SOCIETY OF ACTUARIES

Article from:

Risk Management Newsletter

November 2004 – Issue No. 3

Guaranteed Annuity Options or a Fine Mess

by Mary R. Hardy

1. INTRODUCTION

The actuarial profession in the UK is under unprecedented external scrutiny currently. The serious financial difficulties faced by Equitable Life (UK), the oldest mutual insurer in England, led to the government commissioning an investigation by a senior law lord. The result is the recently published *Penrose Report*. As a consequence of criticisms of the profession in the *Penrose Report*, the government then asked a senior economist, Sir Derek Morris, recently retired chairman of the competition commission to review the way the UK profession sets standards and monitors performance. Although the word 'crisis' is not being publicly bandied about, there is a lot of discomfort around Staple Inn, the headquarters of the Institute of Actuaries.

The solvency problems, which brought Equitable Life (UK) to close its doors to new business, and which nearly broke several other companies, arose from an obscure rider to some insured defined contribution pension contracts issued in the 1970s and '80s. The rider was an annuitization guarantee, called the guaranteed annuity option (GAO), and the risk management challenges that this option created are the topic of this article.

The most significant contributions to the discussion of risk management of these options are Wilkie et al (2003), Ballotta and Haberman (2003) and Boyle and Hardy (2003), where more details about the results in the next couple of sections can be found.

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2. THE GAO

The GAO was attached to with-profit and unit-linked, single-premium and annual-premium contracts. Although most of the contracts affected were with-profit, we will look at a single premium unit-linked version here, as it is more transparent and therefore easier to describe and model than the with-profit version. A unit-linked contract is very similar to a variable annuity contract in the United States, or a segregated fund contract in Canada; premiums (after deduction for expenses) are invested in a fund similar to a mutual fund, with certain guarantees on death and possibly maturity.

Suppose the policyholder's fund at maturity is denoted $F(n)$. The GAO rider guaranteed an annuity rate g such that that the pension after annuitization would be no less than $F(n)/g$. Typically, for a 65-year-old male, $g = 9$. Now, without this guarantee, the pension would depend on the annuity value $\ddot{a}^{(12)}_{65}$ at maturity, which would obviously vary with interest rates, as well as being updated from time-to-time to allow for improvements in mortality. For a cost-neutral annuitization, the amount of pension would be $F(n) / a^{(12)}_{65}$.

So, if $a^{(12)}_{65}(t)$ is the market value of the unit annuity at time t, then the payoff of the option at maturity at time n, say, is

$$\max\left(\frac{F(n)}{g} - \frac{F(n)}{a^{(12)}_{65}(n)},\ 0\right) a^{(12)}_{65}(n)$$

Now, $F(n)$ is the accumulated fund; if the original premium is P, and letting S_t denote the market value of the investment fund at t, then

$$F(n) = P\,\frac{S_n}{S_0}$$

So that the payoff formula can be rearranged to:

$$P\,\frac{S_n}{S_0}\ \max\left(\frac{a^{(12)}_{65}(n) - g}{g},\ 0\right)$$

This is a quanto interest rate option. A quanto option is one that is measured in units different from standard cash units; in this case the payoff is in units of the final fund value. The option itself depends on mortality and interest. We will focus on the equity and interest rate risk, though the cost of mortality improvement has also proved a significant nondiversifiable risk factor.

We can see the experience of the option cost over the last 25 years in Figure 1. This gives the cost of the option per $100 maturity proceeds at retirement for a male age 65 using an up to date mortality table (PMA92(C20)). In the mid-'90s, actuaries began to be aware of the potential liability, and in the late '90s, the true cost of falling interest rates became evident. The figure does not show the cost of the spectacular equity returns in the 1990s.

3. VALUING THE OPTION

3.1 Using Jamshidian's formula for options on coupon bonds

Given that several companies have substantial GAO liability risk, there has been some discussion of how to manage the risk now that it is better understood. Many companies have used reinsurance through banks. The modern actuarial approach to risk management might be to project the liabilities under ***P***-measure, and use a discounted tail measure as a capital requirement. This approach is explored in Wilkie, Waters and Yang (2003). Pelsser (2003) discusses the use of swaptions, though this only manages the interest rate risk, not the equity or mortality parts of the liability.

