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NEW TEXTBOOK ON LIFE CONTINGENCIES

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Jordan's Life Contingencies is being replaced! This session, designed for non-students, will explain the need for a completely new book with a more modern approach; the new concepts of the book which will enhance actuarial education; and the applications of the new concepts. Actuaries who cannot believe that Jordan's text could ever become outdated should attend this session to hear the reasons for replacing it.

MR. JAMES J. MURPHY: This morning you will be hearing from two speakers; Jim Hickman, Professor of Business and Statistics at the University of Wisconsin who is one of the authors of the text; and Bob McKay, a consultant with Hewitt Associates in Lincolnshire, Illinois. Bob has been active in the E&E Committee for some time, serving as chairman of our Part 5A Committee. He then took over as Chairman of the Task Force that worked with Jim and the other authors reviewing the textbook. During that period he also became a General Officer of the E&E Committee and is now an Examination Vice Chairman of our E&E Committee. So he is quite familiar with the book and its development.

The presentations we will be making today will be in three parts. First, Bob will give a little background on the book, some of the whys and wherefores of the new book relative to Jordan, and the issues that received special attention from his Task Force. Following that, Professor Hickman will get into the book and a few "Big Ideas" from the book. Interspersed throughout Jim's presentation will be some time for questions and comments on each of the major ideas. Following that, Bob will come back to talk about the practical uses of the textbook. There should be some time left at the end for further general questions and answers on the material.

MR. ROBERT J. McKAY: I would like to welcome all of you to this session. It is clear from the size of the crowd that most of us have not forgotten that one of the foundations of the actuarial profession, and one of the things that separates us from others, is the ability to understand and to apply actuarial mathematics.

I would like to spend a few minutes reviewing the conclusions of the Contingencies Task Force, and addressing some of the issues which arose during the project. This should give you an idea of why the book was written and what it's all about. I hope we will encourage you to become familiar with it.

Even if you don't read the entire book or thoroughly study it, a familiarization with the text will allow you to ask the right questions and assign the right projects to the students working for you. Those students, after all, are tomorrow's actuaries.

The Contingencies Task Force unanimously endorsed the inclusion of the stochastic or statistical approach to contingencies on the Society's syllabus.

The new approach will provide actuaries with a deeper understanding of contingencies. It certainly did for me as Chairman of the Task Force.

After all, when you think of it, insurance and pensions are really nothing more than applied statistics. An insurance company sells policies at the same price to a group of policyholders having similar characteristics. The premium is based on the expected value of the payout to those individuals. If everybody died exactly according to the mortality tables, there would be no need for insurance.

It is because some people will die earlier than expected and because some will die later that the insurance concept is necessary. In other words, the time until any individual dies is a random variable--the mean or expected value has traditionally been considered in actuarial mathematics; however, this random variable also has a variance. The larger the variance, the larger the need for insurance and the larger the risk to the insurer.

We have always recognized this variance, albeit using very crude techniques. For example, in determining reserves, we use conservative interest rates, loaded mortality tables and age setbacks. Parenthetically, age setbacks have gotten us into trouble recently in the great unisex debate!

The difference is that now tools have been developed to allow us to analyze and measure the variance. This new text uses these techniques, which were not available 30 years ago when Jordan was writing his book.

Many actuaries feel that our profession is in danger of stagnating and being left behind by other professions. This new text should stimulate actuarial research and help our profession move forward. Most members of the Task Force felt that the treatment of mortality as a random variable was very enlightening; however, they also wanted to see what would happen if interest, lapse, salary progression, etc., were also treated as random variables.

Research in these areas will speed up, now that Actuarial Mathematics is being published. By modernizing the foundation of our profession, the authors are helping us get into the twentieth century before it's over.

Another advantage of Actuarial Mathematics is that it expresses actuarial concepts in statistical language. This will help us communicate with and be understood by other professions, and by the management of our client organizations who use statistical language and who rely on statistical analysis daily.

In our view, Actuarial Mathematics is a gradual evolution of the topic, although after reviewing the time chart that Jim Murphy showed at the general session yesterday, it is clear that the evolution is long overdue.

What were the major issues that the Task Force struggled with? I am sure that the authors struggled with the same issues when they were writing the book.

One obvious question was, "Aren't you making actuaries into statisticians? -- Why not leave statistics to the statistician and let the actuary do her or his own thing?"

We feel that contingencies and statistics should be brought closer together. After all, as I said, life contingencies is applied statistics. As Jordan said on page 7 on his text, the number of lives surviving to age one is a random variable. Unfortunately, at the time Jordan wrote the book, the concepts of mathematical statistics were just beginning to be published.

The approach in the text should provide all actuaries with a much better understanding of what contingencies are all about. This will become increasingly important in the era of unisex mortality, possible uni-age mortality, lower profit margins, more public disclosure, government reporting for pension actuaries, etc.

Tools developed in the nineteenth century which were primarily designed to minimize the computations involved in actuarial calculations are no longer appropriate in the late twentieth century when, for instance, every student at the Carnegie-Mellon Institute must have a personal computer.

In the past few years, the Education and Examination Committee has updated the mathematical core of the earlier exams to reflect what is currently being taught to undergraduates in mathematics and in business studies. The stochastic approach ties in very well with this updated core. In fact, students with today's background may be taken aback by the computational nature of Jordan.

I am sure that most people in this room feel that Jordan is a very good, very well written textbook. Most of us refer to it from time to time when we have questions on what formula is appropriate in a particular situation. But Jordan was written 30 years ago. If you lay Jordan and Actuarial Mathematics side by side, which one provides a better basis, a better foundation, for today's students?

The Task Force faced this very question. It is a truly important question for the education of tomorrow's actuaries. We concluded that Actuarial Mathematics is the better textbook in 1983. Our recommendation was approved after being reviewed by the Education and Examination Committee, the Education Policy Committee of the Board of Governors, and finally unanimously by the full Board of Governors of the

Society.

The textbook is a more generalized approach to contingencies, so it will be of interest not only to life actuaries but also to pension actuaries, to health actuaries and to casualty actuaries. As an example of the more generalized approach, the book incorporates risk theory with contingencies. They really are one topic of actuarial mathematics, and this is the first time to my knowledge that they have been combined. Too often when students see different topics in different texts or study notes, they don't relate them to each other.

A critical question that most of you would have, if you have actuarial students working with you or reporting to you, is, "Will the student be able to study from this text, or is it a very theoretical paper which may be read only by eight or ten people in the world?"

In answer to that question, I should report that no member of the Task Force had difficulty understanding it. The book has a great number of illustrative charts, graphs and tables to demonstrate points. These should give the student a feel for the underlying theory. There are a wealth of examples, and many of these are carried through from chapter to chapter.

Another aid to self-teaching is that the order of certain topics has been changed. For instance, in studying Jordan, many students get hung up on chapters 7 and 8, the cash value and population theory material. That has now been moved to Volume 2 so as not to break up the flow of the important theory. These points are extremely important. I mention them here to let you know that any of you could pick up Actuarial Mathematics and get a good understanding of the new approach without having to go back to a university to get your doctorate in statistics. It is a very readable book, and I urge you to at least glance through it.

The new text always uses the assumption of uniform distribution of deaths when approximating functions. This produces consistent results, and more accurate results in the era of high interest rates than did some of Jordan's approximations. For example, annuity values payable monthly are no longer approximated by Woolhouse's formula as they were in Jordan.

One of the members of the Task Force has recently been involved in the valuation of a Mexican pension plan using a 35% interest rate. The Jordan approximation for annuity values falls apart in this case because Jordan is based on an approximation with a truncated interest function.

