RECORD OF SOCIETY OF ACTUARIES
1983 VOL. 9 NO. 4

THE PRACTICAL APPLICATION OF RISK ANALYSIS TECHNIQUES IN HEALTH INSURANCE

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A teaching session dealing with the use of probability distributions and other risk analysis tools in the cost and analysis of health insurance products.

MR. ROBERT G. MAULE: Our topic today is the practical application of risk analysis techniques. We're going to talk about practical applications rather than theory. As a consultant I've seen a lot of companies over the last few years in the group insurance arena and I've seen companies losing money. More and more I've come to believe that we've got to model our business better. We've got to use risk analysis techniques to get a handle on our business, to make realistic models of it. I think this modeling is essential, or else you may have to just get out of the business as it is constituted today. It's a tough business to manage.

We're going to discuss a number of techniques and because this is risk analysis, we'll be talking about probability distributions. The first major topic is individual distributions and we want to talk about multinomial distributions. See Appendix #1. In a non-technical sense, a multinomial distribution is a die, but not necessarily a six-sided die, maybe an n-sided die. And on each face of the die is a number - the outcome. When you roll the die, there is a certain probability that any particular face will come up. Now, one of the faces will come up, so the sum of the probabilities is one.

Example 0 in Appendix #1 is an example of a simple multinomial distribution. It says that the probability of no claims is 50%. The probability of a $50 claim is 10% and the probability of a $100 claim is 40%. The sum of the probabilities is 1.0. The expected value of the claims for this particular die - this is an unusual die, it has three sides - is $45. We could figure out a number of statistics about this distribution. We've already determined what the expected claims are, we could determine what the variance of the expected claims is and so forth, then use those later in some statistical applications. Now look at Appendix #2. Included amongst the data on that page is a multinomial distribution - it might be a little hard to find, but it's in columns 5 and 6. Column 5 is a series of probabilities and column 6 is a series of outcomes. We start with zero claims, with a probability of 25% and so forth, until you go down to the bottom of the distribution where there is a $429,000 claim, with a very small probability. This is a multinomial distribution and this is real data. We develop this type of distribution through a research effort to get probability distributions of group health claims for common sets of benefits that are generally offered for the United States average. That is, cost levels and utilization levels reflect U.S. averages. The cross products of all of those frequencies and amounts are the child $0 deductible claim cost as we saw it for the national average on January 1, 1982. We're going to refer to this distribution quite a bit, so a natural question is how is such a distribution developed? You will see that distributions like these are the cornerstone of most of what we'll discuss this afternoon. But where do you get them?

How many of you in your companies study the distribution of claims by amount per individual? How many of you tabulate and make up tables indicating after a certain deductible, only x% of the claims remain? It is a very time consuming process. It would take a long time to pass all of your claims files for all of your comprehensive plans through, testing what claims were in excess of a prescribed deductible. That is the kind of information that's generally available from a claim system because companies are interested in pooling limits. Usually they're interested in $100 deductibles and $1,000 deductibles, so routinely they will determine figures like out of a total of $100 million of claims, $2.5 million exceeded some amount, say $10,000, in which they were interested. Suppose you have that data. How do you build one of these distributions? Developing them from the practical data is art and science because the data is very likely not pure, it has all kinds of problems. We've looked at data from many companies in all geographic areas and we've seen wide differences in utilization patterns and in the secular differences that occur because of geography and other reasons - lots of practical problems. Suppose that finally you get some check points so you can say that for a child or for an adult, the claims in excess of deductibles $100, $200, $500, $1,000, and so forth are given percentages of the aggregate claims. Once you have that - how do you build this table? You can do a lot of graphical trial and error, or as is indicated in Example 0 you can analyze the data. We will not spend a lot of time on this analysis, because it's one of the more theoretical considerations, but it's not as hard as it looks. The exhibit shows that the claims in excess of deductible D, C(D), is the sum of the difference between claims t, the aggregate amount of claims, and D, assuming that t is greater than D. We will sum or integrate from D to infinity. Take a claims amount in excess of the deductible and weight it by the probability that claim t can occur. That will be the value of the claims in excess of the deductible. The formula is straightforward. Now, here's a little trick, and it can be helpful. Differentiate with respect to D, and solving the resulting integral gives - (1 - F(D)). F(D) is the cumulative distribution function for this particular probability distribution. What we have is C′(D), thus, the derivative is a function of the cumulative distribution of probabilities. We can approximate the derivative. Suppose you have two points, D and D*, and you have data and you've found out what the claims in excess of those two amounts are. Take the difference, divide it by the difference in the amounts, that's the approximation to the derivative: -[1 - F(D)] and you can get F(D). Do this calculation enough times and you'll get enough points in the cumulative distribution.

Now to begin to understand these results, you can write down the column of frequencies. You've got to do a lot of manipulation, but this is a technique that we've used and it works well. Let's test it out - let's go back to Appendix #2. Let's try to find out what the probability is that claims will be in excess of approximately $10,000. If you go to column 1, and look at $10,000, you will see that the annual claims in excess of $10,000, in column 2, is $25.67. If you look at it for $15,000, you will see $18.19, and you'll see I have that as a difference in the calculation in Example 0. Now the difference in the deductibles themselves is $5,000. If you compute the approximation you get -.0015 and that should equal - (1 - F(D)) -- somewhere around $10-15,000. You get into an averaging problem in this example because I've chosen a big interval here for approximating the derivative. Look at column 8 in Appendix #2. This column shows the probability that claims are equal to or in excess of the amount in column 6. If you look around $12-15,000 and move over to column 8 and you'll see numbers like .0017 and .0013. Those two numbers bound the .0015, so the calculation worked here.

What we have shown is that if you collect routine types of data, claims in excess of a certain deductible, you can develop tables like this. The fundamentals are columns 5 and 6. Everything proceeds from columns 5 and 6.

Columns 1 and 2 show calculated values at several deductible levels. From the distribution in columns 5 and 6, if you want to know what the value of the claims are in excess of $10,000, go down the amount column, column 6, until you find the first claim that's in excess of $10,000. From there on, subtract $10,000 from every claim and weight these amounts by the frequencies that occur for each of the amounts above $10,000. Add up the total and the result is what is shown in column 2 for $10,000, $25.67. So, the fundamental tool here is the distribution itself, the die. All other values on the table are derived from column 6.

We have been addressing how to get columns 5 and 6 from your own new data. There are other ways such as graphical techniques, this is simply one that works well. You've got to choose enough points and use some good judgment, but this method will produce the distribution. Obviously, a true, complete distribution of claims would be extremely long. To handle this we want to group amounts. For instance, let's group all claims between $1,000 and $1,200 and call them all the weighted average, say $1,105, and so forth. This will make the resulting distribution manageable. We want to do this grouping so that we don't get too rough around deductible levels that we're interested in. We could group everything between $1,000 and $10,000 and call them all $5,600 claims, but then if someone is interested in a $3,000 deductible we have a problem. We don't have a sufficiently refined distribution to give us good answers there. So, the balance is to have enough claim amounts but to have few enough to make the whole thing manageable, while having enough so that we can make realistic calculations around deductibles that are of practical interest to us.

Let's discuss each of the columns - we've discussed columns 5 and 6 so far. The product of columns 5 and 6 is in column 7, that's simply the expected value of the claims for that particular claim amount. Column 7 is just the cross product. Column 8 is the probability that claims exceed a given amount. That's just the sum of the probabilities summed from the bottom of the table. To calculate the annual cost of the claims in excess of a certain deductible amount you simply find the first amount in column 6 that is greater than the deductible and find the corresponding amount in column 9. From that amount subtract the deductible times the corresponding column 8 probability. The result is the annual claim cost for amounts above the deductible. This is how the amounts in column 2 are determined.

A question has been asked: "How do you find out the frequency of no claims if you're running off the claim file?" You have to build a model of claims cost that take into account not only submitted claims, but the unsubmitted claims because you usually are dealing with a deductible plan and some people accumulate expenses that they never submit so you never see that eligible expense. The way we have handled this problem is from studies that we make in modeling the expected values of claims costs. We consider the total frequencies that certain types of procedures will occur, the average cost of these procedures, and the resulting claim costs from these procedures. We've built another model which is literally a manual which includes the probabilities and costs of benefits for hospital inpatient and outpatient, surgical inpatient and outpatient, outpatient physician, and approximately 20 other benefits. These items are split out by age and sex for the U.S. average. This manual produces what the total charges would be in the environment that we're in. If you actually have a true $0 deductible, you'll probably get more utilization than even those statistics show. You have to make an approximation and if you have nothing but a $100 deductible plan, you'll have to back up and add some claims that you never saw to get the $0 amount.

What kinds of things can you do with this information once you've developed it? Look at Example 1. We can price a variety of different plans. Example 1 is for a child - the cost or the price of pure claims for $100 deductible, 80/20 coinsurance, and $1,000 out-of-pocket limit. Using Appendix #2 we want to find out what the out-of-pocket limit is and that's $4,600 of claims. This calculation is off to the right. You end up paying 100% of the amount in excess of $4,600. It's just a simple little equation. One thousand dollars out-of-pocket equals $100 deductible, plus 20% of x-100 and x turns out to be $4,600. What we've got to do next is to find out what is the cost of claims between $100 and $4,600, because we're going to have to pay 80% of those, and then we'll pay 100% of those claims in excess of $4,600. From the distribution, the claims for a $100 deductible plan is $13.23. This is the monthly amount from column 4. We don't have a $4,600 amount, so you need to interpolate between the $4,000 and the $5,000 amounts. That's what I do in the next step in Example 1. It's a straight line interpolation - you can get fancier if you wanted to, but you probably don't need to - $3.48, those are the claims in excess of $4,600. We're going to pay 80% between $100 and $4,600, so that's the next calculation, .80 x (13.23 - 3.48) and we're going to pay 100% of the amount of the amount in excess of $4,600, that's the $3.48. This all adds up to $11.28.

One question that arises is what is the effect of coinsurance or what is the loading factor when you have such an out-of-pocket maximum? We ended up with a price of $11.28 for the plan above. For a straight comprehensive plan at 80% of the $13.23, the cost would be $10.58. We next took the ratio of $11.28 to $10.58 and multiplied that ratio by the 80% coinsurance so we have an effective coinsurance, if you want to look at it that way, of 85%. Another way of stating this is it is about a 7% load. You can do this kind of pricing for all kinds of deductibles and graded coinsurances and you can let your imagination run away with the kinds of comprehensive plans you could price once you have this table.

Let's now consider Example #2. We're going to price a minor benefit - a $500 supplemental accident plan. How do you do it? There's an observation that I've seen, that accident claims seem to form a fairly constant percentage of total claims at just about every claim level. For adults, it is around 10% of the claims from accidents and for kids it is about 12%. This means we could take this distribution and split it into two pieces roughly. Simply multiply every figure on it by .9 and call it health claims, and multiply every figure on it by .1 and call it the accident claims, realizing that the sum of the two sheets are your total plan. That's in effect what we do here. First, look at the grand total and determine what is the value of the first $500 of benefit. That's C(0) minus C(500) and if you look in the table, you'll see it's $18.85 - 8.21 = $10.64. We'll reimpose the deductible of $100. We'll assume that the standard that we want to measure against is a $100 deductible plan. What we're really doing is just waiving any deductibles on the first $500 of expense, and we'll reimpose the deductible after $500 of expense. There are different ways to do a supplemental accident plan, but this is one of them. Between $500 and $600, we won't have any cost, and from $600 and above we will. For $600 and above, $7.89 is the number. The regular plan is a $100 deductible, 80/20 comprehensive plan and thus it's just 80% of the value in the table at $100 (80% of $13.23 is $10.58). Under the modified plan, I'll take 90% of the standard plan, because that's the non-accident portion, and 10% as the accident portion. Combining these two pieces, the first $500 of accident, $10.64, and we'll pay 80% of everything after $600, and that's 80% of $7.89 plus 90% of the $10.58, all adds up to $11.22. The extra cost is $.64. That's about a 6% load on the standard plan of $10.58. The plan design is for an accident claim. The plan will pay up to the first $500 flat, $0 deductible benefit. But, for a claim above $500, we'll reimpose the $100 deductible. After that point, it will look like the $100 deductible, 80/20 plan. I just picked this plan, there are other ways to price it and there are other benefit structures we could have used, but the point is that you have a tool to do some modeling and pricing. You don't have to look at six accident claims, or something like that, and pull your hair trying to make a wild guess. Once you've got a tool like this, it has enormous predictive value.

MR. BRADFORD S. GILE: I'm Brad Gile with Wisconsin Insurance Department. I've made use of the mathematical form that you set down here, the function of C(D) and the derivative of cumulative distribution. I made use of it when I had to price the health insurance risk sharing plan. What I did in that case, because I didn't have company data at hand, was to take a group rate manual of a company with whom I was very familiar and instead of setting up a big table like this and interpolating, I used a formula tool. One of the things I did notice when I was with that company more than 10 years ago was that a particular functional form fit the company's data remarkably. It's of the form

$$e^{-(ax+t)^{1/2}}$$

where x is your amount of covered expense. I found it interesting.

MR. MAULE: Very interesting and worthwhile comment. In that form, e to the square root, you've got something that goes down more slowly - it has bigger claims out there with higher probability than maybe a standard distribution, like e^{-x}. We're going to come back to that point a little bit later. There are some mathematical functions that do seem to fit some of these distributions pretty well.

Pooling charges. Everybody is interested in pooling charges. Look at Example 3. From this data, pooling charges are expressed as a dollar amount per month, per child, for three different amounts, $15,000, $25,000 and $50,000 and all you have to do is look in the table and you have $1.52, which is 8% of all charges. You've got to watch this because this is just 8% of all charges, not of all claims. Claims are something else, what we're looking at here is eligible expense and not all of them result in benefits. It will turn out that your pooling charges, probably because you have no coinsurance at those levels, are going to be a higher percentage of claims. If claims are based on a $100 deductible, 80/20 coinsurance plan, they'll be less than the eligible charges, so the percentages of benefits will be higher than those shown.

The next example is really important and I think it's very interesting. It has been remarkable to me how few people in the health insurance industry, up until recent times, have understood what I call the leveraging effect of deductibles. We've got this distribution and its centered on January 1, 1982. Suppose you wanted to do some pricing centered on January 1, 1983. If you did your calculations on January 2, 1982, you would not have any data to tell you what the numbers are. How do you use this table to get some answers? Let's assume that the base trend assumption is 15% and we want to know what the cost of a $100 deductible plan one year from now. Go to columns 9 and 6 and find the first claim in column 6 that exceeds $100 ... it's $136.27. If you go over to column 9, the $207.86 is the sum of the expected value of that claim and every claim greater than that. This is the annual value on January 1, 1982, $207.86. It is the sum of the total of claims that exceed $100. We want to take $100 deductible out of every claim. We could go down column 6 from $136.27 on and from each claim we could subtract $100 and we could weight all of the positive differences by the frequencies that are shown and add them up and get the right number. There's an easier way with the table. Since you know that $207.86 is the sum of all the claims in excess of $100 right now, they certainly will be when they're inflated by 15%. So, inflate them all by 15%. Then, subtract $100 from "all of them". "All of them" are the sum of the frequencies from $100 on, and that's in column 8 - that's .4907. That's the calculation that's shown in Example 4. The result is $189.97 for January 1, 1983. The January 1, 1982 value is $158.79, and you can see that if you go over to the $100 amount in column 2. What's the rate of increase? It's 20% - if you divide $189.97 by $158.79 you get 20%. So, what's the lesson here? For a child, if there's a 15% price trend rate you're going to get inflation on a $100 deductible, 80/20 coinsurance plan at 20%. The leveraging we got here was that every claim that was over $100, stays over $100, gets quite a bit more over $100, when we subtract the fixed $100 from it, we get a leveraging of that difference.

