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The Efficient Policyholder Approach to Pricing Guaranteed Minimum Withdrawal Benefit Riders

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ABSTRACT

THE GUARANTEED MINIMUM Withdrawal Benefit (GMWB) rider, despite its many attractive design features, poses a pricing challenge because of the flexibility afforded the policyholder in choosing when to start withdrawals. Policyholder behavior is consequently a very significant (and virtually unknown) factor in determining the cost of the guarantee.

The proposal in this paper is that the product should be priced on the assumption that the policyholder is financially efficient. In other words, the policyholder will choose the start date that maximizes the cost to the insurer.

PRICING THE GMWB RIDER

The Challenge

The GMWB rider allows a policyholder to make a series of withdrawals from a variable annuity fund, with the guarantee that the total value of the withdrawals will never be less than a stipulated amount (usually the net deposit made by the policyholder).

In addition, the rider affords a sizable window period (typically several years) for commencing the withdrawal process. It is this feature that poses the policyholder behavior challenge: By the very nature of the underlying equity funds supporting the rider, the cost to the insurer will vary considerably, depending on the actual start date chosen by the policyholder.

This is a product that will probably grow in popularity as its benefits become more well-known. Risk management protocol requires that the insurer have a firm understanding of the financial exposure associated with those benefits. Unfortunately, as has been pointed out by experts in this product area (e.g. see Mary Hardy's text Investment Guarantees), it is virtually impossible to model policyholder behavior.

Choosing the correct pricing approach for a product so dependent on the behavior of the customer is therefore certainly not a trivial matter. Recent experience in the equity markets (and the attendant effects on life insurers marketing variable products) amply demonstrate the importance of this issue.

The position taken by this paper is in line with the general consensus of the experts: Anticipating the policyholder's actual choice is a virtual impossibility.

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Hence the challenge: The insurance industry should either develop a pricing philosophy that adequately protects the insurer's financial and capital interests (despite the inability to model policyholder behavior), or seriously consider withdrawing the GMWB product from the market.

Balancing Risk Management Protocol and Marketing Needs

It is (hopefully) no longer the view of the industry that risk management is necessarily at odds with marketing needs. Professionals are becoming aware that risk managers bring very valuable skills to bear on otherwise unsolvable business problems. The case of the pricing philosophy for GMWB riders is one such situation.

First, a very clear statement of the seemingly conflicting views posed by the risk manager and the marketer in this case.

Marketer: *Bring to market a very useful Variable Annuity rider, at a cost that can be supported vis-à-vis the competition and the perception of the customer.*

Risk Manager: *Ensure the financial self-sufficiency of said Variable Annuity rider, so the insurer adequately accounts for the investment risks inherent in the product.*

This situation does not have to degenerate into an untenable tug-of-war. Resolution requires all parties concerned to agree that:

- It is in the best interest of the insurer to be in this business and market this product;
- It is not in the best interest of the insurer to price the product in a manner that results in sub-optimal protection against adverse equity-market developments;

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The Philosophy of the Efficient Policyholder

The philosophy of the efficient policyholder addresses the issue in a uniquely intuitive and financially sound manner: Rather than second-guess the actions of the policyholder under various permutations of emerging economic conditions, impute a financially efficient knowledge base to the policyholder.

Based on this financially efficient base, the policyholder automatically chooses the optimal (from the policyholder's point of view) starting point in exercising the benefit. Note that this optimal choice for the policyholder will be the worst possible choice from the point of view of the insurer.

Put another way, the pricing exercise consists of repeatedly finding the cost of the product at various withdrawal starting points, and choosing the maximum of all such possible prices.

Admittedly, such an approach readily satisfies the requirements of a purist risk manager. But does it meet the needs of the marketer, or of the enlightened risk manager whose view is more attuned with optimizing the company's operations, including its acquisition of new business?

Objections to the Philosophy

Objections to this approach will more likely come from the marketing team, and will center around the argument that the resulting cost will be excessive, relative to what the market is able to sustain.

If the comparison is made between what is currently charged on a typical GMWB rider today (80 basis points) and the corresponding expected cost under the Efficient Policyholder approach (170 basis points), the argument certainly seems valid.

A second objection may come from the pricing and IT professionals. The grounds for this objection will be that implementing such an approach would be very difficult (very nearly impossible) operationally.