Another feature of the text, which may be controversial to those who hear about it third hand, is that commutation functions have been de-emphasized. In Jordan and in earlier texts, they were an integral part of the text and were used to develop much of the theory. Commutation functions were necessary in the days before Apple computers and IBM PC's in order to minimize the calculations.

Today, they are much less important. They are presented in the new text as an alternate method of determining empirical values, but unlike Jordan they are included in a separate section of each chapter so as not to interfere with the theory. Many Jordan-taught actuaries think in terms of commutation functions. I am one of them myself. This is a hindrance to our understanding of contingencies.

Much of the material included in Jordan is no longer as important as it once was, for example, the family income benefit. Conversely, many things have happened since Jordan was written, for example, the boom in the pension industry. The new text updates material and changes the emphasis on many topics.

The Education and Examination Committee in the late 1970's felt that a new text would be necessary whether or not the stochastic approach was adopted. I am sure that C. Wallace Jordan did not expect his textbook to last 30 years. I know that the five authors do not expect Actuarial Mathematics to be on the syllabus of the Society of Actuaries in the year 2013.

One question already addressed, which I would like to re-emphasize to this group, is, "Can you read the textbook?" One of the reasons that it is traumatic to remove Jordan from the syllabus is that it is probably the best written textbook we have ever had. One of the reasons that Jordan is so well written is that the author really does not get deeply into the underlying theory. He presents a certain formula or concept and then spends the rest of the chapter manipulating it and changing it.

The mathematics in the new text is nothing more than a student would find in an undergraduate course today. Certainly it is consistent with what we are now including on the Society's syllabus. Finally, most students today are expecting and are really prepared for the stochastic approach to contingencies. The traditional approach is almost an embarrassment to the profession.

I would like to emphasize again that the text's wealth of charts, graphs, tables, examples, and exercises will make it an excellent teaching tool. I think this is critical in our profession, which relies so much on self-study. The textbook at the very foundation of our profession must be readable, and this one is.

One comment I have heard a number of times from people who were not involved in the Task Force, is that Jordan is tried and true, while this new stochastic approach might be a fad which might not be around much longer than the hula hoop. These people ask, "Why don't you stick with Jordan and introduce the probabilistic or stochastic approach as a study note to supplement Jordan?"

Well, we think 30 years is a long time to keep one textbook, and 100 years is a very long time to keep one approach. One thing missing from Jordan is the development of a theoretical foundation so that actuaries can understand contingencies and adapt them to problems of the 1980's.

In today's changing world, that is critical.

Jordan presents many formulas in many different ways, and in that respect, it is an excellent actuarial handbook. But today, when computers can store every formula in Jordan, we need more than that. The stochastic basis is ignored completely in Jordan, except for the one sentence which I mentioned. This is despite the fact that the only reason insurance is necessary in today's society is the random variation of the time until somebody dies. If it were not for this variation or fluctuation, there would not be any actuaries.

In order for any profession to survive and flourish, it must continue to develop, react and change. It is obvious that future actuaries will need different training than we received. This new text will form the basis for this training.

MR. JAMES C. HICKMAN: I should like to start by recording the fact that 1983 is the 500th anniversary of the birth of Martin Luther. There are two reasons why I want to record this fact. First, I do not want anyone to think that actuaries leave unnoted the great events in the march of western civilization. Second, I have a bit of sympathy for Luther. He also challenged the orthodox wisdom of his age, and although my challenge will be much more evolutionary than revolutionary, I feel a heavy burden. As a conservative, I have always believed the burden should be on the innovator, and marching in a distinguished line that contains people like King, and Spurgeon, and Jordan, the weight falls very heavily upon me this morning.

Let me tell you how I am going to organize my presentation. I will repeat a few of the things that Bob McKay has already told you and then begin a section called the Big Ideas. Every now and then in that Big Ideas section I will stop for your questions and comments.

The topic is models. As a child you undoubtedly built models, perhaps of airplanes and ships. Some of our engineering colleagues still make models in order to understand the performance of physical objects. You and I primarily build intellectual models. We build them with pencil and paper, or in computers. We build those intellectual models to help us design and manage insurance and pension systems. Model builders must be very humble, because all of their models, be they the balsa wood models you made as a child or the intellectual ones that you make on the computer, are only crude approximations of that complex world out there that we are trying to understand. We have to update models all the time as our tools get better and as the environment changes.

The topic of actuarial mathematics is intellectual models, either those that have proved to be useful in the design and management of insurance and pension systems or those which hold the distinct potential of having that ability. Keep that word model before you because that will be the theme that runs through all of this presentation. We will give an example of a model that is used in actuarial science.

Actuarial science, as you and I have known it, largely rests on the work of Edmund Halley. This is the Halley of comet fame who lived about the time of Newton and who constructed the famous Breslau life table. It was the first life table that directly influenced current practice.

The life table is an enormously useful model, but it is only a model. It does not capture all of the reality of the mortality process, and it certainly does not capture all the reality of the systems that we have built on it.

Why do we need a new book on life contingencies? The basic reason is that the world has changed. It does not stand still for us. Mathematics has developed. It is not the same subject that it was when King wrote his contingencies text in the last part of the 19th century. In particular, probability and statistics have developed. In the time line that Jim Murphy showed you yesterday at the General Session, you saw that crescendo of activity after World War II as ideas in probability and statistics found their way into the undergraduate curriculum. Mathématics, the tool we use in our model building, has changed, and we need to update our models for that change. Pensions and insurance systems have changed over the years since we solidified or created life contingencies. Social insurance did not exist. Many of the risks that we now regularly insure have only recently been recognized, or may not have existed until recently. Economic and social changes have weakened the stability of life insurance companies writing 20 Payment Life, Ordinary Life, and Endowment at 65. We live in a world that is vastly different. We have volatile interest rates, changes in mortality, changes in regulation.

Risk has also explicitly entered decision making. MBA's now regularly talk about risk and its measurement. Fifty years ago we talked about the diversification of investments. It was something to which we would tip our hat, but it was not a quantified concept. Now, thanks to the great insights into finance by persons like Markowitz and Sharpe, even beginning finance students worry about the quantification of risk in managing investment portfolios. In medicine, in defense, in all of those areas where we are called upon to make decisions, we now examine not only expected results but the degree of reliance that we can put in those expected results. Uncertainty, measured perhaps by variance, has now entered the mainstream of thought. Computational techniques have also undergone a revolution. In many ways, computation dominated early actuarial science. It also dominated early statistics, for that matter. Pick up a textbook of the 1920's and 30's and see the pages devoted to helping you solve simple regression problems, let alone multiple regression problems. Many, many pages were devoted to helping you do that awesome arithmetic.

The world has changed, however. Not only is computation many orders of magnitude faster, but it is cheaper. Computations are almost the only product or service I know of that have become cheaper in real terms. No longer are we slaves to arithmetic. No longer should simplicity of computation be a ruling principle.

Yet in many ways, computational techniques have been in the traditional foundation of actuarial science. Some of the developments that formed part of the books by King, Spurgeon, and Jordan were motivated by a desire for simplicity, rather than for realism. We do not need to do that any more. The need for a new text centers on the fact that the world has changed in all of these ways.

Notice, first of all, that the title of the new text is not Life Contingencies. This is true because the subject is not simply life contingencies. It is what we hope, at least as viewed here in 1983, can be identified as the mathematical foundation of actuarial science. Among the key features is the probability approach. We build on the development of the mathematics of probability, and the entry of risk into decision theory. Therefore, the foundation of these new study materials is a probabilistic approach. The second key feature is integration with Risk Theory. Risk Theory is a subject which used to be pushed off to the side. It is an essential proposition of the authors that Risk Theory is at the heart of actuarial science, and that life contingencies is an attempt to build models for the risk of the uncertainty of the time of death. We think that intellectually and educationally, it makes sense to integrate these topics.