What about $10,000? That's the second part of example 4. Go down column 6 until you get the first claim that's over $10,000, ($12,036). Be careful as the claim before it, $9,652, when inflated by 15%, will exceed $10,000. We can't use the values in that row, we've got to move up one row because now when we inflate we're going to end up with the $9,652 claim going over $10,000. So, we move over, in that row to the $48.66 in column 9 - that's the sum of all the claims that are in excess of $9652. A year from now, every one of the claims will be in excess of $10,000, if we assume the 15% trend rate. Multiply $48.66 by 1.15 and subtract from it $10,000 times the sum of the frequencies for claims that are now in excess of the $9,652 and you get $32.77 shown in the example. The untrended value over in column 2 for $10,000 is $25.67. The resulting leveraged trend rate is 128%. At the $10,000 level we get almost twice the base trend rate. These distributions are a great tool for predicting what trending will be for all kinds of deductibles.

Does this work? This is an interesting mathematical exercise. Let me give you an example. I know a company that took its entire claim file for one year that was two years past - so they knew what the trends had been. They knew their overall portfolio trends for some 18-month period. They took all the claims from that past file and they trended them all forward just like this instead of dealing with a unit claim. They used exactly the same process, subtracted out the fixed deductibles, went through the deductible calculation, produced a string of values and then they compared it with their actual current data. The resemblance was marked, in other words, it was a very accurate predictor at different deductible levels of what the trend is. This is a powerful tool.

Question: How are the points in the table chosen?

The amounts that are shown on the left, the 0, 50, 100, those are arbitrary points, points in which we are interested, which we have chosen in putting this table together. We tried to choose distribution amounts so that we would get amounts within the different deductible categories, but we didn't try to exactly fix the average right in between or anything like that. In fact, in using this table, if you found that one interval was too wide, you distrusted it, say, between 50 and 100, where we have $73.81, you might redistribute the frequencies and the amounts so it all composites down to the same cost and then do your projections. You need to do this in some cases to take out some of the anomalies.

We're going to introduce ourselves into a new area and move on to Example #5. Suppose you've always had $100 deductible per person plans, and all of your family units have two children in them, and now you've been told that you're going to have to have a plan that has a one only deductible maximum for the children. This is kind of an arbitrary plan feature that we've constructed here, but you'll recognize it. A two person deductible maximum is fairly common. And you've got to price it. All of your data shows what happens when you have a $100 per person deductible and everybody's got to satisfy the deductible. Now, in this two child dependent unit all they have to do between them is satisfy the amount of one deductible of $100. What's the added cost? How do we determine it? The technique is generic for a whole group of problems for evaluating family out-of-pocket limits. What we do is get the value of the $100 deductible. Now, looking at the distribution of deductibles in Example #5, and there's a 2% chance of a $34 claim and $34 is under $100, so we'll get the full benefit of that in the value of the deductible. We're not valuing the claims now, we're valuing the deductible. The chance of a $74 claim is 24%. All the other claims on that page are in excess of $100 so all we can get in value of the deductible is $100 and all the frequencies from that point on, sum up to 49% and we get $100. The value of the deductible is the cross product of those four frequencies and amounts and the total is $67.44. If you want to check it against the claim distribution, simply take the $0 annual figure and subtract the $100 annual figure for the total claims and that ought to be the value of a $100 deductible, and in fact it is $67.44. For our next step we've got to be concerned about all possible combinations of claims of the two children. We want to evaluate what the worth of the more limited deductible is in this situation. All we have to look at are those claims of $100 or less - we don't have to look at two $10,000 claims, because we know in that case, we'll only have $100 joint deductible under the limitation of deductible that we're talking about. We're going to convolute the distribution with itself - we're going to take all combinations of results, and there are 16. We can have 0 and 0, both of the children incur no claims with probability .25 x .25 or .0625 and so forth. I've combined all the amounts in the example, that's why you don't have 16 numbers written down because there are pairs where I've added the two frequencies. We now have the joint probability distribution of the valued deductible for two children. The sum of the frequencies add up to 1.0 and the expected value is $144.88 - that's two times the value of each single distribution (we're assuming independence here, so that's exactly what we expect) and that's the tool that will now give us our result as to what it's going to cost us to have a one-time only deductible. What we do next is go over to the modified amount column and we say the value of the $34 is still $34, $68 is $68, and $74 is $74, but from $100 on we now have only $100 of value of deductible.

To do this generically, you have to take family size with one child, two, three, four, five, and six, then you have to convolute the distributions together for all of these combinations and look at the results for all of them. Regardless of the benefits, you use precisely the same method. Instead of looking at $100, you might be looking at two times one deductible of $200, maybe three times, two-and-a-half times, and then you'd weigh your results finally by the proportion of families that have one, two, three, four, five, six, etc., children and you get your results. You don't have a problem if there's a single adult or two adults, it is the first dependent child that introduces the problem. You have to use other distributions here other than just the child distribution. The technique is more complicated because you have to deal with more situations, but not fundamentally different and it comes right out of these distributions.

We get the value of the modified deductible at $89.96 between the two of them. It was in total before $134.88, so that's 67%. We've reduced the value of each deductible, if you like to look at it that way, by 33%, and what I did at the bottom of the example is calculate the effect on a comprehensive plan. I took the total annual claim cost for a child, $226.20, subtracted 67% of the standard deductible for a single child and multiplied by 80% so there's a comprehensive plan now under this modified deductible arrangement. Right below it, being divided into it, is the standard value of the comprehensive plan resulting in a 14% load of the child rate. You can do this for all kinds of combinations.

Next we're going to discuss aggregate distributions. So far we have generated individual distributions and we've learned that we can do a variety of interesting and practical things with them. Our next topic is quite practical, also. We're going to ask the question, if in a portfolio of risks, we have a group that has 1,000 lives or 100 lives - what is the distribution of aggregate claims? Similar to the individual distribution we developed, we want to determine the probabilities and the corresponding claim amounts, but we want it for the aggregate claims in a calendar year for all of the people in the group. One method of determining the aggregate distribution, once you have the individual distributions, is to convolute the distributions. For this to work we need to make an assumption of independence, which probably is a reasonable assumption for health insurance claims - at least for medical claims, and that's what we're working with right now.

You will recall, we convoluted a small, simple distribution when we evaluated a deductible limitation. You can do the same thing for any number of lives, however, convoluting realistic claim distributions a large number of times is a very large task. It's not easy. We spent a long time writing a program that, by brute force, went through the process. You might say that's ridiculous because you have, say, 25 values in the string of your probability distribution, you convolute it together 1,000 times -- 25^{1000} is an awfully big number - computers are fast, but they're not that fast. You've got to use some simplifying techniques. You can use grouping techniques, you can throw out values that are of no material worth and you can use variance preserving techniques and write a program that gives you a very good estimate or approximation of what the true distribution is. That's probably the best way to approach the problem because what we're really dealing with is a multinomial situation where we take 1,000 dice, throw them on the floor, count up all the faces and that's the aggregate claims for the year.

Appendix #3 talks about some of the problems in developing an aggregate claims distribution. It's just some general material that you might find interesting.

Appendix #4 shows what some real aggregate distributions look like. The first table, labeled Aggregate Distributions on page 1, was done for 1,000 employees, with approximately 600 or 700 spouses and another 1,200 kids. It's a typical composition. The original results came out in dollar amounts because the distributions that we were using had dollar amounts. This distribution is derived directly from that original output. To get the distribution on page 1 you divide all linear values by the mean. Other values are divided by the square of the mean. The result is what we call a normalized distribution where 1.0 is the mean. The mean is 1.0 and not $1,000,000 or $1,500,000 for this 1,000 employee group. It's easier to read when we want to talk about attachment points and other uses. This is a real distribution that was actually used in practice, it's not up-to-date, but the principle is still there.

The amount, as a fraction of the mean, is in the first column. So if you have 1,000 employees, what's the chance that the actual experience claims are 49% of the mean? The probabilities are listed in the second column. The answer is that there's no chance. That probably makes sense to you. The cumulative probabilities are shown in the third column. Stop Loss Premium, which is the expected value of all the subsequent claims in excess of the claim in that particular row, is in column 4. The variance of the Stop Loss Premium and the standard deviation of the Stop Loss Premium are in the next two columns.

These are aggregate distributions and there are several of them in Appendix 4. We're going to discuss them in some detail. Let's look up the Stop Loss Premium at the 125% attachment point for this first group of 1,000 employees. We get the claim level, line 84, 1.2506 - that's close to 1.25. The Stop Loss Premium is $.208 \times 10^{-2}$. The pure Stop Loss Premium for 1,000 employee group and 125% attachment point, at the time this distribution was constructed, is approximately 1/4%. We're not finished with that 1/4%, but that's where it is right now. Before we go on, let's turn to examples 6 and 7. The first comment I want to make is you've got to be careful when you start using statistical methods and you do a careful job of evaluating say three out of the four factors. But the fourth factor may have a very large impact on the overall result, so that your results are totally worthless. When you're looking at purely statistical distributions, that can happen in this business.

Suppose we're looking at 1,000 groups of 1,000 lives and we know that they are identical in age/sex characteristics and in the same location and the same industry. From our rating manuals they are the same groups, we cannot distinguish them for pricing purposes - they have all the rating manual characteristics in common. But what's the truth about those 1,000 groups? It's been my experience that they aren't all the same, that some of them are consistently better, some are consistently worse. You find that out if you keep them for 10 years, but without experience it's very difficult to know which groups are good and which are not. We know from general arguments that these groups will be different. We know the socio-economic class, level of education, level of income, things that in many of our rating manuals aren't very well reflected, affect the aggregate level of claims, i.e. the mean level of claims for a group. One of the standard problems that we face in assigning credibility and in developing Stop Loss Premiums is that data we think is homogeneous really isn't homogeneous. It's made up of separate subclasses that have separate internal means and within those subclasses there is statistical fluctuation going around those submeans. What we're talking about is what we call the inherent level of a group. In example 6 I just picked an example and actually this distribution is one that we've used for 100 life groups, because we've seen this kind of experience for 100 life groups, that is, 5% of them are really at 70% of your typical manual mean, with all those typical manual characteristics. The average mean is 1.0 and groups fluctuate between .70 and 1.30.

Now, if you look at the graph in example 6 we're going to ask the question, if we've got that kind of distribution in 10,000 groups and we're looking at the deviation from the mean of these 10,000 groups, what is the character of that distribution? What we've shown there are graphs for each of the separate inherent level categories. What we do is merge them all together when we look at experience and the result is that we see a spread of variance that is wider than any single statistical spread that we would get. This is crucial. To point out how crucial it is, suppose you have exactly two kinds of cases, their mean is either .5 or 1.5 and the average is 1.0 and you don't know how to distinguish them. You really don't know those two numbers, the .5 and the 1.5, but that's what they really are. Now you offer these 10,000 life groups aggregate Stop Loss coverage at 125% of expected. What is expected? One point zero. What's the attachment point? One point two-five. How do you do? You fare poorly!

For a 1,000 life group the inherent level might start at 90% of expected, with a small probability, and go up to 105% or 106% of expected. We looked at the Stop Loss Premium in the first distribution. Now turn to the next two pages. This has what we call the uncertainty distribution. It has the assumption in it that not all these 1,000 life groups are the same, that the average 1,000 life group is made up proportionally of groups that really have means of 90% of expected, 95% of expected, and so forth. What we're really looking at is the merged experience of all these different levels of groups. What do we see at 1.25? - .0047. It's on page four, line 113 and the fourth column is the Stop Loss Premium and it's .4714, etc., x 10^{-2} or about 1/2%. The original value was 1/4%. Under this uncertainty assumption, the Stop Loss Premium has become twice as big. If you'd used a mathematical technique, a perfectly adequate one that does really reflect the distribution, you're going to get the wrong answer because you're going to be assuming the mean is one. The mean is some fixed amount, but you don't reflect the underlying differences. So when you run an aggregate Stop Loss run there is always a mix of different inherent level cases.

QUESTION: Can you tell us, again, where these values in this aggregate distribution came from?

MR. MAULE: By convolution, using a program that in effect just started multiplying individual distributions together -- but truncating after a certain amount of time. If you multiply one of those individual distributions that has 25 lines in it by itself, you've got a lot of lines. So you have to combine amounts at certain stages, but do it judiciously so you don't disturb the distribution at important points. Important points are like the 125% attachment point and so forth. It took us five years to write the program to do that with all of it's optimizing - it's a complicated task. I don't know whether it would take so long these days, because computers are faster and the languages that are available are more powerful.

This leads to an important point here. These aggregate distributions have been tied to experience. When we use these uncertainty distributions to the convolutions, and when we look at 2,000, 3,000, and 4,000 cases over two years and do an actual-to-expected test and there is a very good fit. Thus, we have a technique that truly models reality, not something that's pie in the sky.

Is there some easier way dealing with this? Well, yes there are some things that can be done, but I won't get into those today. It turns out that a log normal distribution, log normal, not

$$e^{-x^2}, \text{ but } e^{-\log x^2}$$

something that goes down much more slowly than e^{-x^2}, fits these aggregate distributions fairly well. You can develop a generalized formula to calculate all kinds of values. Both techniques are useful, but it's handy to have these aggregate distributions where you can be looking at pages of output and can visually see how the claims behave in excess of any given point, without having to mathematically calculate all the values.

Let's go to examples 8 and 9. Now, what can we do with these aggregate distributions? We'll show a couple of techniques - there are lots of things that can be done with them. I've taken the first distribution and shrunk it down for practical purposes, to about 7 points. Claims of .84, .91, .96, and so forth, each with the probability shown there. The sum of the cross products of the claims levels and probabilities equals 1.0, as it should. Now let's try to solve some problems.

First problem: We have a 1000 employee group. Assume it's a non-dividend group with a retroactive premium arrangement. We will use: if the expected pure claims cost is 1.0, then charge them .9, with the proviso that you'll get 1.0 at the end of the year, if the experience justifies it. Standard retro provisions. This could be worked for a distribution for 100 lives or a situation where it was more applicable. I've seen retros offered in a non-dividend situation like this. Let's see what the effect is. We charge .9 for some of the groups which end up experiencing .84 and we keep the .9 - we get .9 to pay all claims. The next level has claims of .91 and they'll pay up to 1.0, so they'll pay us the .91. The next is .96, they'll pay us the .96. At 1.0, they'll pay us the one. One point zero six is the next claims level, but we can only charge 1.0. We're cut off at 1.0 from there on out, we can't charge any more than the maximum premium. What is the total contribution that we received to pay all the claims? We know the total claims here are 1.0 for all of these groups. Well, just cross multiply the premiums by the probabilities and you'll get .966. So this retro agreement cost us .034 or 3.4%, it isn't cheap. We've seen companies do this, lose money on retro arrangements.

Now, move over a couple columns in the example and we'll talk about dividend cases. Case 1: We have a 5% margin. We charge them $1.05, we expect our claims will be 1.0. So what contributions do we get? We only get $.84 in the first claim level. We charge them $1.05, so we give them a dividend back of $.21. All we get to pay claims is the $.84 that they had. And so it goes until we get up to $1.06 as the actual claim amount. We've only got $1.05, we're down $.01 there, we're down $.06 in the next line and so on. Our weighted contribution is .975 - that's 2.5% less than the dollar that we need. What is the risk charge for no carry-over claims under this experience rated arrangement? It's 2.5%.

What if we carry forward the losses. How much of them will you recover? We think we can recover 75% of the losses. That means that the risk charge has to be about 25% of the 2.5%, or .63%. Now, if this calculation is done with a 10% margin then the risk charge with no carry-over claims drops to 1.4%. If we assume, again, we can get 75% recovery of claims, we need .35%. The point is given aggregate distributions of claims, for groups of different sizes, we can do a lot of risk charge calculation work.