Responding to the Objections

PRICE SUSTAINABILITY

How much is a good or service supposed to cost, really? Economists have struggled with the concept for centuries and are still not sure.

For purposes of this discussion, it can be presumed that the market determines the price. There is latent danger in such a supposition, of course: The floor on any market price must necessarily be the actual cost of production (otherwise the company falls for the well-worn joke about selling below cost and making up the difference on volume).

The above statement about the floor on market price is not as trite as it might at first appear: There are many respectable professionals who believe that the life insurance industry violated this rule with respect to term products, during the last two decades of the 20th century. Worse, the indications are that the industry fell prey to a spiraling effect, each company outdoing the other in ever-lower term premium scales, all the time knowing that the mortality tables and expense schedules could not sustain those prices.

The need for rational approaches to pricing insurance products has been highlighted by the recent financial crisis that came very close to engulfing the life insurance industry along with the banks. Now may be the very best time to re-think our view of how products (especially those with equity-related guarantees) are priced.

What would a rational approach to pricing the GMWB rider entail? At a minimum it would have to insist on the following:

- The pricing approach must ensure that the benefit is adequately covered in a self-financing manner
- The approach must account for benefit coverage even under worst-case scenarios
- The approach must pass the scrutiny of prudent risk management professionals

The efficient policyholder approach satisfies all the above requirements. Satisfying the above requirements is tantamount to meeting the basic floor limitation on market price: It must at least equal production cost.

> "Rather than second-guess policyholder actions under various economic conditions, impute a financially efficient knowledge base to the policyholder."

One of the dangers of an industry caught in the frenzy of under-pricing is that no company wants to be the first to step forward and do what is economically sane. Each company walks in lock step with every other company, following the instinctive herd concept that "100,000 lemmings can't be wrong."

The most important point of this paper (and it cannot be emphasized too strongly) is that any insurance price that deviates too far from what is considered prudent in terms of risk management, potentially spells disaster for the insurer.

IMPLEMENTING THE PHILOSOPHY

The second objection to the efficient policyholder approach, that it is difficult to implement operationally, is much easier to answer. As demonstrated in the remaining sections of this paper, nothing could be further from the truth.

General

This section illustrates how the pricing philosophy can be readily implemented. The illustration is done using *Mathematica,* but it could just as easily be implemented in whatever software platform the user sees fit.

Mathematica is used here primarily because of the ease with which it can illustrate mathematical concepts.

Assumptions

The illustration is built around the following product design:

- The GMWB rider provides a seven-year withdrawal period for taking equal monthly payments from the variable annuity fund
- The window of opportunity for starting withdrawals is the seven-year period starting from the effective date of the rider
- The policyholder is presumed to have purchased one share of the underlying equity, worth 1,000.00 at inception
- The guarantee is the initial net fund value of 1,000.00, with the assumption that no partial surrenders of any kind are made between the initial deposit date and the commencement date for withdrawals
- The monthly withdrawals are assumed to be a fixed percentage of the original net fund value of 1,000.00
- No assumptions about expense charges are included in the calculations (presumably if the expense charge is considered as a percentage of the fund, the effect of incorporating expenses could be obtained by appropriate adjustments to the risk-free rate assumption)
- All considerations of lapses and/or mortality are ignored

THE *MATHEMATICA* IMPLEMENTATION PROCESS

Initial Fund Value

$$S_0 = 1000;$$

Fund Accumulation Function

The formula below gives the value of the fund at time n, assuming that withdrawals commenced at time m (< n). Here $\mathcal{R}$ represents the underlying fund value based on equity movements, and $\mathcal{P}$ represents the fixed percentage withdrawn each period. The derivation of this formula is shown in the appendix.

Note: In this regard, $S_0 = \mathcal{R}[[0]]$.

$$\mathcal{V}[n\_, m\_, \mathcal{R}\_, \mathcal{P}\_] := \mathcal{R}[[n]]\left(1 - S_0\,\mathcal{P}\left(\sum_{j=m}^{n}\frac{1.}{\mathcal{R}[[j]]}\right)\right)$$

Stock Accumulation Matrix

In this section, a matrix is created for developing the equity movements underlying the variable annuity fund. It will have 1,000 rows (because 1,000 simulations will be run), and 360 columns representing 360 monthly time periods.