A third feature is an attempt to introduce more of an economic foundation. We think that actuaries should be proud of what they do. They should believe that they are doing good for someone and not simply shoving papers around. We want to try to build an economic foundation for financial security institutions. We want to provide a basis for what actuaries do. What role do actuaries perform in the economy? What good is it to build insurance and pension systems and to manage risk? We think it does a lot of good. We think the world is better off because of what actuaries do. We want this economic foundation to be part of basic actuarial education.

The economic foundation also supports fundamental concepts like premiums and reserves. They are a consequence of a decision rule, an economic decision rule. We want to make it clear that there is an economic decision rule behind these basic actuarial concepts. You might adopt another, but you would have to defend the different rule.

Another feature is an attempt to be more consistent in assumptions. Since we are freed from the labor of having to do everything by pencil and paper, we can be more consistent in the assumptions that we use. We don't necessarily need to start with the uniform distribution of deaths and shift to another assumption because the arithmetic gets too complex. Some of the standard old assumptions need to appear, if for no other reason then to provide continuity with the past, but they no longer need to be in the mainstream of actuarial education. Consistent assumptions about fractional probabilities of death are now quite feasible.

In describing the new book and what it covers and does not cover, I will use the word boundaries rather than limitations. Your authors team has imposed boundaries on itself. One such boundary was to keep the mathematics consistent with what students learn in Part 1 and Part 2. For example, moment generating functions are in, characteristic functions are out. Riemann integration is in, Stieltjes integration is out. These may be scare words for some of you, but they illustrate the knife that separates advanced mathematical ideas from the type of ideas that we are covering on Part 1 and Part 2. No one can achieve the division perfectly, for Part 1 and Part 2 are also dynamic. Topics that will receive greater emphasis in these examinations may not be reflected in these new materials. That is why we have to keep loose in all of our educational ventures.

A second self-imposed boundary is that the estimation of parameters is not covered. That division is essentially the division that the Society of Actuaries made between Part 4 and Part 5 . The estimation of life tables, and the parameters that define them, are Part 5 issues. For the parameters that determine the other processes, such as interest processes, we are in a period of very rapid intellectual development. At this time the modeling of these processes is not in basic actuarial education.

A third boundary relates to computation. How actuarial results are computed is going to depend very much on technology, and more particularly on the technology available to the people who are doing the computing. For Actuarial Mathematics we are going to stick to models. We want to indicate some of the computational aspects, but these are not texts on computing.

A fourth boundary relates to practice and regulation. First, the study of these fields is not a goal of basic actuarial education. Secondly, these are very rapidly changing areas. Thirdly, the authors are not experts. Certainly we are not going to ignore practice and regulation. For example, there will be in the second volume a chapter on non-forfeiture values which we hope is up-to-date at the time that it appears. In addition, there will be material on expense considerations, pension funding and the actuarial mathematics of some new life insurance products. However, these texts will not be study notes on current regulation and practice.

Finally, although some readers may not believe this, we tried to avoid topics that are simply puzzles. Now "puzzle" may be difficult to define. Puzzles are fun. One person's puzzle may be another's basic model. Lots of us get our intellectual jollies through puzzles. Yet I think one has to be careful in stressing puzzles because they may not take us very far. Let me give you examples.

Like a purple cow, no one ever saw a stationary population or ever hopes to see one. We introduce stationary population ideas and the ideas of stable populations and dynamic populations. They are introduced not to solve average age at death problems, but to study the characteristics of funding methods for life insurance, social insurance, and pensions. The attempt to avoid puzzles does not mean that you cannot have a little fun along the way, particularly when it helps to reinforce topics in Part 1 and Part 2.

Does anybody have a clarifying question or comment at this point?

MR. E.J. MOORHEAD: I have come to this session with a completely open mind. I was brought up on Spurgeon, however, and I consider Jordan to be wildly and dangerously innovative. My question is this. Consider, if you will, two actuaries. One is a modern actuary of the kind that you have have been speaking so highly of. The other is a Spurgeon actuary like me. Let us suppose that they are in companies that are identical in all respects, and that both of them are working on, say, a premium for single premium deferred annuities. Does it follow that the modern actuary, since he or she will be taking into account some uncertainties that the old fashioned actuary did not, will necessarily come out with higher premiums for a single premium deferred annuity?

MR. HICKMAN: I would not use the word "necessarily", given the risk some of our colleagues have found in that product. I think the answer is that they may well have higher premiums, but not necessarily so. We cannot get into the minds of those actuaries and tell them what to do. What any particular actuary does will be influenced by that person's attitude towards risk, the company's position, and its ability to manage the risk. All that we can hope to do is to make actuaries better aware of the risks that are out there, and help them to measure those risks more precisely. It would be improper and impossible to predict how each of these newly trained people might react. We hope that they will be able to recognize the risks more completely, and that they will be in a better position to provide a rational premium structure for the product.

I would like to make another remark at this point. Last week we had an Actuarial Research Conference in Madison. Nathan Keyfitz, who has been a frequent visitor at these meetings, was there. Nathan's principal point in his keynote address was that in the past, technological change came at roughly the same rate as the generations passed. A person could reasonably expect to live out most of his working lifetime with one set of technology. That is no longer true. We must organize society and personal expectations to retrain people, including actuaries, once and maybe twice in their working lifetimes. The intellectual, computer and technological revolutions have made it impossible to have that comfortable concordance between the succession of the generations and technological change. Although I would like to think that what we have written would stand the test of time, it will not. The next student generation will have to be educated somewhat differently from the present one.

Now let us move on to the Big Ideas. When I talk about the Big Ideas, I want to make sure that you understand that none of us claim that these ideas are original with any of us on the textbook team. Like most ideas, they have been around a long time. We do believe that they will receive more emphasis in these materials than they have in earlier books. No one is putting down a flag and claiming that we are the first to think of these Big Ideas. Those many topics in Actuarial Mathematics that are common with earlier intellectual traditions are not going to receive much emphasis here. They are important, but it is my assignment today to stress some of the new ideas.

We start with the good old life table, that versatile tool of many disciplines, actuarial science, demography, biostatistics, and reliability engineering. It was the invention of Edmund Halley and remains one of the most fundamental ideas in actuarial science. One cannot discard the life table. Instead, one needs to point out its several interpretations. If you put your feet on the desk and read any of that great trilogy of King, Spurgeon, and Jordan, you will find a bit of waffling on interpretations of the life table. It is our proposition that we should start by stating that a life table can be viewed as a way of describing the distribution of the random variable time until death. You could describe that distribution with a probability density function, a cumulative distribution function, or a force of mortality. All of those are equivalent ways of describing the same distribution. A life table could be the starting place for modeling anything that can die, be it a box car, a person, or a space capsule.

Secondly, the life table can be interpreted as showing the expected progress of a random survivorship group. If you started a group of new-borns into life, subject to random time until death, you could look upon a life table as describing not the certain but the expected progress of that group. You could also talk about variability around that expected group size. You could even drop the probabilistic interpretation and simply state that a life table describes the progress of a deterministic survivorship group, one that marched through life with the Grim Reaper taking off members at exactly the right time. Interpretations two and three, somewhat muddled, have been at the root of quite a bit of earlier expositions of actuarial practice. Each of these ideas is developed in chapter three. The role of the life table as describing or defining the distribution of time until death is a very important one and helps link actuarial science to closely related disciplines.

The life table will still be a foundation stone of actuarial science. The point is that multiple interpretations will be developed. Emphasis will be placed on the role of the life table in describing the distribution of time until death. However, consistent with one of those boundaries that we mentioned before, the questions of estimating the parameters of this distribution will not be covered in this text.