The third aggregate distribution in Appendix #4 is for 35 employees plus spouses and children. I won't spend too much time on it, but you'll find its distribution characteristics are dramatically different than the 1,000 employee distribution.

One of the useful things that comes out of these distributions is what amount of risk charge or profit load should we make for a pure stoploss coverage at a 125% attachment point. We've just been looking at the 1/4% that was the pure risk premium or the 1/2% which is the one we should use as it is the one with the uncertainty distribution. Who likes the idea of a risk charge of 2% of the pure stoploss premium. It's way too little, of course. What should the charge be? What we do in example 9 is look back at the first distribution and ask how much profit do you want from this group of 1,000 lives. Now, suppose you said 2%, that's .02 versus a mean of 1.0. How many standard deviations is that? What is the standard deviation of this distribution? If you look up in the upper left-hand corner of the page you'll see the standard deviation and it is .1140. If you want profit of .02, it is about 18 percent of one standard deviation. If standard deviation is an adequate measure of risk for purposes of setting profit margins, then we can use 18% of the standard deviation at the 125% attachment point. We see that the standard deviation at 1.25 is .02 and 18% of .02 is about .004 and when we add that to the .002, which was our basic premium, we end up with a total of .0057 and what's the total? Well, it's 271% of the pure risk premium. I don't think that surprises anyone. But, we have a way of using the aggregate distributions to get a consistent method for setting profit margins for different kinds of coverages.

Something I have observed in using these distributions over the years in both the individual and the aggregate areas is that companies were badly undercharging for specific excess coverage. Their premiums were 2/3 or 1/2 of what they should have been -- the same thing for aggregate protection. The tendency over the years has been to charge a flat premium for all size cases. Start looking at these distributions and you get different kinds of values.

Now in the paper measuring statistical risk, Appendix #3, I talk about some of the secular problems that you've got to take into account when you start pricing with these distributions. I won't spend much time on it, there's other things that you've got to look at in pricing aggregate stop loss. You've got to look at blowing the expected claims on which you base the overall attachment point and that can result in some serious problems. Or secular changes in frequencies that you don't expect where all of a sudden all utilization across the country goes up 10%, as has happened in the past few years. Those are non-statistical sudden impacts that need to be taken into account in any pricing structure.

We have individual distributions and we have aggregate distributions, both by inherent level, and the statistical distributions around means as tools to work with. Now we're going to discuss another generic area and that's simulation. I'd sure be interested on your comments on this, so please come to the microphone and tell us if you're doing some work in this area. As an example of the ways simulation can be used I'd like to discuss the most recent project we're doing in my office. As a practical application, I want to discuss how this arose. I've spent time in a lot of companies across the country, Blue Cross/Blue Shield organizations, commercial companies, and HMO's, over the last few years and I've been really frustrated with our lack of understanding of the business that we're in and our lack of formulating models to try to manage the business. This work is kind of an outgrowth of that frustration.

Any model is going to have to look at real world situations and there is a great danger that you do a nice job of modeling three out of four variables, as I mentioned before. What are some of those real world situations? Well, a secular economic cycle is one of the very strong realities that we have, surges in utilization followed by depressions of utilization, and we have sudden unexpected shifts, created by things like cost shifts. We've seen these for the last 20 years - they've had a most dramatic and serious impact on the bottom lines of companies. You can't develop a model of the group business without considering the agents or the representatives who write it, or who they write it on, and under what circumstances. The basic question that I was thinking of that first lead to this was for a 100 life group, what credibility should we give it in rating? Should you give no credibility? Is this too small a group to give any credibility to? Straight manual rating, average rating, 25%, 50% - I don't know what the right answer is but we're trying to find out some of the right answers and some of the things that we've found out are a little surprising. What we've developed is a simulation model and we'll go through some of the items that it addresses. There is a rough outline of this model in Appendix #5.

What the model does is take an inforce file and add new business to that inforce file under certain arrangements and it will run for 10 years. We want to simulate, and when you simulate, you just recreate the world and we use Monte Carlo techniques. We take distributions like the ones we've discussed earlier and then using Monte Carlo techniques simulate whether an event occurred or not. We simulate for a 10-year period. We do that for however many replications we want, look at the results, and try to get a distribution of the aggregate results, try to find out optimal credibility to use. This model allows us to put trend rate in year by year. It allows us to establish what a market manual rate is. Sometimes the market in a certain type of coverage might be optimistic, it might be 95%, that is your competitors are 5% below where they should be and we'll put them in at 95% then. Maybe they're above at 105%. The target loss ratio in the marketplace is used. One of the things that the model is going to test is this - you've got a nice book of business and everything is great with one exception: your expenses are 50% more than your competition's and you're trying to recover them. So when they have a 90% loss ratio and you are using an 80% loss ratio, you're bumping up your premium, not because your claims are high but your expenses are high - you have problems. You have the problem of having manual rates that are out of the marketplace, and we've seen a number of companies in that situation. The model starts us out as a company at our manual rate. The model considers the inherent level distribution - we've talked about that topic. The random level distribution is now just the distribution of the aggregate claims. Lapse rates are in here. One of the things that we've observed is that you can increase premium rates by 40% and still be 10% below competition and lose the case. Why? Because they're mad - they didn't expect a 40% rate increase and it doesn't matter that they're still 10% below what the true cost is.

They leave you and go somewhere with a 2% increase above what you were going to charge them because they're mad. The level of rate increase independent of the absolute level of the rate is a real factor out in the marketplace, but so is the rate. The model uses a rather complicated table with the option of inputting arbitrary values depending on what the level of the rate increase is, what the experience loss ratio is, and what the market premium level is. Margins are used for dividend cases. How creditable is the dividend, 100%, 50% and so on. We also consider the way the rates are created - do you use two years of prior experience, one year? Given your experience on this case and your method of giving it credibility what is the projected rate. It will project the rate and then it will simulate what the actual claims for that particular group are in the next year. Distribution of the brokers - the concept here is that you have all kinds of people out there marketing your coverages. We have said that there are three classes of brokers - those that are friendly to you, they have a good working relationship with your home office and field force. Therefore, they'll go out of their way to give you good business and keep bad business away from you. Indifferent brokers - they really don't care. Basically, it's your rate level, if you're low they like you, if you're high they don't like you. Not indifferent brokers - that is, unfriendly. What we mean is that they're probably friendly to someone else, just like your friendly broker is friendly to you but perhaps not to your competitors. So they give you the bad business. Some companies have 100% indifferent brokers. Other companies have very high proportion, or relatively high proportion of friendly brokers. Within these constraints, your friendly broker can, in good conscience, only go so far, after all if your rate is 18% above the marketplace for a 1,000 life group, there is no way he can write the case for you. But, maybe your rate is 4% higher, and because of your service and the long-term relationship, you get the case - even though you're not the lowest in the bid. That's a market tolerance for friendly brokers. Similarly, for the unfriendly broker, if you're low enough, it would be remiss of him and maybe dangerous if you didn't get the case, as much as he'd like to give it to one of his friendly companies. That last item refers to the number of years of data that the broker provides. The broker has two years of data. He gives you one. He knows about three years, he knows about that other year - he knows, but he gives you two. In other words, he knows more than you know, so we simulate based on what he tells you. He has that knowledge and he makes some rational judgments based upon that and now he's sending the case to you and you only have one year's experience to look at.

There are a lot of variables in the model and there probably should be more in it, but we begin to run and when you get something that's as complicated as this, you've got to put in ranges of parameters and see how sensitive the outcome is to any one particular parameter. One of the things that we seem to be finding out is that maybe giving a bit higher credibility than most of us would ever thought was proper isn't such a bad idea.

Now that's kind of a provisional conclusion, which surprises me.

MR. GREGORY W. PARKER: One of the guiding principles when we were developing this model was not that we were trying to determine true mathematical credibility. Most of the models we have seen before try to minimize error or maximize a particular variable. Our approach is to try to maximize profit. What good is it to be accurate, to have a more theoretically correct credibility, if your competitor is writing all of the cases -- because of your credibility. One of the things that we discovered is that even if the marketplace is assigning an incorrect credibility factor, you don't want to deviate too

much from that factor. It will likely cost you business.
MR. MAULE: The model will look at the amount of premium and claims that you get under the different credibility factors. You can evaluate how your overall portfolio performs. What happens if your manual rate is too high over a period of years, what happens to the class of your business? Does it steadily worsen so that you get into a spiral? What is the optimal situation? Now the optimal situation may not be particularly positive. What I find in most of these runs is that risk charges, that you need to charge under the different financing arrangements, are generally higher than what is charged in the marketplace. And frankly, I have almost come to the opinion that traditional financing mechanisms are unlikely to be profitable over the long run because we simply don't have enough margin and enough risk charge in them. And that's probably why companies that 10 years ago that didn't want anything to do with ASO are now encouraging it for their larger groups.

MR. PARKER: Appendix #5 is labeled example 10. This is just one page, one replication, with non-typical input assumptions. This particular run used a market credibility of 100% and our own credibility was 100% also. The first column shows simply the premium that the portfolio developed.

The model has expected claims of $1.00 a year per case in year 1. The model then produces a premium based upon the experience that was provided to you by your broker and then as the groups generate experience through the years, the cases that remain in force have the premiums produced via the credibility formula that you have input using a combination of manual rate and experience rate. About midway across the page you see a column headed "Pol", that's the number of policies that enter your portfolio each year. This replication had 111 policies the first year, at the end of the second year 209 policies, which meant that you wrote some policies and you lapsed some off. If you want to see whether you are losing your good business or your not so good business the second section of the output shows the experience of the lapsed policies. As we go across the output page, we see the premiums that have been developed each of the 10 years in the simulation and the claims that were actually incurred. The third column shows the adjusted claims which are nothing more than credibility weighted claims. The next item is the loss ratio. This run was targeted at an 80% loss ratio so you can see the resulting 79.1% loss ratio means that this particular replication was slightly favorable. The next column shows how your manual rates have changed over the 10 years relative to a norm of 1.0. The fact that after 10 years the manual rate is down to .77 says that you are writing better than average business, which when you are giving a 100% credibility isn't real surprising. The case with an inherent level of 130% will probably not like your rate, they want a carrier that's going to assign a very small amount of credibility to his high level of experience.

The next four columns are the per unit costs, nothing more than the premiums, claims, and adjusted claims divided by the number of policies.

And the last column shows the experience refund under the particular dividend formula that is input.

MR. MAULE: This gives you an idea of what we are trying to do and I consider it an extremely practical application of risk analysis techniques and we are very hopeful that we will learn something from this that we didn't know before.

MR. PARKER: Just one final comment. For those of you who attended the session this morning on the new life contingencies text, you will recall that one of the four interesting questions that the Committee members mentioned at the end of the presentation was: how much premium do we need to charge to make sure that our claims are going to be covered x% of the time, I believe 95% was the example that they used. If you take a look at the aggregate distribution that Bob handed out, you'll find that that particular type of distribution and the way it's presented is exactly the kind of information needed to answer that question. The aggregate distribution can be produced not only in the life insurance setting, in which the new life contingencies text operates, but also in the areas of medical, dental, and long term disability. This is just one additional application that was highlighted this morning. These techniques will provide a great deal of insight and solution.

Editor's Note: The tables on pages 1884 and 1889–96 are the best reproduction possible from the only copy available when this issue of the *Record* went to press. If specific numbers needed by the reader are illegible, they can best be obtained directly from the moderator or the recorder.

Appendix #1

Example #0

Multinomial Distribution

Amount	Probability
0	.50
50	.10
100	.40

Expected = (.50) (0) + (.10) (50) + (.40) (100)

= 0 + 5 + 40

= 45

Analysis of Data

$$C(D) = \int_D^\infty (t - D)\, f(t)$$

$$C'(D) = -\int_D^\infty f(t)$$

$$= -(1 - F(D))$$

$$\frac{C(D^*) - C(D)}{D^* - D} \simeq -(1 - F(D))$$

Example $\frac{C(15000) - C(10000)}{15000 - 10000} = \frac{18.19 - 25.67}{5000}$

$= -.0015$

(true .0013 → .0018)

Appendix #1

Example #1

Price Comprehensive Plan

100 Deductible 80/20 Coinsurance

1000 = out-of-pocket

C(100) = 13.23

1000 = 100 + (.20)(x - 100)
x = 4600

C(4600) = .4 (3.76) + .6 (3.30)

= 3.48

Price = .8 (13.23 - 3.48) + 3.48

= 11.28

(Notice a straight 80/20 plan costs 10.58. This plan has an effective coinsurance of

$$\frac{11.28}{10.58} \times .80 = 85\%$$)

Appendix #1

Example #2

Price Minor Benefit

$500 Supplemental Accident

Assume Accident Claims are approximately 10% of all claims for all claims levels.

$$C(0) - C(500) = 18.85 - 8.21$$

$$= 10.64$$

$$C(600) = 7.89$$

Regular plan is $100 Deductible, 80/20 Comprehensive

$$\text{Price} = .80\ (13.23) = 10.58$$

Modified Plan

$$\text{Price} = .9\ (10.58) + .10\ [10.64 + .8\ (7.89)]$$

$$= 11.22$$

Extra Cost is .64 (+ 6%)

Appendix #1

Example #3 and Example #4

Example #3: Pooling Charge

Amount	Charge	% All Charges*
15,000	1.52	8%
25,000	.91	5%
50,000	.39	2%

*not benefits

Example #4: Trend Projections

Assume 15% base trend

$100 Deductible

207.86 (1.15) - 100 (.4907) = 189.97

120% of 158.79

$10000 Deductible

48.66 (1.15) - .00231765 (10000) = 32.7

128% of 25.67

Appendix #1

Example #5

Evaluation of Deductible Limitation

Limit is for 1 deductible amount.
Example uses 2 children.

Distribution of Deductible

(f)	(A)
.25	0
.02	34
.24	74
.49	100

Expected = 67.44

Convolute

		Modified Amount
.0625	0	0
.0100	34	34
.0004	68	68
.1200	74	74
.2450	100	100
.0096	108	100
.0196	134	100
.0576	148	100
.2352	174	100
.2401	200	100
1.0000	Expected is 134.88	Expected is 89.96

$$\frac{89.96}{134.88} \doteq 67\%$$

Effect on comp plan $\frac{[226.20 - .67\ (67.44)]\ .80}{(226.20 - 67.44)\ .80} = 114\%$

Appendix #1

Example # 6 and Example #7

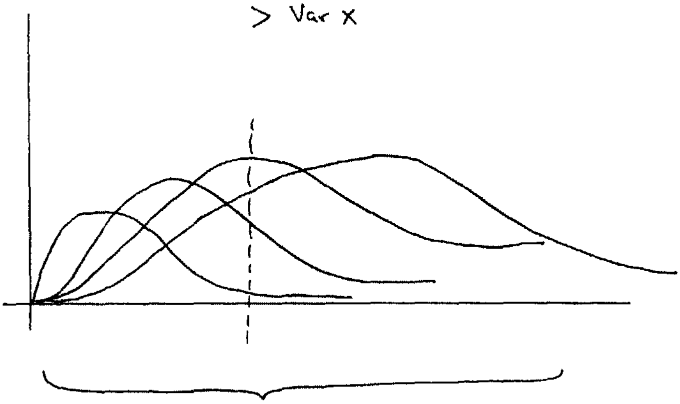
Example #6: Bayesian Problem

Inherent Level	Proportion
.7	.05
.8	.10
.9	.15
1.0	.40
1.1	.15
1.2	.10
1.3	.05

Weighted = 1.0000

Note: $\sum_i \alpha_i (\text{Var } \lambda_i x) = \sum_i \alpha_i \lambda_i^2 \text{ Var } X$

$> \text{Var } X$



spread out claims

Example #7

1000 employees, 125% Attachment

.0020869 (.21%)

Appendix #1

Example #8 and Example #9

Example #8: 1000 Employees

Claims Level	Probability	Retro Non-Dividend	Dividend 5% Mar.	Dividend 10% Mar.
.84	.14	.90	.84	.84
.91	.14	.91	.91	.91
.96	.18	.96	.96	.96
1.00	.18	1.00	1.00	1.00
1.06	.15	1.00	1.05	1.05
1.11	.10	1.00	1.05	1.10
1.21	.11	1.00	1.05	1.10
Weighted = 1.0		Wt. = .966	Wt. = .975	Wt. = .986
		3.4%	2.5%	1.4%

1/4 non-recovery

$$\frac{2.5}{4} = .63\%$$

Example #9:

1000 employees. Want 2% claims under full coverage.

