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Quantitative Modeling of Operational Risk

By Leyla Korkut, Mengxue Wang, Raymond T. Perkins III, Siyi Luo, Vincent Hong Chen

Mentors: Dr. Boning Tong, Vincent Hong Chen, Dr. Avery Ching

CHAPTER 1 BACKGROUND

1.1 Definition of Operational Risk (Op Risk)

Operational risk can be defined in many ways. ***Each definition has its own nuances and characteristics however they often share some common ground***. An Operational Risk ("Op Risk") definition typically has the following attributes:

a. Op Risk has many risk types (i.e., human error, system problems, etc.)
b. Low frequency of occurrence
c. Often times Op Risk is not directly measurable and can be very volatile
d. Typically, analysis is qualitative as opposed to quantitative which is a more subjective approach
e. The same type of Op Risk could be very different from one organization to another

These attributes make any attempt to quantify Op Risks very difficult. This is especially true for companies that require a single methodology to be applied across subsidiaries existing in different countries and jurisdictions. This article is going to present a quantification model that conquers most Op Risk modeling issues.

1.2 Why quantify Operational Risk

1.2.1 Economic Capital framework

Economic Capital offers an interpretation of risks inherent in the company in economic terms. Op Risk is simply one of the important inherent risks that a company has. Op Risk quantification is unquestionably required to complete a comprehensive Economic Capital framework.

1.2.2 Enterprise risk appetites

Companies must thoroughly consider which risks and how much of those risks the company wants to take. This is often referred to as a risk appetite. With regards to Op Risk, risk categories have to be predefined based on a company's experience, current operations, new initiatives, and business strategies. Like other types of risk, the quantification of Op Risk provides an economic basis for informing the enterprise risk appetite.

1.2.3 Independent qualitative assessment

The Op Risk quantitative model opens the door for management to reflect their independent considerations and quantitative assessment. Especially for multi-national operations, Op Risks could be very different across countries even when they are under the same parent company.

CHAPTER 2 MODELING

2.1 Handling operational risk space

The large number of types of Op Risks poses a modeling issue and potentially leads to modeling error which can reduce model stability. Thus, it is important to prioritize each type of Op Risk based on its significance to the organization. Additionally, similar types of Op Risks can be grouped together in order to decrease the number of classes for modeling. For the example coming up shortly, Op Risks are grouped into four classes/categories:

Risk Category	Risk Subcategories
Compliance Risk	Financial Compliance, Transaction Compliance, Transaction Risk
Human Risk	Fraud Risk, People Risk, Systems Risk
Acts of God Risk	Extreme Weather Events
Institutional Risk	Business Risk, International Risk, Legal and Regulatory Risk

Note that, the risk grouping is subjective and can vary widely from industry to industry, company to company, and even subsidiary to subsidiary. Any change to the classification may lead to a significant change in the resulting numerical level of Op Risk.

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2.2 Data

In general, data used for Op Risk modeling is the incurred losses by risk type. Data collection is often challenging due to inconsistencies across industries, companies, and countries. Discussion of overcoming these challenges will be presented later in this article.

Data can be generalized into external data and internal data. The model described is going to use both external data and internal data to take advantage of the extensive coverage of data points in the external data and utilize the internal data to capture enterprise specific risk characteristics. External data can be acquired from outside vendors, while internal data is often collected from subsidiaries by running a risk self-assessment or historical loss inventory.

Here are several steps to overcome some of the data challenges often found in practice:

- Risk self-assessments can be designed in such a way that enables grouping into the classes defined in 2.1. Internal data can also be enhanced by studying the historical loss events in each subsidiary.
- Definition of the risk types in external data can be different and some data may be estimates. Data filtering is often required and the filtered data can then be grouped based on the classes defined in 2.1.
- Since the external and internal data may come in different formats, data transformation is necessary. The approach taken in the example below is to transform the external and internal data into appropriate and consistent data types upon which an empirical distribution can be built.

Before discussing data transformation, the concept of data Severity Points must be introduced:

Severity Points are used to form a consistent framework across different data types and different risks. In this framework data of different formats are able to coexist on the same data axis which allows for building an empirical distribution. Severity Points are determined by the frequency of occurrence, severity, and individual risk rating. Severity Points are the central point of all risk data around it, or alternatively, they represent nearby risk data for modeling purposes.

Transformation of Random Variables[1]:

Since the external and internal data sets have different scales, in order to match the maximum empirical data point in the external data of the same risk category with the maximum in the internal data, the transformation assumes X, the external loss, is a random variable and follows a Weibull[2] distribution.