NOTE: For purposes of this illustration, only 168 columns are strictly needed.

Number of Simulations

```
rows = 1000;
```

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Number of (Monthly) Time Periods

```
columns = 360;
```

Define a Standard Normal Random Variable

```
ndist = NormalDistribution[0,1];
```

Declare a Matrix to Hold the Generated Instances of the Variable

```
StandMat= Table[0,{i,rows},{j,columns}];
```

Proceed to Fill the Matrix

```
Do[StandMat[[i,All]] = Table[Random
[ndist],{columns}];,{i,rows}]
```

Declare the Periodic Factor

$$\Delta t = \frac{1}{12};$$

Adjust the Standard Normal Matrix

```
StandMat *= √Δt
```

Declare a Matrix to Hold the Running Total of the Above Standard Matrix

```
RunStandMat = Table[0,{i,rows},
{j,columns}];
```

Proceed to Fill the Matrix

```
Do[prev = 0.; Do[RunStandMat[[i,j]] =
prev + StandMat[[i,j]];
prev = RunStandMat[[i,j]];,{j,columns}];,
{i,rows}]
```

FUND ACCUMULATION MATRIX

This subsection defines a matrix of fund values, using the above results as a base.

First, the fund accumulation function is given, based on the variables:

t = time since rider effective date
ω = accumulative adjusted standard normal
s = assumed fund return volatility
r = risk-free rate

The formula is the familiar Geometric Brownian Motion stock process found in standard textbooks on derivatives pricing.

Fund Accumulation Formula

$$fS[t\_,\omega\_,s\_,r\_] := S_0 e^{(r-\frac{s^2}{2})t+s\omega}$$

Declare Fund Accumulation Matrix

```
AccumMat = Table[0,{i,rows},{j,columns}];
```

Declare Risk-Free Rate and Volatility

```
rate = 0.04; σ = 0.25;
```

Proceed to Fill Fund Accumulation Matrix

```
Do[Do[AccumMat[[i,j]] =
fS[jΔt,RunStandMat[[i,j]],σ,rate];,
{j,columns}];,{i,rows}]
```

APPLICATION OF THE EFFICIENT POLICYHOLDER ASSUMPTION

Withdrawal Period (Years)

```
years = 7;
```

Withdrawal Rate

$$WR = \frac{1}{years}\Delta t;$$

Window of Opportunity for Starting Withdrawals (Months)

$$WO = \frac{years}{\Delta t};$$

Period for Withdrawals (Months)

$$Per = \frac{years}{\Delta t};$$

Vector of Possible Prices Based on Withdrawal Starting Point

```
VecPrice = Table[0,{i,WO}];
```

Accumulated Withdrawals as of Time n

```
AccWith[n_, 𝒫_] := n 𝒫 S0
```

Determination of Whether a Claim is Payable, and How Much

```
DetClaim[n_, m_, ℛ_, 𝒫_] := If[𝒱
[n, m, ℛ, 𝒫] ≤ 0, Max[0, S0 - AccWith
[n - m, 𝒫]], 0]
```

Calculation of Payoff and Cost

This is the heart of the calculation process that determines the price (on a net single premium basis) of the GMWB benefit.

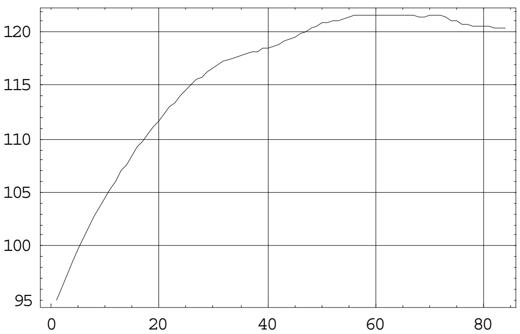
```
Do[Do[Do[RR = AccumMat[[i, Range
[Per + WO]]];
VecPrice[[j]] += e^(-rate (j+k-1) Δt)
DetClaim[j + k - 1, j, RR, WR] /
/ rows;
 If[DetClaim[j + k - 1, j, RR, WR]
 > 0, Break[]];, {k, Per}];,
{i, rows}];, {j, WO}]
```