The next group of chapters contains much traditional material. You will see formulas for premiums and reserves for single premium life, single premium annuities, and annual premium life. What is different? The difference is that these important numbers will be defined in terms of expected values. An economic principle will be used. The process is carried out for many, many different kinds of insurance and annuities in the text. The process will call for the student to formulate a loss variable. A "loss variable" measures the present value of the losses created by the random nature of time until death. We will illustrate with a continuous whole life insurance:

$$L = v^T - \bar{P}(\bar{A}_x)\,\bar{a}_{\overline{T|}}$$

"L" stands for the loss variable. The symbol v^T is the traditional interest function. $\bar{P}(\bar{A}_x)\,\bar{a}_{\overline{T|}}$ is the whole life premium rate times an annuity certain for a random length of time. The symbol "T" on v^T and on the southeast corner of the a stands for the random variable time until death. You will notice that the v^T captures the present value of the pay-out, and the negative term captures the present value of the pay-in. When T is small, the loss is going to be a large, positive number. It will become zero and then turn negative, i.e., into a gain function. We want students to think about the gains and losses of being in this risk business.

The next step is to apply an economic principle. There are many economic principles. However, the principle of equivalence is one that has actuarial roots and is used in many decision theory courses, often under the title of the "actuarial principle".

The second part of this process is, then, to apply the equivalence principle. This means imposing a condition that the expected value of the loss variable be zero.

$$E[L] = 0$$

E, as an expected value operator, means simply that you average the values of the loss variable over all of its possible values with the weights being provided by the probability distribution of time until death. A consequence of the principle of equating the expected value of future losses to zero is the traditional net premium formula.

$$\bar{P}(\bar{A}_x) = \bar{A}_x \div \bar{a}_x$$

One could have used as an example here almost any of the benefits that one might imagine. The process would still be the same. The student will formulate a loss variable, then apply the equivalence principle, and the result will be our usual premium formula.

If it all comes out the same, why do it? The reason is that by the formulation of the loss variable you have rooted yourself in an explicit economic principle. You also have available a model that will give you the ability to measure the risk of this venture. We can talk about the variance of the loss variable. The variance of the loss variable will capture at least part of the risk that the insurance organization is assuming. It measures the variability of losses, and if we can add up a bunch of these individual loss variables into a total loss variable, we can build a little model for a risk enterprise. This will not be for just one contract, but many contracts. Thanks to our ability to compute variances and other moments, we can approximate the distribution of those losses to quantify or measure the amount of risk that the issuing organization is assuming. Those of you who have read a little modern finance will recognize that our colleagues in finance have been quite active with these ideas. They compute variances of rates of return as a device to convey to the manager the uncertainty that is contained in a particular investment portfolio.

There are other aspects of this. Reserves come tumbling out of this same formulation. They are determined by taking expected values of loss variables, not for survival from age x, but given survival until age (x + t).

A reserve becomes an expected value of the same old loss variable, taking account of the fact that the life has now lived longer. This has the same advantages. Reserves become a consequence of a specific economic principle, the expected value or equivalence principle. Reserves gain intellectual roots in economics and in statistics. Also, variances are available. We could compute variances of those L's conditional on living to age (x + t) and be able to learn a little bit more about the degree of uncertainty built into the estimation of reserves.

Now it would be nice, for all kinds of reasons, if we could do even a better job. It is true that at this level, the uncertainty that we measure is mortality uncertainty. You may object and say that this is the least of our problems. You would be correct. However, this is basic actuarial education. Besides, we have not proceeded as far in modeling other processes as we have the mortality process. The self-imposed boundaries require us not to put ourselves beyond the frontier. In turn, this caused us to stress primarily the mortality risk.

To summarize, risk is now introduced naturally as the variance of loss variables. Net premiums and reserves follow from a specific economic decision rule. Actuarial mathematics becomes integrated with earlier study in Parts 1, 2, and 3, in statistics, probability, and decision theory. So the second of these Big Ideas, not new with us but receiving greater stress here than formerly, is that premiums and reserves can be defined in terms of expected values, and that there are advantages to this definition.

UNIDENTIFIED SPEAKER: Is there going to be any mention in your text of the other types of contingencies that go beyond the frontier that you mentioned, the kinds of things that students ought to be thinking about even though you do not try to quantify or describe them in this text? Students should appreciate that the mortality risk is only a piece of the picture.

MR. HICKMAN: The answer to the question is yes. We do mention other risks. The one that comes to everybody's mind is modeling the interest rate variation. Now for a moment I may say some things that not everyone will understand, but I want to try to answer that question. The questioner presumably has in mind the research, largely of the last decade, on modeling interest rate variations. These models are usually constructed with time series analysis using members of the ARIMA class that students are beginning to learn in Part 3. This research uses autoregressive, integrated, moving average models, and combines these interest rate models with life contingencies. Harry Panjer and Phelim Boyle of Waterloo, and several Europeans, have made some progress on that venture. I find these ideas extraordinarily interesting. The references are in the new textbook. However, the results are not picked up because they are still on the research frontier, and we do not have Parts 1, 2, and 3 to the place where everybody could follow them. In addition, the management of interest rate risk is intrinsically more difficult than managing mortality risk because of the simultaneous movements of interest rates for various types of investments.

With respect to some other uncertainties, when we get to the multiple decrement model, we put other decrements into the same framework that we have used in earlier chapters. Decrements due to disability, withdrawal, and so on are encompassed in the same model and the same principles are applied. Salary scales are mentioned. To the best of my knowledge, distribution theory for salary scales is even more primitive than that for interest rates. We only mention this problem.

The next Big Idea should probably be called a basic idea. Multiple life mathematics has been an important part of life contingencies. It tends to be hard. Examiners love the subject because they think that it separates the sheep from the goats. Is there anything new to be said? We think so. Multiple life functions start from an observation that relates to the earlier study of actuaries. The joint life status is the one that concerns two or more lives, where the status destructs when the first one of them dies. Stop and think about that for a moment. That is the same idea that statisticians call the first order statistic. As for the last survivor status, the one that destructs when the last one dies, statisticians have called that the largest order statistic. In fact, our Part 2 students spend a considerable amount of time learning the properties of order statistics. In the following formulas, I have illustrated only two lives. $T_j = \min\left[T(x_1), T(x_2)\right]$ Joint Life

$$T_s = \max\left[T(x_1), T(x_2)\right] \quad \text{Last Survivor}$$

The capital T denotes a random variable; $T(x_1)$ is for the life that has lived until (x_1) and the second life has lived until (x_2). When we talk about T_j, the j standing for the joint status, we are talking about the distribution of the first order statistic. When we talk about T_s, the random variable associated with the last survivor status, we are talking about the largest order statistic. We have simplified this in the formulas by only putting in two lives. If you are well grounded in Part 2, it is pretty much a matter of writing out the formulas to go beyond this insight.

You recognize, of course, that order statistics are taught in Part 2. Those of you who have studied order statistics recently worked with random samples, where the random variables were independently and identically distributed. While there is not much difficulty in making them no longer identically distributed, they must still be independently distributed. The initial expedition into multiple life functions exploits this interrelationship with order statistics.

What are the advantages? One advantage of the approach is that some of the basic assumptions made in the calculations and in the concepts are made explicit. The independence assumption between the times until death of the various lives now comes out front, rather than being suppressed. Also, we relate actuarial mathematics to other disciplines. We do not discard all that stuff we learned in Part 2, but we use it again to develop actuarial mathematics.