The transformed variable Y is defined as Y=X/S, where S represents a scalar derived from

$$\frac{\textit{maximum severity point in the external data set}}{\textit{maximum severity point in the internal data set}}$$

By following the transformation process, Y also follows the Weibull distribution with new parameters defined by the original parameters as follows:

$$\lambda' = \lambda S^{\kappa}$$

$$\kappa' = \kappa$$

2.3 Model selection and calibration

2.3.1 Model Selection

Model selection is a circular process. The modeler has prior knowledge of the risks being modeled and typically has a good understanding of various statistical models. Sufficient considerations have to be given to the types and quality of collected data. Modelers should be fully aware of data limitations and select appropriate candidate models. Typical candidates are exponential, Gamma, log-normal, Weibull, and extreme value distribution.

The circle starts again to confirm and review the candidate models. Peer review and management discussion can effectively detect problems and produce a smaller group of candidates.

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After management and peer reviews, external data are fit to the candidate models and fitness tests are performed. These tests will determine the final model with a specific set of parameters for each specific risk category. The resulting distributions in the example are denoted (i: risk category):

$$D(i) \sim Weibull(\lambda_i, \kappa_i), \qquad i = 1,2,3,4$$

2.3.2 Model fitting and calibration with External Data (Global data)

Fitting and calibration is another iterative process that follows these steps:

a. Fit data to candidate models

b. Perform fitness tests and select model with the best fitting score

c. Use the resulting distributions, to simulate a loss distribution

d. Plot the loss distribution

e. Compare with the empirical distribution

f. Explain the shape and some key percentiles of the simulated distribution to see how well it reflects the characteristics of the risks being modeled

g. If step f is not satisfactory, go back to step b and use the model with the next best score

The resulting distribution can be improved sometimes with better modeling. Exhibit 1 and 2 demonstrates the remarkable improvement in the example when modeled with a Weibull-Weibull two regime-switching[3] distribution rather than a single Weibull distribution:

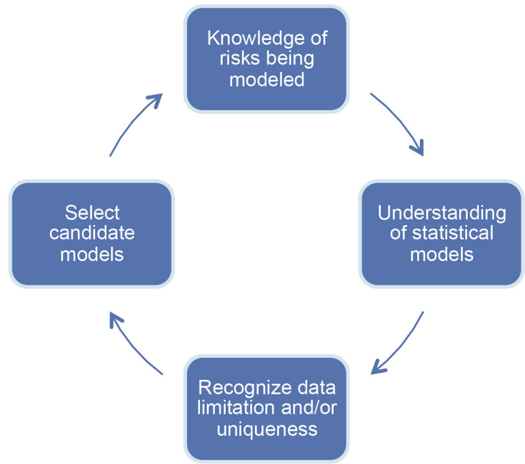


Exhibit 1
Fitted single distribution vs Underlying data

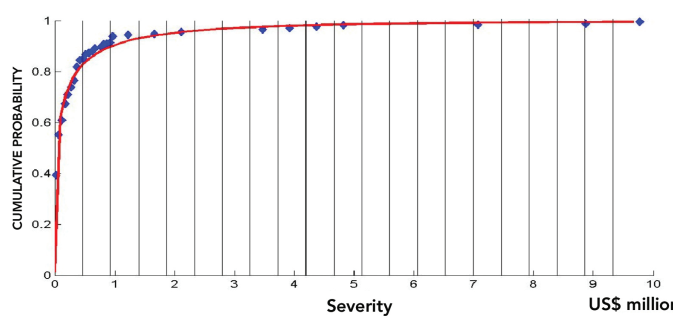
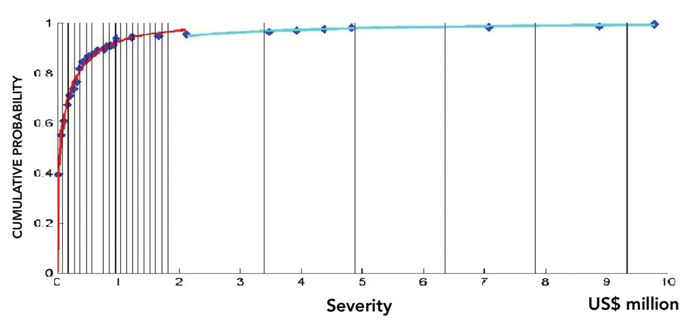


Exhibit 2
Fitted Two-Regime Switch distribution Vs Underlying data



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$$\begin{cases} D_1(i) \sim \text{Weibull}(\lambda_{1,i}, \kappa_{1,i}) & if\ p < 0.9 \\ D_2(i) \sim \text{Weibull}(\lambda_{2,i}, \kappa_{2,i}) & if\ p \geq 0.9 \end{cases}$$

2.3.3 Model Calibration with Internal Data

Once the general distributions of each risk category are identified, internal data are used to recalibrate the general distributions to country specifics. The recalibration process is used to calculate the specific parameters for each of the general distributions using the internal data. The process is similar to 2.3.2 except that only the parameters are altered to achieve the best calibration. The resulting distributions are:

$$D(i,j) \sim Weibull(\lambda_{i,j}, \kappa_{i,j}), \qquad i = 1, 2, 3, 4; j = 1, 2, 3, \ldots.$$