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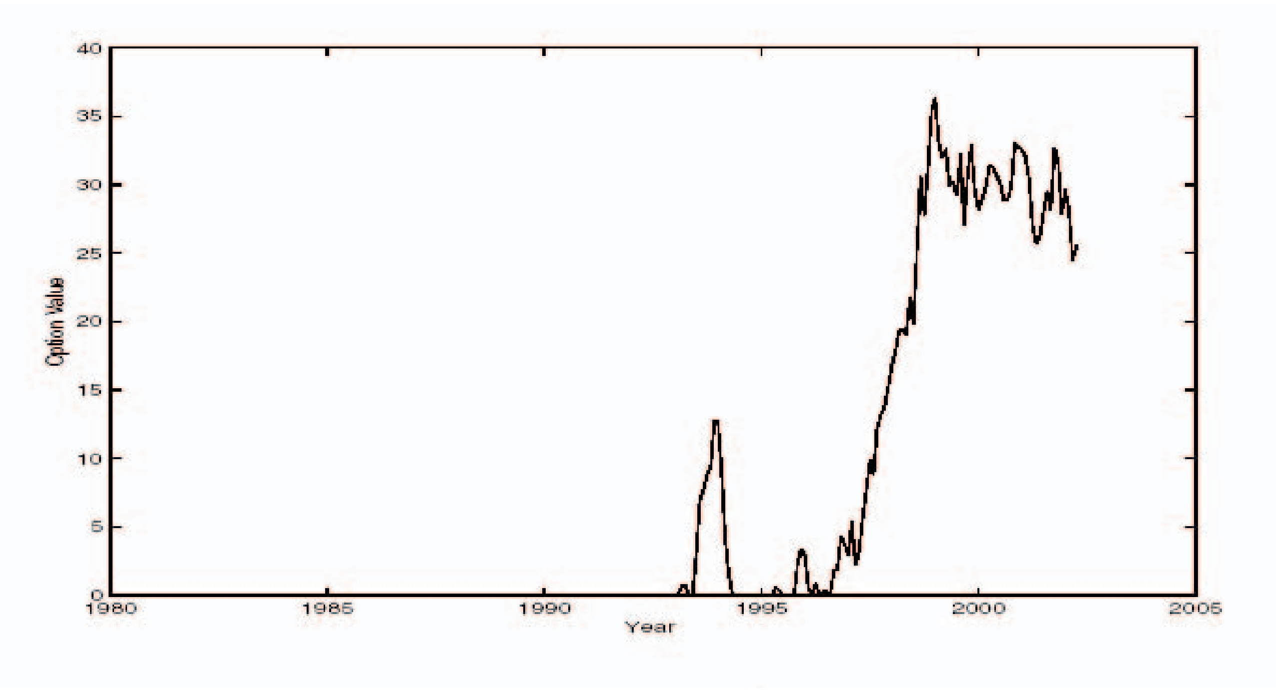
Figure 1: Emerging GAO Cost Per $100 Maturity Proceeds



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From an option pricing viewpoint, the GAO is easier to price than to hedge. We will demonstrate an approach to pricing here.

Assume annual pension payments in arrear, and letting $D(t, T)$ be the price at t of a pure discount bond maturing at T, then we have

$$a_{65}(n) = \sum_{j=1}^{\omega-65} {}_jp_{65}\, D(n, n+j)$$

Using $D(t, n)$ as numeraire means that for any option payoff at n, say, the value at $t < n$ is

$$V(t) = D(t, n) E_Q [V(n) \mid \mathcal{F}_t]$$

Q here represents the forward measure.

The payoff of the GAO at maturity, assuming survival, is

$$V(n) = P \frac{S_n}{S_0} \max\left(\frac{a_{65}^{(12)}(n) - g}{g}, 0 \right)$$

Assume $P = S_0$, for simplicity. The guarantee applies only to lives who survive to annuitize. The value at t of the payoff for a life age $x < 65$, $x+(n-t) = 65$, allowing for survival is then

$$G(t) = {}_{65-x}p_x\, D(t, n)\, E_Q\left[S_n \frac{(a_{65}(n) - g)^+}{g} \,\Big|\, \mathcal{F}_t \right]$$

