjusted current service benefit for year t, to determine the accrued earned pension at the end of year t. For each retired member, the pension being paid is increased by the factor ${}_{t-1}r$. Active members reaching retirement age retire at the end of the year. The liability is the sum of the present values of the increased benefits for active and retired members.

G. Formulas

It follows from simplifying assumptions 2, 3, 4, and 5 that the analysis of surplus changes will not include gains or losses from postretirement mortality, disability, or early retirement. The analyzed surplus changes will include the combined gains and losses from investment income, indexing, and pre-retirement terminations and mortality.

Assume there is no change in liability assumptions during the year. Because of simplifying assumptions 2 and 5, the yearly change in liability can be written down by tracing changes for a cell.

First, for a cell at the ages of retired members,

$$ {}_tL_x^i = \left[{}_{t-1}L_{x-1}^i(1 + i) - {}_t^RP_x\left(1 + \frac{i}{2}\right) \right](1 + {}_{t-1}r). \tag{1} $$

Then, for a cell at the younger ages of active members,

$$ \begin{aligned} {}_tL_x^i = {} & \left[{}_{t-1}L_{x-1}^i(1 + i) - ({}_t^TL_x^i + {}_t^DL_x^i) \left(1 + \frac{i}{2}\right) \right] (1 + {}_{t-1}r) \\ & + {}_tC_x^i\left(1 + \frac{i}{2}\right). \end{aligned} \tag{2} $$

The total for active and retired combined is then

$$
\begin{aligned}
{}_tL^i = {} & \left[{}_{t-1}L^i(1 + i) - \left({}^T_tL^i + {}^D_tL^i + {}^R_tP\right)\left(1 + \frac{i}{2}\right)\right](1 + {}_{t-1}r) \\
& + {}_tC^i\left(1 + \frac{i}{2}\right).
\end{aligned} \tag{3}
$$

The formula for asset growth is

$$
{}_tA = {}_{t-1}A(1 + {}_th) - ({}^T_tP + {}^D_tP + {}^R_tP)\left(1 + \frac{{}_th}{2}\right) + {}_tC^i\left(1 + \frac{{}_th}{2}\right), \tag{4}
$$

or more simply,

$$
{}_tA = {}_{t-1}A(1 + {}_th) + ({}_tC^i - {}_tP)\left(1 + \frac{{}_th}{2}\right). \tag{5}
$$

In a conventional pension plan balance sheet the surplus, ${}_tS = {}_tA - {}_tL$, may be traced by subtracting Equation (3) from Equation (4). If this is done, with current service contributions having the same assumptions as liability, and after terms are rearranged, then

$$
\begin{aligned}
{}_tS = {} & \overbrace{{}_{t-1}S(1 + {}_th)}^{(a)} + \overbrace{\left[{}_{t-1}L^i + \frac{1}{2}({}_tC - {}_tP)\right]({}_th - i)}^{(b)} \\
& \underbrace{- {}_{t-1}r\left[{}_{t-1}L^i(1 + i) - ({}^T_tL + {}^D_tL + {}^R_tP)\left(1 + \frac{i}{2}\right)\right]}_{(c)} \\
& + \underbrace{({}^T_tL^i - {}^T_tP)\left(1 + \frac{i}{2}\right)}_{(d)} + \underbrace{({}^D_tL^i - {}^D_tP)\left(1 + \frac{i}{2}\right)}_{(e)}.
\end{aligned} \tag{6}
$$

where
(a) is previous-year surplus with interest
(b) is excess interest on the mean liability
(c) is year-end cost of indexing benefits
(d) is gain on terminations
(e) is gain on pre-retirement deaths.

If the plan provides payments on termination and pre-retirement death that are equal to the liability held, then the last two terms are zero and the formula simplifies to

$$ {}_tS = {}_{t-1}S(1 + {}_th) + \left[{}_{t-1}L^i + \frac{1}{2}({}_tC^i - {}_tP)\right]({}_th - i) $$
$$ - {}_{t-1}r\left[{}_{t-1}L^i(1 + i) - {}_tP(1 + \frac{i}{2})\right]. \qquad (7) $$

Equation (7) may be thought of as the formula for surplus gain or loss resulting from economic change. It may be noted that component (a) may be positive or negative depending on surplus or deficiency. Also, the factor of ${}_th - i$ in component (b) is of similar magnitude to the factor of ${}_{t-1}r$ in component (c). These observations lead to an approximation: over a period the economic component of pension plan gain and loss through rate of return and indexing will be at a low level if, for this period, the average of $h - r = i$.

Given the simplifying assumptions, the above formulas are general.

III. THE MODEL PENSION PLAN AND ITS VALUATION

The model plan consists of a stationary population of working and retired members, who belong to a career-average pension plan in which all benefits are indexed.

The relationships among salaries, accrued benefits, indexing, and liability during the course of a year were described in the preceding subsection II-F "Adjustments for Indexing." Further details of the model plan are described under the following headings:

A. Mortality
B. Structure
C. Benefits
D. Comparison
E. Implicit Net Rate of Interest for Valuation of Liability
F. Methodology.

A. Mortality

The mortality of retired workers in North America has improved in a striking manner in the past 40 years, as can be seen in the reducing mortality rates in the sequence of mortality tables GA51, GAM71, GA83. The model plan assumes that during the period of study and beyond, the mortality of retired lives is according to a modification of the GAM71 Table. Under the modification the mortality rate at each age is 90 percent of the male rate of the underlying table plus 10 percent of the female rate.

B. Structure

Each year 172 entrants enter the plan at age 30. Each year at each age between 30 and 64, 5 percent of the members leave through termination and death. Members retire at age 65. The entrants equal the decrements, and the population is stationary.

In 1971 the average salary at age 30 is $7,312. The average salary at each age is 1-1/2 percent higher than the average salary at the next lower age in the same year. The average salary for age x in the year t is the average salary for age x in year $t-1$, increased by ${}_{t-1}r$. This establishes a grid of average salaries; for example,

$$
\begin{aligned}
{}_{71}Sal_x &= 7{,}312\ (1.015)^{x-30} \\
{}_{72}Sal_x &= 7{,}312\ (1.015)^{x-30}\ (1 + {}_{71}r).
\end{aligned}
$$

The rate ${}_{t-1}r$ used to increase average salaries is the annual rate of increase in the CPI.

C. Benefits

Each year an active member earns an annual pension of 2 percent of annual salary. New entrants begin earning a pension immediately.

Accrued earned pensions are increased annually by the appropriate annual rate of increase in the CPI.

Pensions paid to retired members cease on death.

If an active member terminates or dies, a payment is made from the plan. The amount of the payment is the amount of the liability which is being funded at the date of termination or death for the accrued earned pension including increases recorded to date. Because the liability being funded provides for future increases in the earned benefit, the payment gives the terminating member a benefit which has a value corresponding to the earned pension with both past and future increases. This is a more liberal benefit on termination than actual plans provide today.

Although more liberal, a case can be made for this level of benefit for terminating members. From the members' point of view, to be fair and effective, an earned benefit should be at the same level for a member who terminates as for a member who has the same record of salary and service and who continues in employment. This level of benefit would encourage portability.

The average salary and average accrued benefit of the members who leave the plan in a year are assumed to be the same as the average salary and average accrued benefit of the members who are at the same age and service and who continue in the plan. This results in a grid of average accrued benefits, simply related to average salaries.

D. Comparison

The model plan, although described as career-average with indexed benefits, can be compared to a common type of final average plan with the same rate of benefit. The average benefits of the two plans are not far apart, as shown by the following comparison of indexing.

COMPARISON OF INDEXING

Type of Benefit	Provision for Indexing: Model Plan	Provision for Indexing: Final Average Plan
Accrued Benefits for Active Members	By CPI, which is a function of time	By an index which is a function of time and age
Benefit on Termination	Average benefits are indexed annually to and after termination	Average benefits are indexed annually to termination
Pension Payments	Benefits paid are indexed annually to and after retirement	Benefits paid are indexed annually to an age near retirement. After retirement very few plans have full indexing, and relatively few have partial indexing.

The model plan has lower indexing for accrued benefits, because the CPI is lower than an index which reflects both time and age, but the model plan has much higher indexing for terminations and higher indexing for pension payments. The model plan is believed to represent reasonably well a final average plan and quite well a final average plan with indexed pension payments.

E. Implicit Net Rate of Interest for Valuation of Liability

The calculation of contributions and liability for the model plan depends on providing for future total rates of return and for increases in benefit according to the rate of increase in the CPI.

This matter has been discussed in subsection II-C, and the main problem is to choose the rate i wisely.

The earlier discussion can be generalized: If

$$ {}_tF = \prod_{s=1}^{t} (1 + {}_sr), \tag{8} $$

$$ {}_tG = \prod_{s=1}^{t} (1 + {}_sh), \tag{9} $$

and A is an annual benefit earned to the end of year t and subject to increases by the index F, without any time lag in the application of the index,* then the liability at the end of the year t for the payment due at the end of n years is

$$ A \cdot \frac{{}_{t+n}F}{{}_tF} \cdot \frac{{}_tG}{{}_{t+n}G} \cdot \text{(probability of survival)}. \tag{10} $$

The total liability for the plan is therefore the sum of products containing factors of the form:

$$ \frac{{}_{t+n}F}{{}_tF} \frac{{}_tG}{{}_{t+n}G} = \left[\frac{1 + {}_{t|n}r}{1 + {}_{t|n}h}\right]^n = \frac{1}{(1 + {}_{t|n}i)^n} \tag{11} $$

where ${}_{t|n}r$, ${}_{t|n}h$, and ${}_{t|n}i$ are the average annual rate of CPI increase, total rate of return, and net rate, respectively, in the n years following year t.

The summation (for n) ranges over the years of future lifetime of members of the plan.

*The calculation formulas do provide for time lag. However, it is simpler, and not misleading, to ignore this in the statistical analysis.

In determining the liability at a particular time t, the problem is to choose a rate i so that the sum of products using i is expected to be close to the sum of products which use the average rates ${}_{t|n}i$. And of course the rates ${}_{t|n}i$ are not known at time t.

To illustrate this determination, the experience of the period 1946–1987 is examined. Balance sheets are prepared for the model plan for periods ending in 1987. Success is anticipated because the economic data for the trials are known in advance and are used in the testing. This is a process of fitting a valuation system to the experience of a period.

The economic data to be used are referred to in subsection II-A and consist of Tables 1, 2, and the CIA Report [2].

Table 3 shows the annual rates of change in the CPI for the years 1924–1987, and they are taken to be the rates ${}_{t}r$ for analysis and calculation. Table 3 also shows 3-year and 9-year average rates.

