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PRINCIPLES OF ACTUARIAL SCIENCE

SOCIETY OF ACTUARIES COMMITTEE ON ACTUARIAL PRINICPLES*

ABSTRACT

The Committee on Actuarial Principles is charged with identifying, circulating and organizing actuarial principles (as distinct from standards) and recommending the resulting statements of principles to the Board of Governors for review and adoption. In October 1991, the Board accepted the Committee's statement entitled "Principles of Actuarial Science." This statement, which constitutes the following paper, is an expression of opinion by the Committee on Actuarial Principles and has been authorized by the Board of Governors. It has not been submitted to the vote of the membership and thus should not be construed as an expression of opinion by the Society of Actuaries.

Editor's note: The Committee believes that continued publication of discussions of Principles of Actuarial Science is appropriate and may prove beneficial. Therefore, written discussions of this paper will continue to be considered for publication. For publication in Volume XLV of the Transactions, *discussions must be received by March 1, 1994.*

PREAMBLE

Actuarial science, the foundation for the actuarial profession, is an *applied science*. As an applied science, its theory is grounded in certain observations about the real world.

The practice of any profession is shaped by the experience of its members, as well as by accumulated scientific knowledge. The practice of the actuarial profession is based on:

- **Principles**—Statements grounded in observation and experience. Principles will be subject to change only if fundamental changes occur in our understanding of the observed world.
- **Methodologies**—Descriptions of applications of the Principles to defined areas of practice. Since Methodologies represent the state of the art, they

Note: This paper has been written by members of the Society of Actuaries. Its application should be limited to the areas of actuarial practice which fall within the purview of that Society. It is not intended to include the areas of property and casualty insurance.

*Arnold A. Dicke, *Chairperson*, Wayne Bergquist, Robert P. Clancy, Robert A. Miller III, Harry H. Panjer, Donald M. Peterson, Charles Barry H. Watson, and Warren Luckner, SOA Staff Liaison.

are likely to change as new techniques are developed in various practice areas.

- **Standards**—Rules of behavior, including, in particular, directives as to when and how professional judgment should be employed. Some Standards are prescriptions of professional conduct and are not usually subject to change. Others involve judgments needed to apply Principles or Methodologies to circumstances of practice and may change as practice circumstances change.

I. Principles

This paper attempts to set forth the fundamental principles that underlie actuarial science. Their statement is intentionally broad and rather formal. The intended audience is practicing actuaries, researchers and others, such as regulators and standard-setting bodies. Care has been taken to define terms with precision. Since actuarial science is rooted in mathematics, the exposition is similar in style to that of a mathematical monograph. To improve understanding, however, numerous clarifying Discussions of both Principles and Definitions have been provided.

The Principles have been grouped according to their basic subject matter. Statistical and Economic and Financial Principles have been borrowed from related disciplines and are limited to those necessary as background to actuarial science. Actuarial Modeling and Risk Management Principles delineate the foundations of the profession common to all practice areas.

The subject matter of this paper is similar to that of the 1989 monograph, *Fundamental Concepts of Actuarial Science*, by C.L. Trowbridge [1]. The purpose and thus the organization and content are somewhat different, although not conflicting. The Trowbridge monograph is intended for a broad spectrum of readers, including nonactuaries, who are interested in the development of the fundamental concepts of actuarial science from its philosophical, economic and sociological roots. On the other hand, this paper is written to meet the practical needs of those for whom precise definition and formal development are essential, including, in particular, those involved in the standard-setting process.

II. Methodologies

Application of the Principles to situations encountered by practicing actuaries leads to new and continually evolving models and techniques. Each such model or technique applies one or more of the Principles to the practice situation. These applications are referred to as Methodologies.

Each practice area has its own Methodologies. It is intended that a series of papers will be developed to formalize the Methodologies pertaining to the various practice areas of Society of Actuaries members.

III. Standards

The proper application of Principles and Methodologies to actuarial practice situations requires the exercise of judgment regarding the choice of methods and assumptions. Standards are statements which define the methods and assumptions believed to be reasonable for particular situations. When adopted and promulgated by professional organizations, regulators, lawmakers, or others in authority, these become Guides to Professional Conduct, Standards of Practice, Technique Papers, Regulations, etc.

The development of Standards is beyond the scope of the work assigned to the Committee on Actuarial Principles.

PRINCIPLES OF ACTUARIAL SCIENCE

The following table gives the organization of the remainder of the paper.

1. Statistical Principles

DEFINITIONS. *Phenomena* are occurrences which can be observed. An *experiment* is an observation of a given phenomenon made under specified conditions. The result of an experiment is called an *outcome*; an *event* is a set of one or more possible outcomes.

1.1 PRINCIPLE (Statistical Regularity). Phenomena exist such that, if a sequence of independent experiments is held under the same specified conditions, the proportion of occurrences of a given event stabilizes as the number of experiments becomes larger.

DEFINITIONS. A phenomenon to which Principle 1.1 applies is said to display *statistical regularity*.

Probability is a measure which takes on values from zero to one and gives the likelihood of occurrence of an event.

A rule which assigns a numerical value to every possible outcome is called a *random variable*. The probability-weighted average of the numerical values taken on by a random variable is called the *expected value* of the random variable.

DISCUSSION. If a phenomenon displays statistical regularity, an estimate of the probability of an event associated with the phenomenon is the proportion of occurrences of the event in a long sequence of experiments. Alternatively, the probability of an event may be estimated subjectively using other criteria.

A random variable is a variable that takes on each of a set of numerical values with a given probability.

DEFINITIONS. A *scientific model* is an abstract and simplified representation of a given phenomenon. A *mathematical model* is a scientific model in which the representation is expressed in mathematical terms.

1.2 PRINCIPLE (Stochastic Modeling). A phenomenon displaying statistical regularity can be described by a mathematical model that can estimate within any desired degree of uncertainty the proportion of occurrences of a given event in a sufficiently long sequence of experiments.

DEFINITION. A model satisfying Principle 1.2 is called a *stochastic model*.

DISCUSSION. A stochastic model cannot predict with certainty the outcome of a single experiment prior to its being carried out.

A stochastic model can estimate the expected value of a random variable, provided the sequence of values that arises from the sequence of experiments converges.

DEFINITION. A *deterministic model* is a simplified stochastic model in which the proportion of occurrences of a given event estimated by the stochastic model is assumed to occur with probability one.

DISCUSSION. The stochastic aspect of a model may not be relevant to a given application; in such situations a deterministic model might be used. A deterministic model is a stochastic model with degree of uncertainty zero.

The uncertainty associated with a stochastic model has two distinct sources:

- the inherent variability of the phenomenon, and
- incomplete knowledge and/or inaccurate representation of the probabilities of alternative sets of outcomes.

Sometimes these are referred to as "process risk" and "parameter risk," respectively. The terms "risk" and "uncertainty," respectively, have also been used; however, in this paper, these terms have been assigned other meanings.

Stochastic models may be based on results obtained from previous experiments or may utilize initial assumptions about the probabilities of various sets of outcomes which may be systematically revised as results of experiments are obtained.

Before a model is used, it must be checked for consistency with available information. This process is commonly referred to as "validation."

DEFINITION. A mathematical model is said to be *valid within a specified degree of accuracy* relative to certain observed results if it reproduces these results within that degree of accuracy.

DISCUSSION. Observed results involving the phenomenon which is represented by a model may not be available or sufficiently voluminous to allow the model to be validated within a specified degree of accuracy. In this case, the usefulness of the model may be established initially by comparison with results of the observation of some similar phenomenon. It would be expected that such "judgmentally validated" models could be validated if sufficient data were available.

Not all observable aspects of the modeled phenomena must be reproduced in order for the model to be valid. For example, a model used in the appraisal of an insurance company may be validated only with respect to a few quantities, such as aggregate reserves and total policy count.

DEFINITION. A mathematical model is *potentially valid* if it produces results that are consistent with available observations of the modeled phenomena and of similar phenomena and is capable of being validated relative to the specified observed results when sufficient data are available.

DISCUSSION. The statistical definitions and principles of this section are important to actuaries for two reasons:

- Most phenomena studied by actuaries are assumed to exhibit statistical regularity. In the real world, "experiments" cannot be replicated precisely. The idealized model which serves as an approximate representation of real world phenomena has the property of statistical regularity.
- Stochastic models (along with other mathematical models) are among the actuary's key tools. They are used to draw conclusions about real world phenomena. Specifically, a stochastic model can be used to make probability statements in connection with a single experiment or multiple experiments.

2. Economic and Financial Principles

DEFINITIONS. An *economic good* is something which has value to a person and which the person may consider exchanging for something else. *Money* is a means of exchange which may be traded for economic goods. The amount of money a person is willing to trade for a good at a specific point in time is the good's *current monetary value* to that person.

2.1 PRINCIPLE (Diversity of Preferences). Different people may assign different current monetary values to the same economic good.

2.2 PRINCIPLE (Time Preference). Money has time value; that is, people tend to prefer receiving money in the present to receiving that same amount of money in the future.

DISCUSSION. Time preference is normally represented by an interest rate, or a system of interest rates, used to discount future receipts or disbursements of money so that they may be compared to amounts of money currently held. The appropriate representations of time preference for a given application will be covered under Methodologies.

DEFINITIONS. A *cash flow* is the receipt or disbursement at a point in time of an amount of money (or of an economic good with a monetary value). A cash flow whose occurrence or amount depends on the occurrence of an event that is not certain to occur is said to be *contingent*. An *asset* is money or economic goods held, or a right to receive future cash flows; an *obligation* is a duty to provide current or future cash flows.

2.3 **PRINCIPLE (Present Value Modeling). For many persons, there exists a mathematical model that can estimate the current monetary value that the person would assign to any future cash flow.**

DEFINITIONS. A model described by Principle 2.3 is called a *present value model*. The estimate of the current monetary value of a future cash flow given by a present value model under a fixed assumption regarding future economic conditions is called the *present value* of the cash flow relative to that assumption. Such a fixed assumption regarding future economic conditions is called a *scenario*.

DISCUSSION. If there is uncertainty regarding future economic conditions, the estimates made by a present value model may represent expected values. Such expected values may be thought of as averages of the present values over various scenarios. In this case, the present value may be thought of as a random variable whose expected value is the current monetary value.

3. Actuarial Modeling Principles

DEFINITIONS. An *actuarial risk* is a phenomenon that has economic consequences and that is subject to uncertainty with respect to one or more of the *actuarial risk variables*: occurrence, timing and severity.

3.1 **PRINCIPLE (Modeling of Actuarial Risks). Actuarial risks can be stochastically modeled based on assumptions regarding the probabilities that will apply to the actuarial risk variables in the future, including assumptions regarding the future environment.**

DEFINITIONS. A model described by Principle 3.1, together with a present value model if applicable, is called an *actuarial model*. The assumptions upon which an actuarial model is based are called *actuarial assumptions*.

DISCUSSION. An actuarial model generally must include a present value model if it is intended to determine economic values. A present value model included in an actuarial model is often based on assumptions concerning aspects of the future economic environment, such as interest rates and inflation rates. The present value model may reflect the preferences of the actuary constructing the model or of the actuary's client. However, in certain situations, the actuary's choice may be constrained by regulations or other Standards.

Historically, actuaries have often used deterministic actuarial models. Such deterministic models may, however, be dynamic in nature, reflecting assumptions about the future environment. For example, surrender rate assumptions for some life insurance and annuity products are often taken to be functions of the market interest rate.

In recent years, models based on nontrivial distributions for actuarial risk variables have been needed in some situations. Actuarial methodologies include such strictly stochastic models, as well as deterministic models. The choice of model and the degree of sensitivity testing required are subject to judgment and are thus matters for Standards.

Recall that the validity of a mathematical model depends upon its ability to reproduce observed results. As time passes and more observations are made, the degree of accuracy of the model may change, or, in the case of a model which was initially validated only judgmentally, it may become possible to determine the degree of accuracy.

3.2 **PRINCIPLE (Validity of Actuarial Models). The change over time in the degree of accuracy of an initially valid actuarial model depends upon changes in:**

- **a. the nature of the right to receive or the duty to make a payment;**
- **b. the various environments (regulatory, judicial, social, financial, economic, etc.) within which the modeled events occur; and**
- **c. the sufficiency and quality of the data available to validate the model.**

DEFINITION. The *actuarial value* of a future cash flow that is contingent upon actuarial risk variables is the present value developed by an actuarial model associated with the actuarial risk variables.

DISCUSSION. Recall that the present value, and hence the actuarial value, of a future cash flow is, in general, a random variable. The actuarial value of any asset or obligation is determined by the actuarial value of the associated cash flows, including money currently held. In general, the component cash flows not only have uncertain values but also are not independent of one another.