What are some of the other Big Ideas? Multiple decrement theory is a theory with many names. It is the study of two random variables, not just one. The variables are time until decrement, which is a continuous random variable, and the cause of decrement, which is discrete. In demography, the causes may be causes of death. In life insurance, they are death or withdrawal. In employee benefit plans, they can be death, withdrawal, disability, accidental death, and as many other causes as are recognized in the benefit plan.

We can also put the life table back together by adding the probabilities of decrement for causes. We have a bivariate distribution, one of the variables being the cause of decrement, and the other being the time of decrement. A whole host of ideas from probability and statistics now are at our command. Many new insights are available to us once we make that fairly simple observation.

What are some of the advantages of making that observation? Once again, risk considerations enter not as simply a footnote, but quite naturally. All of the ideas that we applied before to determine premiums and reserves are still applicable, the only difference being that the expected values are now taken not only over the time of decrement, but also over that second variable, cause of decrement. Those variances now are picking up both the randomness of the time of decrement and the randomness of the cause of decrement. It all flows quite naturally. It is not something new, but is simply a generalization, and in many ways a fairly modest generalization.

In this brief presentation, I have stressed the ideas that occur in chapters 3 to 10. The Risk Theory chapters, chapters 1, 2, and 11 through 13, have already been exposed and discussed. Most of the Big Ideas we have talked about today are from the chapters that are just now hitting the streets. Chapters 3 through 10 might be called the Life Contingencies part, or perhaps the Basic Life Contingencies part.

MS. ANNA RAPPAPORT: Is it relatively straightforward to use commercial statistical packages and build on them to do computations, or are there going to have to be some actuarial packages made available so that people can do these computations without a great deal of programming?

MR. HICKMAN: Standard statistical packages, by and large, will not work. I am talking about MINITAB, SAS, BIOMED. The reason that they do not work is because they do not have the interest function, the v^t, built into them. They are primarily directed not towards computing expected values but to doing inference. Somebody will undoubtedly make some money by developing actuarial software that will do the variance calculations stressed in Actuarial Mathematics. You will not be able to pick up the standard statistical packages and do them.

MS. RAPPAPORT: Will our students be able to pick up some kind of computational tools that will enable them to use these concepts, or are they going to have problems doing the computations?

MR. HICKMAN: Yes, they will be given some computational tools. There is an illustrative life table in the back of our book, augmented by some of the special functions that enter into the calculation of variances. There are a host of numerical problems in which students will compute variances and actually use them, although in a textbookish fashion. And the illustrative life table is designed to be realistic. The textbook will not, however, contain a set of computer programs, nor will any of the problems require the student to write a program. There will be considerable stress put on recursion relationships, in part because they lead to insights and in part because they are easy to compute. But the book will take no stand on what is the easiest way to do the arithmetic. There is plenty of work to be done.

UNIDENTIFIED SPEAKER: Since the textbook seems to concentrate primarily on expected values and variance, that would indicate that it assumes the normal distribution. Is there any attempt to introduce other distributions and how to recognize when some might apply?

MR. HICKMAN: This excellent question has to do with the stress on variance. After all, a normal distribution is specified by its mean and its variance, but other distributions may require more parameters to be specified. The stress on mean and variance, therefore, seems to imply a concentration on normality.

The answer to the question is, of course, in the Risk Theory part of the text. These sections will be integrated with the other material. Rather considerable stress is placed on other distributions and ways of identifying these distributions. It is true that in the Life Contingencies part, emphasis is placed on variances, and some of the problems do require you to use an extended central limit theorem (that the sum of not identically distributed but independent random variables is approximately normal) to solve problems. There are also some problems which ask students to compute medians, and means, and to make direct probability statements that are illustrated with non-normal distributions.

On the general topic of distribution of total claims, the non-normality part is mainly what you will see in chapters 11 and 13. Chapters 3 through 10 will occasionally contain formulas to compute higher order moments, which might be useful with other distributions. There are no exercises on, say, fitting from the Pearson family or on distributions using higher moments. However, higher order moments are sometimes called for, and the formulas for them are exhibited.

We will introduce additional ideas in the second volume. Expected expenses and profit objectives are introduced by using augmented loss variables. This is similar to what you saw before. We have added a term for the present value of expenses, and offset it with a gross premium instead of a net premium. The development now becomes quite natural.

$$L = v^T + \text{PV Expenses} - G\bar{a}_{\overline{T|}}$$
$$E[L] = 0$$

We have gone from models using the life table to multiple decrement models, and now to loss variables with expenses. However, the same principle, the equivalence principle, is used in determining premiums and reserves. We calculate the expected value of the augmented loss variable, set it equal to zero, and solve for the gross premium G. We have determined a G to provide for benefit payments and expenses, but we also can compute variances of the loss variables quite easily. This will be pointed out to students to help them get some idea as to the variability of the results. The important point is that the development is a natural extension of what was done earlier.

The same process of using these augmented loss variables, augmented in the sense of those expense items and loading items, produces asset shares rather than reserves. Thus, asset shares emerge as conditional expected values, as reserves did, conditional upon survival a while longer. Variances of asset shares can also be computed. The main advantage is continuity with the ideas introduced elsewhere, an attempt to be explicit on the economic foundations, and an attempt to make at least a first halting step towards the quantification of uncertainty or variability.

There is also a fair amount of stress on recursion formulas. That is a fancy name for the formulas which relate successive reserve or asset share values. We emphasize the fact that these are the foundation of insurance accounting, pension accounting, and gain and loss analysis. Here again, this is not a new idea; in fact, it constitutes a significant amount of the educational work on the higher exams. We are trying to introduce it with very simple examples at a basic level to show that these ideas in actuarial mathematics carry over naturally into analyzing gains and losses, and serve as the foundation for most of our accounting. Somewhat the same thing can be said with respect to non-forfeiture values. All of the paraphernalia that we developed in the multiple decrement chapter is used as an introduction to non-forfeiture values. It is our belief that this early theoretical development, rooted in the multiple decrement material, will help students understand some of the mathematical, philosophical, and technical problems in this perplexing area.

There are many other things that I am not going to tell you about because of time limitations. There are other chapters that will go into additional areas. I must confess that we have sometimes called one of these chapters "junky premiums". It covers those more complicated benefits, everything from variable life (equity-based variable life) and variable annuities, to income endowments. There is a chapter still in the works which will extend the multiple life material introduced earlier but not developed to its full extent. There is also a chapter on population dynamics, covering not only stationary populations but also stable and dynamic populations, integrating these into pension planning, pension funding, and life insurance funding. Our goal in all of this work is to provide a basic intellectual foundation, as of 1983, for actuarial science. There are sound practical reasons for this updating, but I can speak more authoritatively on the educational and intellectual reasons for making the shift. I find them quite compelling, and I do not apologize for this challenge of the orthodox approach.

MR. McKAY: We've talked for over an hour about this new text, but there are only half a dozen people in this room who have actually looked through the book. Chapters 3 - 10 can be obtained for $23, through the Society's office.

I mentioned earlier that while the contingencies Task Force was reviewing Actuarial Mathematics, several important issues were addressed. Probably the most important was whether the new book offers anything more for the practicing actuary than does Jordan. In other words, does the new book offer techniques, approaches and insights which will be of use and interest to all of us here, or is it merely an interesting academic exercise?

About a month or two after the Task Force had completed the bulk of its work, I contacted the members and asked them if they had used the new techniques or were intending to use them in their actual day-to-day work. Remember that the members of the Task Force were practicing actuaries, the majority of whom had no prior involvement with the Society's Education and Examination Committee.

Most of the members responded with examples of how they had used, or intended to use, the new approaches. Here are four examples which demonstrate how this new approach or the new approaches can answer questions that most of us have been trying to ask for a number of years but have not had the tools or the language to ask.