Display All Possible Prices, In Start Date Order

```
VecPrice
{94.9323,96.12,97.3166,98.4641,99.6454,10
0.713,101.729,102.741,103.663,104.547,105.2
85,106.062,106.997,107.625,108.458,109.278
,109.852,110.438,111.107,111.644,112.295,11
2.964,113.396,114.095,114.649,115.003,115.5
83,115.776,116.358,116.683,116.935,117.254,
117.498,117.598,117.746,118.016,118.225,11
8.135,118.45,118.528,118.74,118.914,119.099
,119.42,119.519,119.826,120.105,120.373,120
.486,120.816,120.836,120.98,121.097,121.29,
121.453,121.509,121.566,121.494,121.609,121
.503,121.588,121.568,121.521,121.617,121.54
7,121.586,121.632,121.481,121.436,121.606,1
21.492,121.505,121.338,121.124,121.08,120.
77,120.724,120.529,120.528,120.562,120.511,
120.447,120.43,120.446}
```

Graph the Price Results

```
ListPlot[VecPrice,Frame→True,PlotJoined→
True,GridLines→Automatic]
```



□ Graphics □

Determining the GMWB Price as the Maximum of All Possible Prices

Inspection of the graph shows approximately where the maximum possible price lies. It can be pinpointed precisely by employing the following routine.

```
Price = 0.;
Do[Price = Max[Price,VecPrice[[i]]];,
{i,WO}]
```

This is the actual price for the rider

```
Price
121.632
```

This shows the withdrawal start month that corresponds to the maximum possible price

```
Position[VecPrice,Price][[1,1]]
67
```

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CONCLUSION

For a product as popular as the GMWB rider, it is important that the insurer be in full control of the risk management ramifications underlying the guarantees: The larger the sales volume, the greater is the potential for severe financial consequences if the pricing proves to be inadequate.

The philosophy proposed in this paper demonstrates a rigorous, defensible, and easily implemented method for ensuring the financial integrity of this product.

Derivation of Fund Accumulation Formula

THIS DERIVATION ASSUMES that withdrawals are made at the beginning of each period.

TIME m

Assume that the fund is being viewed at time m, immediately after the first withdrawal has been taken. The Fund will be identically equal to the indicative equity value, less the withdrawal:

$$R[[m]] - PS_0 \equiv R[[m]] - PS_0 \frac{R[[m]]}{R[[m]]}$$

TIME m+1

Next, assume that we proceed to time m + 1. The fund value would have changed based on market movements, and the second withdrawal is taken. We now have the following result:

$$R[[m]](1+r1) - PS_0 \frac{R[[m]]}{R[[m]]}(1+r1) - PS_0 \equiv$$
$$R[[m+1]] - PS_0 \frac{R[[m+1]]}{R[[m]]} - PS_0 \frac{R[[m+1]]}{R[[m+1]]}$$

TIME m + 2

At time m + 2, the fund value would again have changed based on market movements, and the third withdrawal taken:

$$R[[m+1]](1+r2) - PS_0 \frac{R[[m+1]]}{R[[m]]}(1+r2) - PS_0 \frac{R[[m+1]]}{R[[m+1]]}(1+r2) - PS_0$$
$$R[[m+2]] - PS_0 \frac{R[[m+2]]}{R[[m]]} - PS_0 \frac{R[[m+2]]}{R[[m+1]]} - PS_0 \frac{R[[m+2]]}{R[[m+2]]}$$

TIME m + 3

Similarly for time m + 3: The fund value changes based on market movements, and the fourth withdrawal is taken, yielding:

$$R[[m+2]](1+r3) - PS_0 \frac{R[[m+2]]}{R[[m]]}(1+r3) - PS_0 \frac{R[[m+2]]}{R[[m+1]]}(1+r3) - PS_0 \frac{R[[m+2]]}{R[[m+2]]}(1+r3)$$

$$- PS_0 \equiv R[[m+3]] - PS_0 \frac{R[[m+3]]}{R[[m]]} - PS_0 \frac{R[[m+3]]}{R[[m+1]]} - PS_0 \frac{R[[m+3]]}{R[[m+2]]} - PS_0 \frac{R[[m+3]]}{R[[m+3]]}$$

TIME n (≥ m)

It then becomes a matter of simple induction to derive the generalized formula for time n ≥ m:

$$R[[n]](1 - PS_0 \sum_{j=m}^{n} \frac{1}{R[[j]]})$$