3.3 **PRINCIPLE (Combinations of Cash Flows). The degree of uncertainty of the actuarial value of a combination of cash flows reflects both the uncertainties affecting each underlying actuarial risk variable and the process of combination.**

4. Risk Management Principles

DEFINITIONS. A person or object involved in an event associated with an actuarial risk is called a *risk subject* or *risk*. *Risk identification* is a process for determining whether a given person or object is a risk subject for a given actuarial risk. *Risk control* is a process that reduces the impact of one or more of the actuarial risk variables associated with the actuarial risk. *Risk transfer* or *risk financing* is a mechanism that provides cash flows that are contingent upon the occurrence of an event associated with the actuarial risk and that tend to offset undesirable economic consequences. A *risk management system* is an arrangement involving one or more of risk identification, risk control, and risk transfer or risk financing.

A *financial security system* is an arrangement for risk financing in which one person assumes the obligation to make a payment (or series of payments), called a *benefit* (*benefits*), that offsets undesirable economic consequences that may be experienced by a second person in return for the payment, by or on behalf of the second person, of one or more amounts, called *considerations*.

DISCUSSION. "Person" may indicate either a human being or a corporate or other entity. The term "financial security system" applies to insurance, annuity, retirement, and health care financing systems.

In general, there is a period of time between the date a consideration is received under a financial security system and the date a benefit is paid. During this period, at least part of the consideration may be invested in one or more types of assets.

For annuities and retirement systems, benefits are provided to reduce the uncertainty regarding the availability of income during part or all of the remaining lifetime of one or more persons. The event referred to in the definitions above is survival to a succession of specified ages following the commencement of income or the date of retirement, and the undesirable economic consequence is outliving one's resources.

DEFINITIONS. An event is said to be *insurable* if:

a. it is associated with a phenomenon that is expected to display statistical regularity;
b. it is contingent with respect to number of occurrences, timing and/or severity;
c. the fact of its occurrence is definitely determinable;
d. its occurrence results in undesirable economic consequences for one or more persons; and
e. its future occurrence, timing and/or severity are neither precisely known nor controllable by these persons.

A person is said to have an *insurable interest* in an insurable event to the extent that the occurrence of the event creates an economic need involving that person.

DISCUSSION. An insurance policy may pay "benefits" related to occurrences that do not fit the definition of "insurable event." For example, a group health plan may pay for elective surgery or annual physicals. In such cases, the "premium" for the plan may contain a pass-through of charges that arise from events other than insurable events; or it may be that, while the event is not insurable on an individual basis, it is insurable on a group basis, since the number of participants who will utilize the benefit each year is not precisely known. While an insurance policy or contract may combine payments resulting from insurable events and other payments, it nevertheless may be desirable to be able to distinguish between them.

The term "economic need" covers a wide range. For example, the future welfare of a person's family is an economic need involving that person, and the increased longevity of a group of retirees could create an economic need for a pension plan sponsor.

Another important aspect of insurance systems is the classification of the risk subjects associated with the actuarial risk. In this context, the term "risk" is commonly used to refer to risk subjects.

4.1 PRINCIPLE (Risk Classification). For a group of risks associated with a given actuarial risk, it is possible to identify characteristics of the risks and to establish a set of classes based on these characteristics so that:

a. each risk is assigned to one and only one class; and

b. probabilities of occurrence, timing and/or severity may be associated with each class in a way that results in an actuarial model which, for some degree of accuracy, is:

(1) valid relative to observed results for each class or group of classes having sufficient available data, and

(2) potentially valid for every class.

DEFINITIONS. A set of classes, a set of characteristics and a set of rules for using the characteristics to assign each risk to a class in such a way that the conditions of Principle 4.1 are satisfied with respect to a given group of risks is called a *risk classification system.* These classes are called *risk classes*, and the rules used for assigning risks to risk classes are called *underwriting rules*.

DISCUSSION. A classification system that cannot be associated with an actuarial model that can be validated relative to observed results when appropriate observed results are available is not a risk classification system.

An actuarial model associated with a risk classification system will reproduce any closely comparable observed values for appropriate groups of classes within a specified degree of accuracy. For example, if the insurable event is the occurrence of death within one year and the classes were determined by current age, policy year, sex, smoker/nonsmoker status, state of health, and occupation, the result would be the association with each age (and policy year, perhaps) of probabilities of death within one year. The model associated with this classification system is a mortality table. In order to be a valid model, the mortality table would have to be consistent with relevant observed rates of death for those groups of classes (such as standard class females aged 20–29 at issue, or all substandard males and females combined with issue ages 20–29) for which sufficient data are available.

A risk classification system is defined at a given point in time. Its continued appropriateness for a specific use depends on the continued availability of a valid associated actuarial model.

It should be noted that different insurable events (that is, different coverages) may require different risk classification systems.

DEFINITIONS. An *insurance system* is a financial security system in which:

a. the actuarial risks to be financed arise from insurable events;
b. the risk subjects are grouped according to a risk classification system;
c. the benefits payable are related to an insurable interest;
d. the actuarial value of benefits payable, developed by an actuarial model associated with the risk classification system, is finite; and
e. considerations are consistent with the actuarial value of the associated benefits.

An insurance system is *mandatory* if all persons in a group or in society are required legally or otherwise to participate; otherwise, it is *voluntary*. It is a *personal insurance system* if the decision to participate is made by each insured individually; it is a *group insurance system* if the decision is made on behalf of a group, although participation may be mandatory or voluntary for the members of the group; and it is a *social insurance system* if all members of society (or a defined subgroup of society) are eligible to participate. The entities to which actuarial risk is transferred in an insurance system (whether private or governmental) are called *insurers*.

DISCUSSION. To say that considerations are "consistent with" the actuarial value of benefits means, in effect, that they are risk-related. Some programs that do not fit the above definition of an insurance system are nevertheless included in the class of financial security systems. Examples include programs in which the considerations charged are not risk-related, as well as programs that make payments that are unrelated to insurable events.

In an insurance system, underwriting rules may be formulated for most actuarial risks so that the actuarial value of benefits is different for different risk classes. In some cases, however, either the actuarial value of benefits associated with a risk class or the uncertainty inherent in the underlying actuarial risk variables is so great that coverage is deemed inappropriate.

DEFINITIONS. A *refinement of a risk classification system* is a risk classification system formed from another by subdividing one or more classes.

If there are actuarial models associated with the original risk classification system and with the refinement such that these models assign the same probabilities of occurrence, timing and/or severity to classes that were not subdivided, but they assign differing probabilities to one or more of the subdivisions of at least one class, the refinement is said to be *more homogeneous* than the original system.

DISCUSSION. For a given set of observed results, the actuarial model associated with a more homogeneous risk classification system may have a reduced degree of accuracy since fewer data points are available for each class of the refinement. For some purposes, it is necessary to ensure that a minimum degree of accuracy is attained.

4.2 PRINCIPLE (Pooling). If the actuarial risk associated with a risk classification system displays statistical regularity, it is possible to combine risk classes so as to ensure that there is an actuarial model associated with the new set of risk classes that is valid within a specified degree of accuracy.

DEFINITION. The process of combining risk classes described in Principle 4.2 is called *pooling*.

DISCUSSION. It is clear from Principle 4.2 that there is a trade-off between pooling and homogeneity in insurance systems. Moreover, increased homogeneity generally leads to increased cost of information. This and other practical factors tend to limit the degree of homogeneity which is achievable. The extent of trade-off chosen is a judgment based on the specific situation. Guidelines for the exercise of judgment fall in the category of Standards and are specifically excluded from consideration here. Statistical techniques and economic concepts such as utility theory may be used to inform these judgments.

The trade-off between pooling and homogeneity is implemented by underwriting rules. Some of the distinctions made by these rules result in classes for which the difference in probabilities remains constant over time. For other distinctions, the probabilities of two or more classes may converge over time. The ability to make such temporary distinctions (based on current health status, etc.) is useful, because it decreases the degree of uncertainty regarding current status and allows insureds to be charged more appropriate initial considerations. Thus, the knowledge that all members of a class had normal blood pressure on a certain day might allow that class to be offered lower considerations for life and health insurance. Typical selection criteria are the results of a current medical examination, current employment status and any history of occurrences of the insurable event.

In some forms of insurance, the selection process is repeated periodically, based on the accumulating information available for each risk. This process is called "renewal underwriting."

DEFINITIONS. The *premium structure* of an insurance system is a set of considerations that reflect the assignment of risks to various risk classes. A *refinement of a premium structure* is a premium structure based on a refinement of a risk classification system.

DISCUSSION. An insurance system may provide for dividends or experience refunds that may be thought of as offsets to considerations. The considerations which define a premium structure for such a system are then net of such dividends or experience refunds.

4.3 **PRINCIPLE (Antiselection). If the premium structure of a voluntary insurance system is based on a risk classification system such that a refinement of the system could result in significant differentials in considerations between risks originally assigned to the same class, there will be a tendency for relatively greater participation by those whose considerations would increase if the refinement were put in place.**

DISCUSSION. One implication of Principle 4.3 is that, if one insurer offers more premium classes than another, and if this results in significant differentials in considerations, antiselection is likely to occur, with the risks that would be required to pay higher considerations in the first company tending to gravitate to the insurer with fewer classes.

Once a premium structure has been determined, another actuarial concept comes into play: the use of emerging experience to modify the premium structure, insofar as allowable, for both new and existing insureds.

DEFINITIONS. The *experience* of a finanical security system is the data obtained in the operation of the system.

Estimates, based on such data, of rates of occurrence or amounts of payment related to an actuarial risk are called *experience rates*.

4.4 PRINCIPLE (Induced Experience). The experience rates for events associated with a financial security system will tend to differ from those for the same events in the absence of any such system.

4.5 PRINCIPLE (Insured Experience). The experience rates for the insurable events of an insurance system will tend to differ from the overall rates of occurrence of the same events among all those subject to a given actuarial risk.

DEFINITIONS. An *experience adjustment* is a change in considerations or benefits applicable to the various risk classes to reflect the experience of the financial security system. *Credibility* is the importance assigned to the experience of a given risk class or group of risk classes relative to other information for the purpose of experience adjustment.

DISCUSSION. The inability to establish "true" underlying rates makes the use of experience adjustments essential.

Experience adjustments may reflect only the current period or may involve a recalculation of the considerations or benefits based on the assumption that the future experience rates of the financial security system will be more like its past experience rates than the rates previously assumed.

DEFINITIONS. The *actuarial value of a financial security system* relative to a given actuarial model is the actuarial value, developed by the model, of the combination of cash flows associated with assets, obligations and considerations of the system. The process of determining such an actuarial value is called a *valuation*. If the actuarial value can be expressed as a function of any variable associated with the financial security system and independent of the actuarial model, that variable is called a *financial parameter* of the financial security system. The amounts by which the values of financial parameters can be changed without reducing the expected actuarial value of the financial security system below zero are called *margins*.

DISCUSSION. Actuaries are often called upon to place a value on future contingent cash flows related to the operations of a financial security system. Because the actuarial value is, in general, a random variable, it may be preferable to state the conditions under which the actuarial value may be expected to fall within a given range.

Actuaries perform valuations in at least three contexts: pricing (or rate-making), reserving and appraisal. Typically, when the actuary performs a valuation, the purpose is to determine the values of one or more of the financial parameters that produce actuarial values in a specified range. In pricing, the parameters are the set of considerations or "premium rates," while in reserving the parameter is called a "reserve." In appraisals, the financial parameter is the price to be paid or received for the right to the cash flows being valued.

A set of financial parameters that is often important is the set of accounting values of the assets of the financial security system. The amount by which the accounting value of assets exceeds the sum of reserves and the accounting value of other obligations is called "surplus."

When setting the financial parameters, actuaries take account of other information in addition to the actuarial value. For example, the financial security system may have to meet certain criteria to be allowed (by regulators, creditors, shareholders, etc.) to continue operations.

Moreover, actuaries take account of the uncertainty inherent in actuarial values.

DEFINITIONS. *Ruin* occurs when a financial security system first fails to satisfy all conditions required to remain in operation. The statement of the conditions under which ruin occurs is called the *ruin criterion*. The probability that ruin will occur within a specified period of time, as calculated using an actuarial model, is called the *ruin probability* of the financial security system relative to that model within that period of time.

4.6 PRINCIPLE (Avoidance of Ruin). For most ruin criteria, there are combinations of values of the financial parameters that will reduce, below a given specified positive level, the ruin probability relative to an actuarial model.

DEFINITION. A measure of the probability that a financial security system is likely to be able to pay all benefits as promised is called the *degree of actuarial soundness* of the financial security system.

4.7 PRINCIPLE (Actuarial Soundness). For most financial security systems, there are combinations of margins that will produce, relative to a valid actuarial model, a degree of actuarial soundness that exceeds a given specified level less than one.