The pension fund median total rates of return are in Table 1 for years 1960–1987. To this series have been added the rates according to investment category in the CIA Report [2], with fractional weightings, for three periods:

For 1924–45: 0.37 (common stock) + 0.63 (long bonds)
For 1946–51: 0.37 (common stock) + 0.58 (long bonds) + 0.05 (T-bills)
For 1952–59: 0.37 (common stocks) + 0.38 (long bonds) + 0.15 (mortgages) + 0.10 (T-bills)

The rates added are believed to be reasonable. However, they may have overstated somewhat the involvement in common stock of the typical pension plan in the early years.

For analysis and calculation, the rates for this series were taken to represent ${}_{t}h$, the total rate of return for pension plans for years 1924–1987. These rates are shown in Table 3, together with 3-year and 9-year average rates.

Net rates of interest were calculated from the corresponding total rates of return and change in CPI and are shown in Table 4, together with 3-year, 9-year, and 31-year average net rates.

In Tables 3 and 4, average rates are shown for odd-numbered years (3, 9, 31) rather than even-numbered years, so that the average rate can be printed at the central year.

Since 1945 the CPI has moved quite smoothly from year to year, while total and net rates have fluctuated greatly.

After the period of post-war readjustment, the CPI rate began in 1961 to increase steadily to a peak in 1974. From 1961 to 1970, the trend in total

TABLE 3

ANNUAL PERCENTAGE RATES OF CHANGE/RETURN

(1)	(2)	CPI		(5)	Total Rate of Return	
Year	CPI for the Year	(3)* 3-Year Average	(4)* 9-Year Average	Total Rate of Return for the Year	(6)* 3-Year Average	(7)* 9-Year Average
1924.....	-2.14			9.10		
1925.....	2.73			13.89		
1926.....	-1.60	0.00		12.43	16.36	
1927.....	-1.08	-0.71		23.03	15.89	
1928.....	0.55	0.72		12.53	10.40	
1929.....	2.72	-0.92	-2.63	-2.82	1.07	6.45
1930.....	-5.82	-4.55	-2.77	-5.60	-8.07	7.05
1931.....	-10.11	-8.04	-2.38	-15.33	-6.27	6.98
1932.....	-8.13	-6.82	-2.11	3.01	2.58	6.33
1933.....	-2.04	-3.01	-1.75	23.75	15.15	4.22
1934.....	1.39	0.45	-2.25	19.78	18.31	5.28
1935.....	2.05	1.59	-1.32	11.73	15.97	5.73
1936.....	1.34	2.45	0.34	16.54	6.95	7.51
1937.....	3.97	1.10	2.02	-6.04	5.24	7.56
1938.....	-1.91	1.52	2.64	6.43	-0.66	5.84
1939.....	2.60	1.67	2.60	-1.96	0.90	4.79
1940.....	4.43	4.55	2.25	-1.57	-0.05	4.27
1941.....	6.67	4.83	2.22	3.46	2.91	4.27
1942.....	3.41	3.70	2.44	7.03	6.63	5.33
1943.....	1.10	1.12	4.25	9.48	7.79	4.85
1944.....	-1.09	0.36	4.90	6.89	10.87	5.45
1945.....	1.10	1.95	4.54	16.46	8.62	6.93
1946.....	5.98	7.17	4.47	2.95	6.99	8.62
1947.....	14.87	9.71	5.27	2.15	2.76	8.69
1948.....	8.48	8.05	4.94	3.17	5.58	7.79
1949.....	1.23	5.23	5.07	11.67	11.11	7.33
1950.....	6.10	5.95	4.98	19.08	12.70	7.65
1951.....	10.73	4.91	4.35	7.64	9.22	8.58
1952.....	-1.73	2.86	3.11	1.64	4.00	8.71
1953.....	0.00	-0.46	2.41	2.81	7.73	7.80
1954.....	0.35	0.23	2.57	19.64	11.02	7.72
1955.....	0.35	1.27	2.04	11.26	11.18	5.72
1956.....	3.15	1.84	1.03	3.24	3.20	5.92
1957.....	2.03	2.61	1.26	-4.31	3.07	7.21
1958.....	2.66	1.99	1.44	10.85	2.21	7.12
1959.....	1.29	1.74	1.61	0.66	6.90	5.92

TABLE 3—*Continued*

(1) Year	(2) CPI for the Year	CPI: (3)* 3-Year Average	CPI: (4)* 9-Year Average	(5) Total Rate of Return for the Year	Total Rate of Return: (6)* 3-Year Average	Total Rate of Return: (7)* 9-Year Average
1960.....	1.28	0.96	1.77	9.50	7.69	5.90
1961.....	0.32	1.06	1.76	13.30	8.16	5.93
1962.....	1.57	1.25	1.91	2.00	7.70	6.17
1963.....	1.86	1.75	2.08	8.10	7.00	5.82
1964.....	1.82	2.22	2.39	11.10	7.52	6.81
1965.....	2.99	2.76	2.76	3.50	3.96	5.36
1966.....	3.48	3.56	2.89	−2.30	2.85	4.05
1967.....	4.20	3.90	3.26	7.60	4.77	5.19
1968.....	4.03	4.29	3.62	9.40	4.45	6.26
1969.....	4.65	3.38	4.43	−3.20	2.37	4.78
1970.....	1.48	3.66	5.44	1.30	3.33	2.82
1971.....	4.87	3.80	6.11	12.50	10.50	4.51
1972.....	5.10	6.39	6.30	18.40	9.25	5.02
1973.....	9.27	8.86	6.91	−2.10	0.40	4.94
1974.....	12.32	10.36	7.32	−12.70	−1.10	6.82
1975.....	9.53	9.22	8.27	13.20	3.56	8.33
1976.....	5.91	8.29	8.97	12.40	11.42	8.94
1977.....	9.46	7.90	9.76	8.70	11.51	7.09
1978.....	8.36	9.20	9.75	13.50	12.37	9.65
1979.....	9.80	9.78	8.88	15.00	15.58	13.60
1980.....	11.19	11.03	8.23	18.30	11.36	13.10
1981.....	12.10	10.84	8.05	1.50	13.29	14.29
1982.....	9.26	8.59	7.46	21.10	13.83	14.76
1983.....	4.55	5.83	6.99	20.00	16.50	13.70
1984.....	3.76	4.22		8.80	17.26	
1985.....	4.35	4.09		23.50	14.87	
1986.....	4.17	4.22		12.80	13.30	
1987.....	4.15			4.40		

*Average rates are shown at the central year.

TABLE 4

ANNUAL IMPLICIT NET RATE OF INTEREST (IN PERCENT)

		Rate of Interest		
(1) Year	(2) Rate for the Year	(3)* 3-Year Average	(4)* 9-Year Average	(5)* 31-Year Average
1924.....	11.49			
1925.....	10.86			
1926.....	14.26	16.36		
1927.....	24.38	16.73		
1928.....	11.92	9.61		
1929.....	−5.39	2.00	9.33	
1930.....	0.23	−3.69	10.10	
1931.....	−5.80	1.92	9.58	
1932.....	12.13	10.09	8.63	
1933.....	26.32	18.72	6.08	
1934.....	18.14	17.78	7.71	
1935.....	9.49	14.15	7.14	
1936.....	15.00	4.40	7.14	
1937.....	−9.63	4.09	5.43	
1938.....	8.51	−2.15	3.12	
1939.....	−4.45	−0.76	2.13	
1940.....	−5.74	−4.41	1.98	5.66
1941.....	−3.01	−1.83	2.00	5.31
1942.....	3.50	2.82	2.82	4.64
1943.....	8.29	6.60	0.57	4.17
1944.....	8.07	10.47	0.52	3.77
1945.....	15.19	6.54	2.29	4.22
1946.....	−2.86	−0.16	3.97	4.62
1947.....	−11.07	−6.34	3.24	4.83
1948.....	−4.90	−2.29	2.72	4.65
1949.....	10.31	5.60	2.15	4.16
1950.....	12.24	6.37	2.54	3.61
1951.....	−2.79	4.11	4.06	3.12
1952.....	3.43	1.11	5.43	2.76
1953.....	2.81	8.23	5.27	3.27
1954.....	19.22	10.76	5.02	2.74
1955.....	10.87	9.78	3.61	2.88
1956.....	0.08	1.34	4.84	3.31
1957.....	−6.22	0.45	5.87	3.81
1958.....	7.97	0.21	5.60	3.33
1959.....	−0.63	5.07	4.24	2.23

TABLE 4—*Continued*

(1) Year	(2) Rate for the Year	Rate of Interest (3)* 3-Year Average	(4)* 9-Year Average	(5)* 31-Year Average
1960.....	8.12	6.66	4.05	2.08
1961.....	12.94	7.03	4.10	1.81
1962.....	0.42	6.37	4.18	1.89
1963.....	6.13	5.16	3.66	2.42
1964.....	9.11	5.18	4.32	2.74
1965.....	0.50	1.16	2.53	2.62
1966.....	−5.59	−0.68	1.13	1.92
1967.....	3.26	0.84	1.87	2.35
1968.....	5.16	0.15	2.55	2.69
1969.....	−7.50	−0.98	0.33	2.76
1970.....	−0.18	−0.32	−2.49	2.73
1971.....	7.28	6.45	−1.51	2.66
1972.....	12.65	2.69	−1.21	2.66
1973.....	−10.41	−7.77	−1.84	
1974.....	−22.28	−10.38	−0.47	
1975.....	3.35	−5.18	0.06	
1976.....	6.13	2.89	−0.03	
1977.....	−0.69	3.35	−2.43	
1978.....	4.74	2.90	−0.09	
1979.....	4.74	5.29	4.33	
1980.....	6.39	0.30	4.50	
1981.....	−9.46	2.21	5.77	
1982.....	10.84	4.83	6.79	
1983.....	14.78	10.08	6.27	
1984.....	4.86	12.51		
1985.....	18.35	10.35		
1986.....	8.28	8.71		
1987.....	0.24			

*Average rates are shown at the central year.

rates was downward, and in net rates downward even more so. This culminated in the difficult years 1973 and 1974 when the battle to restrain inflation was at its peak, the CPI increasing strongly, total return rates being negative, and net rates highly negative.

From 1975 to 1987 there has been a strong recovery. The CPI rate has decreased after 1981. Total and net rates have been strongly positive.

In view of these variations, it is perhaps surprising that the net rates for long periods, such as 31 years, have been steadily positive, as shown by the final column in Table 4.

A summary of the experience of the period 1945–1987 is given by these rates:

Period	Average Net Rate
1945–1959.........	3.23%
1960–1987.........	2.93
1945–1987.........	3.03%

In the testing of the period 1946–1987, the initial calculations for the model plan take the implicit net rate of interest for valuation of liabilities and contributions to be 3 percent.

F. Methodology

The year 1971 is arbitrarily chosen as the year for the first calculations, which are made in steps:

1. Lives and valuation factors by age
2. Average salaries and average accrued benefits by age, for 1971
3. Current service contribution, payments, and liability, by age, for 1971
4. Totals for 1971.