.02 = 154% of Variance (.013)

= 18% of std. dev. (.114)

At 125% Attachment point

.0021 + 1.54 (.0004) = .0027 129%

.0021 + .18 (.02) = .0057 271%

Appendix #2

Health Cost Guidelines
January 1, 1982

CLAIMS PROBABILITY DISTRIBUTION

TIME: 11:24:35
DATE: 3/29/82

EXPERIENCE LEVEL: 100.% M AND R
TYPE OF COVERAGE: MEDICAL - ALL COVERAGES(NON-MAT)
INSURED TYPE: CHILD
CLAIM CENTER DATE: 01/01/82
AREA FACTOR: 1.0000

(1) CALENDAR YEAR DEDUCT. D	(2) ANNUAL CLAIMS COST SX(D)= (9)-D×(8)	(3) %/ OF $0 SX SX(D)÷SX(0)	(4) MONTHLY CLAIMS COST SX(D)÷12	(5) ANNUAL FREQUENCY F(T)	(6) TOTAL ANNUAL CLAIM A(T)	(7) ANNUAL CLAIMS COST F(T) × A(T)	(8) PROBABILITY THAT CLAIMS ≥ A(T)	(9) ANNUAL COST OF CLAIMS ≥ A(T)
0	226.20	100.0000	18.85	0.25000000	0.00	0.0000	1.00000000	226.2000
50	189.02	83.5626	15.75	0.02000000	34.07	0.6813	0.75000000	226.2000
100	158.79	70.1968	13.23	0.23927272	73.81	17.6608	0.73000000	225.5187
150	137.80	60.9206	11.48	0.25872728	136.27	35.2555	0.49072728	207.8579
200	126.90	56.1001	10.57	0.03800000	181.69	6.9041	0.23200000	172.6024
500	98.54	43.5619	8.21	0.02622222	215.75	5.6575	0.19400000	165.6983
1000	79.48	35.1350	6.62	0.07444445	266.85	19.8657	0.16777778	160.0408
1500	69.68	30.8048	5.81	0.02466666	334.98	8.2630	0.09333333	140.1751
2000	61.97	27.3942	5.16	0.01200000	420.15	5.0418	0.06866667	131.9121
2500	56.16	24.8276	4.68	0.00605797	522.35	3.1644	0.05666667	126.8703
3000	51.93	22.9588	4.33	0.01750525	652.94	11.4298	0.05060870	123.7060
4000	45.07	19.9250	3.76	0.00165901	817.59	1.3564	0.03310345	112.2761
5000	39.55	17.4840	3.30	0.01127053	1021.99	11.5183	0.03144444	110.9198
7500	31.42	13.8892	2.62	0.00248970	1283.16	3.1947	0.02017391	99.4014
10000	25.67	11.3490	2.14	0.00286731	1606.79	4.6072	0.01768421	96.2067
15000	18.19	8.0413	1.52	0.00325876	2009.91	6.5498	0.01481690	91.5995
20000	13.81	6.1065	1.15	0.00310359	2498.19	7.7534	0.01155814	85.0497
25000	10.87	4.8062	0.91	0.00171169	3122.74	5.3452	0.00845455	77.2964
30000	8.78	3.8808	0.73	0.00109842	3917.62	4.3032	0.00674286	71.9512
35000	7.35	3.2512	0.61	0.00203492	4939.61	10.0517	0.00564444	67.6480
40000	6.24	2.7591	0.52	0.00065238	6131.93	4.0004	0.00360952	57.5963
45000	5.34	2.3601	0.44	0.00063949	7721.69	4.9379	0.00295714	53.5960
50000	4.68	2.0711	0.39	0.00054622	9652.10	5.2722	0.00231765	48.6580
60000	3.65	1.6135	0.30	0.00046402	12036.74	5.5853	0.00177143	43.3858
70000	2.97	1.3109	0.25	0.00037408	15102.69	5.6496	0.00130741	37.8005
80000	2.48	1.0943	0.21	0.00028571	18850.04	5.3856	0.00093333	32.1509
90000	2.06	0.9091	0.17	0.00021493	23619.26	5.0765	0.00064762	26.7653
100000	1.81	0.7997	0.15	0.00014784	29524.19	4.3649	0.00043269	21.4888
250000	0.49	0.2186	0.04	0.00010436	37018.57	3.8633	0.00028485	17.3239
				0.00006774	46329.82	3.1384	0.00018049	13.4607
				0.00004429	57911.37	2.5649	0.00011275	10.3223
				0.00002658	72675.55	1.9317	0.00006846	7.7574
				0.00001868	90838.51	1.6969	0.00004188	5.8257
				0.00001040	119231.93	1.2400	0.00002320	4.1288
				0.00000569	147620.48	0.8400	0.00001280	2.8888
				0.00000435	198732.93	0.8645	0.00000711	2.0489
				0.00000276	429119.74	1.1844	0.00000276	1.1844

Appendix #3

MEASURING STATISTICAL RISK WITH A RISK ANALYZER SYSTEM

An important actuarial function is assessment of financial risk in a given insurance situation. Financial risk arises because of uncertainty. Uncertainty itself derives from a number of inherently different sources. Among these are:

1. An actuary's best estimate of the probability of a future event is not likely to be exactly correct. Thus, even though true inherent probabilities reasonably may be assumed to exist in a situation, it is not likely that the actuary will exactly determine these probabilities.

2. An actuary can err in judgment and make arithmetical errors in assessing probabilities. From time to time, he will make mistakes.

3. Certain events, which impact financial outcome, probably cannot be assigned probabilities. Such events, sometime termed "acts of God" or catastrophes, are generally assumed to be unpredictable.

4. Often secular influences, which can and do impact financial results, are also largely unpredictable. Such influences, such as economic depression or recession, generally fall outside the actuary's capability of making credible predictions.

5. Finally, even if the actuary could be certain that he had considered all factors and that he had made entirely correct assessments of the probabilities, there is the risk of pure statistical fluctuation. As a simple example, there is a measurable probability that a "perfect coin" will show a run of 100 heads in 100 tosses.

We do not claim ability to provide special insight in dealing with the first four items on this list, but we have developed tools which do provide considerable insight into the fifth item.

The need to know the probability distribution of possible aggregate outcomes arises quite frequently in actuarial work. To give substance to this statement, the following is an abbreviated list of situations in which the assessment of the degree of statistical fluctuation is important.

1. What is the true underlying net cost, and what contingency and profit margins are appropriate in setting a premium for aggregate stop loss coverage for medical care coverage (or for dental, long term disability, group life & ADD, etc.)? This example might involve 10,000 insured lives, reinsurance of claims above $25,000 and an attachment point of 125% of expected aggregate retained claims. Given claims cost distributions for an individual, the

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fundamental actuarial problem is to determine the possible aggregate outcomes and the related probabilities.

2. For an overall portfolio of insurance (group or individual life, long term disability, etc.), what is an appropriate retention limit so that unacceptable variations in total retained cost would occur with suitably low probabilities and net reinsurance costs would be minimized?

3. What are appropriate amounts of earmarked surplus so that, for given confidence levels, it could be expected that surplus would absorb adverse results in varous lines of business?

4. Considering only risk related to statistical variation, what mechanisms can be employed by the actuary in suggesting consistent profit and contingency margins for various types of products?

Although the circumstances of these examples initially appear to be quite different, each case calls for determination of all possible aggregate outcomes and their related probabilities.

In the past, mathematical techniques, often approximate and generally limited to specific point estimates, have been employed to answer some of these questions. Often a mathematical distribution, such as the normal distribution, is assumed to be appropriate, even though it is known that such a distribution is likely to produce inaccurate results. Not until the capability of computers was greatly increased, could the problem, which is simple in concept, be dealt with on a cost efficient computational basis.

Mathematically, the problem is to determine the overall distribution from the convolution of individual multinomial frequency distributions. The problem can be simplified as follows: assume that there is a die with m faces, and that when this die is rolled each face can be expected to surface with a given probability. A number will be attached to each face (the financial outcome) and this number will be recorded if that face of the die surfaces. Suppose a number (say N) of such identical dice are rolled together, and the total of the numbers appearing on the dice is determined. If this process is repeated an infinite number of times, what is the distribution of the sum of the faces of the dice? From such a distribution, many useful statistics can be determined, such as stop-loss values, measures of variation of outcomes, and so forth. Such statistics provide valuable insight and assistance to the actuary in addressing the kinds of problems mentioned above.

Over the last few years we have developed and refined what we have come to term the "risk analyzer." This computer program essentially determines all the various combinations of results when an m face die is rolled n times, or equivalently, when n m-faced dice are rolled together. The output of this program lists the possible outcomes (e.g., aggregate claims) in ascending order, together with the respective probabilities. Also, "stop-loss" theoretical premiums at all aggregate claims values are calculated. With an accurate picture of the distribution of aggregate results, an

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actuary has recourse to a powerful tool,. together with his judgment, in analyzing questions which involve statistical variation of overall results.

An example of both the input and output of the risk analyzer system is included with this presentation. In this example, the question arises as to what the distribution of aggregate claims would be for 100 adults insured under a comprehensive major medical plan. The input, a probability distribution of claims for an individual adult, is shown in Exhibit 1. From this exhibit it can be seen that there is 43% probability of no claim, a 25% probability of a $36 claim, and so forth. The sum of the cross products of the frequencies and the amounts of a claim is $251.19, the expected annual claims cost for an adult under this program.

Mathematically, the answer to the question is obtained by considering all possible combinations of multiplications of the various frequencies for the 100 individuals. Each such multiplication would be assigned to the related aggregate claim amount. After this mathematical task was performed, overall results would be listed in order of numerical value. The risk analyzer system does just this. Exhibit 2 shows a portion of the output.

This output is a concrete illustration of the information that is developed by the system. For example, the pure claim cost for stopping the loss at 125% of expected annual claims (1.25 x $25,119 = $31,398) is about $1,529 (this cost actually relates to a stop loss level of $31,423, slightly in excess of the $31,398).

In our experience, use of the Risk Analyzer system has proved to be quite cost efficient. Even when a variety of different dice are convoluted together a larger number of times (say 10,000), the system cost is quite reasonable.

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Exhibit 1

Individual Claims Distribution (Die)
Comprehensive Medical Coverage*

(1) Face Number of the die	(2) Probability that the face will surface on a toss	(3) Financial Outcome (amount of claims paid)
1	.42819	$ 0
2	.26080	36
3	.11083	116
4	.08277	227
5	.02871	437
6	.04380	1,012
7	.01843	1,813
8	.02012	3,191
9	.00461	6,810
10	.00129	11,322
11	.00041	17,841
12	.00003	47,010
13	.00001	94,320

Sum of probabilities = 1.00000

Expected Annual Cost [(.42819 x 0) + (.26080 x 36) + etc.] is $251.19

* Although adequate for presentation of concepts, this particular claims distribution is out-of-date.

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Exhibit 2

Sample Risk Analyzer Output

LOC	AMOUNT (1)	PROBABILITY (2)	CUMULATIVE (3)	STOP-LOSS PREMIUM (4)	VARIANCE OF STOP-LOSS (5)	STANDARD DEVIATION (6)
1)	7542.8315308	.0000000010	.0000000010	17576.1724199	88020276.5019500	9381.9121984
2)	9538.5771429	.0064598544	.0064598553	15580.4264091	88020276.4609300	9381.9121961
3)	11123.8417680	.0011316689	.0075915242	14005.6014715	87717336.6113200	9365.7533923
4)	11830.4556033	.0196234890	.0272150132	13502.6350461	87611498.9492100	9360.1014369
5)	13441.3068258	.0465382124	.0737532256	11741.8662480	86367430.3496000	9293.4078972
6)	15543.9964259	.0794569208	.1532101464	9793.4567806	83027853.5117100	9111.9621100
7)	17263.3322283	.0641502293	.2173603757	8337.5407017	78251791.1611300	8845.8042480
8)	18836.8015727	.0818559421	.2992163178	7106.0812447	72949919.7056200	8542.2432511
9)	20774.9995941	.1113046187	.4105209366	5748.0339328	65516451.9484400	8094.2233846
10)	22403.5044015	.0822457778	.4927667144	4669.9916691	57494986.4770500	7595.7215400
11)	24302.9552957	.0854761421	.5782428565	3807.8737090	50595260.7529200	7113.0345871
12)	26176.1591207	.0797257629	.6579686194	3017.9366139	43201861.7941880	6572.7971056
13)	27990.4650640	.0603848216	.7183534410	2417.8405231	36926547.1113270	6076.7217404
14)	29854.1340624	.0538571865	.7722106275	1932.4314094	31540649.9390960	5616.1062970
15)	31423.0705224	.0484570781	.8206677055	1529.4864825	26811699.4133280	5178.0014843
16)	33190.3076088	.0371248335	.8577925391	1212.5638002	22834864.3917220	4778.5039316
17)	34898.2583268	.0288245176	.8866170567	969.6804646	19637729.4997550	4431.4477881
18)	36654.8906579	.0259724253	.9125894820	778.5083199	16927454.7521960	4114.2988165
19)	38467.0718594	.0203724278	.9329619098	612.8347633	14648027.7576280	3826.2209212
20)	40282.8524244	.0158624652	.9488243750	498.4154751	12774061.7413930	3574.0819437
21)	42188.1749481	.0122792549	.9611036298	396.9625365	11230529.2602530	3352.0932054
22)	43934.1343683	.0091461433	.9702497732	325.9391485	9967879.4443960	3157.1948695
23)	45397.9362568	.0036677624	.9739175356	282.3908592	9103899.6160480	3017.2689116
24)	46937.4929560	.0059197068	.9798372424	250.0601652	8461116.6930060	2908.7988746
25)	48475.3708633	.0043768789	.9842141213	214.0114580	7648133.8423880	2765.5250186
26)	50230.8238736	.0033791280	.9875932493	185.5235472	6938496.0346980	2634.1025103
27)	51991.0856488	.0022402584	.9898335077	163.6744915	6331166.3473810	2516.1804051
28)	53749.1592023	.0017558336	.9915893413	145.8010500	5792616.9899980	2406.7856136
29)	55559.8201499	.0012814250	.9928707663	130.5709266	5296614.7540580	2301.4375407
30)	57342.3239124	.0009535033	.9938242696	117.8656933	4856721.0631480	2203.7969650
31)	59078.9291828	.0006899399	.9945143095	107.1412340	4464385.9248040	2113.8557010
32)	60760.4432396	.0006253781	.9951396876	97.9166934	4125959.4494472	2031.1227066
33)	62537.8980717	.0005432205	.9956829141	89.2779597	3794356.2676846	1947.9107442
34)	64145.7494886	.0004058712	.9960887853	82.3237066	3519124.3953550	1875.9329400
35)	65997.1542759	.0005307462	.9966195695	75.1138134	3270038.3190697	1797.2307364
36)	67871.0377467	.0003858550	.9970054244	68.7623799	2980625.6096801	1720.6468579

(1) Amount - the list of all possible financial outcomes (aggregate claims for the 100 lives) in ascending order.