DISCUSSION. One way to define the degree of actuarial soundness is as the complement of the ruin probability, where ruin is defined to be the failure to pay benefits as promised.

Note that actuarial soundness is defined relative to a financial security system. It may be quite different for a subsystem. For example, a governmental pension plan may be designed to be funded through participant contributions, but may enjoy a governmental guarantee of solvency. This system may be analyzed with or without taking account of the guarantee; the degree of actuarial soundness could differ significantly.

Principle 4.7 requires the actuarial model to be valid, which in turn means the model reproduces observed results within a specified degree of accuracy. This requirement applies to the modeling of the assets as well as the obligations. Both assets and liabilities must be validly modeled before a conclusion can be reached regarding the actuarial soundness of a financial security system.

In practical situations the level of margins, and thus the degree of actuarial soundness attainable, may be constrained by market conditions.

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GLOSSARY

PRINCIPLES OF ACTUARIAL SCIENCE

1.1 PRINCIPLE (Statistical Regularity). Phenomena exist such that, if a sequence of independent experiments is held under the same specified conditions, the proportion of occurrences of a given event stabilizes as the number of experiments becomes larger.

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a. the nature of the right to receive or the duty to make a payment;
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(2) potentially valid for every class.

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4.4 **PRINCIPLE (Induced Experience). The experience rates for events associated with a financial security system will tend to differ from those for the same events in the absence of any such system.**

4.5 **PRINCIPLE (Insured Experience). The experience rates for the insurable events of an insurance system will tend to differ from the overall rates of occurrence of the same events among all those subject to a given actuarial risk.**

4.6 **PRINCIPLE (Avoidance of Ruin). For most ruin criteria, there are combinations of values of the financial parameters that will reduce, below a given specified positive level, the ruin probability relative to an actuarial model.**

4.7 **PRINCIPLE (Actuarial Soundness). For most financial security systems, there are combinations of margins that will produce, relative to a valid actuarial model, a degree of actuarial soundness that exceeds a given specified level less than one.**

GLOSSARY

DEFINITIONS

Note: In this glossary, the parenthetical reference at the end of each paragraph gives the relative location of the definition(s) in the main body of the paper: "p. P4.1" indicates that the definition can be found preceding *Principle 4.1, and "f. P4.1" indicates that the definition can be found* following *Principle 4.1.*

The assumptions upon which an actuarial model is based are called **actuarial assumptions.** (f. P3.1)

A model described by Principle 3.1, together with a present value model if applicable, is called an **actuarial model.** (f. P3.1)

An **actuarial risk** is a phenomenon that has economic consequences and that is subject to uncertainty with respect to one or more of the **actuarial risk variables**: occurrence, timing and severity. (p. P3.1)

The **actuarial value** of a future cash flow that is contingent upon actuarial risk variables is the present value developed by an actuarial model associated with the actuarial risk variables. (f. P3.2)

The **actuarial value of a financial security system** relative to a given actuarial model is the actuarial value, developed by the model, of the combination of cash flows associated with assets, obligations and considerations of the system. (p. P4.6)

An **asset** is money or economic goods held, or a right to receive future cash flows; an *obligation* is a duty to provide current or future cash flows. (p. P2.3)

A *financial security system* is an arrangement for risk financing in which one person assumes the obligation to make a payment (or series of payments), called a **benefit (benefits)**, that offsets undesirable economic consequences that may be experienced by a second person in return for the payment, by or on behalf of the second person, of one or more amounts, called *considerations.* (p. P4.1)

A **cash flow** is the receipt or disbursement at a point in time of an amount of money (or of an economic good with a monetary value). (p. P2.3)

A *financial security system* is an arrangement for risk financing in which one person assumes the obligation to make a payment (or series of payments), called a *benefit (benefits)*, that offsets undesirable economic consequences that may be experienced by a second person in return for the payment, by or on behalf of the second person, of one or more amounts, called **considerations.** (p. P4.1)

A cash flow whose occurrence or amount depends on the occurrence of an event that is not certain to occur is said to be **contingent.** (p. P2.3)

Credibility is the importance assigned to the experience of a given risk class or group of risk classes relative to other information for the purpose of experience adjustment. (f. P4.5)

The amount of money a person is willing to trade for a good at a specific point in time is the good's **current monetary value** to that person. (p. P2.1)

A measure of the probability that a financial security system is likely to be able to pay all benefits as promised is called the **degree of actuarial soundness** of the financial security system. (f. P4.6)

A **deterministic model** is a simplified stochastic model in which the proportion of occurrences of a given event estimated by the stochastic model is assumed to occur with probability one. (f. P1.2)

An **economic good** is something which has value to a person and which the person may consider exchanging for something else. (p. P2.1)

The result of an experiment is called an *outcome*; an **event** is a set of one or more possible outcomes. (p. P1.1)

The probability-weighted average of the numerical values taken on by a random variable is called the **expected value** of the random variable. (f. P1.1)

The **experience** of a financial security system is the data obtained in the operation of the system. (p. P4.4)

An **experience adjustment** is a change in considerations or benefits applicable to the various risk classes to reflect the experience of the financial security system. (p. P4.5)

Estimates, based on such data, of rates of occurrence or amounts of payment related to an actuarial risk based on a set of data are called **experience rates.** (p. P4.4)

An **experiment** is an observation of a given phenomenon made under specified conditions. (p. P1.1)

If the actuarial value can be expressed as a function of any variable associated with the financial security system and independent of the actuarial model, that variable is called a **financial parameter** of the financial security system. (p. P4.6)

A **financial security system** is an arrangement for risk financing in which one person assumes the obligation to make a payment (or series of payments), called a *benefit (benefits)*, that offsets undesirable economic consequences that may be experienced by a second person in return for the payment, by or on behalf of the second person, of one or more amounts, called *considerations*. (p. P4.1)

An event is said to be **insurable** if:

a. it is associated with a phenomenon that is expected to display statistical regularity;
b. it is contingent with respect to number of occurrences, timing and/or severity;
c. the fact of its occurrence is definitely determinable;
d. its occurrence results in undesirable economic consequences for one or more persons; and
e. its future occurrence, timing and/or severity are neither precisely known nor controllable by these persons. (p. P4.1)

A person is said to have an **insurable interest** in an insurable event to the extent that the occurrence of the event creates an economic need involving that person. (p. P4.1)

An **insurance system** is a financial security system in which:

a. the actuarial risks to be financed arise from insurable events;
b. the risk subjects are grouped according to a risk classification system;
c. the benefits payable are related to an insurable interest;
d. the actuarial value of benefits payable, developed by an actuarial model associated with the risk classification system, is finite; and
e. considerations are consistent with the actuarial value of the associated benefits.

An insurance system is **mandatory** if all persons in a group or in society are required legally or otherwise to participate; otherwise, it is **voluntary.** It is a **personal insurance system** if the decision to participate is made by each insured individually; it is a **group insurance system** if the decision is made on behalf of a group, although participation may be mandatory or voluntary for the members of the group; and it is a **social insurance system** if all members of society (or a defined subgroup of society) are eligible to participate. (f. P4.1)

The entities to which actuarial risk is transferred in an insurance system (whether private or governmental) are called **insurers.** (f. P4.1)

The amounts by which the values of financial parameters can be changed without reducing the expected actuarial value of the financial security system below zero are called **margins.** (p. P4.6)

A **mathematical model** is a scientific model in which the representation is expressed in mathematical terms. (p. P1.2)

Money is a means of exchange which may be traded for economic goods. (p. P2.1)

A *refinement of a risk classification system* is a risk classification system formed from another by subdividing one or more classes. If there are actuarial models associated with the original risk classification system and with the refinement such that these models assign the same probabilities of occurrence, timing and/or severity to classes that were not subdivided, but they assign differing probabilities to one or more of the subdivisions of at least one class, the refinement is said to be **more homogeneous** than the original system. (f. P4.1)

An *asset* is money or economic goods held, or a right to receive future cash flows; an **obligation** is a duty to provide current or future cash flows. (p. P2.3)

The result of an experiment is called an **outcome**; an *event* is a set of one or more possible outcomes. (p. P1.1)

Phenomena are occurrences which can be observed. (p. P1.1)

The process of combining risk classes described in Principle 4.2 is called **pooling**. (f. P4.2)

A mathematical model is **potentially valid** if it produces results that are consistent with available observations of the modeled phenomena and of similar phenomena and is capable of being validated relative to the specified observed results when sufficient data are available. (f. P1.2)

The **premium structure** of an insurance system is a set of considerations that reflect the assignment of risks to various risk classes. (p. P4.3)

The estimate of the current monetary value of a future cash flow given by a present value model under a fixed assumption regarding future economic conditions is called the **present value** of the cash flow relative to that assumption. (f. P2.3)

A model described by Principle 2.3 is called a **present value model.** (f. P2.3)

Probability is a measure which takes on values from zero to one and gives the likelihood of occurrence of an event. (f. P1.1)

A rule which assigns a numerical value to every possible outcome is called a **random variable**. (f. P1.1)

A **refinement of a premium structure** is a premium structure based on a refinement of a risk classification system. (p. P4.3)

A **refinement of a risk classification system** is a risk classification system formed from another by subdividing one or more classes. (f. P4.1)

A set of classes, a set of characteristics and a set of rules for using the characteristics to assign each risk to a class in such a way that the conditions of Principle 4.1 are satisfied with respect to a given group of risks is called a **risk classification system.** These classes are called **risk classes,** and the rules used for assigning risks to risk classes are called *underwriting rules*. (f. P4.1)

Risk control is a process that reduces the impact of one or more of the actuarial risk variables associated with the actuarial risk. (p. P4.1)

Risk identification is a process for determining whether a given person or object is a risk subject for a given actuarial risk. (p. P4.1)

A **risk management system** is an arrangement involving one or more of risk identification, risk control, and risk transfer or risk financing. (p. P4.1)

A person or object involved in an event associated with an actuarial risk is called a **risk subject** or **risk**. (p. P4.1)

Risk transfer or **risk financing** is a mechanism that provides cash flows that are contingent upon the occurrence of an event associated with the actuarial risk and that tend to offset undesirable economic consequences. (p. P4.1)

Ruin occurs when a financial security system first fails to satisfy all conditions required to remain in operation. (p. P4.6)

The statement of the conditions under which ruin occurs is called the **ruin criterion**. (p. P4.6)

The probability that ruin will occur within a specified period of time, as calculated using an actuarial model, is called the **ruin probability** of the financial security system relative to that model within that period of time. (p. P4.6)

The estimate of the current monetary value of a future cash flow given by a present value model under a fixed assumption regarding future economic conditions is called the *present value* of the cash flow relative to that assumption. Such a fixed assumption regarding future economic conditions is called a **scenario.** (f. P2.3)

A **scientific model** is an abstract and simplified representation of a given phenomenon. (p. P1.2)

A phenomenon to which Principle 1.1 applies is said to display **statistical regularity**. (f. P1.1)

A model satisfying Principle 1.2 is called a **stochastic model.** (f. P1.2)

The rules used for assigning risks to risk classes are called **underwriting rules**. (f. P4.1)

A mathematical model is said to be **valid within a specified degree of accuracy** relative to certain observed results if it reproduces these results within that degree of accuracy. (f. P1.2)

The process of determining the actuarial value of a financial security system is called a **valuation.** (p. P4.6)

DISCUSSION OF PRECEDING PAPER

R. STEPHEN RADCLIFFE:

"Principles of Actuarial Science" is an important addition to the actuarial literature. It provides the intellectual foundation for our science. The writers are to be congratulated for doing such an excellent job with a very difficult subject. All actuaries owe a debt of gratitude to them for this fine work. I know from first-hand experience how much work and how many hours were required to produce the finished document. Thanks to all of the members of the Society of Actuaries Committee on Actuarial Principles.

I have written this discussion to present some background and perspective on the development of these principles. I have observed the process since its beginning in the early 1980s.

In 1984, the Society of Actuaries (SOA) appointed its first Task Force on Actuarial Principles. That task force recommended that (1) it was appropriate for the Society to articulate principles, (2) well-articulated principles evolve through discovery, research and practice, and (3) standards of practice should be founded on principles. This task force also developed a procedure for the articulation and adoption of principles by the Board of Governors.

In 1986, a Committee on Valuation Principles was formed. That committee had difficulty in completing its task of writing principles because of a confusion between standards and principles. At the time, there were no standards or principles, and no one had made the distinction between the two. Additional confusion was caused by poor definition of terms. The committee was not able to resolve issues because of this imprecise communication. As a result, the committee was not successful.