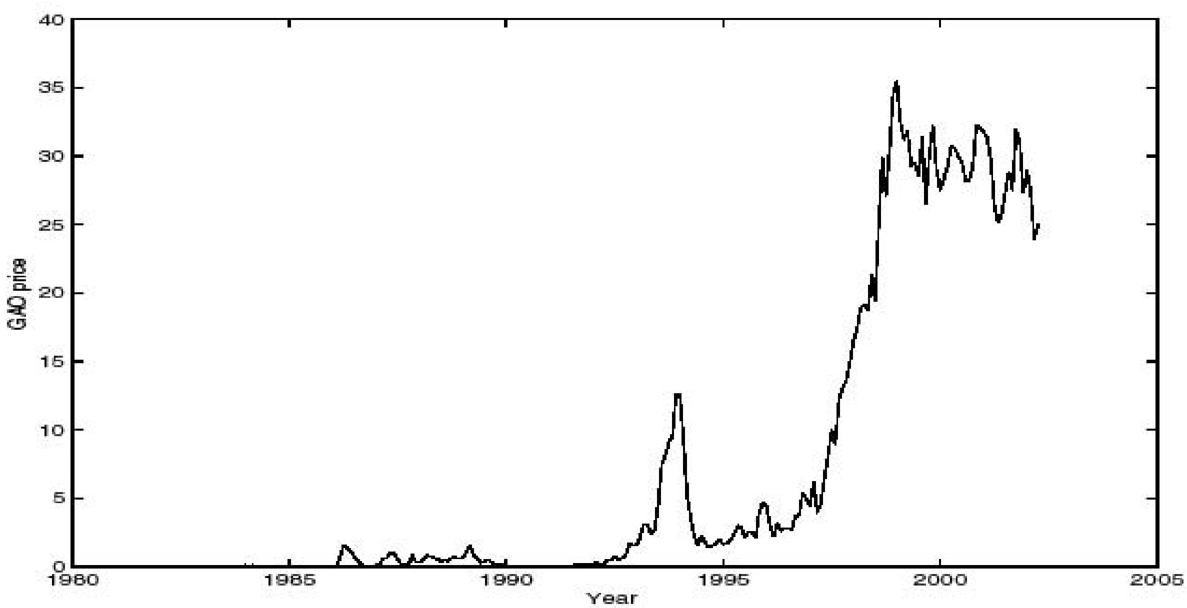
If we assume further that S_t is independent of $D(t, T)$—that is, that interest rates and stocks are independent, then we can simplify further. Recall that under the Q-measure, the discounted value at $t < T$ of S_T must be S_t, so:

$$G(t) = {}_{65-x}p_x\, D(t, n)\, E_Q[S_n]\, E_Q\left[\frac{(a_{65}(n) - g)^+}{g} \,\Big|\, \mathcal{F}_t \right]$$

$$= {}_{65-x}p_x\, S_t\, E_Q\left[\frac{(a_{65}(n) - g)^+}{g} \,\Big|\, \mathcal{F}_t \right]$$

So, we have effectively eliminated the quanto problem, and we are left with an (undiscounted) interest rate option. In Boyle and Hardy (2003) the annuity is treated as a coupon bond, and we use Jamshidian's formula for valuing options on coupon bonds in terms of options on pure discount bonds (Jamshidian 1989). In order to apply this, we use the Hull-White (or

Figure 2: GAO Option Value, 10-years to Maturity, % of Fund



extended Vasicek) single-factor interest rate model. The interest rate model is fitted to the term structure at the valuation date. In Figure 2 we show the resulting option prices for a contract valued at the dates given on the x-axis, assuming a life age 55, that is, assuming 10 years to maturity. At the more recent dates the values are very similar to Figure 1 as they should be. The option is deep in the money, and because the graph is shown in units of the fund S_t, the cost is unaffected by discounting[1]. Notice though, that the option price gives some slightly earlier warning that the option might cost money.

Although we have an option price, we don't really have a hedge. It is well known in the area of interest rate options that single-factor models don't give very good hedges. While they might adequately model the Q-measure distribution of losses at maturity, they do not adequately model the Q-measure process over the term of the contract. An accurate representation of the process is required for the dynamic hedge. For the dynamics of the interest rate process to be sufficiently accurate a model with at least two stochastic factors is required, but modelling such a complex option with a two-factor model would be very difficult.

However, we can indicate roughly what the hedge looks like by using a much simpler model for interest rates.