The data for 1971 at the valuation rate of interest, 3 percent, are shown in Table 5.

The model plan has a stationary population and uses the same index for increases in salary and accrued benefits. These features simplify the relations between three items in two successive years:

TABLE 5

MODEL PENSION PLAN DATA FOR 1971

Age at Beginning of Year	Lives at Year-End	Average Salary	Average Accrued Benefit	Current Contribution at 3%	Payments at 3%	Year-End Liability at 3%
30......	172	7,312	73	53,227		54,025
31......	163	7,422	222	105,727	2,701	160,177
32......	155	7,533	372	105,006	8,009	263,315
33......	147	7,646	525	105,290	13,165	363,507
34......	140	7,761	680	103,578	18,175	460,824
35......	133	7,877	838	102,872	23,041	555,331
36......	126	7,995	998	102,170	27,766	647,093
37......	120	8,115	1,160	101,473	32,354	736,176
38......	114	8,237	1,325	100,780	36,808	822,640
39......	108	8,360	1,492	100,093	41,131	906,547
40......	103	8,486	1,662	99,410	45,326	987,958
41......	98	8,613	1,834	98,732	49,397	1,066,929
42......	93	8,742	2,009	98,058	53,345	1,143,519
43......	88	8,873	2,186	97,389	57,175	1,217,783
44......	84	9,007	2,366	96,725	60,888	1,289,777
45......	80	9,142	2,549	96,065	64,487	1,359,552
46......	76	9,279	2,735	95,409	67,976	1,427,162
47......	72	9,418	2,923	94,758	71,357	1,492,658
48......	68	9,559	3,114	94,112	74,631	1,556,089
49......	65	9,703	3,308	93,470	77,803	1,617,505
50......	62	9,848	3,505	92,832	80,874	1,676,954
51......	58	9,996	3,705	92,199	83,846	1,734,481
52......	56	10,146	3,908	91,570	86,722	1,790,133
53......	53	10,298	4,114	90,945	89,505	1,843,954
54......	50	10,453	4,323	90,325	92,196	1,895,989
55......	48	10,609	4,535	89,708	94,797	1,946,279
56......	45	10,768	4,751	89,096	97,312	1,994,867
57......	43	10,930	4,969	88,488	99,741	2,041,793
58......	41	11,094	5,191	87,885	102,087	2,087,097
59......	39	11,260	5,417	87,285	104,353	2,130,819
60......	37	11,429	5,645	86,690	106,539	2,172,996
61......	35	11,601	5,877	86,098	108,647	2,213,667
62......	33	11,775	6,113	85,511	110,681	2,252,866
63......	32	11,951	6,352	84,927	112,641	2,290,631
64......	30	12,131	6,594	84,348	114,529	2,326,997
65......	29		6,594		192,985	2,197,999
66......	29		6,594		188,893	2,069,342
67......	28		6,594		184,440	1,941,410
68......	27		6,594		179,610	1,814,612
69......	26		6,594		174,388	1,689,386
70......	26		6,594		168,744	1,566,213
71......	25		6,594		162,671	1,445,597
72......	24		6,594		156,207	1,328,018
73......	22		6,594		149,427	1,213,893
74......	21		6,594		142,394	1,103,586
75......	20		6,594		135,142	997,439
76......	19		6,594		127,670	895,801
77......	18		6,594		119,950	799,065
78......	17		6,594		111,968	707,645
79......	15		6,594		103,771	621,925

TABLE 5—*Continued*

Age at Beginning of Year	Lives at Year-End	Average Salary	Average Accrued Benefit	Current Contribution at 3%	Payments at 3%	Year-End Liability at 3%
80......	14		6,594		95,429	542,224
81......	13		6,594		87,029	468,785
82......	11		6,594		78,683	401,740
83......	10		6,594		70,493	341,122
84......	9		6,594		62,556	286,862
85......	8		6,594		54,968	238,794
86......	7		6,594		47,808	196,662
87......	6		6,594		41,140	160,138
88......	5		6,594		35,010	128,836
89......	4		6,594		29,445	102,333
90......	3		6,594		24,455	80,177
91......	3		6,594		20,043	61,907
92......	2		6,594		16,201	47,050
93......	2		6,594		12,908	35,143
94......	1		6,594		10,122	25,752
95......	1		6,594		7,792	18,483
96......	1		6,594		5,880	12,968
97......	1		6,594		4,342	8,874
98......	0		6,594		3,132	5,905
99......	0		6,594		2,201	3,809
100......	0		6,594		1,503	2,371
101......	0		6,594		993	1,417
102......	0		6,594		632	806
103......	0		6,594		385	432
104......	0		6,594		223	215
105......	0		6,594		120	97
106......	0		6,594		59	38
107......	0		6,594		26	13
108......	0		6,594		10	3
109......	0		6,594		3	0
110......	0		6,594		0	0
30–64....	2,864			3,271,250	2,310,005	48,528,089
65–110...	447				3,011,848	23,564,887
Total	3,311			3,271,250	5,321,853	72,092,976

Note: Totals do not always agree with totals by column because of rounding in figures by age.

Liability

$$ {}_{t}L^{i} = {}_{t-1}L^{i} \cdot (1 + {}_{t-1}r) \qquad (12) $$

Current Service Contribution

$$ {}_{t}C^{i} = {}_{t-1}C^{i} \cdot (1 + {}_{t-1}r) \qquad (13) $$

Payments

$$ {}_{t}P = {}_{t-1}P \cdot (1 + {}_{t-2}r) \qquad (14) $$

A verification of these relations is given in the Appendix.

These relations are used to obtain the three items for years 1945–1987.

When a period for testing has been selected, the asset value for the model plan at the beginning of the period is fixed. Asset values at market value for later years are obtained by applying Equation (5), in which ${}_{t}h$, the total rate of return for year t, is the total rate from Table 3.

Surplus at the end of the year is the excess of assets over liability.

In order to demonstrate balance sheets for a period under differing conditions, the term "track" is used. A "track" is the definition of assets at the beginning of the period, together with payments, liabilities, contributions and special payments, and refunds during the period.

IV. FINANCING

To understand the financing difficulties that the model plan would have had in the period 1946–1987 and to devise a financing system that in retrospect would have relieved most of those difficulties, annual statements are developed for a number of tracks.

Track 1 (Table 6) is the starting point; it illustrates the fluctuations in surplus for the period 1960–1987. This illustration leads to the thought that the liability should be defined in terms of an asset fluctuation reserve, so that the liability will vary with asset values and fluctuations will be relieved. This thought process is discussed below under the heading "Valuation in a Modern Economy." The method is then used and extended to other periods and other funding situations and illustrated by other tracks.

- Track 2 (Table 9) illustrates the method for 1960–1987.
- Track 3 (Table 10) illustrates how the method can be applied to a partially funded situation.
- Track 4 (Table 11) illustrates the method for 1946–1987, when refunds and special payments are provided for and when starting assets are at 80 percent of the 3 percent liability.

- Track 5 (Table 12) illustrates the method for 1946–1987, when refunds and special payments are provided for and when starting assets are at 120 percent of the 3 percent liability.
- Track 6 (Table 13) illustrates the method for 1946–1987, when contributions, payments, and liability are reduced from a 3 percent basis to a 4 percent basis, refunds and special payments are provided for, and starting assets are at 80 percent of the 4 percent liability.

The features of the tracks are summarized in the following table.

Track	Table No.	Period	Basis of Contributions and Payments	Liability*	Starting Assets as Percentage of Unadjusted Liability	Refunds and Special Payments Provided for
1	6	1960–87	3%	3%	100%	No
2	9	1960–87	3	3 adj	100	No
3	10	1960–87	3	3 adj	50	No
4	11	1946–87	3	3 adj	80	Yes
5	12	1946–87	3	3 adj	120	Yes
6	13	1946–87	4	4 adj	80	Yes

*If liability is adjusted and expressed in terms of an asset fluctuation reserve, it is shown as "adj."

The illustrations are discussed below.

A. Track 1 (Table 6)

The period 1960–1987 is used for the illustration. In the period the CPI rate increases markedly and then retreats. It includes the unprecedented years 1973 and 1974. It does not include a world war or a major depression. It should be a good period for studying fluctuations.

The starting value of assets is at 100 percent of the 3 percent liability, and assets are at market value. Liabilities are at 3 percent. Surplus in terms of the 3 percent liability goes from a high of +26 percent to a low of −17 percent, a range of 43 percent.

B. Valuation in a Modern Economy

From Tables 1, 3, and 4, the explanation for the track 1 financing is evident. Negative rates of return on pension plan assets are unusual, even when assets are at market value. Two successive years of negative rates are doubly unusual but did occur in 1973 and 1974. For the same two years the combined increase in the CPI was at next to its highest level in 60 years,

TABLE 6

ANNUAL STATEMENTS BY YEAR—TRACK 1 (IN THOUSANDS)

Year	Current Contribution at 3%	Payments at 3%	Liability at 3%	Assets	Surplus	Surplus as Percentage of Liability at 3%
1959			54,202	54,202	0	0
1960	2,491	4,061	54,901	57,706	2,805	5
1961	2,523	4,114	55,604	63,685	8,081	15
1962	2,531	4,167	55,782	63,307	7,525	13
1963	2,571	4,180	56,658	66,760	10,103	18
1964	2,619	4,245	57,712	72,454	14,742	26
1965	2,666	4,324	58,762	73,302	14,540	25
1966	2,746	4,403	60,519	69,978	9,460	16
1967	2,842	4,535	62,625	73,539	10,914	17
1968	2,961	4,693	65,255	78,639	13,384	21
1969	3,080	4,890	67,885	74,342	6,457	10
1970	3,224	5,087	71,042	73,433	2,392	3
1971	3,271	5,323	72,093	80,432	8,339	12
1972	3,431	5,402	75,604	93,079	17,475	23
1973	3,606	5,665	79,460	89,086	9,626	12
1974	3,940	5,954	86,826	75,886	−10,940	−13
1975	4,425	6,506	97,523	83,684	−13,838	−14
1976	4,847	7,308	106,816	91,448	−15,369	−14
1977	5,133	8,004	113,129	96,408	−16,721	−15
1978	5,619	8,477	123,831	106,372	−17,459	−14
1979	6,089	9,279	134,184	118,898	−15,285	−11
1980	6,685	10,055	147,334	136,979	−10,354	−7
1981	7,433	11,040	163,820	135,400	−28,420	−17
1982	8,333	12,275	183,642	159,611	−24,031	−13
1983	9,104	13,761	200,648	186,412	−14,236	−7
1984	9,519	15,035	209,777	197,057	−12,720	−6
1985	9,877	15,719	217,665	236,836	19,171	9
1986	10,306	16,310	227,133	260,763	33,630	15
1987	10,736	17,020	236,605	265,815	29,210	12

being exceeded only by the combined increase in 1980 and 1981. So for 1973 and 1974 the unusual combination of negative rates of return and record high increases in CPI gave rise to the sudden drop in the surplus of the model pension plan.