In 1988, a new Task Force on Actuarial Principles was formed, and it presented a report that distinguished between standards and principles. It also recommended that a joint committee be formed with the Casualty Actuarial Society (CAS) to write fundamental principles. During the period 1989 to 1991, several drafts of the principles document were circulated to the memberships of the SOA and the CAS. In the meantime, Charles Trowbridge wrote a monograph on the fundamental principles of actuarial science. This monograph was used extensively by the committee in preparing the document. The monograph was presented at the 1989 Centennial Celebration.

Finally, after many drafts had been prepared and many valuable suggestions had been received from both SOA and CAS members, a final statement on principles was completed. The statement was presented to and accepted by the Board of Governors of the SOA in 1992. The committee also recommended that for posterity the statement be published as a paper in the *Transactions*.

The original reason for articulating principles was to sort out the confusion about actuarial terms and definitions used in valuation methodologies. However, during the process of articulating these principles, the committee discovered that the principles had many more fundamental uses. They even have the chance of identifying the common ground of all actuaries around the world.

All actuaries evaluate the financial consequence of risk. Therefore, they must share some common intellectual foundation. Finding that common ground will define the bond that pulls the various actuarial disciplines toward a center that will be more powerful and effective for the profession as a whole. Principles create a precise language for facilitating the discussion of the many issues facing actuaries.

Principles define the actuarial paradigm. The definitions, assumptions, axioms, and theorems describe our science; they provide the intellectual foundation for what we do. It is possible to operate without principles, but on a temporary and superficial basis only. Without good principles, our standards will be without basis and will collapse under close scrutiny or challenge. It is for these reasons that we must discover and articulate the principles of our science. This is our only hope for claiming to be a true science—a science that is useful in solving business problems.

If we actuaries really want to claim that we have a science, then we must have principles. Principles are a prerequisite for membership in the scientific community; they define the uniqueness of the actuary's work. With boundaries clearly defined through principles, there is less danger of other professions laying claim to what is uniquely in actuaries' domain.

Principles are useful in other ways as well. They categorize and inventory the tools available to the actuary. They provide the skill set for solving the problems of evaluating the financial consequence of risk. Principles provide a precise and common language for actuaries to use. Paul McCrossan said in a recent article that, "Actuaries share a common language, but are separated by it." With well-articulated principles, that separation should disappear. Principles can eliminate the confusion caused when actuaries from different disciplines use different notation and terminology for the same basic ideas. During the discussion stage of articulating principles, many important issues are identified, debated and clarified. It is no doubt hard work that can sometimes cause difficult disagreement. However, it is worth the effort when the final principle that satisfies all constraints is found. The solution exists—it just may be difficult to find. These important discoveries are the strength of our profession. Actually, the principles so discovered do no less than define our profession. The paradigm describes the true identity of the actuary.

Principles also can help define and guide our research efforts. The principles are the building blocks of the profession. As the principles are put together, some holes in the logic may be discovered that need to be addressed through research. Principles also define the current boundaries of our paradigm and show where research might extend those boundaries. For instance, our current paradigm is focused on the evaluation of insurance risk. Once that boundary is clearly drawn, it becomes more apparent how to extend the boundary to include the evaluation of all risk. A complete set of principles defines the territory of the actuaries' domain. However, it also clarifies the ways to extend that domain.

As stated in the SOA's strategic premise for education, principles can drive the education effort. An understanding of fundamental principles and methodologies should be a crucial part of the education of actuaries.

This paper has defined three categories of actuarial principles: (1) fundamental principles, (2) methodologies and (3) standards. Fundamental principles are the basic elements of the general science; they define the tools available to the actuary. The fundamental principles are not expected to change much over time, and they will not change unless the basic paradigm changes. Methodologies also are referred to as specific principles and define how to use the tools for a specific practice or discipline or a specific actuarial task. Methodologies usually are based on techniques that reflect the state of the art.

Standards are behavioral principles that have been officially recognized by an appropriate body. Standards define what tools an actuary ought to use in a given circumstance. The key word is "ought." Principles avoid the use of words like "should" and "ought." This distinction is at the heart of why it is important to differentiate between standards and principles. In their research efforts, learned bodies are in search of truth, a search that sometimes takes them past described boundaries. In developing standards, professional bodies, on the other hand, are in the business of ensuring that actuarial practice stays within the boundaries. For this reason, the articulation of standards and principles must be separated. Only the standards should be involved with the discipline process. All principles should be completely divorced from the discipline process. This point is important to clarify because some worry that strict adherence to principles could inhibit the research efforts of the learned bodies. Actually, we would like the principles process to encourage research. Principles, especially methodologies, define what the actuary is capable of doing. Standards define what the actuary should do in light of what he or she is capable of doing.

Many different actuarial organizations have been working separately on actuarial principles. The Working Agreement between these organizations has developed a framework for the articulation of principles by the various organizations. The Working Agreement Task Force has adopted the following resolution on principles:

> "There should be no conflict or inconsistency among the basic actuarial principles developed by the organizations. To facilitate that outcome, the distribution of discussion drafts of basic actuarial principles developed by an organization should provide adequate opportunity for comments by the actuaries in the other organizations. Under normal circumstances, the CAS and SOA will have the responsibility to manage the development of actuarial principles."

The CAS and SOA have taken different routes in developing principles. While the SOA was working on fundamental principles, the CAS was working on principles specific to its practice. The SOA, in its work, labeled these specific principles as methodologies. The CAS used an inductive approach to discover its principles. By reviewing actuarial practices, the CAS derived methodologies by induction. The SOA approached the problem from another direction by using deductive reasoning. The SOA articulated fundamental principles from which methodologies, or specific principles, could be deduced. Nevertheless, both groups have developed valuable information.

A joint effort on principles between the two organizations should be quite productive. We will be able to compare and contrast our models for validity, completeness and robustness. We will discover what the models have in common and what differences they have. Some models might even be a subset of one another with no inconsistency or discrepancy at all. We all will benefit from a deeper understanding of our models after such a rigorous comparison. Actually, this is an interesting time to be comparing models; new paradigms are being discovered and old paradigms are being challenged.

There are some dangers to be avoided when articulating principles. Articulating principles should be an intellectual process. However, sometimes principles can be misused to substantiate political positions. Principles also can be misapplied by the legal profession. Unfortunately, this must be kept in mind when writing principles. The writers must carefully define the purpose of principles and their intended application. It is important to make sure that these principles are not misused or misinterpreted by our profession or the legal profession.

Principles also take a long time to develop fully. This has caused some problems with those who would like to see them developed more quickly. Furthermore, principles have not been developed in a timely fashion with respect to standards. Standards should be derived from principles, but in many cases the standards have been developed before the corresponding principle. That is just an unfortunate consequence of our rush to get standards in place, but it is not a good argument for not writing the principles. The worst that can happen is that we might have to adjust some standards in the future to conform to a principle.

In conclusion, it is difficult to imagine a case for not pursuing the articulation of principles. Without principles, we are without a science and therefore without a basis for being a true profession. If properly done, principles will define our reason for being. They can be the foundation for the strategic planning for the future of the actuary and the profession. We cannot launch a future for the actuary without a firm foundation. It is in this spirit that we could encourage this endeavor, no matter how difficult it may be. It is a hard job—not an impossible one. It may take a long time, but it is crucial to develop a good foundation for our science in these critical and changing times.

WILLIAM J. SCHREINER:

The Board of Governors of the Society has shown excellent judgment in publishing this paper in the *Transactions*, where it can be commented on by interested parties. The paper is the third public exposure of the evolving conclusions of the Committee on Actuarial Principles. The Committee has been diligent in its efforts, and the document has benefited greatly from the exposure process; however, I find the paper to be extraordinarily disappointing.

I think that the paper offers a view of the actuarial profession that misrepresents the nature of actuarial expertise and is overly insurance-company-oriented. The paper's foremost defect is a characterization of the actuary, throughout the paper, as one who estimates the outcome of future events.

(In earlier versions of the paper, the work of the actuary was said to "predict" future results, rather than "estimate" them. This cosmetic change, however, neither masks nor corrects the erroneous thrust of the paper.)

The source of the paper's error is its steadfast championing of stochastic modeling. The Stochastic Modeling Principle (1.2) is stated early and is the foundation for most of the so-called actuarial principles in the paper. This principle states that:

> "A phenomenon displaying statistical regularity can be described by a mathematical model that can estimate within any desired degree of uncertainty the proportion of occurrences of a given event in a sufficiently long sequence of experiments."

Statistical regularity is defined in Principle 1.1 as requiring "independent events held under the same specified conditions." Unfortunately, *no* phenomena studied by actuaries exhibit the statistical regularity required to achieve the definition of stochastic modeling in Principle 1.2, and consequently, most of the purported principles that follow are built on an imaginary foundation.

Statistical regularity is never achieved because the human contingent events studied by actuaries are never repeated under the identical conditions required by the hypothesis. (For example, the mortality experience of 36-year-old white males in New York City in 1996 will be influenced by conditions different from those that influenced the mortality experience of 36-year-old while males in New York City in 1966.) If statistical regularity does not apply to the contingent events with which actuaries concern themselves, it must follow that the models actuaries operate *cannot* "estimate within any desired degree of uncertainty the proportion of occurrences of a given event" and therefore that the Stochastic Modeling Principle does not apply to their work. For the careful reader, this fact is hinted at in the penultimate bullet of Section 1, where it is indicated that, "[m]ost phenomena studied by actuaries are assumed to exhibit statistical regularity." Of course, assuming statistical regularity does not make it so. In particular, assuming statistical regularity where it does not exist in actuarial work cannot lead to actuarial principles because, as the paper indicates, principles should be "grounded in observation and experience."

Let me make it clear that I do not object to the use of faux stochastic models by actuaries; my objection is only to any representation or implication by the actuarial profession that the use of such models produces results that estimate future events "within any desired degree of uncertainty." Indeed, I applaud all efforts to make it known that the special work of the actuary is the construction or operation of models of future contingent events. However, because these events occur in our messy human environment, we must recognize that it is not possible to find situations in which "independent experiments" are conducted under "the same specified conditions," and therefore, by definition, actuarial models cannot provide "valid" or "accurate" estimates of future results. Thus, we must exert great care to avoid the suggestion in any forum that actuarial expertise implies any special knowledge about how the future will play out.

In any event, the test of actuarial models is not one of accuracy; the test is one of usefulness. Effective actuarial models are those that are useful for gaining insights into the consequences of future contingent events, but to suggest that "accurate" or "valid" results are achieved by such models is, at best, misleading. I believe that such suggestions have no place in a document that attempts to describe actuarial principles.

The simple rule of thumb I suggest that the reader follow is: any "Principle" that contains the word(s) "estimate," "probability," "stochastically modeled," "accuracy," or "valid" cannot be an actuarial principle, because the statement assumes something that is not true for contingencies studied by actuaries. Also, note that there are four Principles that fail a basic test of principleship—reliability—because they only "tend" to be true (Principles 2.2, 4.3, 4.4, and 4.5). By my count, this leaves only three unhelpful general statements (Statistical Regularity, Diversity of Preference, and Combinations of Cash Flows) as survivors from the original troop of 15.

I believe that Section 4, "Risk Management Principles," represents an area of actuarial activity (insurance schemes) rather than an area of actuarial principles. All the "Principles" in this section fail the "accuracy"/"valid" or "tend" tests indicated above. In addition, the section includes some very strange expositions of actuarial methodology. Consider the following quotation:

> "A classification system that is not associated with an actuarial model that can be validated relative to observed results when appropriate observed results are available is not a risk classification system."

Working one's way through the double negative, it turns out that the effective meaning of this statement is that there is no such thing as a risk classification system on the face of the earth (since our models cannot produce stochastic validity). This, of course, is not the point the authors intended. But apart from this serious technical quibble, the paper's suggestion that one does not have a risk classification system unless results can be "validated" ignores the practical fact that no risk classification system seeks to isolate all characteristics (even if they were knowable) affecting the possible result. In fact, a risk classification system is just a special type of actuarial model, and common to all actuarial models, the test of any particular system is its usefulness. Clearly, it is the rationale of marketplace practicality, rather than a mathematical concept of validity, that serves to explain why an individual's smoking history was ignored for risk classification purposes for most of the twentieth century by life insurers and why it continues to be ignored by annuity writers.

Another peculiarity in this section is the discussion in connection with Principle 4.3, Antiselection, about the likelihood of an individual to choose an insurer relative to the number of premium classes offered by the insurer:

> ". . . if one insurer offers more premium classes than another, and if this results in significant differentials in considerations, antiselection is likely to occur, with the risks that would be required to pay higher considerations in the first company tending to a gravitate to the insurer with fewer classes."

Actually, all risks will gravitate to the insurer with the lower premium. For example, if one insurer charges $6 to all insureds, there will be few takers for the insurer that offers 91 rate categories from $10 to $100. The source of the difficulty here is the presence of a number of unstated assumptions underlying the discussion, with the consequence that a plain reading of the text is problematic.