3.2 Using a lognormal assumption for $a_{65}(t)$

There is substantial autocorrelation in the values for $a_{65}(t)$. Nevertheless, in order to give an indication of what the hedge might look like, we assume that $a_{65}(t)$ follows a lognormal process; we also continue to assume that the annuity is independent of equity performance. Note that I am not advocating this approach! I am just using it to illustrate what this hedge might look like. With different assumptions the hedge would look broadly similar.

With these assumptions we can express the option value as

$$G(t) = {}_{65\text{-}x}p_x\, D(t,n)\, E_Q[S_n]\, E_Q\left[\frac{(a_{65}(n)-g)^+}{g}\,\middle|\,\mathcal{F}_t\right]$$

$$= \frac{S_t}{g}\;\frac{{}_{n\text{-}t}p_x}{D(t,n)}\; E_Q\left[D(t,n)\,(a_{65}(t)-g)^+\,\middle|\,\mathcal{F}_t\right]$$

and now the expectation term is a simple option on the risky asset $a_{65}(t)$. The resulting option cost is

$$G(t) = {}_{65\text{-}x}p_x\, S_t \left\{\frac{a_{65}(t)}{g}\,\Phi(d_1) - \Phi(d_2)\right\}$$

where

$$d_1 = \frac{\log(a_{65}(t)) - \log g + \sigma_a^2\,(n-t)/2}{\sigma_a\sqrt{n-t}},$$

$$d_2 = d_1 - \sigma_a\sqrt{n-t}$$

and σ_a is the volatility of $a_{65}(t)$.

We can hedge this in three parts: an annuity part invested in $a_{65}(t)$, H_t^a, say, a bond part in a pure discount bond maturing when the policyholder reaches age 65, H_t^b, and an equity part invested in the same assets as the premium, H_t^s. Each part is determined, as usual, by differentiating the bond price with respect to the different assets.

The result is

$$H_t^a = {}_{65\text{-}x}p_x\,\frac{S_t}{g}\; a_{65}(t)\;\Phi(d_1)$$

$$H_t^b = -H_t^a$$

$$H_t^S = {}_{65\text{-}x}p_x\; S_t \left\{\frac{a_{65}(t)}{g}\,\Phi(d_1) - \Phi(d_2)\right\}$$

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[1] *We know that the undiscounted value of a deep in-the-money option is more or less the current moneyness.*

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> "Perhaps the first lesson from the GAO story is that insurers need to be very careful about all financial guarantees, and that (almost) no guarantee is really cost-free."

Which just says that we take the entire option value and put it in the equity fund, and in addition short sell some bonds maturing at the retirement age, and use the proceeds to buy the annuity asset. Even when the option value is relatively small, the positions taken in the bond and annuity might be substantial. For example, assume $\sigma_a = .025$, $g = 9$, and consider a life age 45 with 20 years to retirement. If the long-term rate of interest is around 10 percent (as it was in the early '80s), then the option price is low, at around 0.2 percent of the initial premium. However, the annuity hedge amounts to around 5 percent of the premium, much more substantial, and the sensitivity to changes in the annuity value is more apparent.

4. SOME THOUGHTS

Even though with some simplifying assumptions, the ongoing challenge of hedging is very difficult. Perhaps the first lesson from the GAO story is that insurers need to be very careful about all financial guarantees, and that (almost) no guarantee is really cost-free. Some insurers were so casual about these guarantees that they did not even record which policies carried the option and which did not. They may have believed that they could mitigate the cost of the guarantees by adjusting the with-profit bonus (dividend) when the policyholder exercised the option—giving with the right hand and taking away with the left. The high court found that approach was not an acceptable interpretation of the concept of a guarantee.

The second lesson might be to emphasize the importance of financial mathematics—the mathematics of financial guarantees—in actuarial education. Under the current plans for the 2005 SOA education redesign, only risk management and investment specialists will learn financial mathematics. But the optimal risk management for GAOs would have been not to offer them in the first place (however fascinating they might be to financial engineers). A deep understanding of the nature of financial guarantees is critical at all stages—product development, marketing, valuation and risk management. Every life insurance actuary needs to be comfortable with the characteristics of financial guarantees and how these are managed. Therefore, every life insurance actuary needs to have a good grasp of modern financial mathematics. We must ensure that actuarial education provides what is necessary.

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