In the early 1970s inflation was beginning to advance rapidly. Central banking authorities sought to restrain it by restricting credit and raising interest rates. The move was successful because the increase in the CPI dropped to a lower level in 1976, only to begin to rise, reaching high levels again in 1980 and 1981. Again interest rates were raised, this time to unprecedented heights. This second attempt was more successful. The annual increase in the CPI came down in 1983 to the 4 percent level.

From 1975 to 1987, the total rates of return have been strongly positive, and CPI increases from 1983 to 1988 have been close to 4 percent, indicating a strong recovery in net rates from 1975 to 1987.

If rapid and large reductions in the market value of assets occur, how should liabilities be treated?

If the rapid reductions are not indicating some uncertainty that expected investment income will be paid and not indicating that a major and general recession in business activity is probable, then the rapid reductions have the effect of reducing the total rate of return on assets in the present but increasing the total rate of return in the future. It is probable that after these rapid reductions, the rate of increase in the CPI will be considerably lower than in the immediate past, and this together with the higher total rate of return in the future indicates a higher net rate of return in the future. Table 3 is evidence that this has happened in the past. It would then be appropriate to increase the implicit net rate of interest used in the calculation of liability and to reduce the liability. The question is, how much reduction in liability is appropriate?

To shed light on this question, a simple simulation has been made to illustrate the large reduction in assets at market value during the early 1970s and the related reduction in liability. Table 6 shows that, for the model plan, the assets in relation to the liability at 3 percent are reduced from 123 percent at the end of 1972 to 87 percent at the end of 1974, a total reduction of 36 percent. The total reduction from 1964 to 1981 is 43 percent.

For the simulation, it is assumed that a sudden reduction in the market value of assets of the model plan occurs at the end of 1971. The reduction is 43 percent of the liability at 3 percent. The relation between assets and liability is achieved by making a number of assumptions, which are described below and illustrated in detail in Table 7:

- The model for assets corresponding to the liability for active lives is a savings bond without coupons and of term 15 years. The model of the liability for active lives is the discount factor for 15 years at the appropriate net rate, multiplied by the life annuity value at age 65 at the net rate of 3 percent.
- The model for assets corresponding to the liability for retired lives is a mortgage in the form of an annuity-certain of term 9 years. The model of the liability for retired lives is also an annuity-certain of term 9 years.
- The annual rate of CPI increase is set at 4.85 percent both when asset values are high and when asset values are low. This rate appears to be consistent with the rates in Table 3.

TABLE 7

VALUATIONS WITH LARGE REDUCTION IN MARKET VALUE OF ASSETS

Assets				Liability				
(1) Total Rate of Return on Assets (h)	(2) Market Value Factor per Unit	(3) Units	(4) = (2) × (3) Asset Value (in 000's)	(5) Future Rate of Change in CPI (r)	(6) Net Rate of Interest (i)*	(7) Factor per Unit Accrued Benefit	(8) Accrued Benefit Units (Annual Payment)	(9) = (7) × (8) Liability Value (in 000's)
Active Lives								
6.00%	0.41727†	153,936	64,233	4.85%	1.10%	9.957‡	6,451	64,233
10.00	0.23939†	153,936	36,851	4.85	4.91	5.713‡	6,451	36,855
Retired Lives								
6.00%	7.006§	3,608	25,276	4.85%	1.10%	8.5739‖	2,948	25,276
10.00	6.047§	3,608	21,818	4.85	4.91	7.3123‖	2,948	21,557
Total Active and Retired Lives								
6.00%	—	—	89,509	4.85%	1.10%	—	—	89,509
10.00	—	—	58,669	4.85	4.91	—	—	58,412

*$i=[(1+h)/(1+r)]-1$

†Factor is v^{15} at rate h.

‡Factor is (v^{15} at rate i)$\times$($\bar{a}_{65}$ at 3%).

§Factor is $\bar{a}_{\overline{9}|}$ at rate h.

‖Factor is $\bar{a}_{\overline{9}|}$ at rate i.

- The reduction in asset values occurs when the market total rate of return increases from 6 percent to 10 percent. The reduction in liability occurs at the same time and when the corresponding net rate increases from 1.10 percent to 4.91 percent.

In this simulation, assets and liabilities are closely related and well matched, and Table 7 shows that the reduction in liability is very close to the reduction in assets. In real life, assets and liabilities inevitably will be less closely related and less well matched than this. However, the real-life differences will be accumulated and combined in later valuations, and year-to-year differences may be expected to offset each other.

In the following demonstrations, the liability is assumed to vary with asset value within the limits which have been set.

At the end of year t, the balance sheet can be written as

$$ {}_tA = {}_tL^{3\%} + {}_tS \tag{15} $$

where ${}_tS$ is positive or negative, or

$$ \text{Assets} = \text{Liability at } 3\% + \text{surplus or } - \text{deficiency} $$

or

$$ {}_tA = 0.80{}_tL^{3\%} + (0.20{}_tL^{3\%} + {}_tS). \tag{16} $$

The term in parentheses can be written as an asset fluctuation reserve, ${}_tAR$, which varies with asset values from year $t+1$ onward and is defined as ${}_tA - 0.80{}_tL^{3\%}$, with a maximum of $0.40{}_tL^{3\%}$ and a minimum of zero.

With this definition of ${}_tAR$, the liability can be written in adjusted form as:

$$ \text{Adjusted Liability} = 0.80{}_tL^{3\%} + {}_tAR. \tag{17} $$

The adjusted liability varies with market value of assets within the limits ${}_tL^{3\%} \pm 0.20{}_tL^{3\%}$, and it is expected to absorb the fluctuations in Table 6.

With the introduction of the asset fluctuation reserve, changes in balance sheet items occur and are defined in detail in Table 8.

C. Testing the Asset Fluctuation Reserve

The asset fluctuation reserve should not be too large. If it were too large, an underfunding might occur from which it would be difficult to recover. This suggests we test the lower limit.

TABLE 8

VARIATION IN BALANCE SHEET ITEMS—TRACK 2

Asset Value ${}_tA$	${}_tAR$	${}_tL$	Surplus
${}_tA > 1.20\,{}_tL^{3\%}$	$0.40\,{}_tL^{3\%}$	$1.20\,{}_tL^{3\%}$	${}_tA - 1.20\,{}_tL^{3\%}$
$1.20\,{}_tL^{3\%} \geq {}_tA \geq 0.80\,{}_tL^{3\%}$	${}_tA - 0.80\,{}_tL^{3\%}$	${}_tA$	0
$0.80\,{}_tL^{3\%} > {}_tA$	0	$0.80\,{}_tL^{3\%}$	$-(0.80\,{}_tL^{3\%} - {}_tA)$

For the model plan, an increase in the valuation interest rate from 3 percent to 4½ percent causes a reduction in liability of 20 percent, so that

$$0.80\,{}_tL^{3\%} = {}_tL^{4\frac{1}{2}\%}. \tag{18}$$

After a reduction in assets and asset fluctuation reserve to zero, does it appear that the net interest rate will exceed 4½ percent? Following the drop in asset values in 1974, the net rate for 1976–1987 is 5.55 percent, so the test is met.

D. Track 2 (Table 9)

For track 2, all items except liability and surplus are the same as track 1: contributions, payments, assets, long-term funding objective at 3 percent, and starting assets.

With the new definitions, fluctuations in surplus have been all but eliminated.

E. Track 3 (Table 10)

If the track 2 method of financing is to be a practical method, it must work well in partially funded situations.

To examine this, an example is taken and illustrated as track 3 in Table 10.

At the end of 1959 the model plan is in force, but the benefit rate is taken to be 1 percent of salary per year of service. The annual data at this time, including assets and liabilities, are one-half of the data for track 2 in Table 9.

At the beginning of 1960, the plan changes so that the benefit rate doubles to 2 percent. The liability, contributions, and payments all double, but the invested assets remain at the 1959 level until augmented by the first of a series of amortization payments at year-end.

The Financial Accounting Standards Board (FASB) requires that unfunded liabilities be amortized over a period not exceeding the expected future lifetime of service of participants. The Ontario Pension Benefits Act requires a limit of 15 years.

TABLE 9

ANNUAL STATEMENTS BY YEAR—TRACK 2 (IN THOUSANDS)

(1) Year	(2) 80% of Liability at 3%	(3) Asset Fluctuation Reserve	(4) = (2) + (3) Liability	(5) Assets	(6) = (5) − (4) Surplus	Surplus as Percentage of Liability at 3%
1959	43,362	10,840	54,202	54,202	0	0
1960	43,921	13,785	57,706	57,706	0	0
1961	44,483	19,201	63,685	63,685	0	0
1962	44,626	18,681	63,307	63,307	0	0
1963	45,326	21,434	66,760	66,760	0	0
1964	46,169	23,085	69,254	72,454	3,200	6
1965	47,009	23,505	70,514	73,302	2,788	5
1966	48,415	21,563	69,978	69,978	0	0
1967	50,100	23,439	73,539	73,539	0	0
1968	52,204	26,102	78,306	78,639	333	1
1969	54,308	20,034	74,342	74,342	0	0
1970	56,833	16,600	73,433	73,433	0	0
1971	57,674	22,758	80,432	80,432	0	0
1972	60,483	30,242	90,725	93,079	2,354	3
1973	63,568	25,518	89,086	89,086	0	0
1974	69,460	6,425	75,886	75,886	0	0
1975	78,018	5,666	83,684	83,684	0	0
1976	85,453	5,995	91,448	91,448	0	0
1977	90,503	5,905	96,408	96,408	0	0
1978	99,065	7,307	106,372	106,372	0	0
1979	107,347	11,552	118,898	118,898	0	0
1980	117,867	19,112	136,979	136,979	0	0
1981	131,056	4,344	135,400	135,400	0	0
1982	146,914	12,697	159,611	159,611	0	0
1983	160,518	25,893	186,412	186,412	0	0
1984	167,822	29,235	197,057	197,057	0	0
1985	174,132	62,704	236,836	236,836	0	0
1986	181,707	79,057	260,763	260,763	0	0
1987	189,284	76,531	265,815	265,815	0	0

TABLE 10

ANNUAL STATEMENTS BY YEAR—TRACK 3 (IN THOUSANDS)