There are other inexplicable aspects of the paper. The paper exhibits an apparent belief that actuarial modeling is confined to present-value calculations (Principle 2.3). No reference is ever made to investigating potential accumulations of funds, a common goal of actuarial calculations. Similarly, I am inclined to believe that actuaries are not interested in people's subjective preference *tendencies* with respect to present and future monetary values (Principle 2.2, Time Preference) as much as they are interested in the objective fact that a dollar can be exchanged in the marketplace today for a promise of more money in the future.

As these comments indicate, I believe the paper fails to achieve the Committee's goal of discovering actuarial principles. However, to the extent that this endeavor has brought the investigation of actuarial principles to the attention of the Society's membership, I believe it has provided a great service to the profession. While we might like to believe that all knowledge about future contingent events is possessed by actuaries, a thoroughly understood recognition of our limitations and the limitations of our tools can be of great assistance in enabling us to properly represent ourselves to our publics. The paper has served this purpose for me, and I hope others will be able to say the same.

SAM GUTTERMAN:

> "Any statistical regularity breaks down once pressure is placed upon it for control purposes."
>
> —*Goodhart's law, by Charles Goodhart, formerly an official in the Bank of England, now a professor at the London School of Economics*

This discussion concerns the implications of the application of the assumption of statistical regularity, as described in Principle 1.1. As the paper's Discussion prior to Principle 2.1 indicates, most phenomena studied by actuaries are assumed to exhibit statistical regularity. But, as also pointed out in the paper, "experiments" cannot be replicated precisely in the real world and validation can be performed only on the basis of historical experience.

Sources of Risk

A significant degree of risk can result from the application of an actuarial model developed on the basis of historical outcomes to future risk. Such risk can result from the following three causes:

- Incomplete knowledge of the underlying actuarial risks
- Insufficient experience to allow for full credibility, due to the inherent variability of the phenomena
- Nonrandom changes in the environment and exposures between the period studied and the period for which the resulting actuarial model will be applied.

The first two classes are referred to in the second Discussion following Principle 1.2 as "parameter risk" and "process risk"; the third is referred to in Principle 3.2b as the change in degree of accuracy of an initially (more appropriately "historically") valid actuarial model resulting from changes in the various environments within which the modeled events occur. I refer to this third class of risk as "environment risk."

Prior to Principle 3.2, the paper indicates that "As time passes and more observations are made, the degree of accuracy of the model may change." In fact, the degree of accuracy can be affected more by the relative degree of environment risk than by a reduction in process or even parameter risk.

Dynamic Structures of Financial Security Systems

I contend that the application of a nondynamic (that is, not responsive to emerging experience) structure of a financial security system, whether in the form of insurance products or financing methods, may be inappropriate if significant environment or parameter risk exists. In many cases, only through the use of dynamic or experience responsive approaches can the financial risk of a financial security system be soundly undertaken or evaluated.

Although the verification of the validity of a model is extremely important for the actuarial soundness of a financial security system, it is often more important that the structure of the system be responsive to emerging adverse (or favorable) experience resulting from parameter, process and environment risks. The inability to react to adverse environment change can adversely affect the actuarial soundness of the system.

The following is a list of a few recent or possible future examples of such environment changes:

- Changes in insurance company income taxation, in which taxes on in-force policies have increased to a level not anticipated at the time of policy issuance
- Fundamental changes in monetary or fiscal economic policy, resulting in significant shifts in interest rates, yield curves or inflation
- Medical breakthroughs or epidemics resulting in significant changes in life expectancy
- New product innovations resulting in cannibalization of existing in-force policies
- Significant changes in tort law
- Unanticipated antiselection
- Significant changes in the medical care financing or delivery system.

Some of these changes can occur without prior warning, while others can be anticipated. Their impact varies among systems or products, depending in part upon the responsiveness of their underlying structure and exposure period. Historically, various risk management techniques have been applied to reduce risk, including those referred to in the paper. Additional alternatives exist, including use of increased margins (the size of which may be related to the perceived degree of environment risk, among other factors), reinsurance and application of a dynamic rather than static structure.

Partially as a result of the high degree of difficulty of estimating future experience and of the risk associated with certain contractual guarantees, actuaries and designers of insurance and other financial security systems have developed dynamic structures and systems in almost all areas in which actuaries practice. Examples of such dynamic systems include participating insurance, nonparticipating insurance with current prices less than guaranteed maximum prices (for example, interest rate credits or insurance charges), surrender or withdrawal charges, use of separate accounts or market-value-adjusted products, experience-rated group insurance, hedging techniques, and pension contribution methods incorporating the amortization of future actuarial gains and losses.

Dynamic Models

Stochastic models are useful if statistical regularity exists. In addition, stochastic models can provide valuable information that enable certain adverse scenarios to be identified that should be anticipated. However, the risk associated with a lack of statistical regularity under adverse scenarios is often quantified assuming only random fluctuations in current best-estimate assumptions, thus not appropriately reflecting environment risk.

Dynamic models are referred to prior to Principle 3.2 as reflecting assumptions about the future environment. Such assumptions, if present, often incorporate only an extrapolation of the present. More appropriately, such models should reflect various environment risks through alternative visions of the future. Alternative assumptions underlying these scenarios can be developed through various techniques, including those of futurism, such as testing the sensitivity of an actuarial model to various adverse scenarios. In addition, these dynamic actuarial models should anticipate the system's corresponding reactions to these alternative futures. Dynamic models should be used in the analysis of both dynamic and nondynamic systems.

Summary

In summary, although dynamic models are referred to briefly in this paper, I believe that sufficiently rigorous treatment has not been given to the use of dynamic models and dynamic structures of financial security systems that have been developed in response to the risk to which they are subject. Recognition of the existence of significant environment risk, as well as parameter and process risks, and the difficulty in estimating the impact of these risks has resulted in an increased use of dynamic structures and an increased use of dynamic models. Such dynamic approaches are fundamental to the sound operation of the financial security systems that actuaries are involved with.

NATHAN F. JONES:

I confine this discussion to the subject of socialization of risk. This is touched upon in at least three of the Discussions in the Principles:

- Preceding Principle 3.2, ". . . in certain situations, the actuary's choice may be constrained by regulations or other Standards."
- Preceding Principle 4.1, "While an insurance policy or contract may combine payments resulting from insurable events and other payments, it nevertheless may be desirable to be able to distinguish between them."
- Preceding Principle 4.2, "Some programs that do not fit the above definition of an insurance system are nevertheless included in the class of financial security systems. Examples include programs in which the considerations charged are not risk-related, as well as programs that make payments that are unrelated to insurable events."

Many members may have read the letter of George Hogeman, a member of the Society, in the *New York Times* of August 10, 1992, concerning the serious problems of Empire Blue Cross and Blue Shield, including

> "If it is public policy that health insurance premiums for the old and unhealthy be kept at an artificially low level so as to match those for the young and healthy, then a public subsidy for such an assessment insurance plan is required. . . . This subsidy should be financed by a tax that is levied across the board, to be paid for by the healthy as well as by the sick."

I do not know how far George's tongue was in his cheek when he wrote that. The Society's published Principles should give the members better guidance in these multiplying and important situations. As a possibility, add to the Discussion preceding Principle 4.2,

> "At least when it may not be apparent to the nonactuary that 'considerations charged are not risk-related,' the actuary, having the opportunity, should point this out—quantitatively if possible, but without value judgment."

This might have reduced the problems now resulting from the lack of full actuarial funding for OASDI—a lack, incidentally, in which I have always concurred and of which actuaries in the 1930s were fully aware. This also illustrates the problems of "definitions" that seem inconsistent, not only with popular usage, but also with definitions accepted in other professions. I would not like to read that an actuary, testifying before a Congressional Committee, had said, "Many of the Empire Blue Cross and Blue Shield benefits are not insurable events."

A final word. The SOA and the CAS are different professional organizations; each has the right to define its Principles, as they have done. The present paper is properly "not intended to include the areas of property and casualty insurance." However, as a member of both organizations, I feel they should constantly work together to harmonize their Principles on problems that both face.

CHARLES S. FUHRER:

I. Definition of Event

The definition (first set of definitions in Section 1.) is: "an *event* is a set of one or more possible outcomes." I think that the words "one or more possible" should be deleted. There is no particular reason to carefully exclude the empty set. What is gained by restricting the outcomes to the possible ones? Are there any impossible outcomes? Do we need to worry about them?

II. Definition of Probability

I think that this definition (first set of definitions after Principle 1.1) also should mention that probability is an additive measure. This means that the probability of the union of two disjoint events equals the sum of their probabilities. This additivity property is standard in the concept of probability. Most people could not conceive of probability without it. Usually this is extended to the property of being countably additive. This means that the probability of the union of a countably infinite sequence of mutually disjoint sets equals the sum of their probabilities. This property allows the use of limits and the study of the convergence of random variables; see [1, p. 30] or [2, p. 11].

III. Discussion of Random Variable

I think that (last sentence of Discussion after Principle 1.1) "A random variable is a variable that takes on each of a set of numerical values with a given probability" should be deleted. This is true only for discrete random variables. A continuous random variable takes on numerical values within each of a set of intervals (of those values) with a given probability. The probability that it takes on a particular numerical value is always zero. A mixed random variable does both. *Random variable* is well-defined in the previous definition section. This discussion of random variables is confusing.

IV. Continuous Risk Classification Schemes

The set of Principles 4.1 through 4.3 and the associated definitions and discussions all pertain to traditional, or what might be called discrete, risk classification systems. I think the "Principles of Actuarial Science" should be expanded to include what I call continuous risk classification. By continuous risk classification, I mean using characteristics of the risks that can be expressed as real numbers and not assigning these characteristics to discrete classes. In a sense, each real number is its own class. Then the probabilities of occurrence, timing, and/or severity are associated with each real number by way of a continuous function. Often this sort of scheme can produce an actuarial model that is valid relative to observed results, using considerably less data than the discrete classification. The following are two examples:

First, consider the problem of estimating health-care costs based on systolic blood pressure (SBP). The traditional approach would classify each of the insureds into ranges of SBP values. Then a cost would be determined for each range. If there were not enough insureds in some range, the ranges would be widened or combined until each range had enough data. Premiums based on this classification scheme would have a distinct disadvantage in that they would take a large jump at each of the range boundaries. The continuous system that I am suggesting would avoid this problem. In the continuous method, cost as a function of SBP would be represented by a continuous function. The family of continuous functions could be selected based on both prior belief of the actuary as well as the fit to the data. The specific shape of the function would be determined by selecting parameters based on goodness of fit to the data. As the amount of data increased, functions that use more parameters can be used. The amount of data needed to fit an n parameter function would be about the same as that needed to determine the costs for n ranges. If the amount of data were very large, some nonparametric methods could be used. This involves calculating values within very short ranges and then smoothing them. Two types of refinements of continuous classifications can reduce antiselection: (1) The number of parameters can be increased, and (2) the function could apply to more characteristics such as diastolic blood pressure.

A second example is age rating. Here the actuary is already effectively using a continuous classification scheme. This is the rationale for the process of graduation. Suppose an actuary was constructing a mortality table by age and there were insufficient data for a particular age. The actuary would not need to combine the data for this age with other, nearby ages. Instead, the answer would be to use a graduation formula that gave little weight to the fit at this age. This is essentially a nonparametric but continuous approach. Also, note that the amount of data needed to determine the three parameters for a Makeham mortality table is far less than what is needed for year-by-year rates. The paper would seem to leave no option in this case but to classify the insureds into wide age ranges. In fact, the discussion (after Principle 4.1) seems to be advocating 10-year age ranges. I believe that the Makeham method is superior.

The language of Principle 4.1 does not actually exclude continuous classifications, if the set of classes referred to can be infinite. Nevertheless, in the discussions, the use of "grouped" (definition of *insurance system*, line b), the use of "subdivided" (definition of *refinement of a risk classification system*), and the use of "more" (discussion after Principle 4.3) would not fit a continuous classification system.

REFERENCES

1. HALMOS, P.R. *Measure Theory.* New York: Springer-Verlag Inc., 1974, and Litton Educational Publishing Inc., 1950.
2. WILKS, S.S. *Mathematical Statistics.* New York: John Wiley & Sons, Inc., 1962.

ROBERT L. BROWN:

I would like to thank this Committee for bringing forth this important statement of principles.

When I think of principles, I think of basic truths, laws or assumptions, that is, some basic or essential qualities or rules concerning the functioning of natural phenomena or mechanical processes. When I read the Principles of Actuarial Science promulgated by the Society of Actuaries, I find such universality.