(2) Probability - the probability that the indicated amount will be experienced.

(3) Cumulative - the probability that total claims will be less than or equal to the indicated amount.

(4) Stop-Loss Premium - the theoretical cost (no margin for expense, profit or contingencies) of paying the portion of claims in excess of the value in the amount column.

(5,6) The variance and standard deviation of the stop loss premium. These statistics provide measures of how widely actual stop loss costs can vary from the theoretical mean of such costs (the stop loss premium).

The system also tabulates the mean of the distribution (100 x \$251.19 = \$25,119), the variance (\$88,020,277), the standard deviation (\$9,382) and other more technical statistical values.

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Aggregate Distributions

INPUT-FILE NAME=LOC21M   OUTPUT-FILE NAME=J00189
FIRST MOMENT = 1.0000002
SECOND MOMENT = 1.0130055
CUMULATIVE PR = 1.0000000000
VARIANCE = 0.0130050
STD.DEV. = 0.1140394574
MOM.COEF.SKEW. = 0.7256930457
MOM.COEF.KURT. = 4.7800335005

1000 EMPS.-PLAN 2-UNLIMITED-UNIT.-NO UNC.

DISTRIBUTION LENGTH= 205

LOC	AMOUNT	PROBABILITY	CUMULATIVE	STOP-LOSS PREMIUM	VARIANCE OF STOP-LOSS	STANDARD DEVIATION	RATIO
1)	0.4922659	.0000000000	0.0000000000	0.5077343	0.1300500E-01	0.1140395	0.25614E-01
2)	0.4990226	.0000000000	0.0000000000	0.5009777	0.1300500E-01	0.1140395	0.25959E-01
3)	0.5056620	.0000000000	0.0000000000	0.4943182	0.1300500E-01	0.1140395	0.26309E-01
4)	0.5132341	.0000000000	0.0000000000	0.4867611	0.1300500E-01	0.1140395	0.26717E-01
5)	0.5199540	.0000000001	0.0000000001	0.4800463	0.1300500E-01	0.1140395	0.27091E-01
6)	0.5281880	.0000000003	0.0000000004	0.4718158	0.1300500E-01	0.1140395	0.27564E-01
7)	0.5352054	.0000000004	0.0000000007	0.4647949	0.1300500E-01	0.1140395	0.27980E-01
8)	0.5410682	.0000000007	0.0000000014	0.4589321	0.1300500E-01	0.1140395	0.28338E-01
9)	0.5495629	.0000000032	0.0000000047	0.4504174	0.1300500E-01	0.1140395	0.28873E-01
10)	0.5569629	.0000000041	0.0000000088	0.4430373	0.1300500E-01	0.1140395	0.29354E-01
11)	0.5650493	.0000000147	0.0000000235	0.4349509	0.1300500E-01	0.1140395	0.29900E-01
12)	0.5726084	.0000000238	0.0000000473	0.4273918	0.1300500E-01	0.1140395	0.30424E-01
13)	0.5788715	.0000000347	0.0000000820	0.4211287	0.1300500E-01	0.1140395	0.30861E-01
14)	0.5873317	.0000001341	0.0000002161	0.4126685	0.1300500E-01	0.1140395	0.31514E-01
15)	0.5949389	.0000001545	0.0000003706	0.4050614	0.1300500E-01	0.1140394	0.32106E-01
16)	0.6035142	.0000005106	0.0000008812	0.3964860	0.1300499E-01	0.1140394	0.32801E-01
17)	0.6112918	.0000006517	0.0000015329	0.3887089	0.1300499E-01	0.1140394	0.33457E-01
18)	0.6177348	.0000009601	0.0000024931	0.3822608	0.1300498E-01	0.1140394	0.34021E-01
19)	0.6261758	.0000028438	0.0000053369	0.3738249	0.1300496E-01	0.1140393	0.34789E-01
20)	0.6342793	.0000037111	0.0000090479	0.3657210	0.1300493E-01	0.1140392	0.35560E-01
21)	0.6424066	.0000080209	0.0000170688	0.3579430	0.1300486E-01	0.1140389	0.36414E-01
22)	0.6509108	.0000123391	0.0000294080	0.3490821	0.1300478E-01	0.1140385	0.37254E-01
23)	0.6581793	.0000147330	0.0000441410	0.3418219	0.1300463E-01	0.1140379	0.38045E-01
24)	0.6671053	.0000419523	0.0000860933	0.3328950	0.1300437E-01	0.1140367	0.39064E-01
25)	0.6754079	.0000415954	0.0001276887	0.3245434	0.1300390E-01	0.1140346	0.40062E-01
26)	0.6838501	.0000863758	0.0002140645	0.3161529	0.1300321E-01	0.1140316	0.41129E-01
27)	0.6922413	.0001279783	0.0003420428	0.3077635	0.1300208E-01	0.1140267	0.42247E-01
28)	0.6998111	.0001323493	0.0004743921	0.3001962	0.1300051E-01	0.1140198	0.43307E-01
29)	0.7087511	.0003339992	0.0008083912	0.2912609	0.1299800E-01	0.1140088	0.44627E-01
30)	0.7172809	.0003207922	0.0011291835	0.2827376	0.1299404E-01	0.1139914	0.45950E-01
31)	0.7256025	.0005416797	0.0016708631	0.2745457	0.1298816E-01	0.1139602	0.47344E-01
32)	0.7343890	.0008450101	0.0025158933	0.2656535	0.1298090E-01	0.1139358	0.48864E-01
33)	0.7426507	.0008098811	0.0033257743	0.2574126	0.1297003E-01	0.1138860	0.50306E-01
34)	0.7516738	.0017037676	0.0050295419	0.2484144	0.1295465E-01	0.1138194	0.52149E-01
35)	0.7605564	.0016801692	0.0067097111	0.2395815	0.1293305E-01	0.1137236	0.53962E-01
36)	0.7688914	.0022536033	0.0089633144	0.2312032	0.1290641E-01	0.1136064	0.55023E-01
37)	0.7778465	.0036181570	0.0125794714	0.2224274	0.1287040E-01	0.1134478	0.57863E-01
38)	0.7861322	.0053461116	0.0154261850	0.2136555	0.1282166E-01	0.1132328	0.60011E-01
39)	0.7950550	.0058080825	0.0217342055	0.2086771	0.1276080E-01	0.1129641	0.62346E-01
40)	0.8049907	.0060232364	0.0277575025	0.1957368	0.1268137E-01	0.1126116	0.64707E-01
41)	0.8135193	.0063980136	0.0341555160	0.1874470	0.1259806E-01	0.1122081	0.67169E-01
42)	0.8227950	.0113632850	0.0455188011	0.1780061	0.1247472E-01	0.1116903	0.69091E-01
43)	0.8321590	.0089028998	0.0544217005	0.1695498	0.1232637E-01	0.1110242	0.72701E-01
44)	0.8418572	.0162241445	0.0706458450	0.1603799	0.1215224E-01	0.1102512	0.75172E-01
45)	0.8513245	.0120840607	0.0834505116	0.1515770	0.1194384E-01	0.1092863	0.78795E-01
46)	0.8590741	.0133871862	0.0968376780	0.1444739	0.1175203E-01	0.1084088	0.81348E-01
47)	0.8680880	.0248253846	0.1216630475	0.1356144	0.1148598E-01	0.1071727	0.84646E-01
48)	0.8768680	.0178346262	0.1394976737	0.1270310	0.1117364E-01	0.1057057	0.87960E-01
49)	0.8882770	.0217632978	0.1671274715	0.1187558	0.1088347E-01	0.1041364	0.91313E-01

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51)	0.9070255	.0258747893	0.2177585906	0.1033803	0.1009971E-01		
52)	0.9187500	.0348051591	0.2526237577	0.9575070E-01	0.9677787E-02	0.0983757	0.10101
53)	0.9285215	.0204011056	0.2611049254	0.8845341E-01	0.9225073E-02	0.0960566	0.10427
54)	0.9383794	.0364620342	0.3180677576	0.8137125E-01	0.8752744E-02	0.0933561	0.10757
55)	0.9401633	.0344535804	0.3530213300	0.7469432E-01	0.8267083E-02	0.0909234	0.11067
56)	0.9552772	.0293784570	0.3823997950	0.6880283E-01	0.7805360E-02	0.0883479	0.11345
57)	0.9652484	.0442640009	0.4266640840	0.6264454E-01	0.7304151E-02	0.0854643	0.11660
58)	0.9754835	.0335534618	0.4602181455	0.5677645E-01	0.6782843E-02	0.0823566	0.11946
59)	0.9853005	.0368840374	0.4970921055	0.5187782E-01	0.6293554E-02	0.0793528	0.12226
60)	0.9952237	.0400607043	0.5371428928	0.4688084E-01	0.5810354E-02	0.0762255	0.12499
61)	1.0051020	.0301358602	0.5672707524	0.4141422E-01	0.5341234E-02	0.0730837	0.12743
62)	1.0152813	.0412430745	0.6005724322	0.3750976E-01	0.4882638E-02	0.0698754	0.13017
63)	1.0256604	.0317051007	0.6403575330	0.3349344E-01	0.4434118E-02	0.0663092	0.13250
64)	1.0361556	.0344128439	0.6747703789	0.2967235E-01	0.4010257E-02	0.0633266	0.13515
65)	1.0463543	.0315973426	0.7085677145	0.2655543E-01	0.3624808E-02	0.0602054	0.13753
66)	1.0562366	.0258022408	0.7321849642	0.2345367E-01	0.3276442E-02	0.0572450	0.13972
67)	1.0669080	.0322009673	0.7644509316	0.2059553E-01	0.2932821E-02	0.0541555	0.14240
68)	1.0776290	.0237225450	0.7881734766	0.1807072E-01	0.2615430E-02	0.0511462	0.14476
69)	1.0877674	.0220239022	0.8109940480	0.1592263E-01	0.2340244E-02	0.0484160	0.14723
70)	1.0983349	.0244728008	0.8354691596	0.1342530E-01	0.2080496E-02	0.0457001	0.14990
71)	1.1089241	.0108275415	0.8522907511	0.1210304E-01	0.1857511E-02	0.0430969	0.15247
72)	1.1195098	.0206978042	0.8729942353	0.1060033E-01	0.1650812E-02	0.0406255	0.15558
73)	1.1304377	.0152037493	0.8881979046	0.9230073E-02	0.1462470E-02	0.0382423	0.15845
74)	1.1401640	.0121469419	0.9003444764	0.8142648E-02	0.1312384E-02	0.0362269	0.16117
75)	1.1508690	.0153502207	0.9157032051	0.7075843E-02	0.1185710E-02	0.0341425	0.16475
76)	1.1623531	.0100634414	0.9286666466	0.6107772E-02	0.1027071E-02	0.0320479	0.16816
77)	1.1736542	.0110000555	0.9376867021	0.5278657E-02	0.9077755E-03	0.0301293	0.17197
78)	1.1844920	.0081668443	0.9458535464	0.4603373E-02	0.8073424E-03	0.0284147	0.17539
79)	1.1947553	.0086710692	0.9527252156	0.4047745E-02	0.7234321E-03	0.0268967	0.17872
80)	1.2059702	.0074427810	0.9605679965	0.3517233E-02	0.6425440E-03	0.0253405	0.18260
81)	1.2174942	.0051485537	0.9657165503	0.3002050E-02	0.5647115E-03	0.0238086	0.18691
82)	1.2283888	.0049930250	0.9707045752	0.2604540E-02	0.5092201E-03	0.0225659	0.18953
83)	1.2396531	.0049305103	0.9751460056	0.2359542E-02	0.4540091E-03	0.0213075	0.19241
84)	1.2506229	.0030501531	0.9782022306	0.2008950E-02	0.4064438E-03	0.0201605	0.19475
85)	1.2616237	.0034297080	0.9810321147	0.1842746E-02	0.3653867E-03	0.0190627	0.19719
86)	1.2732966	.0023405772	0.9839728919	0.1632027E-02	0.3242454E-03	0.0180068	0.19868
87)	1.2850154	.0023472090	0.9863199049	0.1444234E-02	0.2887740E-03	0.0168935	0.19995
88)	1.2965202	.0017340005	0.9880597874	0.1208743E-02	0.2577606E-03	0.0160552	0.20033
89)	1.3076200	.0013094520	0.9894492402	0.1154305E-02	0.2310183E-03	0.0151992	0.20013
90)	1.3194128	.0014363302	0.9900855784	0.1024882E-02	0.2055304E-03	0.0143363	0.19957
91)	1.3312962	.0009617188	0.9918672972	0.9215716E-03	0.1825510E-03	0.0135112	0.19809
92)	1.3427077	.0009049519	0.9927574441	0.8287651E-03	0.1627402E-03	0.0127570	0.19636
93)	1.3544171	.0008140554	0.9935726045	0.7454053E-03	0.1444579E-03	0.0120191	0.19417
94)	1.3661012	.0005737831	0.9941463076	0.6688670E-03	0.1280564E-03	0.0113162	0.19145
95)	1.3775741	.0006060403	0.9947530274	0.6016747E-03	0.1135585E-03	0.0106564	0.18874
96)	1.3897400	.0004989961	0.9952520234	0.5378246E-03	0.9976465E-04	0.0099862	0.18549
97)	1.4013274	.0003980203	0.9956548503	0.4828511E-03	0.8860194E-04	0.0093804	0.18225
98)	1.4129938	.0004207903	0.9960871646	0.4321120E-03	0.7737400E-04	0.0087462	0.17906
99)	1.4253154	.0003547870	0.9964204136	0.3837103E-03	0.6736201E-04	0.0082075	0.17555
100)	1.4375113	.0003369517	0.9967635653	0.3401261E-03	0.5858502E-04	0.0076520	0.17219
101)	1.4493305	.0002072805	0.9970506250	0.3018456E-03	0.5094603E-04	0.0071412	0.16845
102)	1.4613966	.0002916417	0.9973417175	0.2662818E-03	0.4416651E-04	0.0066458	0.16586
103)	1.4737731	.0002876678	0.9978043653	0.2353818E-03	0.3744867E-04	0.0061645	0.16202
104)	1.4860627	.0002473593	0.9980567406	0.2040019E-03	0.3263645E-04	0.0057128	0.15998
105)	1.4984521	.0002241144	0.9980305590	0.1774471E-03	0.2742045E-04	0.0052840	0.15735
106)	1.5107350	.0002154587	0.9983010137	0.1553437E-03	0.2385807E-04	0.0048845	0.15449
107)	1.5231765	.0001964901	0.9984462218	0.1328057E-03	0.2024805E-04	0.0045051	0.15203
108)	1.5355322	.0001814767	0.9986749685	0.1142531E-03	0.1724858E-04	0.0041551	0.15047
109)	1.5479499	.0001662470	0.9988459455	0.9705744E-04	0.1461808E-04	0.0038258	0.14936
110)	1.5603108	.0001472062	0.9989951517	0.8359324E-04	0.1237765E-04	0.0035162	0.14807
111)	1.5729437	.0001427708	0.9991359225	0.7087345E-04	0.1042638E-04	0.0032243	0.14714
112)	1.5852929	.0001113410	0.9992473135	0.6020276E-04	0.8811043E-05	0.0029603	0.14656
113)	1.5977844	.0001157844	0.9993631034	0.5080057E-04	0.7425537E-05	0.0027250	0.14617
114)	1.6110389	.0000978432	0.9994609986	0.4235886E-04	0.6141540E-05	0.0024883	0.14617
115)	1.6237917	.0000819216	0.9995429181	0.3548514E-04	0.5144367E-05	0.0022802	0.14652
116)	1.6367507	.0000751077	0.9996180258	0.2956174E-04	0.4356740E-05	0.0020873	0.14738
117)	1.6497236	.0000617548	0.9996797856	0.2460567E-04	0.3654251E-05	0.0019116	0.14851