(1) Year	(2) Amortization Payment	(3) PV of Future Amortization Payments at 6%	(4) Invested Assets	(5)* Funded 80% Liability at 3%	(6) Asset Fluctuation Reserve	(7)† = (5) + (6) + (3) Total Liability	(8)† = (4) + (3) Total Assets	(9)† = (8) − (7) Surplus
1959			27,101	21,681	5,420	27,101	27,101	0
1960	2,790	25,933	30,821	23,175	7,646	56,754	56,754	0
1961	2,790	24,699	36,013	24,724	11,289	60,712	60,712	0
1962	2,790	23,391	37,872	25,913	11,959	61,263	61,263	0
1963	2,790	22,004	42,055	27,723	13,861	63,588	64,060	471
1964	2,790	20,535	47,796	29,741	14,871	65,147	68,331	3,184
1965	2,790	18,977	50,572	31,828	15,914	66,719	69,549	2,830
1966	2,790	17,325	50,561	34,555	16,006	67,886	67,886	0
1967	2,790	15,575	55,436	37,640	17,796	71,011	71,011	0
1968	2,790	13,719	61,624	41,229	20,395	75,343	75,343	0
1969	2,790	11,752	60,662	44,906	15,756	72,414	72,414	0
1970	2,790	9,668	62,365	49,099	13,266	72,032	72,032	0
1971	2,790	7,458	70,770	51,708	19,062	78,228	78,228	0
1972	2,790	5,115	84,429	56,391	28,038	89,544	89,544	0
1973	2,790	2,632	83,408	61,462	21,946	86,040	86,040	0
1974	2,790	0	73,719	69,460	4,258	73,719	73,719	0
1975			81,231	78,018	3,213	81,231	81,231	0
1976			88,691	85,453	3,238	88,691	88,691	0
1977			93,411	90,503	2,908	93,411	93,411	0
1978			102,971	99,065	3,906	102,971	102,971	0
1979			114,987	107,347	7,640	114,987	114,987	0
1980			132,352	117,867	14,485	132,352	132,352	0
1981			130,703	131,056	0	131,056	130,703	−353
1982			153,923	146,914	7,009	153,923	153,923	0
1983			179,586	160,518	19,068	179,586	179,586	0
1984			189,631	167,822	21,809	189,631	189,631	0
1985			227,865	174,132	53,533	227,665	227,665	0
1986			250,418	181,707	68,711	250,418	250,418	0
1987			255,015	189,284	65,731	255,015	255,015	0

*Column (5) = 0.80 (liability at 3% − column (3)).

†Columnar additions and subtractions are not always exact because of rounding in 000's.

In the example, a 15-year period is used, and the annual amortization payment is $27{,}101 \div a_{\overline{15}|}^{6\%} = 2{,}790$, where 6 percent is a rate of interest which would be considered a reasonable rate on an asset purchased in 1960.

During the amortization period at year t, the present value of future amortization payments (${}_tPV$) is an asset of the plan, so that with an unadjusted liability,

$$\underset{\text{(assets)}}{{}_tA} = \underset{\text{(invested assets)}}{{}_tIA} + {}_tPV = {}_tL^{3\%} + \underset{\text{(surplus)}}{{}_tS}. \tag{19}$$

The liability can be written in adjusted form by rewriting the right-hand side first as

$$0.80({}_tL^{3\%} - {}_tPV) + [0.20({}_tL^{3\%} - {}_tPV) + {}_tS] + {}_tPV \tag{20}$$

and then as

$$0.80({}_tL^{3\%} - {}_tPV) + {}_tAR + {}_tPV \tag{21}$$

where the asset fluctuation reserve, ${}_tAR$, varies with invested asset values from year $t+1$ onward and is defined as ${}_tIA - 0.80({}_tL^{3\%} - {}_tPV)$ with a maximum of $0.40({}_tL^{3\%} - {}_tPV)$ and a minimum of zero.

Table 10 shows that funding is satisfactory over the period. A comparison of Table 10 with Table 9 shows that the funding for tracks 3 and 2 are similar.

F. Track 4 (Table 11) and Track 5 (Table 12)

Another test for the method of financing is the making of adjustments for surplus and deficiencies, and there are many ways in which these adjustments can be defined. One definition has been arrived at empirically, and the financing of the model plan with this definition is illustrated for the period 1946–1987 with two levels of assets in 1945 as starting points. The illustrations are track 4 in Table 11 and track 5 in Table 12.

For track 2, negative surplus or deficiency arises when the asset value is lower than 80 percent of the liability at 3 percent, and the asset fluctuation reserve is zero.

The Ontario Pension Benefits Act and regulations required for many years that experience deficiencies be liquidated by special annual payments over a five-year period. The first payment would be $1/a_{\overline{5}|}$ times the deficiency, or 0.24 times the deficiency at 6 percent.

TABLE 11

ANNUAL STATEMENTS BY YEAR—TRACK 4 (IN THOUSANDS)

(1) Year	(2) Refund (−) or Special Payment (+)	(3) 80% of Liability at 3%	(4) Asset Fluctuation Reserve	(5)* = (3) + (4) Liability	(6) Assets	(7)* = (6) − (5) Surplus
1945	0	25,541	1	25,542	25,542	0
1946	887	25,821	419	26,241	26,241	0
1947	1,166	27,366	0	27,366	27,095	−271
1948	1,969	31,435	0	31,435	29,130	−2,305
1949	1,657	34,101	0	34,101	33,117	−984
1950	181	34,520	3,743	38,263	38,263	0
1951	292	36,626	3,652	40,278	40,278	0
1952	1,398	40,556	642	41,197	41,197	0
1953	970	39,854	1,911	41,765	41,765	0
1954	0	39,854	8,497	48,351	48,351	0
1955	0	39,994	12,254	52,248	52,248	0
1956	0	40,134	12,314	52,447	52,447	0
1957	0	41,398	7,408	48,806	48,806	0
1958	0	42,238	10,301	52,539	52,539	0
1959	0	43,362	8,023	51,384	51,384	0
1960	0	43,921	10,700	54,621	54,621	0
1961	0	44,483	15,706	60,189	60,189	0
1962	0	44,626	15,116	59,741	59,741	0
1963	0	45,326	17,580	62,906	62,906	0
1964	0	46,169	22,002	68,171	68,171	0
1965	0	47,009	21,861	68,870	68,870	0
1966	0	48,415	17,233	65,648	65,648	0
1967	0	50,100	18,780	68,880	68,880	0
1968	0	52,204	21,338	73,542	73,542	0
1969	0	54,308	15,100	69,408	69,408	0
1970	0	56,833	11,602	68,435	68,435	0
1971	0	57,674	17,135	74,809	74,809	0
1972	0	60,483	25,938	86,421	86,421	0
1973	0	63,568	19,000	82,568	82,568	0
1974	1,907	69,460	2,642	72,103	72,103	0
1975	2,008	78,018	3,393	81,411	81,411	0
1976	1,738	85,453	5,177	90,630	90,630	0
1977	1,511	90,503	6,527	97,031	97,031	0
1978	1,049	99,065	9,063	108,128	108,128	0
1979	0	107,347	13,570	120,917	120,917	0
1980	0	117,867	21,501	139,367	139,367	0
1981	2,307	131,056	9,075	140,132	140,132	0
1982	0	146,914	18,427	165,341	165,341	0
1983	0	160,518	32,769	193,287	193,287	0
1984	0	167,822	36,716	204,538	204,538	0
1985	0	174,132	71,943	246,075	246,075	0
1986	0	181,707	89,478	271,185	271,185	0
1987	0	189,284	87,411	276,695	276,695	0

*Columnar additions and subtractions are not always exact because of rounding in 000's.

TABLE 12

ANNUAL STATEMENTS BY YEAR—TRACK 5
(IN THOUSANDS)

(1) Year	(2) Refund (−) or Special Payment (+)	(3) 80% of Liability at 3%	(4) Asset Fluctuation Reserve	(5)* = (3) + (4) Liability	(6) Assets	(7)* = (6) − (5) Surplus
1945	0	25,541	12,770	38,311	38,311	0
1946	0	25,821	12,678	38,500	38,500	0
1947	0	27,366	11,086	38,452	38,452	0
1948	0	31,435	7,443	38,878	38,878	0
1949	0	34,101	8,246	42,346	42,346	0
1950	0	34,520	14,552	49,072	49,072	0
1951	0	36,626	14,995	51,621	51,621	0
1952	0	40,556	10,772	51,328	51,328	0
1953	0	39,854	11,356	51,210	51,210	0
1954	0	39,854	19,797	59,651	59,651	0
1955	0	39,994	19,997	59,990	64,821	4,830
1956	−1,255	40,134	20,067	60,200	64,173	3,973
1957	0	41,398	18,629	60,026	60,026	0
1958	0	42,238	21,119	63,357	64,977	1,620
1959	0	43,362	20,543	63,904	63,904	0
1960	0	43,921	21,960	65,881	68,330	2,449
1961	−2,159	44,483	22,242	66,725	73,562	6,838
1962	−1,546	44,626	22,313	66,938	71,835	4,897
1963	−1,918	45,326	22,663	67,989	74,062	6,073
1964	−2,715	46,169	23,085	69,254	77,851	8,597
1965	−2,010	47,009	23,505	70,514	76,879	6,365
1966	0	48,415	24,208	72,623	73,473	850
1967	0	50,100	25,050	75,150	77,299	2,149
1968	0	52,204	26,102	78,306	82,752	4,446
1969	0	54,308	24,016	78,324	78,324	0
1970	0	56,833	20,633	77,466	77,466	0
1971	0	57,674	27,295	84,969	84,969	0
1972	−1,854	60,483	30,242	90,725	96,597	5,872
1973	0	63,568	28,962	92,530	92,530	0
1974	0	69,460	9,432	78,892	78,892	0
1975	164	78,018	9,234	87,252	87,252	0
1976	163	85,453	10,167	95,620	95,620	0
1977	209	90,503	10,650	101,153	101,153	0
1978	0	99,065	12,693	111,758	111,758	0
1979	0	107,347	17,745	125,092	125,092	0
1980	0	117,867	26,439	144,306	144,306	0
1981	1,104	131,056	12,885	143,941	143,941	0
1982	0	146,914	23,040	169,954	169,954	0
1983	0	160,518	38,305	198,823	198,823	0
1984	0	167,822	42,739	210,561	210,561	0
1985	0	174,132	79,382	253,514	253,514	0
1986	0	181,707	90,853	272,560	279,575	7,015
1987	0	189,284	94,642	283,926	285,455	1,529

*Columnar additions and subtractions are not always exact because of rounding in 000's.

Deficiencies will be better avoided if special payments begin when the asset value is lower than 90 percent of the liability at 3 percent, rather than 80 percent. With these guides, special payments are defined. ${}_tA$ is the year-end asset value before any special payment or refund is made. If ${}_tA < 0.9{}_tL$, then the special payment for that year is $0.24(0.9{}_tL - {}_tA)$.

Surplus arises when the asset value is greater than 120 percent of the liability at 3 percent, and the asset fluctuation reserve is 40 percent of the liability at 3 percent.

Large surplus funds are unnecessary, but small surpluses provide a comforting margin. Refunds begin when the asset value exceeds 130 percent of the liability at 3 percent, rather than 120 percent.