However, at the same time as these "Principles of Actuarial Science" were being promulgated by the SOA, another set of "Principles of Actuarial Science" was being promulgated by the CAS. Unfortunately, the two sets of principles were not completely in accord. The principles promulgated by the CAS were really Standards of Practice and, it could be argued, were somewhat specific to the U.S. in the early 1990s. While they were extremely well-written and valuable, having two sets of principles that are not totally in accord can prove, and already has (for an actuary at a regulatory hearing) proven, embarrassing.

Could the Committee please outline what is being done to mend this tear in our principle "fence-building," and what processes are being put in place to see that such an occurrence is not repeated?

BEDA CHAN:

The committee beats Hawking [9, p.vi], who wrote ". . . each equation I included in the book would halve the sales. I therefore resolved not to have any equations at all. In the end, however, I *did* put in one equation. . . ." It is indeed a challenge to express concepts and principles solely verbally without resorting to equations. We congratulate the Committee for its success in an excellent summary of the actuarial discipline and trust that the paper will become a seminal reference for generations to come. The purpose of this discussion is to provide a time capsule of equations and graphics that arise in the application of these principles.

Since the time of Cài Lún (?-121 A.D., inventor of paper), paper has been the medium of transmission of ideas and concepts—verbally (as in the current paper), symbolically (as in any mathematics-physics-engineering technical report), or graphically (as in Hawking's book). In this discussion, we show how the actuarial paradigm can evolve when the application of actuarial principles migrate to the silicon medium in the language of Computer Algebra Systems (CAS) [8], which now commonly integrates symbolic, graphic, and verbal modes. We illustrate with notebooks done in Derive, Student Edition [1], the most bang-for-the-buck $50 CAS that runs on any PC. (Manual says Intel 8086, 512K, MD-DOS 2.1, if you can find one, and up.) For further examples on CAS applied to actuarial science, see the pioneering paper of Chan [6].

To read the rest of this discussion, imagine multimedia: You are viewing the four enclosed notebooks on the screen. The narrator's script is the four paragraphs below. Inside CAS notebooks, you can branch out and try your what-if enquiries. Assume kindly that we have anticipated your questions, and dream that CAS will incorporate multimedia capabilities—coming soon to a CPU near you.

"We begin with an illustration from risk theory. According to [3, §12.6], the ruin probability is a linear combination of exponentials when the claim amount distribution is a combination of exponentials and gamma $(2,\beta)$'s. The idea generated some interests, for example, [2] and [14], but not as much as one would expect due probably to the tedium in cracking partial fractions. Not any more with CAS. Notebook Xgamma2β.mth solves the ruin probability problem for gamma $(2,\beta)$ claims as long as the crucial denominator (on line 13 of notebook) factors over the rationals. The form of θ that allows the denominator of line 13 to factor over the rationals is given on line 15. It is proved in ratgamma.mth, which is not included in this discussion to limit its length.

"Notebook aidsmle.mth works over Panjer's classic [12] on AIDS. The novel use of CAS here is to use its array capability to blanket search the root of the likelihood equation. For example, lines 28–29 tell us μ is in the interval (0.44, 0.45). One may sequentially refine the blanket, as in bisection.

"The advantage of an array programming language (as APL should allude to) becomes more decisive for optimizations over parameter pairs. To find the maximum likelihood estimate of a Pareto (α,θ) model fitted to wind related catastropic losses in 1977, as found in [10, p.64], we use APL to blanket the $\alpha\theta$ plane with a grid of points, calculate the loglikelihood function on those points, and zero in on the target. Derive is not used because it is slower than APL in number crunching. Our MLE of $\hat{a} = 1.4557$ and $\theta = 5.1137$ ($lnL = -117.736$) differs from that found in [10, p.116] ($\hat{a} = 5.084$, $\theta = 30.498$, $lnL = -119.583$). Our loglikelihood surface plot by Derive explains why. The slippery slope causes traditional numerical packages to miss the peak when starting from afar, such as the two-moments-matching $\alpha = 4.809$, $\theta = 29.421$. By the way, the part of the $\alpha\theta$ plane that gets a loglikelihood tent over its head is the 95% likelihood based confidence region for the parameter pair (α,θ).

"So CAS can mince algebraic expressions, cook numerical recipes [13], and serve palatable plots. Can it prove (new) theorems? Yes, if the user has the wisdom to guide it. Our last notebook proves the folk result of recursive formula for compound Poisson distribution [3, (11.4.15)], found, for example, in [11] and revisited by many authors including the discussant [4]. The notebook recursiv.mth established the pattern that an analytical proof can easily follow. Historically this was not how (11.4.15) got discovered; but [5] and [7, p.23], for example, were indeed motivated by CAS and then analytically proved."

1: "Notebook Xgamma2ß.mth, introduction: When X is gamma(2,1) and θ=19/72, th~
~e right side of (12.6.9) in Actuarial Mathematics simplifies to:"

2: $$\frac{684\,(2-r)}{91\,(91\,r^{2}-146\,r+19)}$$

3: "which is partial fractioned into"

4: $$\frac{57}{130\,(13\,r-19)}+\frac{57}{70\,(1-7\,r)}$$

5: "The ruin probability is then (57/70) EXP(-u/7) - (3/130) EXP(-19u/13). Th~
~e prohibitive algebric work has been done by Derive, Student Edition where~
~ partial fraction is executed by Expand r."

6: ""

7: "Notebook Xgamma2ß.mth illustrates that when X is gamma(2,ß), the right si~
~de of (12.6.9) in Actuarial Mathematics can always be partial fractioned o~
~ver the rationals for appropriate choices of θ."

8: $$MX\,(r) := \left[\frac{\beta}{\beta-r}\right]^{\alpha}$$

9: $$\mu := \frac{\alpha}{\beta}$$

10: $$\frac{\frac{\theta}{1+\theta}\,(MX\,(r)-1)}{1+(1+\theta)\,\mu\,r-MX\,(r)}$$

11: $$\frac{\beta\,\theta\left[1-\left[\frac{\beta}{\beta-r}\right]^{\alpha}\right]}{\left[\beta\left[\frac{\beta}{\beta-r}\right]^{\alpha}-\alpha\,r\,(\theta+1)-\beta\right](\theta+1)}$$

12: $\alpha := 2$

13: $$\frac{\beta\,\theta\,(2\,\beta-r)}{(2\,\beta^2\,\theta-\beta\,r\,(4\,\theta+3)+2\,r^2\,(\theta+1))\,(\theta+1)}$$

14: "For α=2 and $\beta \in$ rational, the necessary and sufficient condition for the above expression to be partial fractioned over the rationals is that θ takes the following form:"

15: $$\theta := \frac{p^2-9\,q^2}{8\,q^2}$$

16: "where p and q are relatively prime positive integers. Loading θ positive is equivalent to p>3q. For proof of line 15, see notebook ratgamma.mth."

17: $$\frac{4\,\beta\,q^2\,(p^2-9\,q^2)\,(2\,\beta-r)}{(\beta^2\,(p^2-9\,q^2)+2\,\beta\,r\,(3\,q^2-p^2)+r^2\,(p^2-q^2))\,(p^2-q^2)}$$

18: "Expand r in Derive, Student Edition would do partial fraction."

19: $$\frac{\beta\,q\,(p^2-9\,q^2)}{p\,(r\,(p+q)-\beta\,(p+3\,q))\,(p+q)}+\frac{\beta\,q\,(p^2-9\,q^2)}{p\,(r\,(p-q)+\beta\,(3\,q-p))\,(q-p)}$$

```
1:   "Notebook aidsmle.mth uses vector coverage to capture mle."

2:   n1 :=

3:   n2 :=

4:   n3 :=

5:   n4 :=

6:   d1 :=

7:   d2 :=

8:   d3 :=

9:   d4 :=

10:  p1 :=

11:  p2 :=

12:  p3 :=

13:  p4 :=

14:  C (n, k) :=

                                                                d1            d2          ~
15:  C (n1, d1) C (n2, d2) C (n3, d3) C (n4, d4) (1 - p1)   (1 - p2)   (1 - p3)~

     ~d3           d4   n1 - d1    n2 - d2    n3 - d3    n4 - d4
     ~   (1 - p4)    p1          p2          p3          p4

                                                                    d1            d2      ~
16:  LN (C (n1, d1) C (n2, d2) C (n3, d3) C (n4, d4) (1 - p1)   (1 - p2)   (1 -~

     ~    d3           d4   n1 - d1    n2 - d2    n3 - d3    n4 - d4
     ~ p3)   (1 - p4)    p1          p2          p3          p4         )

            d1 LN (1 - p1) + (n1 - d1) LN (p1) + d2 LN (1 - p2) + (n2 - d2) LN (p~
17:  LN (ê                                                                        ~
```

```
2) + d3 LN (1 - p3) + (n3 -  ) LN (p3) + d4 LN (1 - p4) + (n4 - d4) LN (p

4)
   C (n4, d4) C (n3, d3) C (n2, d2) C (n1, d1))

18:  d1 LN (1 - p1) + (n1 - d1) LN (p1) + d2 LN (1 - p2) + (n2 - d2) LN (p2) +

     d3 LN (1 - p3) + (n3 - d3) LN (p3) + d4 LN (1 - p4) + (n4 - d4) LN (p4) +

     LN (C (n4, d4)) + LN (C (n3, d3)) + LN (C (n2, d2)) + LN (C (n1, d1))
```

19: $p1 := EXP \left[- \frac{3}{8} \mu\right]$

20: $p2 := EXP \left[- \frac{3}{4} \mu\right]$

21: $p3 := EXP \left[- \frac{3}{2} \mu\right]$

22: $p4 := EXP \left[- \frac{5}{2} \mu\right]$

```
                5 µ / 2               3 µ / 2              3 µ / 4
23:  d4 LN (ê          - 1) + d3 LN (ê          - 1) + d2 LN (ê          - 1) + d1 LN

        3 µ / 8
      (ê        - 1) + LN (C (n1, d1)) + LN (C (n2, d2)) + LN (C (n3, d3)) + LN

                       µ (3 n1 + 2 (3 n2 + 2 (3 n3 + 5 n4)))
      (C (n4, d4)) - ─────────────────────────────────────────
                                        8

     d    ┌            5 µ / 2               3 µ / 2               3 µ / 4
24:  ──   │d4 LN (ê          - 1) + d3 LN (ê          - 1) + d2 LN (ê          - 1) + d
     dµ   └

          3 µ / 8
     1 LN (ê        - 1) + LN (C (n1, d1)) + LN (C (n2, d2)) + LN (C (n3, d3))
```

$$+ \text{LN}(\text{C}(n4, d4)) - \frac{\mu\,(3\ n1 + 2\ (3\ n2 + 2\ (3\ n3 + 5\ n4)))}{8}\Bigg]$$

25: $$\frac{5\ d4\ \hat{e}^{5\ \mu\ /\ 2}}{2\ (\hat{e}^{5\ \mu\ /\ 2} - 1)} + \frac{3\ d3\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{3\ d2\ \hat{e}^{3\ \mu\ /\ 4}}{4\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ d1\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{3\ n1 + 2\ (3\ n2 + 2\ (3\ n3 + 5\ n4))}{8}$$

26: $$\frac{5\ 2\ \hat{e}^{5\ \mu\ /\ 2}}{2\ (\hat{e}^{5\ \mu\ /\ 2} - 1)} + \frac{3\ 9\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{3\ 6\ \hat{e}^{3\ \mu\ /\ 4}}{4\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ 1\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{3\ 10 + 2\ (3\ 14 + 2\ (3\ 21 + 5\ 3))}{8}$$

27: $$\frac{5\ \hat{e}^{5\ \mu\ /\ 2}}{\hat{e}^{5\ \mu\ /\ 2} - 1} + \frac{27\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{9\ \hat{e}^{3\ \mu\ /\ 4}}{2\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{213}{4}$$

28: $$\text{VECTOR}\left[\frac{5\ \hat{e}^{5\ \mu\ /\ 2}}{\hat{e}^{5\ \mu\ /\ 2} - 1} + \frac{27\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{9\ \hat{e}^{3\ \mu\ /\ 4}}{2\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{213}{4},\ \mu,\ 0.4,\ 0.5,\ 0.01\right]$$

29: $$\left[\frac{6471}{1396}, \frac{13427}{3764}, \frac{4031}{1580}, \frac{1831}{1156}, \frac{45}{68}, -\frac{41}{188}, -\frac{4797}{4532}, -\frac{8885}{4772}, -\frac{9481}{3604}, -\frac{5643}{1676}, -\frac{20575}{5052}\right]$$

30: $$\frac{5\ 4\ \hat{e}^{5\ \mu\ /\ 2}}{2\ (\hat{e}^{5\ \mu\ /\ 2} - 1)} + \frac{3\ 15\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{3\ 10\ \hat{e}^{3\ \mu\ /\ 4}}{4\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ 1\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{3\ 9 + 2\ (3\ 18 + 2\ (3\ 20 + 5\ 5))}{8}$$