The first question came from an actuary at a life insurance company that specializes in writing ordinary life annuities. In light of the Norris decision and possible Congressional action, he feels that his insurance company probably will be forced into a unisex pricing structure for its annuity products. He, therefore, wanted to know what the increased risk to his company would be due to changing to a unisex mortality basis for pricing annuities.

As you know, annuities for males are traditionally priced, not surprisingly, using male mortality experience, while annuities for females are priced using female mortality experience. When you combine the two mortality tables or the two mathematical distributions, the variance of the mortality rates will increase.

Most discussions about developing unisex mortality tables or bases center around combining the male and female rates. However, statisticians will tell you that when this is done, the risk increases. Insurance companies must be aware of this increased risk if they intend to remain in business in the 21st century.

Based on a straightforward application of one of the formulas in Actuarial Mathematics and using the assumption of a 60%/40% male/female mix, he concluded that the variance would increase 93%--almost doubling! This result means that additional margins will be needed by insurance companies when they offer annuities using a unisex mortality assumption.

A second question came from an individual who consults with insurance companies on pricing their products, setting reserves, etc. If insurance were invented today, I think this particular question would be asked of actuaries by the executives and boards of directors of insurance companies and by the regulators. Because insurance developed before the statistical language was common, we don't hear this question frequently. I think in the next few years we will begin to start hearing it.

The question he asked was, "If I offer a life insurance policy, what gross premium is required so that the probability that the insurance company will lose as a result of random mortality fluctation is under 5%?" Or, what premium is required so that 95% of the time my premium will cover random mortality fluctuations?

Again, using formulas straight out of Actuarial Mathematics, the appropriate premiums can be developed. The actuary involved concluded that if the resulting premiums were competitive, they would be used. If they were uncompetitive, the results would at least give an idea of the vulnerability of the pricing structure.

The third question came from a pension actuary. One of the most common questions I get as a pension actuary is, "What payments should my plan expect to pay out over the next 10, 20 or 30 years based on my current retirees?" The investment manager for the pension plan uses this information to set investment strategy. In particular, it gives the investment manager an idea of the degree of liquidity required at various points in time. This question arose quite frequently in the last couple of years with the tremendous interest in bond immunization for retired lives.

The solution to the question is, of course, very simple. All of us here today can run out a fixed group of retirees looking at the probabilities of mortality and come up with a pretty good estimate of the payout.

The problem with this approach is that if you have a hundred groups of retirees, the average payout should equal the pattern you're going to be showing your client using this approach. However, in some individual cases the actual payout will be significantly different from that shown by the Jordan approach. Again, the problem is that traditionally we ignore the variance and focus entirely on the mean or expected value.

A student who has read Actuarial Mathematics would be more inclined to ask the question, "What is the expected payout over the next 30 years, and what is the 90% or 95% confidence interval of the payouts?" In other words, what range of payouts could be expected?

This additional information, the confidence intervals, will give the investment manager much better data for determining what degree of liquidity is needed or might be needed. Without this information, the investment manager is making decisions based on techniques developed a hundred years ago in order to minimize mathematical computations.

The last question or last approach did not come from a member of the Task Force but from one of the members of the Board of Governors of the Society, John Montgomery, the actuary for the California Department of Insurance. He is involved in determining the future direction of financial reporting. At the same time that the new text was being written, John and other individuals with similar interests were moving along a path which was in some ways parallel to that of the authors.

Their study concluded that level 2 contingent liabilities should be developed using probabilistic or stochastic concepts. I recently talked to John, and he was very pleased with the introduction of the new text because it helped him express ideas that he had had for a number of years on the variability of reserves. Thus, he is immediately putting to use some of the new information that has been reflected in Actuarial Mathematics.

We find it encouraging that the individual members of the Task Force, who volunteered primarily out of a sense of professional responsibility, have immediately put some of the information that they learned to use in their day-to-day work.

It is inevitable that as students study the text, they will find more uses for the information beyond those that we have talked about today and beyond those contemplated by the authors. In fact, the five authors have said all along that they do not know all of the uses for the new approaches and concepts. However, by making them available to our students, we cannot help but stimulate research, stimulate practicing actuaries and prevent our profession from stagnating.

MR. MURPHY: Are there any questions now, general or specific, for either Bob or Jim?

MR. GORDON LEAVITT: You have said now that reserves should be viewed as expected values. I have enough trouble explaining to other people what a life insurance reserve is, the figure in the Annual Statement. How do you bridge the gap between somebody who understands it the old way and the new way of understanding it?

MR. HICKMAN: Let me start. It is true that the ideas of insurance reserves are subtle. Insurance company and pension fund accounting are not simply cash flow accounting. There is much more to it than that.

Expected values, however, are increasingly entering business education. It would be false to say that they have rubbed off on all students, but everybody who goes through a business school in North America today will have been exposed to the idea of expected values. They would have had them in statistics, finance, and insurance classes. This is the language that is already being used in business education. It is already being used to some extent by our colleagues in finance. The language of expected values may not make explaining reserves harder, because you have a more precise framework in which to explain what is going on. However, it would be false to say that simply spouting the words "expected value" will now cause everybody to say "Aha! Now I see!" You are still going to have to put some ideas on paper, and talk and visit, to help get the idea across that reserves are an estimate of what we will need to balance the future flow of payments.

MR. McKAY: Jim Murphy mentioned yesterday in the General Session that Actuarial Mathematics uses a new language to develop familiar formulas for q_x, annuities, reserves, insurance formulas, etc. Since you can get to the same formulas, an actuary who is brought up on the new book and an actuary who was brought up on Jordan can communicate with each other. They can come up with the same result. The difference is that the person who studies Actuarial Mathematics can then take that information further. Not only can that person calculate the reserve, for example, but also the variance of that reserve or the variance of the premium.

Thus, the two actuaries can still get to the same level and communicate with each other, but one of them can go farther with the information. At a minimum, the new actuary is going to be asking some new questions and will know the limitations of the information he or she has.

MR. HICKMAN: I think that the reserve issue is a good one. We want to get across the idea that there is variability in what we are trying to estimate.

UNIDENTIFIED SPEAKER: I think you will have to consider that when a figure appears in an Annual Statement, you are looking backward at it, and to call it an expected value is incongruent, to say the least.

MR. HICKMAN: That may be the conventional wisdom of the past, but I think that accounting is moving rapidly to a stochastic approach. You saw it here in the John Montgomery example. Also, the Trueblood report of a decade ago from the American Institute of Certified Public Accountants gives you some of the thought patterns that are coming in accounting too.

MR. CHARLES ORMSBY: I would like to ask if the new text includes mathematical tools or techniques for answering the question of what retention limits a company should adopt for individual ordinary insurance?

MR. HICKMAN: It does much more than anything we have had before, because of the integration of life contingencies and risk theory.

However, it would be false to claim that there is a numbered equation that everybody can pick out and use to answer the important question you propose. I think that there are many more helpful tools in the texts than we have had in basic actuarial education up until now.

MR. ORMSBY: How about cost benefit analysis in the underwriting area?

MR. HICKMAN: The answer is no. This textbook is basic actuarial mathematics. The question of cost benefit analysis in underwriting is a beautiful decision theory question, which interests me greatly, but we ruled it out explicity. There are some words in there about how you select a life table, and the importance of matching your selection standards with what you have assumed. But as far as the mathematics and details of a cost benefit analysis in underwriting, the answer is no.