If ${}_tA > 1.30{}_tL$, then the refund for the year is $0.24({}_tA - 1.2{}_tL)$.

Track 4 in Table 11 begins at the end of 1945 with asset value at $1,000 above 80 percent liability at 3 percent. In the immediate post-war period, the high inflation rates cause several years of special payments. In the late 1970s, there is another series of special payments, but deficiencies are avoided.

Track 5 in Table 12 begins at the end of 1945 with asset value at 120 percent of the liability at 3 percent. In 1956–1972, there are a number of refunds, followed by a few special payments from 1975 to 1981. Deficiencies are avoided.

Over the entire period the funding is stable, and the higher special payments for track 4, and higher refunds for track 5, have brought the two to very similar funding conditions at the end of the period.

G. Track 6 (Table 13)

Sometimes a pension fund is underfunded and becomes subject to deficiencies and special payments. To illustrate this, track 6 (Table 13) is defined for the period 1946–1987.

Track 6 has contributions, payments and liability at 4 percent, and all of these are lower than those for track 4.

The same formulas for refunds and special payments apply as for tracks 4 and 5, but with adjustment to 4 percent.

The track begins at the end of 1945 with asset value at $1,000 above 80 percent liability at 4 percent.

As for track 4, a series of special payments are required in the post-war period. Then in the 1970s a long series of special payments are required, some of them greater than annual contributions. The magnitude and the continuation of these special payments over a long time would have been

TABLE 13

ANNUAL STATEMENTS BY YEAR—TRACK 6
(IN THOUSANDS)

(1) Year	(2) Current Contribution at 4%	(3) Payments at 4%	(4) Refund (−) or Special Payment (+)	(5) 80% of Liability at 4%	(6) Asset Fluctuation Reserve	(7)* = (6) − (5) Liability	(8) Assets	(9)* = (8) − (7) Surplus
1945 ...			0	22,005	1	22,006	22,006	0
1946 ...	1,142	2,208	829	22,247	155	22,402	22,402	0
1947 ...	1,210	2,232	1,122	23,578	0	23,578	22,973	−605
1948 ...	1,390	2,366	1,862	27,084	0	27,084	24,572	−2,512
1949 ...	1,508	2,717	1,655	29,380	0	29,380	27,814	−1,567
1950 ...	1,526	2,948	455	29,742	2,277	32,018	32,018	0
1951 ...	1,619	2,984	589	31,556	2,080	33,636	33,636	0
1952 ...	1,793	3,166	1,561	34,942	0	34,942	34,365	−577
1953 ...	1,762	3,506	1,216	34,338	441	34,779	34,779	0
1954 ...	1,762	3,445	0	34,338	5,423	39,761	39,761	0
1955 ...	1,768	3,445	0	34,458	8,009	42,467	42,467	0
1956 ...	1,774	3,457	0	34,578	7,554	42,132	42,132	0
1957 ...	1,830	3,469	339	35,668	3,384	39,052	39,052	0
1958 ...	1,868	3,579	0	36,392	5,094	41,485	41,485	0
1959 ...	1,917	3,651	483	37,360	3,142	40,502	40,502	0
1960 ...	1,942	3,748	28	37,842	4,643	42,484	42,484	0
1961 ...	1,967	3,797	0	38,326	7,857	46,183	46,183	0
1962 ...	1,973	3,845	0	38,449	6,767	45,216	45,216	0
1963 ...	2,004	3,858	0	39,052	7,897	46,949	46,949	0
1964 ...	2,041	3,918	0	39,779	10,401	50,180	50,180	0
1965 ...	2,078	3,991	0	40,503	9,487	49,990	49,990	0
1966 ...	2,141	4,064	0	41,714	5,226	46,939	49,939	0
1967 ...	2,215	4,185	24	43,165	5,320	48,485	48,485	0
1968 ...	2,308	4,331	0	44,978	5,947	50,925	50,925	0
1969 ...	2,401	4,513	1,301	46,791	1,728	48,519	48,519	0
1970 ...	2,513	4,695	1,952	48,966	0	48,966	48,906	−61
1971 ...	2,550	4,913	815	49,691	3,632	53,323	53,323	0
1972 ...	2,674	4,986	0	52,111	8,499	60,610	60,610	0
1973 ...	2,811	5,229	1,121	54,769	3,297	58,066	58,066	0
1974 ...	3,071	5,495	4,537	59,846	0	59,846	52,958	−6,888
1975 ...	3,449	6,005	4,415	67,219	0	67,219	61,640	−5,579
1976 ...	3,778	6,744	4,007	73,625	0	73,625	70,140	−3,485
1977 ...	4,001	7,387	3,603	77,976	0	77,976	76,313	−1,663
1978 ...	4,380	7,824	3,140	85,353	726	86,079	86,079	0
1979 ...	4,746	8,564	2,199	92,488	4,598	97,086	97,086	0
1980 ...	5,211	9,280	920	101,552	9,780	111,332	111,332	0
1981 ...	5,794	10,189	4,429	112,916	88	113,004	113,004	0
1982 ...	6,496	11,329	2,615	126,578	7,541	134,119	134,119	0
1983 ...	7,097	12,700	194	138,300	16,674	154,974	154,974	0
1984 ...	7,420	13,876	191	144,592	17,470	162,062	162,062	0
1985 ...	7,699	14,508	0	150,029	42,509	192,538	192,538	0
1986 ...	8,034	15,053	0	156,555	53,159	209,714	209,714	0
1987 ...	8,369	15,708	0	163,083	48,358	211,442	211,442	0

*Columnar additions and subtractions are not always exact because of rounding in 000's.

stressful for the plan sponsor in comparison to the steady payments of track 5 and relatively steady payments of track 4.

Although irregular, the special payments have kept the plan funded.

H. Appraisal

The illustrations of tracks 3, 4, and 5 have shown an evident stability in the economic component of funding over long and different periods in the recent past and a variety of funding situations. This stability is in contrast to the funding of many actual defined-benefit pension plans with nonindexed or partially indexed benefits, which plans have accumulated large surpluses in the 1970s and 1980s.

The illustrations have depended on having economic data for these periods. Will the concepts of funding which have been used perform well in the future? Time will tell, but some comments can be made.

The value of the illustrations is not limited by the model. Other stationary populations would have shown the same relationship between assets and liabilities. Nonstationary populations would have required different weightings for the economic factors by calendar year.

The concepts have led to a method of funding in which assets and liabilities are related to each other in a simple and practical way and have provided for the unruly character of common stocks in the pension plan portfolio.

The method which is used can be significantly refined; for example, the size of the asset fluctuation reserve can be made to depend on the mix of assets.

The funding controls of contributions, special payments, and refunds and the setting of liability can be powerful. Although probably not required, a limit on inflationary increases in benefits can be imposed.

Good performance in the future will require that funding objectives be set so that funding margins can be built up, watched over, and made available to absorb fluctuations.

SUMMARY AND CONCLUSION

A model defined-benefit pension plan has been used to examine the financing of defined-benefit plans with fully indexed benefits for the years 1946–1987. These years are characterized by increases of inflation to high levels and then retreats from these levels and by much variation in rates of return on assets.

The model plan is a career-average plan which increases all accrued earned benefits according to the CPI.

It has been found that the financing for the period selected could have been done at the long-term implicit net rate of interest of 3 percent. Also, yearly progress is easier to understand and fluctuations are more easily absorbed, if the pension plan balance sheet uses market value for assets and a flexible funding objective consisting of a fixed conventional liability and a variable asset fluctuation reserve.

ACKNOWLEDGMENT

The author thanks Julius Vogel and Darryl E. Leach for their helpful comments on an early draft of the paper.

REFERENCES

1. BERIN, BARNET N., AND RICHTER, ANTHONY B. "Constant Replacement Ratios in Retirement," *TSA* XXXIV (1982): 9–27.
2. CANADIAN INSTITUTE OF ACTUARIES. "Report on Canadian Economic Statistics, 1924–1987." Ottawa, Ontario: CIA, July 1988.
3. DUNCAN, ROBERT M. "A Retirement System Granting Unit Annuities and Investing in Equities," *TSA* IV (1952): 317–44.
4. FURNISH, JEFFERY J. "Pension Plans in an Inflationary Environment," *TSA* XXXIV (1982): 29–55.
5. IBBOTSON, R.G., AND SINQUEFIELD, R.A. *Stocks, Bonds and Inflation. The Past (1926–1976) and The Future (1977–2000)*, Financial Analysts Research Foundation. Charlottesville, Va.: University of Virginia, 1977.
6. RAPPAPORT, ANNA M. "The Future of Retirement—An Update," *The Actuary* 24, no. 2 (February 1990): 1, 3–4.
7. *Report of the Task Force on Inflation Protection for Employment Pension Plans, 1988.* Toronto, Ont.: Queen's Printer for Ontario, Publications Services Section.
8. RICHMOND, GEROLD, AND ROSEN, MARK L. "Indexing Pensions—Protecting Post-retirement Purchasing Power," *TSA* XXXIV (1982): 157–93.

APPENDIX
FORMULAS AND CALCULATIONS

Formula (1)

For ages $x > 64$, the average accrued annual pension increases each year

$$ {}_t ab_x = {}_{t-1}ab_{x-1}(1 + {}_{t-1}r). \tag{A1} $$

An approximate equation is

$$ \bar{a}_x(1 + i) - \left(\frac{1 + p_x}{2}\right)\left(1 + \frac{i}{2}\right) = p_x \times \bar{a}_{x+1}. $$

Each year the increased accrued annual pension is paid to survivors, so that

$$ \begin{aligned} {}_{t-1}L^i_{x-1} &= {}_{t-1}n_{x-1} \times {}_{t-1}ab_{x-1} \times \bar{a}_x \\ {}^R_t P_x &= \left(\frac{{}_{t-1}n_{x-1} + {}_t n_x}{2}\right) \times {}_{t-1}ab_{x-1} \\ {}_t n_x &= {}_{t-1}n_{x-1} \times p_x. \end{aligned} \tag{A2} $$

under simplifying assumption 2.

$$ {}_t L^i_x = {}_t n_x \times {}_t ab_x \times \bar{a}_{x+1}. \tag{A3} $$

On substitution these six equations verify Formula (1).

Formula (2)

This formula is self-evident from the simplifying assumptions. It is worthwhile to verify that the details of the model plan conform to it.