Appendix #4, Page 3

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INPUT-FILE NAME=LUC2IHP     OUTPUT-FILE NAME=JUU135
FIRST   MOMENT =          1.0000002
SECOND  MOMENT =                              1.0220419
 CUMULATIVE PR =....   0.9999999989
 VARIANCE      =              0.0220414
 STD.DEV.      =       0.1484634664
NUM.COEF.SKEW. =          0.5503636001
MOM.COEF.KURT. =........   3.9630103578

    1000 EMPS. PLAN 2-UNLIMITED-UNIT.

DISTRIBUTION LENGTH.....   275
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LOC	AMOUNT	PROBABILITY	CUMULATIVE	STOP-LOSS PREMIUM	VARIANCE OF STOP-LOSS	STANDARD DEVIATION	RATIO
1)	0.3960972	.0000000000	0.0000000000	0.6039030	0.2204134E-01	0.1484634	0.36498E-01
2)	0.4013552	.0000000000	0.0000000000	0.5986450	0.2204134E-01	0.1484634	0.36819E-01
3)	0.4057626	.0000000000	0.0000000000	0.5942370	0.2204134E-01	0.1484634	0.37092E-01
4)	0.4121447	.0000000000	0.0000000000	0.5878556	0.2204134E-01	0.1484634	0.37495E-01
5)	0.4176673	.0000000000	0.0000000001	0.5823330	0.2204134E-01	0.1484634	0.37850E-01
6)	0.4237369	.0000000002	0.0000000002	0.5762633	0.2204134E-01	0.1484634	0.38249E-01
7)	0.4294066	.0000000002	0.0000000005	0.5705938	0.2204134E-01	0.1484634	0.38624E-01
8)	0.4341033	.0000000004	0.0000000009	0.5658970	0.2204134E-01	0.1484634	0.38949E-01
9)	0.4404550	.0000000014	0.0000000023	0.5595453	0.2204134E-01	0.1484634	0.39392E-01
10)	0.4461642	.0000000016	0.0000000039	0.5538360	0.2204134E-01	0.1484634	0.39740E-01
11)	0.4525854	.0000000035	0.0000000074	0.5474149	0.2204134E-01	0.1484634	0.40265E-01
12)	0.4584334	.0000000072	0.0000000146	0.5415666	0.2204134E-01	0.1484634	0.40699E-01
13)	0.4632869	.0000000165	0.0000000311	0.5367133	0.2204134E-01	0.1484634	0.41067E-01
14)	0.4696406	.0000000280	0.0000000591	0.5303596	0.2204134E-01	0.1484634	0.41559E-01
15)	0.4757161	.0000000412	0.0000001003	0.5242821	0.2204134E-01	0.1484634	0.42041E-01
16)	0.4822478	.0000000939	0.0000001942	0.5177524	0.2204134E-01	0.1484634	0.42571E-01
17)	0.4882707	.0000001412	0.0000003354	0.5117290	0.2204134E-01	0.1484634	0.43072E-01
18)	0.4936944	.0000001738	0.0000005092	0.5063054	0.2204134E-01	0.1484634	0.43559E-01
19)	0.5003993	.0000004992	0.0000010083	0.4996009	0.2204134E-01	0.1484634	0.44118E-01
20)	0.5067019	.0000005197	0.0000015280	0.4932985	0.2204138E-01	0.1484634	0.44682E-01
21)	0.5131590	.0000010977	0.0000026257	0.4868413	0.2204137E-01	0.1484634	0.45274E-01
22)	0.5194890	.0000016312	0.0000042569	0.4805113	0.2204135E-01	0.1484633	0.45871E-01
23)	0.5252195	.0000017499	0.0000060068	0.4747808	0.2204133E-01	0.1484632	0.46824E-01
24)	0.5320652	.0000045749	0.0000105817	0.4679351	0.2204124E-01	0.1484631	0.47305E-01
25)	0.5385567	.0000044899	0.0000150715	0.4614437	0.2204123E-01	0.1484629	0.47706E-01
26)	0.5451346	.0000081177	0.0000231893	0.4548659	0.2204114E-01	0.1484626	0.48456E-01
27)	0.5517056	.0000124678	0.0000356571	0.4482950	0.2204100E-01	0.1484621	0.49166E-01
28)	0.5579106	.0000124191	0.0000480763	0.4420823	0.2204080E-01	0.1484615	0.49857E-01
29)	0.5650607	.0000282457	0.0000763220	0.4349985	0.2204050E-01	0.1484604	0.50668E-01
30)	0.5715134	.0000253156	0.0001016376	0.4284863	0.2204007E-01	0.1484590	0.51437E-01
31)	0.5779232	.0000394668	0.0001411044	0.4220741	0.2203952E-01	0.1484571	0.52217E-01
32)	0.5832166	.0000429895	0.0001840939	0.4167866	0.2203864E-01	0.1484550	0.52878E-01
33)	0.5902603	.0000641411	0.0002482349	0.4097102	0.2203762E-01	0.1484518	0.53700E-01
34)	0.5968750	.0000711207	0.0003193556	0.4031311	0.2203658E-01	0.1484486	0.54663E-01
35)	0.6026974	.0001235941	0.0004429497	0.3973100	0.2203400E-01	0.1484412	0.55460E-01
36)	0.6094593	.0001512336	0.0005941833	0.3905519	0.2203235E-01	0.1484329	0.56413E-01
37)	0.6164950	.0002141801	0.0008083634	0.3835208	0.2202913E-01	0.1484220	0.57434E-01
38)	0.6234235	.0002473150	0.0010556784	0.3765970	0.2202480E-01	0.1484077	0.58488E-01
39)	0.6306277	.0003166842	0.0013725626	0.3692011	0.2201941E-01	0.1483900	0.59640E-01
40)	0.6374606	.0004187671	0.0017911296	0.3626720	0.2201165E-01	0.1483639	0.60749E-01
41)	0.6449509	.0004363465	0.0022274761	0.3550923	0.2200286E-01	0.1483336	0.61946E-01
42)	0.6524205	.0006064791	0.0028339552	0.3480465	0.2199180E-01	0.1482963	0.63180E-01
43)	0.6595060	.0007900093	0.0036307065	0.3405021	0.2197726E-01	0.1482471	0.64528E-01
44)	0.6669006	.0008216033	0.0044523698	0.3332088	0.2195909E-01	0.1481800	0.65902E-01
45)	0.6747619	.0012538015	0.0057001713	0.3253600	0.2193608E-01	0.1481003	0.67415E-01
46)	0.6824522	.0012010127	0.0069011840	0.3177916	0.2190708E-01	0.1480130	0.68994E-01
47)	0.6875540	.0014375024	0.0083446864	0.3106069	0.2187701E-01	0.1479088	0.70415E-01
48)	0.6975426	.0021599470	0.0105046334	0.3027174	0.2183506E-01	0.1477646	0.72133E-01
49)	0.7045609	.0014761244	0.0119807578	0.2957405	0.2179190E-01	0.1476208	0.73671E-01

51)	0.7170902	.0028491832	0.0156835550	0.2834551	0.2169822E-01	[illegible]	[illegible]
52)	0.7262811	.0038494351	0.0201808081	0.2740695	0.2161708E-01	0.1470275	0.78845E-01
53)	0.7325290	.0027222830	0.0220062911	0.2662960	0.2158370E-01	0.1467777	0.80290E-01
54)	0.7390947	.0046662763	0.0273207670	0.2611586	0.2145694E-01	0.1464700	0.82358E-01
55)	0.7484126	.0048299053	0.0317560727	0.2526131	0.2133557E-01	0.1460670	0.84393E-01
56)	0.7580249	.0059092811	0.0377459538	0.2448616	0.2120577E-01	0.1456220	0.86603E-01
57)	0.7642904	.0050755222	0.0428214560	0.2374859	0.2108621E-01	0.1451420	0.88705E-01
58)	0.7772007	.0062179938	0.0490394490	0.2303733	0.2091735E-01	0.1446203	0.90790E-01
59)	0.7849333	.0089894024	0.0579484522	0.2220081	0.2072399E-01	0.1439563	0.93315E-01
60)	0.7888764	.0075502854	0.0655142561	0.2141339	0.2051047E-01	0.1432148	0.95783E-01
61)	0.7971164	.0104653770	0.0769448351	0.2058151	0.2026555E-01	0.1425571	0.98465E-01
62)	0.8094426	.0052163058	0.0817179389	0.1996797	0.2005946E-01	0.1416314	0.10046
63)	0.8104001	.0096457407	0.0913636796	0.1941907	0.1980707E-01	0.1409506	0.10251
64)	0.8191159	.0128336040	0.1037972036	0.1862130	0.1950417E-01	0.1398720	0.10503
65)	0.8263254	.0116962330	0.1154955166	0.1798101	0.1929015E-01	0.1388049	0.10728
66)	0.8348162	.0127160510	0.1282123670	0.1722940	0.1894465E-01	0.1376403	0.10995
67)	0.8433934	.0173202080	0.1455803956	0.1646916	0.1856770E-01	0.1362636	0.11279
68)	0.8525877	.0154512800	0.1609916756	0.1569636	0.1814430E-01	0.1347011	0.11560
69)	0.8611591	.0159042220	0.1769758976	0.1497722	0.1772111E-01	0.1331207	0.11832
70)	0.8696326	.0202460830	0.1972727406	0.1427982	0.1720236E-01	0.1314624	0.12163
71)	0.8765521	.0159419460	0.2132140006	0.1357984	0.1680315E-01	0.1296270	0.12378
72)	0.8862669	.0176958100	0.2309105226	0.1295716	0.1635532E-01	0.1278879	0.12623
73)	0.8923698	.0155960410	0.2484573636	0.1248646	0.1599613E-01	0.1264758	0.12810
74)	0.9002038	.0212394150	0.2676967786	0.1169745	0.1552597E-01	0.1248032	0.13050
75)	0.9094671	.0225282730	0.2902258516	0.1121910	0.1495273E-01	0.1222819	0.13320
76)	0.9167735	.0148167950	0.3100410460	0.1070051	0.1440793E-01	0.1203658	0.13539
77)	0.9254655	.0233345610	0.3315704076	0.1010098	0.1392748E-01	0.1180148	0.13780
78)	0.9350796	.0277085500	0.3611647576	0.9459901E-01	0.1330840E-01	0.1153278	0.14060
79)	0.9447300	.0268129940	0.3879777316	0.8842869E-01	0.1266195E-01	0.1125253	0.14319
80)	0.9540290	.0280757030	0.4120534546	0.8274259E-01	0.1204893E-01	0.1097494	0.14557
81)	0.9637023	.0249301730	0.4431491676	0.7705523E-01	0.1140794E-01	0.1068002	0.14805
82)	0.9702740	.0110209920	0.4525126196	0.7330442E-01	0.1097196E-01	0.1047447	0.14951
83)	0.9761549	.0218599670	0.4743725666	0.7016512E-01	0.1050450E-01	0.1024055	0.15042
84)	0.9853020	.0285932490	0.5029650356	0.6531511E-01	0.9996490E-02	0.0999824	0.15345
85)	0.9953940	.0267805000	0.5317463956	0.6033854E-01	0.9363703E-02	0.0967662	0.15519
86)	1.0052288	.0237396940	0.5554880496	0.5573563E-01	0.8756725E-02	0.0935774	0.15712
87)	1.0143711	.0271451050	0.5826311946	0.5166973E-01	0.8211703E-02	0.0906161	0.15842
88)	1.0239331	.0247594680	0.6073590546	0.4767885E-01	0.7657608E-02	0.0875089	0.16061
89)	1.0330711	.0173141350	0.6287047076	0.4404121E-01	0.7140449E-02	0.0845405	0.16213
90)	1.0406609	.0260953820	0.6507981645	0.4125970E-01	0.6743433E-02	0.0821184	0.16352
91)	1.0485310	.0163602400	0.6671504675	0.3849431E-01	0.6355462E-02	0.0795456	0.16450
92)	1.0571635	.0207360920	0.6870945595	0.3562105E-01	0.5908613E-02	0.0766675	0.16587
93)	1.0659873	.0221030190	0.7099981785	0.3206710E-01	0.5492401E-02	0.0741141	0.16712
94)	1.0760371	.0215743150	0.7315724935	0.2995265E-01	0.5044602E-02	0.0710258	0.16842
95)	1.0865761	.0216809770	0.7532534705	0.2712364E-01	0.4600601E-02	0.0678572	0.16976
96)	1.0970104	.0212020390	0.7744561005	0.2459406E-01	0.4196869E-02	0.0647456	0.17102
97)	1.1070308	.0152684440	0.7897245445	0.2228767E-01	0.3834782E-02	0.0619256	0.17206
98)	1.1191044	.0104770710	0.8002024155	0.2070471E-01	0.3541656E-02	0.0599504	0.17280
99)	1.1209602	.0135672620	0.8150896775	0.1942472E-01	0.3373818E-02	0.0580811	0.17362
100)	1.1313115	.0162808130	0.8321784405	0.1752508E-01	0.3061496E-02	0.0555308	0.17469
101)	1.1421203	.0137482500	0.8479267485	0.1571110E-01	0.2762544E-02	0.0525549	0.17583
102)	1.1522466	.0134365470	0.8613632755	0.1417115E-01	0.2505964E-02	0.0500546	0.17684
103)	1.1610248	.0110072200	0.8723745155	0.1287030E-01	0.2287406E-02	0.0476268	0.17773
104)	1.1713954	.0113274550	0.8856497605	0.1162817E-01	0.2078773E-02	0.0455436	0.17883
105)	1.1796040	.0095572273	0.8932551958	0.1006917E-01	0.1917005E-02	0.0437636	0.17968
106)	1.1879455	.0079875447	0.9012827405	0.9773505E-02	0.1765705E-02	0.0419478	0.18047
107)	1.1975300	.0048869930	0.9111242555	0.8831905E-02	0.1603911E-02	0.0400485	0.18160
108)	1.2075140	.0089529694	0.9200822029	0.7944571E-02	0.1451242E-02	0.0380450	0.18268
109)	1.2164369	.0083073632	0.9284659661	0.7071602E-02	0.1300367E-02	0.0360604	0.18369
110)	1.2301101	.0086352146	0.9373504707	0.6236264E-02	0.1150079E-02	0.0340012	0.18530
111)	1.2396750	.0024649105	0.9402748912	0.5635906E-02	0.1049694E-02	0.0323941	0.18622
112)	1.2461450	.0065025057	0.9467112564	0.5250540E-02	0.9034060E-03	0.0313602	0.18731
113)	1.2562201	.0047239656	0.9513007225	0.4714321E-02	0.8064104E-03	0.0294002	0.18845
114)	1.2656906	.0056870619	0.9571405035	0.4255007E-02	0.6075063E-03	0.0284101	0.18980
115)	1.2775708	.0049869122	0.9620974957	0.3785501E-02	0.7165498E-03	0.0267604	0.19131
116)	1.2865141	.0040659032	0.9661614604	0.3331104E-02	0.6420465E-03	0.0255396	0.19276
117)	1.2978507	.0031692777	0.9693271506	0.3014030E-02	0.5647908E-03	0.0241824	0.19397