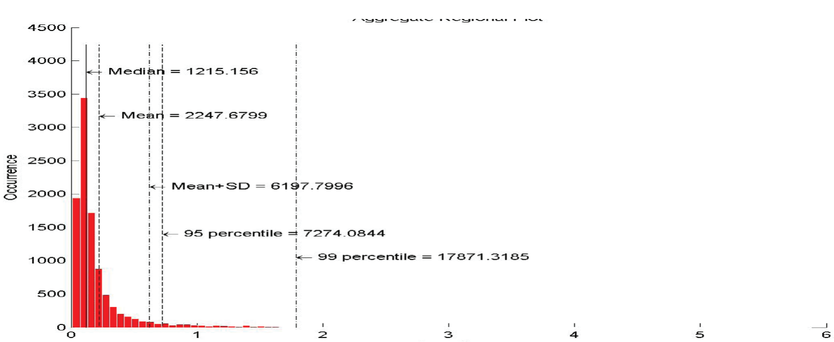
Or

$$\begin{cases} D_1(i,j) \sim \text{Weibull}(\lambda_{1,i,j}, \kappa_{1,i,j}) & if\ p < 0.9 \\ D_2(i,j) \sim \text{Weibull}(\lambda_{2,i,j}, \kappa_{2,i,j}) & if\ p \geq 0.9 \end{cases}$$

$i = 1, 2, 3, 4; j = 1, 2, 3, \ldots.$ i: risk category; j: country

The resulting distributions not only reflect the unique characteristics of the risks of the country, but also the general descriptions of the risks.

Exhibit 3
Aggregate Regional Plot



2.4 Simulation and Aggregation

Monte-Carlo simulation is used to generate the distribution for each of the risk categories. ***Managers can reflect their perceptions of the future into the simulated results by introducing a certain degree of skewness during the standard procedure.*** This feature allows for the inclusion of management discretion and stress/scenario testing.

Pearson correlation was employed in the example for aggregation purposes. The external data was used to calculate correlation of the different risk types. Gross Domestic Product (GDP) growth rate of different countries was used to calculate the correlation between the countries. According to Chernobai, Jorion, and Yu, ("*The Determinations of Operational Losses*") GDP growth rates are inversely correlated to a firm's operational risk. The methodology shown here is to start modeling with actual/estimated loss events and then to develop the statistical distributions. Any potential internal inconsistencies found when using GDP growth rates to calculate correlations between the countries must be well understood.

Along with Pearson correlation, Cholesky Decomposition[4] is used for aggregation. The aggregation takes the procedure as:

$$A(i,j) \xrightarrow{\rho\ (i:\,risk\ categories)} A(j) \xrightarrow{\rho\ (j:\,country\ subsidiaries)} A$$

A plot of the aggregated non-skewed example is given in Exhibit 3. The 99th percentile is 179 million, which is reasonable given the operations in the countries being studied.

The results demonstrate the following achievements:

- The model is generalized to capture industry risk profile of each individual risk type
- Modeled specifically each of the enterprise prioritized Op Risk categories independently
- Satisfied enterprise risk appetites

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- Allowed for management's risk view
- Distinguished country risk specifics

2.5 Pros and Cons

ADVANTAGES	DISADVANTAGES
• Populates a full loss distribution due to Op Risk • Associates loss amount with the probability of occurrence • Easily rolls-up to and is consistent with the economic capital framework • Can quickly and easily update the model with new information (data and other analytics)	• The classification of risk categories is subjective; the results may change significantly when the classification is changed • Assuming the joint distribution to be normally distributed may be incorrect • Not easy to understand for non-technical audiences

ENDNOTES

1 Transformation of Random Variable: http://math.arizona.edu/~jwatkins/f-transform.pdf

2 Weibull distribution: http://en.wikipedia.org/wiki/Weibull_distribution

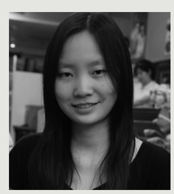
3 Regime Switch Model: http://dss.ucsd.edu/~jhamilto/palgrav1.pdf

4 See details at: http://en.wikipedia.org/wiki/Cholesky_decomposition

Leyla Korkut, is at CHG & Associates Company in Washington, DC. She can be reached at *leylakrkt@gmail.com.*

Mengxue Wang, is at Hong Kong University of Science and Technology in Hong Kong. She can be reached at *wmengxue1234@gmail.com.*

Raymond T. Perkins III, is a full-time Ph.D student at Princeton University in Princeton, N.J. He can be reached at *raymondp@princeton.edu.*



Siyi Luo, is a full-time Ph.D Student at Boston University in Boston, Mass. She can be reached at *sluo0418@gmail.com.*

Vincent Hong Chen, is risk director at MetLife Asia Pacific Limited, in Hong Kong. He can be reached at *vchen4@metlife.com.*