For ages $30 < x \leq 64$, the average accrued annual pension increases each year

$$ {}_t ab_x = {}_{t-1}ab_{x-1}(1 + {}_{t-1}r) + 0.02\ {}_t Sal_x. \tag{A4} $$

New entrants are at age 30. Decrements are through death and termination, so that

$$_{t-1}n_{x-1} - {}^{T}_{t}d_x - {}^{D}_{t}d_x = {}_{t}n_x$$

$$_{t-1}L^{i}_{x-1} = {}_{t-1}n_{x-1} \times {}_{t-1}ab_{x-1} \times v^{65-x}\,\bar{a}_{65}$$

$$^{T}_{t}L^{i}_{x} = {}^{T}_{t}d_x \times {}_{t-1}ab_{x-1} \times \frac{v^{64-x}\,\bar{a}_{65}}{1+\frac{i}{2}} \tag{A5}$$

$$^{D}_{t}L^{i}_{x} = {}^{D}_{t}d_x \times {}_{t-1}ab_{x-1} \times \frac{v^{64-x}\,\bar{a}_{65}}{1+\frac{i}{2}} \tag{A6}$$

$$_{t}C^{i}_{x} = {}_{t}n_x \times (0.02)_{t}Sal_x \times \frac{v^{64-x}\,\bar{a}_{65}}{1+\frac{i}{2}} \tag{A7}$$

$$_{t}L^{i}_{x} = {}_{t}n_x \times {}_{t}ab_x \times v^{64-x}\,\bar{a}_{65}. \tag{A8}$$

On substitution, these seven equations verify that the yearly components of the model plan satisfy Formula (2).

Formulas (12), (13), (14)

Verification of these three formulas that apply to the model plan follows. The structure of the model plan has features that simplify the calculations:

(1) Average salaries at each age increase each year according to the same index that is used to increase average accrued benefits.
(2) Average salaries are simply related by age.
(3) The age distribution is stationary.
(4) Single payments on death or termination before 65 are the liabilities held.

Thus for average salaries,

$$\begin{aligned} {}_{t}Sal_x &= {}_{t-1}Sal_x\,(1 + {}_{t-1}r) \\ &= ({}_{t-1}Sal_{30}) \times 1.015^{x-30}\,(1 + {}_{t-1}r) \end{aligned} \tag{A9}$$

Equation (A4) may be repeatedly applied and Equation (A9) made use of, so that for ages $30<x\leq 64$,

$$_{t}ab_x = 0.02\left(\sum_{y=31}^{x} {}_{t}Sal_y + \frac{1}{2}\,{}_{t}Sal_{30}\right) \tag{A10}$$

Equation (A1) may be repeatedly applied and Equation (A9) made use of, so that for ages $x > 64$,

$$\begin{aligned} {}_tab_x &= 0.02\left(\sum_{y=31}^{64} {}_tSal_y + \frac{1}{2}\,{}_tSal_{30}\right) \\ &= {}_tab_{64} \end{aligned} \tag{A11}$$

From (A9), (A10), and (A11), it is clear that

$${}_tab_x = {}_{t-1}ab_x(1 + {}_{t-1}r) \tag{A12}$$

For ages $30 \le x \le 64$, the lives at the end of the year,

$$n_x = 172(0.95)^{x-30}$$

and

$$n_{64} = 172(0.95)^{34} = 30.$$

For ages $x > 64$,

$$n_x = 30\frac{\ell_{x+1}}{\ell_{65}},$$

where the ℓ's are taken from the GAM71 (Modified) mortality table. Because of the stationary population, the values of n_x are the same for all years t.

Liability

The formulas for the liability by age are (A3) and (A8). Because of the stationary populations and Equation (A12), when totals are taken, it follows that

$${}_tL^i = {}_{t-1}L^i(1 + {}_{t-1}r). \tag{12}$$

Current Service Contribution

The formula is in (A7). (Note that for entry age 30, the factor 1/2 should be applied.) Because of the stationary population and Equation (A9), when totals are taken, it follows that

$${}_tC^i = {}_{t-1}C^i(1 + {}_{t-1}r). \tag{13}$$

Payments

The liabilities released on terminations and deaths among active lives are in (A5) and (A6). In the model plan the payments made in these events are the liabilities released.

Also

$$ {}^{T}d_x + {}^{D}d_x = 0.05\, n_{x-1}, $$

So that

$$ {}^{T}_{t}P_x + {}^{D}_{t}P_x = 0.05 n_{x-1} \times {}_{t-1}ab_{x-1} \times \frac{v^{64-x}\,\bar{a}_{65}}{1 + \dfrac{i}{2}} \tag{A13} $$

for age range $30 < x \leq 64$.

For age range $x > 64$, the payments to retired lives for the stationary population can be written from (A2)

$$ {}^{R}_{t}P_x = \frac{n_{x-1} + n_x}{2} \times {}_{t-1}ab_{64}. \tag{A14} $$

Then

$$ {}_{t}P_x = {}^{T}_{t}P_x + {}^{D}_{t}P_x + {}^{R}_{t}P_x. $$

When totals are taken, from (A12),

$$ {}_{t}P = {}_{t-1}P(1 + {}_{t-2}r). \tag{14} $$

DISCUSSION OF PRECEDING PAPER

ROBERT J. MYERS:

Mr. Maynard has written a truly monumental paper showing how it is possible fiscally and administratively to index the benefits in a defined-benefit pension plan for changes in the Consumer Price Index (CPI). The key is in using a valuation interest rate equal to the real interest rate, after allowing for the effect of inflation (say, 3 percent) and also valuing the assets at market value at all times.

The paper does not go into the policy reasons for such an approach. In my opinion, the major weakness of defined-benefit pension plans in North America is the absence of indexing. As a result, pensions that appear to be quite adequate at the time of retirement (or even more than adequate) will most certainly "wither on the vine" and probably become inadequate after the pensioner has been on the roll for many years. The same situation also occurs for vested deferred pensions for those who withdraw from service before retirement, although here there is even more "withering" because of the longer period involved from when the flat pension amount is determined.

The only answer to this undesirable situation is indexing, or else the "excess interest" method after utilizing a valuation interest rate for the plan that is based on the real interest rate (rather than the going market nominal rate). The objection may well be raised that the cost of the plan will then be greatly increased. The answer thereto is that, if this is all that can be afforded, it would be far better to pay lower benefits initially, but have them automatically increased over time as inflation occurs. The indexing method described by Mr. Maynard would do this exactly. The excess interest method, if properly applied, would quite probably give a reasonably close approximation to this desired result (and might be considered "safer" by some plan sponsors).

For the sake of the record, I would point out that I expressed policy views similar to the foregoing in Chapter 11 of my book *Indexation of Pension and Other Benefits*, Pension Research Council, Wharton School, University of Pennsylvania (Homewood, Ill.: Richard D. Irwin, Inc., 1978).

D'ALTON S. RUDD:

I was taken by Mr. Maynard's paper, as in the late 1950s I had to set up a plan almost the same as the model indexed career-average plan of this paper. A very large German company was establishing a pension plan for its then-small Canadian operation and the local executives were given strict instructions. The plan must be career-average; accrued benefits must be indexed; and third (the variation from the paper), accrued benefits to date must be fully insured but only as far as indexing to date was concerned. I was working in the group annuity division of a large Canadian life insurance company and the plan was set up as a fully insured group annuity with the employer funding for the additional cost of indexing after the year-end. No doubt, the plan has not survived in this form. In my own firm, I have two small plans (less than 100 lives) that are final average earnings with partially indexed benefits and use the "net rate of return" concept.

In Ontario the consulting actuary deals with three different "official" net rates of return when dealing with future payments subject to both inflation and discounting at interest. In the courts when dealing with tort cases requiring the present value of lost future earnings, the judges got tired of listening to two actuaries and two economists arguing about future interest rates and inflation. For some years now the statute has required a net discount of 2.5 percent. In computing minimum transfer values for payment of commuted values of accrued benefits on termination of employment under pension plans as required by statute, the Canadian Institute of Actuaries specifies 3.5 percent for fully indexed benefits subject to modification in the first five years to grade off the initial difference one way or the other between the 3.5 percent and average current differences between new money returns and inflation. On the other hand, under the Family Law Act of Ontario and similar provisions in other provinces, in obtaining the value of a spouse's interest in a fully indexed pension plan, 3 percent is still used under an exposure draft, and the profession is still arguing over whether the rates should differ or the rates must be the same despite the different purposes.

I often say that actuaries are the only professionals who know they're always wrong as they deal with future interest rates and inflation. My concern with excellent papers such as Mr. Maynard's is that the past cannot predict the future. The world has changed very significantly since World War II. Inflation in Canada during the Korean War was choked off very quickly by requiring a 50 percent down payment through credit controls (I had just graduated and was trying to buy a car!). Such an approach cannot be applied today.

As was demonstrated by the huge global currency movements in September 1992, no central bank can protect its currency value through its foreign currency reserves. The central banks must use short-term interest rates that will move independently of inflation. In a country such as Canada, whether we are going to have inflation in the original sense of the word by printing money will be dependent upon who wins the next election and appoints the governor of our central bank. The CPI also is affected in a country with large exports and imports by the value of our dollar, which is also affected not only by internal monetary and fiscal policy but also by political factors both domestic and international.

The demonstrations in the paper seem to indicate successful navigation for the plan through the trials of the past.

With the requirements of the Ontario Pension Benefits Act in mind, Tables 11 and 12 bring in the concept of deficiency payments. I would assume that a "solvency valuation" (market value of assets and liabilities) has in effect been required. Of course, there is one problem in Canada: it is very difficult to get any refunds out of a pension plan.

Nevertheless, Mr. Maynard has demonstrated an interesting technique that I believe can be adapted to various regulatory requirements, at least in Canada. His calculations have been made on the basis of the 3 percent overall net return, which is effectively derived from the past data, which in turn develops the experience funds. Nevertheless, wise plan sponsors will hedge their bets on the future by techniques such as a maximum on the amount of indexing recognized in any one year with or without a carry-forward and limiting the guarantee of future indexing to a fixed date in the future. Also, it might be wise to force the investment manager to follow a strict formula and not try to second-guess the future stock market and future returns on short-, medium- and long-term bonds.

ROBERT L. BROWN:

I want to start by thanking Mr. Maynard for his thought-provoking paper. It is especially pleasing to see one of our semiretired members taking his scarce time to continue to produce high-quality research material for the profession.

Canada has had an ongoing debate about the ability of defined-benefit plan sponsors to offer and fund indexed retirement-income benefits for their plan participants. The debate has focused on two particular issues: affordability and variance. I would suggest that these two issues have been raised with equal concern and emotion. Employers have stated that they might be willing to extend indexation to retirement-income benefits if only they could be told with some level of assurance just what the cost would be. However, they state emphatically that they are not willing to "sign a blank check"; that is, the problem of the *variance* of the benefit level is at least as important as the cost itself.

In that regard, the author has provided an important model that could be used to minimize the variance associated with the indexation of benefits. However, I would like to ask a more basic question (maybe the answer lies in the paper, but I could not sort it out clearly on my own). Is there any more variance for a defined-benefit pension plan with indexed retirement-income benefits than for a plan with no indexation? Perhaps we could consider two possible indexation formulas here: full indexation to the cost of living and the Province of Ontario-proposed formula of 75 percent of CPI less 1 percent, and then compare them to a nonindexed plan on variability of surplus.