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```
INPUT-FILE NAME=LF2118      OUTPUT-FILE NAME=J00195
FIRST   MOMENT =          0.4999985
SECOND  MOMENT =                                1.0076495
 CUMULATIVE PR =       1.0000000000
 VARIANCE      =                 0.0076523
 STD.DEV.      =       0.0874776615
NOM.COEF.SKEW. =           0.1871470208
NOM.COEF.KURT. =           3.043142733

   1000 EMPS.-PLAN 2-$15,000 RET.-UNIT.-NO UNC

 DISTRIBUTION LENGTH=       230
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LOC	AMOUNT	PROBABILITY	CUMULATIVE	STOP-LOSS PREMIUM	VARIANCE OF STOP-LOSS	STANDARD DEVIATION	RATIO
1)	0.5261093	.0000000000	0.0000000000	0.4738892	0.7652341E-02	0.0874777	0.16148E-01
2)	0.5298573	.0000000000	0.0000000000	0.4701411	0.7652341E-02	0.0874777	0.16277E-01
3)	0.5336225	.0000000000	0.0000000000	0.4663759	0.7652341E-02	0.0874777	0.16408E-01
4)	0.5372517	.0000000000	0.0000000000	0.4627468	0.7652341E-02	0.0874777	0.16537E-01
5)	0.5406903	.0000000000	0.0000000000	0.4591082	0.7652341E-02	0.0874777	0.16668E-01
6)	0.5447417	.0000000000	0.0000000000	0.4552567	0.7652341E-02	0.0874777	0.16809E-01
7)	0.5485920	.0000000000	0.0000000001	0.4514056	0.7652341E-02	0.0874777	0.16952E-01
8)	0.5524372	.0000000000	0.0000000001	0.4475612	0.7652341E-02	0.0874777	0.17098E-01
9)	0.5561942	.0000000001	0.0000000002	0.4438043	0.7652341E-02	0.0874777	0.17243E-01
10)	0.5599099	.0000000001	0.0000000003	0.4400885	0.7652341E-02	0.0874777	0.17388E-01
11)	0.5637478	.0000000002	0.0000000004	0.4362506	0.7652341E-02	0.0874777	0.17541E-01
12)	0.5676598	.0000000003	0.0000000007	0.4323389	0.7652341E-02	0.0874777	0.17708E-01
13)	0.5716046	.0000000004	0.0000000011	0.4283939	0.7652341E-02	0.0874777	0.17863E-01
14)	0.5755197	.0000000006	0.0000000017	0.4244788	0.7652341E-02	0.0874777	0.18028E-01
15)	0.5793177	.0000000008	0.0000000025	0.4206808	0.7652341E-02	0.0874777	0.18190E-01
16)	0.5831469	.0000000013	0.0000000039	0.4168496	0.7652341E-02	0.0874777	0.18358E-01
17)	0.5872342	.0000000022	0.0000000061	0.4127643	0.7652341E-02	0.0874777	0.18539E-01
18)	0.5913171	.0000000031	0.0000000092	0.4086814	0.7652341E-02	0.0874777	0.18724E-01
19)	0.5951399	.0000000042	0.0000000134	0.4048586	0.7652341E-02	0.0874777	0.18901E-01
20)	0.5989959	.0000000066	0.0000000199	0.4010045	0.7652341E-02	0.0874777	0.19083E-01
21)	0.6029448	.0000000093	0.0000000293	0.3970537	0.7652341E-02	0.0874777	0.19273E-01
22)	0.6069378	.0000000141	0.0000000434	0.3930610	0.7652341E-02	0.0874777	0.19469E-01
23)	0.6111202	.0000000219	0.0000000653	0.3888782	0.7652341E-02	0.0874777	0.19678E-01
24)	0.6151145	.0000000271	0.0000000924	0.3848859	0.7652341E-02	0.0874777	0.19882E-01
25)	0.6189713	.0000000404	0.0000001328	0.3810272	0.7652340E-02	0.0874777	0.20083E-01
26)	0.6228777	.0000000548	0.0000001875	0.3771208	0.7652340E-02	0.0874777	0.20241E-01
27)	0.6269803	.0000000858	0.0000002733	0.3730182	0.7652339E-02	0.0874777	0.20515E-01
28)	0.6311688	.0000001179	0.0000003912	0.3688297	0.7652339E-02	0.0874776	0.20748E-01
29)	0.6353010	.0000001643	0.0000005555	0.3646974	0.7652337E-02	0.0874776	0.20985E-01
30)	0.6393982	.0000002218	0.0000007769	0.3606003	0.7652336E-02	0.0874776	0.21221E-01
31)	0.6433447	.0000002897	0.0000010666	0.3566537	0.7652334E-02	0.0874776	0.21456E-01
32)	0.6473908	.0000004212	0.0000014878	0.3526077	0.7652330E-02	0.0874776	0.21702E-01
33)	0.6516610	.0000006089	0.0000020967	0.3483375	0.7652326E-02	0.0874776	0.21968E-01
34)	0.6559594	.0000008175	0.0000029142	0.3440391	0.7652320E-02	0.0874775	0.22243E-01
35)	0.6601534	.0000010555	0.0000039698	0.3398451	0.7652311E-02	0.0874775	0.22517E-01
36)	0.6642820	.0000014305	0.0000054003	0.3357166	0.7652300E-02	0.0874774	0.22794E-01
37)	0.6683273	.0000017938	0.0000071941	0.3316713	0.7652286E-02	0.0874773	0.23072E-01
38)	0.6724532	.0000025357	0.0000097298	0.3275454	0.7652266E-02	0.0874772	0.23362E-01
39)	0.6767442	.0000034572	0.0000131870	0.3232545	0.7652234E-02	0.0874771	0.23672E-01
40)	0.6810688	.0000045153	0.0000177024	0.3189303	0.7652202E-02	0.0874769	0.23993E-01
41)	0.6853160	.0000057628	0.0000234652	0.3146828	0.7652155E-02	0.0874766	0.24317E-01
42)	0.6894514	.0000071700	0.0000306352	0.3105474	0.7652094E-02	0.0874762	0.24641E-01
43)	0.6935544	.0000093329	0.0000399681	0.3064445	0.7652017E-02	0.0874758	0.24970E-01
44)	0.6979903	.0000131613	0.0000531294	0.3020909	0.7651911E-02	0.0874752	0.25330E-01
45)	0.7023206	.0000165594	0.0000696888	0.2976708	0.7651770E-02	0.0874744	0.25705E-01
46)	0.7066972	.0000210163	0.0000907051	0.2933025	0.7651598E-02	0.0874734	0.26088E-01
47)	0.7110149	.0000261014	0.0001168065	0.2889882	0.7651362E-02	0.0874721	0.26477E-01
48)	0.7151847	.0000311158	0.0001479223	0.2848159	0.7651082E-02	0.0874705	0.26863E-01
49)	0.7194878	.0000419508	0.0001898731	0.2805534	0.7650726E-02	0.0874684	0.27270E-01

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50)	0.7230902	.0000340000	0.0002044170	0.2701030	[illegible]	[illegible]	[illegible]
51)	0.7283304	.0000859946	0.0003095743	0.2716727	0.7649682E-02	0.0074623	0.28156E-01
52)	0.7326494	.0000745242	0.0003841035	0.2673597	0.7648941E-02	0.0074502	0.26610E-01
53)	0.7369005	.0000994258	0.0004807274	0.2630196	0.7648046E-02	0.0074531	0.29070E-01
54)	0.7413343	.0001144219	0.0006086495	0.2586740	0.7646935E-02	0.0074408	0.29562E-01
55)	0.7458305	.0001549835	0.0007684520	0.2541805	0.7645534E-02	0.0074307	0.30079E-01
56)	0.7503997	.0001408150	0.0009572678	0.2496108	0.7643765E-02	0.0074206	0.30622E-01
57)	0.7540241	.0002193434	0.0011768117	0.2451946	0.7641671E-02	0.0074106	0.31166E-01
58)	0.7591520	.0002655234	0.0014401356	0.2408718	0.7639194E-02	0.0074025	0.31715E-01
59)	0.7635031	.0003270847	0.0017671403	0.2364471	0.7636148E-02	0.0073851	0.32245E-01
60)	0.7680566	.0003874400	0.0021549004	0.2314815	0.7632445E-02	0.0073639	0.32901E-01
61)	0.7726473	.0004900002	0.0026457800	0.2274007	0.7627901E-02	0.0073379	0.33544E-01
62)	0.7771720	.0005384595	0.0031842400	0.2226076	0.7622510E-02	0.0073070	0.34144E-01
63)	0.7817097	.0006848208	0.0038690080	0.2183646	0.7616135E-02	0.0072705	0.34870E-01
64)	0.7863210	.0007759586	0.0046452074	0.2137713	0.7608425E-02	0.0072263	0.35591E-01
65)	0.7907050	.0006013652	0.0055263426	0.2093275	0.7599650E-02	0.0071760	0.36305E-01
66)	0.7952375	.0010447711	0.0065711637	0.2049002	0.7589454E-02	0.0071175	0.37040E-01
67)	0.7998932	.0013083075	0.0078794712	0.2002750	0.7577063E-02	0.0070463	0.37835E-01
68)	0.8046300	.0014651450	0.0093426178	0.1955755	0.7562204E-02	0.0069614	0.38667E-01
69)	0.8091423	.0016205095	0.0109463266	0.1910559	0.7545804E-02	0.0068666	0.39445E-01
70)	0.8137201	.0016451476	0.0128053241	0.1865710	0.7527034E-02	0.0067505	0.40344E-01
71)	0.8182040	.0021075602	0.0149658044	0.1820425	0.7505526E-02	0.0066345	0.41210E-01
72)	0.8229407	.0026131408	0.0175764852	0.1774855	0.7480360E-02	0.0064891	0.42146E-01
73)	0.8277656	.0024972946	0.0205262796	0.1727455	0.7450655E-02	0.0063172	0.43131E-01
74)	0.8324775	.0031715497	0.0236478295	0.1681303	0.7417687E-02	0.0061200	0.44119E-01
75)	0.8370307	.0034836500	0.0271818795	0.1636050	0.7381003E-02	0.0059179	0.45098E-01
76)	0.8415604	.0039285915	0.0311100718	0.1592701	0.7342045E-02	0.0056857	0.46098E-01
77)	0.8463125	.0047374430	0.0358475639	0.1546741	0.7245716E-02	0.0054150	0.47168E-01
78)	0.8512090	.0052637842	0.0411113531	0.1499531	0.7242244E-02	0.0051014	0.48297E-01
79)	0.8559512	.0054692400	0.0465806512	0.1454059	0.7184602E-02	0.0047624	0.49411E-01
80)	0.8606322	.0062146652	0.0527453164	0.1409429	0.7122225E-02	0.0043933	0.50533E-01
81)	0.8653082	.0067704965	0.0595600149	0.1364570	0.7052865E-02	0.0039013	0.51686E-01
82)	0.8701088	.0074894440	0.0670560140	0.1319487	0.6977054E-02	0.0035208	0.52857E-01
83)	0.8749036	.0085425074	0.0750480614	0.1274508	0.6892249E-02	0.0030196	0.54078E-01
84)	0.8798070	.0069400284	0.0846394248	0.1229170	0.6794306E-02	0.0024503	0.55316E-01
85)	0.8846652	.0095594049	0.0941488346	0.1185445	0.6701725E-02	0.0018641	0.56535E-01
86)	0.8895734	.0101414583	0.1043402430	0.1142744	0.6590456E-02	0.0012309	0.57740E-01
87)	0.8940603	.0104959372	0.1152446301	0.1100820	0.6480748E-02	0.0005520	0.58945E-01
88)	0.8990242	.0124200132	0.1282204433	0.1056060	0.6365130E-02	0.0797618	0.60227E-01
89)	0.9040800	.0130612664	0.1412697122	0.1013171	0.6232106E-02	0.0789437	0.61511E-01
90)	0.9089280	.0130041748	0.1550938870	0.9712034E-01	0.6095082E-02	0.0780710	0.62758E-01
91)	0.9136914	.0138855534	0.1689744404	0.9309535E-01	0.5954541E-02	0.0771657	0.63962E-01
92)	0.9184806	.0155197022	0.1844491431	0.8910046E-01	0.5806840E-02	0.0762027	0.65166E-01
93)	0.9234725	.0166064263	0.2011006714	0.8504478E-01	0.5646717E-02	0.0751448	0.66597E-01
94)	0.9284050	.0175671133	0.2186931047	0.8103964E-01	0.5479270E-02	0.0740221	0.67612E-01
95)	0.9335400	.0175174857	0.2362106704	0.7716826E-01	0.5307050E-02	0.0728599	0.68783E-01
96)	0.9383115	.0184755030	0.2546061734	0.7344809E-01	0.5134595E-02	0.0716557	0.69907E-01
97)	0.9433150	.0197627940	0.2744404675	0.6971035E-01	0.4952077E-02	0.0703710	0.71030E-01
98)	0.9482127	.0184729049	0.2943219523	0.6616534E-01	0.4764958E-02	0.0690612	0.72008E-01
99)	0.9531057	.0217501001	0.3150030525	0.6265157E-01	0.4581407E-02	0.0676867	0.73126E-01
100)	0.9583961	.0221985618	0.3372016142	0.5900240E-01	0.4381607E-02	0.0661945	0.74162E-01
101)	0.9634977	.0215435532	0.3587446675	0.5570115E-01	0.4184224E-02	0.0646856	0.75119E-01
102)	0.9684529	.0211283119	0.3804729794	0.5252362E-01	0.3991084E-02	0.0631810	0.76001E-01
103)	0.9734100	.0222037753	0.4026767546	0.4945254E-01	0.3794511E-02	0.0616402	0.76851E-01
104)	0.9783613	.0223033455	0.4249601502	0.4649498E-01	0.3600211E-02	0.0600684	0.77604E-01
105)	0.9834743	.0240040541	0.4494064043	0.4355495E-01	0.3412583E-02	0.0584170	0.78350E-01
106)	0.9887055	.0237553024	0.4720170071	0.4067287E-01	0.3214677E-02	0.0566901	0.79037E-01
107)	0.9938368	.0230442190	0.4954171062	0.3796773E-01	0.3023801E-02	0.0549898	0.79643E-01
108)	0.9988820	.0224965113	0.5100656175	0.3542456E-01	0.2840256E-02	0.0532940	0.80178E-01
109)	1.0038977	.0225741652	0.5419357627	0.3301132E-01	0.2662152E-02	0.0515960	0.80644E-01
110)	1.0089087	.0230047006	0.5644494913	0.3068598E-01	0.2487263E-02	0.0498725	0.81055E-01
111)	1.0143226	.0240554000	0.5894302413	0.2835400E-01	0.2308843E-02	0.0480504	0.81424E-01
112)	1.0196314	.0217663851	0.6110692764	0.2617370E-01	0.2130254E-02	0.0462413	0.81695E-01
113)	1.0247044	.0219178030	0.6329870794	0.2420052E-01	0.1982080E-02	0.0445206	0.81902E-01
114)	1.0297924	.0204063240	0.6538943038	0.2233517E-01	0.1832212E-02	0.0428043	0.82040E-01
115)	1.0348932	.0204942157	0.6740976145	0.2056794E-01	0.1689133E-02	0.0410991	0.82125E-01
116)	1.0402650	.0220452670	0.6969428004	0.1882140E-01	0.1540334E-02	0.0393234	0.82158E-01
117)	1.0450411	.0194024402	0.7165858506	0.1714242E-01	0.1411307E-02	0.0375684	0.82094E-01