MR. McKAY: Those were both excellent examples of the type of things that we will now be able to work on. This book appears in the Associateship syllabus and gives the mathematical theory of actuarial science. The Fellowship syllabus is designed to give practical applications. With the language that is now available, and with the new techniques, the practical questions can be approached using modern mathematical tools. The next step we see is to start working on some of the Fellowship topics to bring them into the 1980's or 1990's. I am also sure that a lot of people here, and other people who are exposed to the book, will come up with valuable ideas that nobody has thought of yet.

MR. FRANK WECK: Going back to that question on the reserves and the Annual Statement, it seems to me that it is very difficult to explain to the layman what a reserve is, if you talk of it in terms of expected values. It seems to be more useful to keep in mind that the reserves are a schedule which measure the funds to be retained year by year in order for the company to carry out its obligations with regard to future risks and payments. If we get beyond that, we get into an area with which people are not familiar, and they come away with a feeling that this is something that can change from time to time. I do not think that this really is the correct way of looking at statement reserves.

MR. MURPHY: In response to your observation, I would like to refer to one of the things I already mentioned as a practical application. It relates to your statement that the reserve is a given which you have, no matter what variations you have experienced from the original assumptions. It is in your Annual Statement, and your assets are behind it. Another element of the life insurance company Annual Statement is surplus. One of the questions the life company has to deal with is how much surplus it should have. The tools that will be available to us through this textbook should help us to answer that question with a better understanding of the reliability and confidence that we can place in the amount of surplus we have. This is one place where reserves as a given, as opposed to the variability concept in the textbook, come together in answering another question.

MR. McKAY: Another member of our Task Force came up with a suggestion for using the new approaches. He works in Quebec, where their auto insurance is mandatory. Much of it is under the program of the provincial government. Every year a reserve is set up for disability claims under automobile insurance. The number is large, amounting to several billion dollars. He suggested that instead of just giving a number like that to the authorities and the financial planners, a range should be given. The statement would be that there is a 95% probability that the number will be between two numbers like 3½ billion and 4½ billion, instead of giving one number that all of us know will not be right. The range is really there, and we need information about it. We should be using it today in our planning.

MR. EDWARD SHUR: I think the direction of the new textbook is a welcome one. This attempt to bring the Society into the 20th century is certainly a good one. No doubt there will be some reluctance or opposition on the part of many of the older members because they do not have particular facilities or knowledge of the developments that are being publicized in the textbook. But I think it would be ironic if the actuarial profession, which has a strong background in mathematics and statistics, did not use these implements, when some other professions which have little or no background in them are beginning to employ them much more.

An example is the medical profession. Anyone who reads the New England Journal of Medicine will find references, actual printed statements, about the use of Bayes Theorem. The medical diagnostician, for example, has a patient with several possible diseases that could have given rise to the observed symptoms. He must identify the one that is most likely present. This is a prime example of the application of Bayes Theorem. Also, many scientific studies are reported in that Journal, with specific indication of the variances that are involved. I think it would be particularly ironic for a profession such as the medical one to use these statistical tools, while the actuarial profession was not using them also to its full ability.

MR. RICHARD SCHREITMUELLER: I have several observations on those four examples that were given, to show that the new techniques are an improvement, or could be. On the first example about the pension pay-out, there is an implication that we really know the underlying distribution of the mortality. I am not sure that we do, or that the person using the results would appreciate that. There is a kind of spurious accuracy, or least the perception of one. Another example is the unisex example, where you have to state the male/female mix of 60/40. We really do not know what the mix is, so that again there is a problem in that area. Then you get into the third example about keeping the risk of loss under 5%. That is true if it depends on the mortality, and if you know these other things, but if you have economic parameters that have a greater impact, where does that leave you? I am not trying to say that the techniques are useless, but I wonder whether we need to have a greater appreciation of these things, or better examples, or both.

My second observation is that we have a challenge to put these things into practice. I think that we have a way to go on that. And finally, I read in the paper this morning that there was a lady who won a Nobel Prize for some very good work that no one appreciated for a long time. I hope that this will not be the same kind of example. I hope that we will appreciate it very soon.

MR. J. EARNEST BOOHER: I would like to point out that there is no item in the Annual Statement that is not an estimated item. Even in the case of cash, you do not know that you are going to get that cash. Your bonds, your stocks, all your assets and all your liabilities are estimated items. Many of the reserves that you talk about are statutory requirements. I would now like to point back to some history of 50 years ago, when interest rates were dropping from 6% down to 3%. It was required at that time to take reserves on policies already in existence and strengthen them to meet obligations if the situation continued. This tool here seems to me to be one which would tell you or assist you in making decisions as to what to do.

MR. JOHN E. FOLEY: I can see where the use of the variance as an estimate of risk on individual random variables would be helpful in the pension field. But where we are also concerned with things like the total present value of future benefits divided by the total present value of future earnings, it seems to me that the variance on that quantity would be a lot more difficult to calculate. Does the book cover anything other than individual random variables, or their sum?

MR. HICKMAN: The specific question that you asked is an excellent question. The answer to it is no. The funding chapter reverts pretty much to expected values. I will call your attention to another item that is on the syllabus now. Behind your question is an important fact, that the calculation of the variance of a quotient of two random variables can be very difficult. The Elandt-Johnson and Johnson textbook gives some approximate formulas for this variance. The basic mathematics is in Elandt-Johnson and Johnson and the new texts. However, we did not carry out the particular example which you mentioned.

MR. McKAY: One of the reasons that we do not carry it out is that we did not have this mathematical foundation before. Pension mathematics and other actuarial mathematics develop from the basic textbook, be it Jordan or Actuarial Mathematics. Because we are bringing the basics more up to date, the other factors will begin to be considered and analyzed.

There were a couple of other questions asked about the economic factors being more important than the variability of mortality. That is probably true. However, we need to start somewhere, and it seems better to get some information about variability than none.

MR. GARY B. ROSEN: I have a comment regarding Mr. Ormsby's question on retention limits in the new textbook.

Earlier this summer, my company considered raising its retention limit from $750,000 to $1,000,000. Using the new Risk Theory study note, we determined that there would be some cost savings to us if we raised our retention limit. However, because of the significant increase in the variance of our retained claims, we decided against raising our retention limit.

Thus, there are techniques in the new textbook which can help you to determine the proper retention limit for your company.

MR. FRANK ALBERT: I have a comment, and two questions. The comment is that the last change that we made took place rather slowly. I can recall clearly when I started as a student, maybe the first or second year that Jordan was out, older actuaries said that the only way to learn life contingencies was King. The first question is this: Under the old model of looking at the life table, correctly or incorrectly, it was frequently viewed that the main question was how many people would die or be expected to die at a certain age. This led directly into the kind of analysis and exposure theory and the determination of mortality rates that we learned in Part 5. Now where the prime question is "time until death", will this lead as well into an exposure theory to develop the parameters?

MR. HICKMAN: It certainly opens up a lot of new insights. You can still estimate the distribution of time until death by estimating those conditional probabilities of death. To estimate the q's, conditioned on living to age x, you break up the distribution into little pieces and estimate each one. But now the ideas that we used to have at the back of the book, namely Makeham, Gompertz, and other distributions, can be revitalized. The estimation of survival functions can be an estimation problem, like in Part 2.

I think that this observation leads to sharper thinking about the estimation of a life table. You can use the actuarial technique and break the problem up into some sub-problems of estimating conditional probabilities. You also can view it as a classical estimation problem, and use maximum likelihood or another estimation method with a family of distributions. I think that the alternative is a little clearer once you have seen the multiple interpretations of the life table.

MR. ALBERT: The second question is when will the book be available in a bound volume?