31: $$\frac{10\ \hat{e}^{5\ \mu\ /\ 2}}{\hat{e}^{5\ \mu\ /\ 2} - 1} + \frac{45\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{15\ \hat{e}^{3\ \mu\ /\ 4}}{2\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{475}{8}$$

32: $$\mathrm{VECTOR}\left[\frac{10\ \hat{e}^{5\ \mu\ /\ 2}}{\hat{e}^{5\ \mu\ /\ 2} - 1} + \frac{45\ \hat{e}^{3\ \mu\ /\ 2}}{2\ (\hat{e}^{3\ \mu\ /\ 2} - 1)} + \frac{15\ \hat{e}^{3\ \mu\ /\ 4}}{2\ (\hat{e}^{3\ \mu\ /\ 4} - 1)} + \frac{3\ \hat{e}^{3\ \mu\ /\ 8}}{8\ (\hat{e}^{3\ \mu\ /\ 8} - 1)} - \frac{475}{8}, \mu, 0.8, 0.9, 0.01\right]$$

33: $$\left[\frac{13271}{5400}, \frac{8841}{4328}, \frac{9323}{5688}, \frac{9999}{8024}, \frac{7413}{8584}, \frac{821}{1672}, \frac{799}{6232}, -\frac{175}{776}, -\frac{317}{556}, -\frac{7343}{8104}, -\frac{6563}{5320}\right]$$

34: $$\frac{5\;14\;\bullet^{5\,\mu\,/\,2}}{2\,(\bullet^{5\,\mu\,/\,2}-1)}+\frac{3\;14\;\bullet^{3\,\mu\,/\,2}}{2\,(\bullet^{3\,\mu\,/\,2}-1)}+\frac{3\;20\;\bullet^{3\,\mu\,/\,4}}{4\,(\bullet^{3\,\mu\,/\,4}-1)}+\frac{3\;3\;\bullet^{3\,\mu\,/\,8}}{8\,(\bullet^{3\,\mu\,/\,8}-1)}-\frac{3\;21+2\,(3\;51+2\,(3\;29+5\;19))}{8}$$

35: $$\frac{35\;\bullet^{5\,\mu\,/\,2}}{\bullet^{5\,\mu\,/\,2}-1}+\frac{21\;\bullet^{3\,\mu\,/\,2}}{\bullet^{3\,\mu\,/\,2}-1}+\frac{15\;\bullet^{3\,\mu\,/\,4}}{\bullet^{3\,\mu\,/\,4}-1}+\frac{9\;\bullet^{3\,\mu\,/\,8}}{8\,(\bullet^{3\,\mu\,/\,8}-1)}-\frac{1097}{8}$$

36: $$\mathrm{VECTOR}\left[\frac{35\;\bullet^{5\,\mu\,/\,2}}{\bullet^{5\,\mu\,/\,2}-1}+\frac{21\;\bullet^{3\,\mu\,/\,2}}{\bullet^{3\,\mu\,/\,2}-1}+\frac{15\;\bullet^{3\,\mu\,/\,4}}{\bullet^{3\,\mu\,/\,4}-1}+\frac{9\;\bullet^{3\,\mu\,/\,8}}{8\,(\bullet^{3\,\mu\,/\,8}-1)}-\frac{1097}{8},\ \mu,\ 0.5,\ 0.6,\ 0.01\right]$$

37: $$\left[\frac{8487}{1352},\ 4.38,\ \frac{389}{152},\ \frac{1823}{2248},\ -\frac{3261}{3752},\ -\frac{8567}{3448},\ -4.03906,\ -\frac{14127}{2552},\ -\frac{15127}{2168},\ -\frac{6359}{760},\ -\frac{20269}{2088}\right]$$

38: $$\frac{5\;4\;\bullet^{5\,\mu\,/\,2}}{2\,(\bullet^{5\,\mu\,/\,2}-1)}+\frac{3\;10\;\bullet^{3\,\mu\,/\,2}}{2\,(\bullet^{3\,\mu\,/\,2}-1)}+\frac{3\;3\;\bullet^{3\,\mu\,/\,4}}{4\,(\bullet^{3\,\mu\,/\,4}-1)}+\frac{3\;0\;\bullet^{3\,\mu}}{8\,(\bullet^{3\,\mu\,/}}$$

$$\frac{\ldots / 8}{\ldots 8 - 1)} - \frac{3\ 8 + 2\ (3\ 29 + 2\ (3\ 20 + 5\ 7))}{8}$$

39: $$\frac{10\ ê^{5\ \mu\ /\ 2}}{ê^{5\ \mu\ /\ 2} - 1} + \frac{15\ ê^{3\ \mu\ /\ 2}}{ê^{3\ \mu\ /\ 2} - 1} + \frac{9\ ê^{3\ \mu\ /\ 4}}{4\ (ê^{3\ \mu\ /\ 4} - 1)} - \frac{289}{4}$$

40: $$\text{VECTOR}\left[\frac{10\ ê^{5\ \mu\ /\ 2}}{ê^{5\ \mu\ /\ 2} - 1} + \frac{15\ ê^{3\ \mu\ /\ 2}}{ê^{3\ \mu\ /\ 2} - 1} + \frac{9\ ê^{3\ \mu\ /\ 4}}{4\ (ê^{3\ \mu\ /\ 4} - 1)} - \frac{289}{4}, \mu, 0\ldots\right.$$

$$\left.\ldots .2, 0.3, 0.01\right]$$

41: $$\left[\frac{34045}{1252}, \frac{2875}{124}, \frac{48397}{2476}, \frac{41477}{2556}, \frac{21787}{1652}, \frac{25905}{2492}, 7.82031, \frac{4373}{804}, \frac{50\ldots}{15\ldots}\right.$$

$$\left.\frac{\ldots 27}{\ldots 56}, \frac{73}{62}, -\frac{1835}{2492}\right]$$

42: $$\frac{5\ 0\ ê^{5\ \mu\ /\ 2}}{2\ (ê^{5\ \mu\ /\ 2} - 1)} + \frac{3\ 5\ ê^{3\ \mu\ /\ 2}}{2\ (ê^{3\ \mu\ /\ 2} - 1)} + \frac{3\ 6\ ê^{3\ \mu\ /\ 4}}{4\ (ê^{3\ \mu\ /\ 4} - 1)} + \frac{3\ 4\ ê^{3\ \mu\ldots}}{8\ (ê^{3\ \mu\ /\ldots}}$$

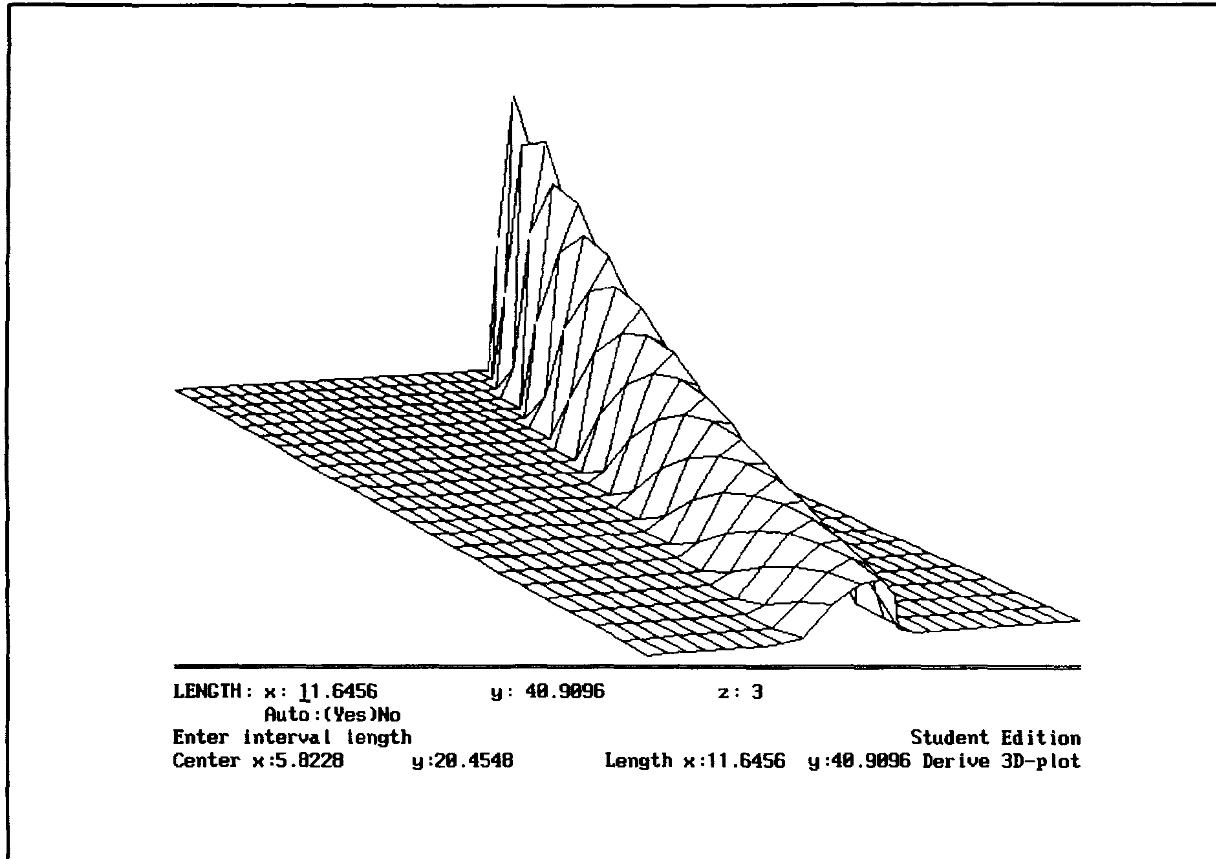
$$\frac{\ldots / 8}{\ldots 8 - 1)} - \frac{3\ 6 + 2\ (3\ 9 + 2\ (3\ 7 + 5\ 1))}{8}$$

43: $$\frac{15\ ê^{3\ \mu\ /\ 2}}{2\ (ê^{3\ \mu\ /\ 2} - 1)} + \frac{9\ ê^{3\ \mu\ /\ 4}}{2\ (ê^{3\ \mu\ /\ 4} - 1)} + \frac{3\ ê^{3\ \mu\ /\ 8}}{2\ (ê^{3\ \mu\ /\ 8} - 1)} - 22$$

44: VECTOR $\left[\dfrac{15\,\hat{e}^{3\mu/2}}{2\,(\hat{e}^{3\mu/2}-1)}+\dfrac{9\,\hat{e}^{3\mu/4}}{2\,(\hat{e}^{3\mu/4}-1)}+\dfrac{3\,\hat{e}^{3\mu/8}}{2\,(\hat{e}^{3\mu/8}-1)}-22,\ \mu,\ 1,\ 1.1,\ 0.01\right]$

45: $\left[\dfrac{2011}{2053},\ \dfrac{257}{305},\ \dfrac{107}{151},\ \dfrac{914}{1583},\ \dfrac{751}{1673},\ \dfrac{199}{616},\ \dfrac{182}{911},\ \dfrac{35}{443},\ -\dfrac{26}{661},\ -\dfrac{3}{21},\ \dfrac{33}{44},\ -\dfrac{315}{1171}\right]$

```
1:   "Notebook windmle.mth prepares the loglikelihood surface sliced at the 95%~
     ~ confidence region level."

2:   x := [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1.5, 1.5~
     ~, 1.5, 1.5, 2.5, 2.5, 2.5, 3.5, 3.5, 3.5, 3.5, 4.5, 4.5, 4.5, 4.5, 6.5, 6.~
     ~5, 7.5, 13.5, 15.5, 20.5, 21.5, 22.5, 22.5, 23.5, 25.5, 30.5, 41.5]

                                                            40
3:   LNL (α, θ) := 40 LN (α) + 40 α LN (θ) - (α + 1)  Σ  LN (ELEMENT (x, n) + θ)
                                                           n=1

4:   LNL (1.4557, 5.1137)

5:   -117.736

6:   MAX (120.736 + LNL (α, θ), 0)

7:   "The 3D plot of the above is attached."

8:   LNL (5.084, 30.498)

9:   -119.583
```


LENGTH: x: 11.6456 y: 40.9096 z: 3
Auto:(Yes)No
Enter interval length
Student Edition
Center x:5.8228 y:20.4548 Length x:11.6456 y:40.9096 Derive 3D-plot

1: "Notebook recursiv.mth derives the recursive formula (11.4.15) in Actuarial Mathematics. Probability generating function of X is denoted by GX and the Poisson parameter of N is μ."

2: GS (s) :=

3: GX (s) :=

4: $GS(s) = EXP(\mu (GX(s) - 1))$

5: $\frac{d}{ds} LN(GS(s) = EXP(\mu (GX(s) - 1)))$

6: "Operators derivative and LN work on both sides of an equation."