If it turns out that there is no more variance in surplus associated with indexation than without, then a key reason for nonindexation of benefits is lost. This is an important public policy item.

Again, I thank the author for his contribution and hope that he can provide an answer to my supplementary question.

HARRY M. SATANOVE:

Mr. Maynard is to be congratulated for a thorough analysis of the volatility of the funding position and level of funding contributions of a defined-benefit pension plan with indexed benefits. In my discussion, I refer to liabilities, surplus and deficit with the traditional meaning, rather than the adjusted values used by the author.

Plan sponsors have generally avoided plans that provide automatic inflation adjustments not only because of the cost of the automatic adjustments but also because of the unknown or uncertain cost. The author has demonstrated that although an indexed plan's surplus/deficit may be volatile from year to year, over the long term the assets can be expected to remain within a corridor of 80–120 percent of the plan's liabilities virtually the entire time. The author then concludes that because the surplus/deficit is confined to a narrow corridor, the funding contribution can be kept reasonably smooth.

It is not surprising that the funding contributions to an indexed plan should be less volatile than what might normally be required to a comparable plan providing nonindexed benefits. In both an indexed plan and a nonindexed plan, the surplus/deficit is equal to the assets minus the liabilities, and in both types of plans, the asset value fluctuates according to the assets' nominal rate of return. Consequently, in a plan whose benefits are not adjusted for inflation either before or after retirement, the asset fluctuations are reflected directly in the surplus/deficit fluctuations. As a result, the surplus/deficit of a nonindexed plan is related to nominal asset returns, so the standard deviation of *nominal* returns is a reasonable proxy of a nonindexed plan's surplus/deficit volatility.

In a plan whose benefits are adjusted for inflation both before and after retirement, the liabilities fluctuate according to inflation. Consequently, in a plan whose benefits are fully indexed, the surplus/deficit is related to the nominal returns of the assets minus the inflation returns of the liabilities, or a net real rate of return. As a result, the standard deviation of *real* returns is a reasonable proxy of an indexed plan's surplus/deficit volatility.

Table 1 shows standard deviations of the figures from the author's Table 3 column (5) [Total Rate of Return for the Year] and Table 4 column (2) [Real Rate of Return for the Year] over the entire period 1924–1987 as well as for each of the 31-year periods used by the author. Although the standard deviation of the real rate of return is greater than the standard deviation of the nominal rate of return over the entire period, the standard deviation of the real rate of return for each 31-year period is less than the corresponding standard deviation of the nominal return in all cases. Consequently, a plan sponsor providing indexed benefits in any one of the 31-year periods would have tended to experience less surplus/deficit volatility and hence less funding contribution volatility than a plan sponsor providing comparable but nonindexed benefits.

The greater volatility of the real rate of return over the entire period is due primarily to real rates of return falling from more than 5 percent in the first 31-year period to less than 3 percent in the last 31-year period. A plan sponsor that established a pension plan with indexed benefits in the early 1930s with a formula based on then-expected real returns would have had generally smooth contribution requirements over the entire period to date, but would have also had an increasing cost as real rates of return dropped.

I do not believe that an indexed plan could be funded by the method suggested by the author. In track 2, special payments to fund the deficit do not commence until the assets fall below 80 percent of the liabilities. Later, in tracks 4 and 5, the author shows the effect of starting the special payments

TABLE 1

STANDARD DEVIATIONS OVER 31-YEAR PERIODS

Year	Nominal Returns	Real Returns
1940	7.58	6.11
1941	7.24	5.77
1942	6.70	5.11
1943	6.31	4.58
1944	5.93	4.17
1945	6.32	4.61
1946	6.93	5.02
1947	7.49	5.22
1948	7.66	5.02
1949	7.25	4.47
1950	6.72	3.90
1951	6.27	3.41
1952	5.98	3.04
1953	6.48	3.51
1954	6.17	3.00
1955	6.27	3.13
1956	6.73	3.55
1957	7.21	4.06
1958	6.92	3.61
1959	6.20	2.63
1960	6.40	2.47
1961	6.27	2.18
1962	6.46	2.25
1963	6.82	2.76
1964	7.21	3.07
1965	7.42	2.94
1966	6.85	2.25
1967	7.29	2.68
1968	7.88	3.05
1969	8.07	3.12
1970	8.20	3.09
1971	8.25	3.01
1972	8.28	3.01
All Years	8.57	9.10

when the funding ratio drops below 90 percent. It can be argued that special payments in fact should start when the assets drop below 100 percent of the liabilities.

The author suggests that the 80 percent level may be a reasonable basis, because if that level has been reached because of declining markets, the markets may be expected to go back up. The author uses as an example the years 1973 and 1974 when the funding ratio of the model plan fell from 123 percent to 87 percent because of falling markets and then gradually returned to positive territory by 1985. The author suggests that the 1974 downturn was temporary and that declining markets can be expected to reverse.

Financial theory suggests that this premise does not hold. If a low point (as in 1974) is recognized by investors to be a temporary blip in a long-term upward trend, then investors would soon push the market back up to its true level. To that extent the market is efficient. An investor in 1974 would not know if 1972 values were the true values with 1974 as a temporary negative blip, or if 1974 values were the true values with 1972 as a temporary positive blip. Consequently, an investor must assume that at any given time the market value is the true value of a portfolio, with no expectation of a reversion to an unknown mean. As a result, it can be argued that after a downturn that leaves a plan in a deficit, the funding position has returned to its true long-term level, and special payments are needed immediately to correct the deficiency. Of course, with an asset valuation method that smooths asset values, both the 1972 and 1974 values can be taken into account in calculating the actuarial value of assets, dampening the effect of such large short-term changes.

(AUTHOR'S REVIEW OF DISCUSSIONS)

JOHN C. MAYNARD:

I am grateful to the discussants for their observations, comments, and questions on the paper.

Mr. Myers has drawn attention to the importance of making increases in pension payments from time to time. He refers to two methods of doing this: indexing and excess interest. The timing of increases is probably better with indexing, while the cost of increases is better aligned to the earnings of the pension fund with excess interest.

Dr. James Hickman has sent me an outline of two pension plans that have used the excess interest method: the Wisconsin Retirement System and the United Presbyterian Church (U.S.). For the first, the retired life fund has been valued at 5 percent, and during the 1980s the retired lives have had increases in benefits that were near to or greater than the rate of inflation. For the second, the fund is valued at 4½ percent; increases have been made since 1963 and they have been approximately equal to the rate of inflation. For both plans the actuary and the investment manager have worked together closely, and undoubtedly this would be applauded by Mr. Rudd.

Mr. Rudd refers to some of the issues he has dealt with in many years of guiding pension plans. He points out that the actuary cannot predict future interest rates and inflation. However, perhaps he would agree that a study of rates in the past may reveal relationships that are likely to recur and this knowledge may help in dealing with events in the future.

Mr. Brown and Mr. Satanove have referred to the cost of benefits and the variability of cost and have sought a comparison of these features between indexed and nonindexed benefits.

To help in answering these questions, a table of average rates (see Table 2) has been prepared from the rates in Tables 3 and 4 of the paper. The periods prior to 1934 have been excluded. The averages for 9-, 18-, 27-, and 54-year periods are shown for total return and net rates. The characteristics of these periods have been commented on in the paper.

TABLE 2

TABLE OF AVERAGE RATES

	Total Rate of Return	Rate of Change in CPI	Net Rate
Nine-Year Periods			
1934–1942	5.84%	2.64%	3.12%
1943–1951	8.69	5.27	3.24
1952–1960	5.92	1.03	4.84
1961–1969	5.36	2.76	2.53
1970–1978	6.82	7.32	−0.47
1979–1987	13.70	6.99	6.27
Standard Deviation	2.88		2.08
Eighteen-Year Periods			
1934–1951	7.27%		3.18%
1952–1969	5.64		3.69
1970–1987	10.26		2.90
Standard Deviation	1.91		0.32
Twenty-Seven-Year Periods			
1934–1960	6.82%		3.73%
1961–1987	8.63		2.78
Standard Deviation	0.91		0.48
Means for all Periods	7.72%		3.26%

Note: The average rates for nine-year periods are geometric averages. For longer periods the average rates are arithmetical averages of the rates for nine-year periods.

The 9-year rates are useful in showing trends, while the 18-year rates show better the long-term rates. The interpretation of results depends somewhat on the division points selected to define periods. If the 18-year period 1961–1978 had been selected, the net rate averages would have appeared differently. However, this need not be a concern. The 1970–1978 period was unusual because all monetary controls were launched to restrict credit and halt the inflation that had been growing for several decades. It is reasonable to combine this period with the following period of recovery rather than the previous period of growing inflation.

The standard deviations represent the variability of the average rates for selected periods. For each of the three periods, the standard deviations for the average net rates are lower than for the average total return rates. The reduction in the standard deviation for net rates when the period is lengthened from 9 to 18 years is interesting. This suggests for the future that the average net rate should be stable and not variable for periods of 20 years and up.

Turn now to cost and variability. The contribution to a pension plan for a block of benefits provides for the benefits to be paid together with some surplus (or deficit), so that:

Contribution = (1) The present value of actual benefits on the contribution date, plus
(2) The present value of surplus (or deficit) remaining after actual benefits have been paid.

For nonindexed benefits, term 1 depends on the average total return rate. For indexed benefits, term 1 depends on the average net rate.

It follows from this that the contribution for indexed benefits can be set with the confidence that the experience over long periods will be less variable than that for nonindexed benefits. Also, the contribution will be lower in relation to benefits because a margin need not be included to avoid deficits.

Mr. Santanove is concerned that special payments do not begin until asset values drop to 80 percent of the liability calculated at 3 percent and, if this happens, that there is an unrealistic dependence on the expectation that asset values will recover. Perhaps the following explanation will assist him.

Market values go up and down. What does a downturn in assets mean? As long as the downturn is not so severe as to indicate some lower probability that future income payments or capital installments will be paid, the downturn means that the yield on investments has increased. This implies that a higher discount rate should be used for calculating the liability. So the liability goes down with the asset values, and because of this, assets and liability are said to be related. As long as the benefit payments are made further into the future than the asset payments, it can be argued that assets and liabilities are also well-matched, and assets and liabilities should move in equal amounts, so that surplus or deficit is unchanged by the movement. If the deficit has not changed, there is no need to start a new series of special payments, and since assets and liability are properly related at the lower level, there is no requirement that asset values increase. What happens if assets drop so low that future payments are impaired? Then a real deficit has occurred and special payments should start. In the paper the dividing line for this situation is taken to be when assets drop below 80 percent of the liability, calculated at 3 percent. It is realized that the above explanation is not complete in some respects.

I thank the discussants for adding scope and interest to the paper.