MR. MURPHY: We are introducing the book in the 1984 syllabus beginning with the May Part 4 exam. We have found in producing major study materials that one of the best editing groups in the business is our students. They find more errors than anybody else. We want to make sure that the material gets sufficient exposure to the student group to help the authors make sure they have done the job they want, and to make sure that in the printing we have not missed too much. I would expect that we will want to expose it through 1984 and maybe part of 1985 on this basis. It will probably be in late 1985 that we will actually publish a hard-bound volume.

There are also some questions that we are still looking at. We want to put some of the material from the second volume on the Part 4 syllabus. It is still basic enough to be introduced there. We are still deciding whether certain subjects should be in the first volume rather than the second. The best estimate for publishing the book is late 1985.

MR. RICHARD P. PETERSON: This book is the first one I have ever seen that attempts to get some social justice into calculations. I challenge any actuary in the world to place a U.S. dollar value on my life, based on all of my insurances, my annuities, my Social Security, my benefits etc., and so on, or that of my joint tenant spouse involved with earned rights that have been accrued. The U.S. Federal Court proceedings are on file with the Society of Actuaries and the American Academy of Actuaries. Again I challenge anybody to place any value on all the life benefits of that and my spouse, and also on liberty in the United States.

MR. PAUL G. SCHOTT: With the introduction of this new book into the syllabus, should there and will there be any change in the risk theory syllabus?

MR. MURPHY: No. The risk theory syllabus that we have now on the exams will be on Part 5. The Course of Reading consists of the chapters that were initially developed for this book. They were ready sooner, so we used them to replace earlier risk theory material. So actually Risk Theory is already there, ahead of the final publication of the book.

MR. McKAY: The new material is chapters 3 through 10 of volume one. The Risk Theory material is contained in chapters 1, 2, 11, 12 and 13. This has been on the syllabus for a year and a half.

UNIDENTIFIED SPEAKER: For the person who is a fairly recent Fellow, and then for someone who has been a Fellow for 15 or 20 years, what should our approach personally be to the new textbook? I cannot imagine that I should go through and read it and work all the problems. On the other hand, I wonder if ignoring it would be very wise. I would appreciate comments from various people on the panel.

MR. MURPHY: First, by attending this session you have taken a big step towards approaching the textbook. Our approach to this kind of session, and to the one day seminars associated with the spring meetings, is to design the programs for practicing actuaries who are not going to be studying this textbook for exam purposes, but who are already Fellows or Associates beyond the Part 4 level. We very much hope that practicing actuaries will seek every opportunity, such as these seminars, to become aware of the book, and of the tools and the power that they present.

The other approach you can take is to encourage your students, who have been and will be getting more powerful tools at their disposal through the syllabus, to approach the questions you give to them with that background. Encourage them to present you with ideas and approaches that take advantage of what they are learning. By coming here, or going to the seminar next spring, you will be better able to encourage this. We think the book is evolutionary, in terms of its comparability with traditional results. We do not expect you to be uncomfortable communicating with your students.

MR. McKAY: One more approach is to read it. If you have gone to one of the seminars, and you have attended this, you will be off to a running start. This book is readable. You may not want to do every problem in the book, but it is something you can pick up and work through. Because of its critical importance to actuaries, I encourage you to do that.

MR. HICKMAN: The question that you ask strikes at the very heart of the whole continuing education effort by the Society. In a certain sense that is the life blood of the Society. All of us in our own personal development, and also in planning Society affairs, have asked the same question that you ask. Our time is limited and it is difficult for us, individually and as people involved in Society affairs, to know just how to organize it. Part of the answer is going to be forced on us. That is, the world is going to force some of these issues on us. As more of those MBA's come out who are used to talking about quantification of risks, and they assume management positions, you will need to communicate with them using this language. None of us can give you a neat answer to the question as to how much time you should spend studying, given your particular job, background and position. It will evolve because of the things that are forced on your company and are compelled by the need to communicate with your own students. I am convinced that much re-education will be required in your practice in your lifetime. This is the way the intellectual and business world is going. We are not going to be able to live out our professional lives simply using the tools that were on the early exams when we went through them. The particular tools you must master I cannot predict. But I am positive that you will have to master some new ones. The probability is quite high that they will be based on at least some of the ideas in the textbook.

MR. GODFREY PERROTT: First of all, I think the new textbook is a great stride forward, but probably only one stride, in what actuaries need to do to stay ahead in the field we regard as our own profession. I offer this example. In 1972, I worked quite hard with a prominent accountant when GAAP was being developed. He was in awe of the tools that actuaries routinely used that might be applied to business problems. In 1982, my wife went through an MBA degree, and I was awed at the tools she was learning in traditional actuarial fields that we ought to be learning. This books starts to address that. It is a real step forward.

MR. CLAUDE PAQUIN: I would like to echo some of the comments made earlier by Dick Schreitmueller about the inexactness of the basic mortality tables, and as a result, the inexactness of the variances that might be derived from them. It might be instructive for us to consider how some of the problems that we tackle are perceived at some levels that we sometimes do not expect. On June 15th, 1983, the United States Supreme Court rendered a decision, the title of which is Jones and Laughlin Steel Corporation versus Pfeifer, with the following citation: ----US----, 103 S.Ct. 2541, 76 L.Ed. 2d 768 (1983). In this case, the United States Supreme Court considered the estimated value of the lost earnings of a disabled worker. The U.S. Supreme Court cautioned against delusive exactness. Now it seems to me that in life we have the choice between total ignorance, what we might consider total enlightenment, and partial enlightenment. I have the impression that the new book will not give us total enlightenment because I believe that it is beyond us. Nevertheless, I think it takes us a good step along the way. It provides better partial enlightenment than we had before, and I believe it is a very progressive step forward.

MR. ROBERT K. DICKSON: I want to address the fact that in pensions, we will not be able to charge more than the expected value to the statement of the employer; in the Annual Statement of insurance companies we will not be able to put another figure in for the expected value as a reserve; and finally we will not be able to charge to group or individual life insurance policyholders more or less than the expected value of what it costs. I wonder, therefore, what will be the practical utility beyond the level of confidence or risk of the new book?

MR. McKAY: The last point is important. The book gives you an idea of the vulnerability or the variability of the calculations. The legislation and regulations behind reserves on financial statements have been developed based on Jordan and Spurgeon mathematics. Now that new approaches are available, the legislators and regulators may say that the old methodology is not appropriate, that there is another way of reporting reserves, that one reserve number is not necessarily best. There is, in fact, a range of possible reserve numbers. The example of reserve considerations by John Montgomery in California shows that regulators are already thinking about new approaches. We have used one type of expected value reserves. Within a number of years, the current way of reporting reserves may change.

MR. HICKMAN: Recall the extended discussion in the early 70's about GAAP reserving. What we called "Delta Reserving" was to build in contingency margins and assumptions. Although the mathematics is a bit different, the ideas are closely related. At that time there was a lengthy dialogue with the accounting profession on the risk nature of insurance, and the necessity for those margins for adverse deviation. This is just another step in that same evolutionary process of helping our public and ourselves understand the risk nature of our business.

MR. MURPHY: You saw the example that Bob gave from John Montgomery. A state insurance department, and the NAIC, are already looking at aspects of Annual Statement requirements that would be related to variability and risk. When we set premiums we set margins in those premiums. These kinds of figures will help you determine what those margins ought to be. I mentioned the surplus example, but there are many others.

I would like to close the session with three comments. First of all, I would like to recognize one of the other authors who is present today, Cecil Nesbitt. Second, I would like to thank you all very much for coming. We appreciate your interest. We hope that you will spread the word about the textbook, and learn about it. And third, I suggest we can all have a little fun with our friends who did not attend today by telling them about the waiver which you all received today for the continuing education requirement which we are putting in for Actuarial Mathematics! Thank you all very much for coming.