7: $\frac{\frac{d}{ds} GS(s)}{GS(s)} = \mu \frac{d}{ds} GX(s)$

8: $\frac{d}{ds} GS(s) = GS(s) \, \mu \frac{d}{ds} GX(s)$

9: $TAYLOR\left[\frac{d}{ds} GS(s) = GS(s) \, \mu \frac{d}{ds} GX(s), s, 0, 3\right]$

10: $s^3 \lim_{s \to 0} \frac{\left[\frac{d}{ds}\right]^4 GS(s)}{6} + s^2 \lim_{s \to 0} \frac{\left[\frac{d}{ds}\right]^3 GS(s)}{2} + s \lim_{s \to 0} \left[\frac{d}{ds}\right]^2 GS(s) + \lim_{s \to 0} \frac{d}{ds} GS(s) = s^3 \lim_{s \to 0} \frac{\mu \left[\left[\frac{d}{ds} GX(s)\right] \left[\frac{d}{ds}\right]^3 GS(s) + 3 \left[\left[\frac{d}{ds}\right]^2 GX(s)\right] \left[\frac{d}{ds} \ldots\right.\right.}{\ldots}$

$$\frac{\Big]^{2} GS(s) + 3\left[\left[\frac{d}{ds}\right]^{3} GX(s)\right] \frac{d}{ds} GS(s) + GS(s) \left[\frac{d}{ds}\right]^{4} GX(s)\Big]}{6} + s^{2} \lim_{s \to 0}$$

$$\frac{\mu \left[\left[\frac{d}{ds} GX(s)\right] \left[\frac{d}{ds}\right]^{2} GS(s) + 2\left[\left[\frac{d}{ds}\right]^{2} GX(s)\right] \frac{d}{ds} GS(s) + GS(s) \left[\frac{d}{ds}\right]^{3} GX\right.}{2}$$

$$\frac{\left.(s)\right]}{} + s \lim_{s \to 0} \mu \left[\left[\frac{d}{ds} GX(s)\right] \frac{d}{ds} GS(s) + GS(s) \left[\frac{d}{ds}\right]^{2} GX(s)\right] + \lim_{s \to 0} \mu\, G$$

$$S(s) \frac{d}{ds} GX(s)$$

11: "Derive did not prove (11.4.15). The reader is asked to compare coefficien‾

‾ts of s^n and obtain (11.4.15). Certainly one may go further down the Tayl‾

‾or series; to see the pattern in this case, however, one needs to go no fu‾

‾rther than the cubic term, as has been done above."

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(AUTHORS' REVIEW OF DISCUSSIONS)

SOCIETY OF ACTUARIES COMMITTEE ON ACTUARIAL PRINCIPLES*:

The Committee thanks Messrs. Brown, Chan, Fuhrer, Gutterman, Jones, Radcliffe, and Schreiner for submitting written discussions of the statement of principles. During the development of the statement, the Committee had the benefit of input from many individuals both before and during the two exposure periods. The published statement evolved in response to that input. These written discussions add a valuable supplement to the statement and

*The members of the Society of Actuaries Committee on Actuarial Principles who prepared this response are: Arnold A. Dicke (chair), Daniel F. Case, Warren Luckner, Donald M. Peterson, Joseph H. Tan, and Charles Barry H. Watson.

provide the opportunity for the Committee to further discuss the ideas presented in the statement.

Mr. Radcliffe provides valuable background and historical perspective on the various efforts within the SOA to identify principles important to actuarial theory and practice. He highlights the many ways in which principles are useful—in categorizing tools available to actuaries, in defining and guiding research, and in driving education efforts. Most importantly, "principles define the actuarial paradigm." His discussion of the different approaches to the development of statements of principles taken by the CAS and the SOA is helpful in identifying the potential for future cooperative efforts between the CAS and the SOA. While Mr. Radcliffe correctly characterizes the SOA approach as using deductive reasoning, we note that we do define principles as "statements grounded in observation and experience." Finally, Mr. Radcliffe states very well the case for pursuing the articulation of principles, and the Committee wholeheartedly agrees with him.

Mr. Schreiner expresses a critical view of the statement of principles. He accurately points out that the entire treatment of stochastic modeling proceeds from the assumption of statistical regularity. Statistical regularity "is defined as requiring independent events held under the same specified conditions," according to Mr. Schreiner, and "*no* phenomena studied by actuaries" satisfy these requirements. "[C]onsequently," he states, "most of the purported principles that follow are built on an imaginary foundation."

The Committee believes Mr. Schreiner has not correctly characterized the definition of statistical regularity given in the statement of principles. A phenomenon to which Principle 1.1 applies is said to "display statistical regularity." Principle 1.1 is stated conditionally and may apply regardless of whether a sequence of independent experiments under the same specified conditions can actually be carried out. In the real world, experiments cannot be replicated precisely. Nevertheless, a phenomenon may display statistical regularity. In order to infer that any particular phenomenon is statistically regular, statistical tests must be used, and the conclusion will be, at best, that the likelihood of statistical regularity is high.

The determination of statistical regularity is made especially difficult by the fact that financial and economic phenomena are changing constantly and are influenced by a number of factors. Within a short time, however, the changes may be minimal and hence approximate statistical regularity may be observed. Over a longer time, the relationships among the various factors may remain relatively constant, although the magnitudes of influence could change.

Unless a phenomenon is assumed to display statistical regularity, it is hard to see how stochastic models can be employed. The actuary who uses such models is acting as if he or she has made this assumption. As noted above, this assumption does not preclude a recognition that the phenomena observed in the past may change with time.

In this regard, Mr. Gutterman gives numerous examples of what he calls "environment risk"—"nonrandom changes in the environment . . . between the period studied and the period for which the resulting actuarial model will be applied." As pointed out above, this risk, and others discussed by Mr. Gutterman, exist even under the assumption of statistical regularity. The fact remains, statistical regularity is a fundamental concept essential for the construction of stochastic models.

Mr. Gutterman provides a very helpful supplement to the discussion in the Committee's statement. His thorough discussion of "environment risk" gives additional explanation of the importance of dynamic modeling and feedback mechanisms such as the experience adjustment defined and discussed in the statement. His discussion will be especially valuable as statements of actuarial methodologies and standards of actuarial practice are developed.

Mr. Schreiner makes the further assertion that a statement that some things "tend" to be true should not be described as a "principle." The statements in question describe likelihoods that are based on observation and experience (for example, people most often prefer to receive a given amount of money now than the same amount later) and so are consistent with "principles" as defined in the statement of principles.

Mr. Schreiner disapproves of the fact that the statement of principles requires, as part of a risk classification system, an actuarial model that could be validated for every class if the requisite data were available. He correctly points out that marketplace practicality strongly influences many actuarial decisions. The Committee notes that Principle 4.1 does not imply that a set of rules that ignores one or more quite significant variables (such as smoking status) cannot be a risk classification system. The Principle does require that a valid, or potentially valid, actuarial model be found. The continued appropriateness of a risk classification system depends on the continued availability of a valid associated model. The Committee believes there are principles that apply to risk classification systems that do not apply to classification mechanisms that have some, but not all, of the features of risk classification systems as defined.

Mr. Schreiner seems to believe that it is possible to have a useful model without testing its results against observed experience (or, in the absence of observed experience, being able to test its results if and when such experience becomes available). We would agree that a close correspondence of a model's results with observed experience does not ensure that the assumptions used in the model are the only assumptions that would have produced such close correspondence. By examining the sensitivity of the results to changes in various assumptions, it is possible to gain some idea of the degree of confidence that one may attach to the assumptions—at least, as far as their relation to the observed experience is concerned. Modelers must never lose sight of the fact that variables that appear to have behaved in certain ways that gave rise to the observed results may not necessarily behave in corresponding ways in the future. Nevertheless, it seems appropriate to believe that there is value in checking a model against observed results. Furthermore, it is difficult to imagine any constructive use for a model whose results could not be checked against some kind of observed experience if that experience were to become available.

Mr. Schreiner criticizes the "apparent belief that actuarial modeling is confined to present-value calculations," ignoring "potential accumulations of funds—a common goal of actuarial calculations." The Committee agrees that the statement treats accumulations and present values rather differently. Accumulations result from the receipt of net future cash flows generated by assets and obligations. The risks relating to such cash flows, including risk of benefit payment, asset default and receipt of considerations, are among the actuarial risks that may be modeled in accordance with Principle 3.1. The model will, of course, have to specify the disposition of excess cash in the future, as well as the treatment of deficiencies; that is, the model will require a reinvestment assumption, whether a single interest rate or an investment and borrowing strategy.

Such models cannot, however, give a current monetary equivalent of the projected future cash flows without the addition of a present-value model. A present-value model will depend on the projected economic environment (which may have been specified in modeling accumulations), but will also include the relative preference of the evaluator for cash flows at different times.

Certain present-value models are in common use; for example, discounting at the average earnings rate of assets on the valuation date, or discounting with a risk-free short-term rate or risk-free yield curve. Each of these models presumably serves a need. None appears to deserve to be singled out as *the* present-value model.

Research is currently under way in this area, largely centered on option-pricing models and related approaches. One goal of this research is to develop models that can estimate market values. As the results of this research become established, it may be useful to develop a Methodology with respect to present-value models.

Mr. Schreiner notes that Section 4 (Risk Management Principles) focuses on a single area of actuarial activity: "insurance schemes." The Committee recognized the need to provide a broad orientation and, throughout the statement of principles, attempted to consider areas of actuarial application other than insurance company work. The Committee regrets if it has not been completely successful in this endeavor. However, the Committee believes that none of the statements are limited in application to insurance company activities solely. In the particular instance referenced by Mr. Schreiner, we fully intended "risk management" to relate to a broad spectrum of risks, including the "risk" of extended life covered through retirement programs, both insured and uninsured.

Mr. Jones has correctly interpreted the Committee's intent for financial security systems in which the considerations are not risk-related—for example, systems in which the insurance rates contain subsidies. The Committee defined "insurance systems" to exclude such financial security systems. Making distinctions is the purpose of definitions. The Committee believes there are principles that hold for insurance systems but do not hold for all financial security systems. Of course, no value judgment should be inferred: a system providing subsidies may be very useful, but it will not function exactly like an insurance system. Confusion between these terms could cause harm if those principles that apply specifically to insurance systems were to be extended uncritically to systems that fail to meet all the conditions of insurance systems.

Mr. Jones suggests that actuaries should speak out when the principles of their science appear to have been misapplied. The Committee agrees with Mr. Jones and hopes that the publication of "Principles of Actuarial Science" will help actuaries better articulate their concerns in such situations.

When the Committee stated that some financial security systems "make payments that are unrelated to insurable events," an example in mind was prepaid service costs, such as the cost of a scheduled annual physical examination. The term "insurable event" is not in such broad use that much confusion is to be expected.

The Committee views Mr. Fuhrer's discussion as making two general points about the statement of principles:

1. The description of probability and statistics is incomplete and extremely selective
2. Principles and definitions are not always stated in the most general terms.

In particular, Mr. Fuhrer believes that the definition of "event" should include the set of no outcomes—the trivial event—and the definition of probability should refer to its property of countable additivity. Were completeness at any level to be desired, Mr. Fuhrer's additions would be essential. The Committee faced the need to trade off completeness of statement against readability and opted to include in the Statistical and the Economic and Financial Sections the minimum information required to develop the ideas of Sections 3 and 4. A different rule, allowing for Mr. Fuhrer's amendments, could certainly have been defended.

In addition, Mr. Fuhrer points out that the statement consistently favors discrete, as opposed to continuous, probability distributions. The Committee is pleased that Mr. Fuhrer has called attention to the possibility of generalization to the continuous case. The Committee does not agree, however, in the context of risk classification that the discrete approach has the disadvantage that Mr. Fuhrer seems to ascribe to it. The discussion following Principle 4.1 indicates that if insufficient data are available for single risk classes, data for groups of classes can be used. That discussion does not say that classes should be combined to form larger classes (with the result that there would be large jumps in the probabilities assigned to adjacent classes).

The Committee agrees with Mr. Brown's general characterization of principles and is gratified that Mr. Brown found that "Principles of Actuarial Science" satisfied that characterization.

The Committee appreciates the kind words of Mr. Chan regarding the statement of principles. The remainder of his discussion appears to venture rather far beyond the Committee's work, and the Committee offers no comments on it.

In closing, the Committee notes that this statement of principles was developed in an attempt to identify principles that serve as the foundation for the various areas of actuarial practice represented by members of the Society of Actuaries. However, the Committee agrees with Messrs. Radcliffe, Brown and Jones that coordination of its work with that of the corresponding committees of the CAS would be very beneficial to the entire actuarial profession.