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```
INPUT-FILE NAME=DOUT4        OUTPUT-FILE NAME=DOUT6
IRST   MOMENT =          1.0000000
SCOND  MOMENT =                                 1.4112239
CUMULATIVE PR =       1.0000000000
VARIANCE      =              0.4112239
STD.DEV.      =       0.6412673870
DM.COEF.SKEW. =            5.007910539
DM.COEF.KURT. =           69.49884235
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IBH79 (56) ADULT 36 CHILD MEDICAL WITH MATERNITY-NORMALIZED   35 ee

DISTRIBUTION LENGTH=   219

DC	AMOUNT	PROBABILITY	CUMULATIVE	STOP-LOSS PREMIUM	VARIANCE OF STOP-LOSS	STANDARD DEVIATION	RATIO
1)	0.1016992	.0000000000	0.0000000000	0.8903008	0.4112239	0.6412674	0.45778
2)	0.1173546	.0000000011	0.0000000012	0.8826454	0.4112239	0.6412674	0.46590
3)	0.1309361	.0000000283	0.0000000295	0.8690639	0.4112239	0.6412674	0.47318
4)	0.1483411	.0000007725	0.0000008020	0.8516589	0.4112239	0.6412674	0.48285
5)	0.1736868	.0000283855	0.0000291874	0.8263132	0.4112238	0.6412674	0.49766
6)	0.1901815	.0000969335	0.0001261210	0.8098190	0.4112230	0.6412667	0.50780
7)	0.2153059	.0006917657	0.0008178866	0.7846970	0.4112180	0.6412628	0.52405
8)	0.2376566	.0011999893	0.0020178759	0.7623453	0.4111897	0.6412405	0.53936
9)	0.2599745	.0024819483	0.0044998242	0.7400925	0.4111220	0.6411886	0.55550
10)	0.2897907	.0062641654	0.0107639896	0.7104104	0.4107274	0.6410362	0.57844
11)	0.3174517	.0069084102	0.0176723998	0.6828494	0.4105096	0.6407102	0.60117
12)	0.3443735	.0112838446	0.0289562444	0.6565998	0.4098771	0.6402164	0.62424
13)	0.3746802	.0158218647	0.0447781091	0.6271706	0.4087505	0.6393360	0.65174
14)	0.4052196	.0190770461	0.0638551551	0.5979985	0.4070756	0.6380243	0.68073
15)	0.4385758	.0278199068	0.0916750620	0.5663980	0.4045652	0.6360544	0.71428
16)	0.4729507	.0282504862	0.1199255482	0.5355378	0.4011330	0.6333506	0.74903
17)	0.5073713	.0340235595	0.1539491078	0.5052451	0.3968368	0.6299496	0.78543
18)	0.5428111	.0365919077	0.1905410154	0.4752612	0.3914872	0.6256894	0.82373
19)	0.5806488	.0430185419	0.2335595574	0.4446732	0.3848551	0.6203669	0.86556
20)	0.6211400	.0470943540	0.2806539113	0.4135590	0.3767387	0.6137905	0.91088
21)	0.6607750	.0402794559	0.3209333672	0.3850070	0.3678544	0.6065100	0.95525
22)	0.7024044	.0517578813	0.3726912486	0.3568186	0.3579423	0.5982828	1.0031
23)	0.7462689	.0444019985	0.4170932470	0.3293021	0.3467257	0.5888342	1.0529
24)	0.7901266	.0460185094	0.4631117565	0.3037371	0.3351456	0.5789176	1.1034
25)	0.8343292	.0432849039	0.5063966604	0.2800053	0.3231960	0.5685023	1.1542
26)	0.8627072	.0461086945	0.5525053549	0.2561257	0.3100616	0.5568317	1.2106
27)	0.9317869	.0406729296	0.5931782845	0.2341620	0.2967665	0.5447628	1.2674
28)	0.9786628	.0338685020	0.6270467864	0.2150927	0.2842747	0.5331741	1.3216
29)	1.0255826	.0359285466	0.6629753330	0.1975738	0.2721330	0.5216627	1.3772
30)	1.0783580	.0352024425	0.6981777755	0.1798072	0.2589282	0.5088459	1.4400
31)	1.1343181	.0330936363	0.7312714118	0.1629172	0.2455380	0.4955179	1.5071
32)	1.1888005	.0263371405	0.7576085523	0.1482762	0.2331396	0.4828453	1.5723
33)	1.2421842	.0251453552	0.7827539075	0.1353364	0.2216692	0.4708175	1.6379
34)	1.2987674	.0246925630	0.8074464705	0.1230440	0.2102253	0.4585033	1.7085
35)	1.3575619	.0201676323	0.8276141028	0.1117229	0.1990801	0.4461840	1.7819
36)	1.4159336	.0197877075	0.8474018103	0.1016604	0.1887717	0.4344787	1.8569
37)	1.4780054	.0171136063	0.8645154166	0.9218839E-01	0.1785753	0.4225818	1.9371
38)	1.5402672	.0157127819	0.8802281985	0.8375287E-01	0.1691051	0.4112239	2.0191
39)	1.6029609	.0129857144	0.8932139128	0.7624359E-01	0.1602757	0.4003445	2.1021
40)	1.6669600	.0124566152	0.9056705280	0.6940972E-01	0.1519494	0.3898069	2.1892
41)	1.7308680	.0099582211	0.9156287491	0.6338131E-01	0.1442635	0.3798204	2.2761
42)	1.7972225	.0101493225	0.9257780715	0.5778289E-01	0.1369021	0.3700028	2.3692
43)	1.8676777	.0087854194	0.9345634909	0.5255358E-01	0.1297053	0.3601462	2.4681
44)	1.9390820	.0075431801	0.9421066710	0.4728113E-01	0.1230031	0.3507180	2.5689
45)	2.0082742	.0062138587	0.9483205297	0.4307536E-01	0.1170218	0.3420845	2.6671
46)	2.0800243	.0060119387	0.9543324684	0.4016735E-01	0.1113034	0.3336216	2.7710
47)	2.1541711	.0051123307	0.9594447991	0.3670125E-01	0.1058584	0.3253589	2.8781
48)	2.2281743	.0045293420	0.9639741411	0.3378004E-01	0.1008484	0.3175664	2.9854
49)	2.3031446	.0039589115	0.9679330526	0.3107917E-01	0.9616109E-01	0.3100985	3.0941
50)	2.3800086	.0036304139	0.9715634665	0.2861437E-01	0.9171994E-01	0.3028530	3.2054
51)	2.4586955	.0029626388	0.9745261052	0.2637679E-01	0.8751590E-01	0.2958309	3.3179
52)	2.5376231	.0027242018	0.9772503070	0.2436620E-01	0.8361296E-01	0.2891590	3.4315
53)	2.6191587	.0023796043	0.9796299113	0.2251129E-01	0.7987767E-01	0.2826264	3.5483
54)	2.7013411	.0022285673	0.9818588786	0.2083722E-01	0.7625775E-01	0.2763832	3.6659
55)	2.7850099	.0018669401	0.9837258187	0.1931938E-01	0.7308865E-01	0.2703495	3.7832
56)	2.8667113	.0016668463	0.9853926649	0.1758976E-01	0.7009025E-01	0.2647456	3.8961
57)	2.9532747	.0015071329	0.9868997978	0.1672529E-01	0.6712909E-01	0.2590928	4.0136
58)	3.0425176	.0012693284	0.9881691262	0.1555619E-01	0.6420594E-01	0.2533467	4.1325
59)	3.1308356	.0010990998	0.9892682260	0.1451131E-01	0.6166185E-01	0.2483181	4.2492
60)	3.2182277	.0010187432	0.9902869692	0.1357344E-01	0.5922381E-01	0.2433800	4.3639
61)	3.3100662	.0010413069	0.9913282761	0.1268141E-01	0.5684602E-01	0.2384240	4.4826
62)	3.4025630	.0008321270	0.9921604031	0.1187931E-01	0.5459392E-01	0.2336534	4.5957
63)	3.4937273	.0008023881	0.9929627912	0.1116461E-01	0.5250962E-01	0.2291458	4.7032
64)	3.5899387	.0006691118	0.9936319030	0.1040756E-01	0.5044110E-01	0.2245910	4.8096
65)	3.6851615	.0005509644	0.9941828675	0.9881167E-02	0.4851389E-01	0.2202597	4.9097
66)	3.7831982	.0004874689	0.9946703364	0.9310874E-02	0.4664330E-01	0.2159706	5.0096

Appendix #5

SIMULATION

ACRDSM3

Input

1. Number of replications
2. Credibility factors - yours
3. Credibility factors - market
4. Trend
5. Market manual
6. Market TLR
7. First year manual
8. Flat manual? Y or N
9. Inherent level distribution
10. Random level distribution
11. Lapse rates
12. Market adjustment to lapse rates
13. Margin
14. Dividend Credibility factor
15. Number of new cases considered each year
16. Distribution of brokers
17. Market tolerance (friendly brokers)
18. Broker tolerance (unfriendly brokers)
19. Number of years of data that broker provides

Manual Calculation

M(n) Prior Year's Actual Claims/Expected Claims

Appendix #5

New Business Logic

1. A case is offered for assessment

2. Inherent level is determined randomly (input item I)
3. Two years of historical experience are produced using inherent level and random level (I&J)

4. Determine adjusted claims for use in quoting a premiu, both for your company and the market (B&C)

   a. There is an option to have the broker give you only one year of data (when he has two) (S)

   b. The market premium is adjusted if the market TLR does not equal your TLR (F)

5. Broker assessment

   a. A broker is classified, randomly, as friendly, indifferent, or unfriendly (P)

   b. The case is broker classified as good, average, or poor

   c. Friendly broker:

      - will consider you as long as your prem/mkt prem $\leq$ 1 + MTOL (Q)

      - sends you all good cases

      - sends you 80% of average cases

      - sends you no poor cases

   d. Indifferent broker:

      - will consider you as long as your prem/mkt prem $\leq$ 1.0

      - sends you 1/3 of all cases

   e. Unfriendly broker:

      - will consider you as long as your prem/mkt prem $\leq$ 1 - UTOL (R)

      - sends you no good cases

      - sends you 20% of average cases

      - sends you all poor cases

Appendix #5

6. If case not written, continue

7. If case is written, then:

   a. Calculate claims for the year (D,I,J)

   b. Calculate adjusted claims for next year's premium (B,D,I,J)

   c. Calculate premiums for the year based on the historical data

   d. Calculate experience refunds (M,N)

   e. Accumulate policy data for printing

Inforce Logic

1. A case is up for renewal

2. Calculate claims for the year using inherent level (I,J) (same as last year) and random level (new this year)

3. Calculate adjusted claims for calculating next year's premium (B)

4. Calculate this year's premium using last year's adjusted claims

5. Determine if policy lapses or not: (K,L)

   a. Determine

      - historical loss ratio

      - rate increase (in excess of trend)

      - renewal rate/market rate

   b. Based on the above three values, determine the probability of lapse this year

   c. Randomly determine if cases lapses or persists

6. Calculate experience refund (M,N)

7. Accumulate policy data for printing, there are separate accumulators for active and lapsed policies

Appendix #5

Example #10

ACTIVE POLICIES

PREM	CLAIMS	ADJ CLM	CL/PRM	POL	MAN	PER UNIT				$ REF
157.21	113.72	113.72	0.723	111	1.00	1.416	1.025	1.025	0.723	21.54
322.58	241.66	241.66	0.749	209	0.90	1.543	1.156	1.156	0.749	[illegible]
488.40	378.90	378.90	0.776	257	0.85	1.702	1.320	1.320	0.776	59.14
717.15	570.30	570.30	0.795	365	0.82	1.965	1.562	1.562	0.795	79.34
985.81	767.15	767.15	0.778	428	0.82	2.303	1.792	1.792	0.778	103.55
1248.60	967.80	967.80	0.775	474	0.79	2.634	2.042	2.042	0.775	145.83
1578.29	1252.87	1252.87	0.794	528	0.77	2.989	2.373	2.373	0.794	152.32
1499.45	1197.67	1197.67	0.799	442	0.76	3.392	2.710	2.710	0.799	146.62
2277.40	1845.17	1845.17	0.810	581	0.76	3.920	3.176	3.176	0.810	191.56
2263.31	1795.70	1795.70	0.793	500	0.77	4.527	3.591	3.591	0.793	214.29
11538.20	9130.94	9130.94	0.791							%1159.2

LAPSED POLICIES

PREM	CLAIMS	ADJ CLM	CL/PRM	POL	MAN	PER UNIT			
0.00	0.00	0.00	N/A	0	1.00	N/A			
18.92	16.57	16.57	0.876	13	0.90	1.455	1.275	1.275	0.876
82.82	71.24	71.24	0.860	48	0.85	1.725	1.484	1.484	0.860
184.41	141.18	141.18	0.766	88	0.82	2.096	1.604	1.604	0.766
319.67	252.75	252.75	0.791	139	0.82	2.300	1.818	1.818	0.791
537.26	432.14	432.14	0.804	207	0.79	2.595	2.088	2.088	0.804
799.77	617.84	617.84	0.773	267	0.77	2.995	2.314	2.314	0.773
1611.34	1275.35	1275.35	0.791	464	0.76	3.473	2.749	2.749	0.791
1739.63	1378.46	1378.46	0.792	431	0.76	4.036	3.198	3.198	0.792
2903.52	2293.87	2293.87	0.790	624	0.77	4.653	3.676	3.676	0.790
8197.34	6479.40	6479.40	0.790						

LOSS RATIOS BY INHERENT LEVEL-ACTIVE

.70		.80		.90		1.00		1.10		1.20		1.30	
0.788	6	0.678	5	0.894	10	0.711	56	0.710	18	0.687	10	0.712	6
0.844	11	0.785	11	0.611	20	0.729	96	0.835	31	0.769	24	0.775	16
0.800	14	0.935	13	0.815	39	0.757	125	0.706	46	0.830	27	0.521	23
0.791	18	0.838	21	0.763	53	0.815	145	0.851	57	0.781	41	0.683	30
0.742	21	0.751	23	0.769	64	0.770	169	0.844	59	0.746	53	0.798	39
0.828	21	0.856	28	0.771	71	0.758	186	0.759	64	0.773	57	0.814	47
0.796	23	0.735	32	0.839	76	0.792	210	0.771	73	0.819	63	0.777	51
0.850	20	0.885	26	0.759	64	0.813	177	0.820	61	0.772	52	0.760	42
0.814	23	0.758	26	0.839	88	0.819	235	0.784	95	0.790	71	0.835	43
0.811	21	0.820	21	0.779	73	0.779	205	0.824	85	0.768	59	0.843	36
0.809	178	0.804	206	0.792	558	0.788	1604	0.798	589	0.781	457	0.797	333

LOSS RATIOS BY INHERENT LEVEL - LAPSED

.70		.80		.90		1.00		1.10		1.20		1.30	
N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
N/A	N/A	N/A	N/A	0.675	3	0.800	4	0.990	4	1.022	2	N/A	N/A
0.900	1	0.709	3	0.800	4	0.977	22	0.671	7	0.874	9	0.500	2
0.622	1	0.800	4	0.765	7	0.769	40	0.760	18	0.738	12	0.813	6
0.973	4	0.745	9	0.862	11	0.769	61	0.806	27	0.809	15	0.774	9
0.849	8	0.738	12	0.878	21	0.786	88	0.752	40	0.873	26	0.856	12
0.752	11	0.781	16	0.803	31	0.781	110	0.798	49	0.733	34	0.710	15
0.812	14	0.731	27	0.766	60	0.802	189	0.747	89	0.854	55	0.822	30
0.800	14	0.758	29	0.817	51	0.792	178	0.802	73	0.772	52	0.795	34
0.797	23	0.843	43	0.757	85	0.762	250	0.784	99	0.838	79	0.862	45
0.803	76	0.784	143	0.784	273	0.782	942	0.779	406	0.816	287	0.821	154