An out-of-sample analysis of investment guarantees for equity-linked products: Lessons from the financial crisis of the late-2000s

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Introduction

• Insurance companies have been selling investment guarantees in many insurance products
  – Segregated funds in Canada, variable annuities in the US
  – Universal and participating policies
  – Other equity-linked insurance and annuities

• Life insurance AND protection against market downturns (and crashes)

• An investment guarantee is a long-term put option that is not typically traded on financial markets
Introduction

• Risk management
  – Model the cost of the guarantee: stochastic models
  – Actuarial approach and dynamic hedging approach

• Actuarial approach
  – Project the value of the guarantee using multiple scenarios of the underlying asset
  – Calculate a reserve based upon tail risk measures

• Dynamic hedging approach
  – Replicate the payoff of the guarantee with stocks and bonds or other available assets: Financial engineering
  – Calculate a reserve for hedging errors
Agenda

- Introduction
- Overview of data and models
- Actuarial approach – Left tail analysis
- Dynamic hedging approach
- Conclusion
Overview of data and models

• Data: log-returns on the S&P 500 Total Return Index
  – Period: February 1956 – December 2010
  – Frequency: monthly

• Classes of models
  – Independent
  – GARCH and extensions
    • Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)
    • Asymmetric power ARCH (APARCH)
    • Exponential GARCH (EGARCH)
  – Regime-switching (RS) and mixtures
  – RS-GARCH models (Gray (1996), Klaassen (2002) and Haas et al. (2004)) and extensions
Overview of data and models

• Distribution of the error term in models
  – Normal (NORM)
  – Student (STD)
  – Normal Inverse Gaussian (NIG)
  – Generalized Error (GED) (a.k.a. Exponential Power)
  – Skewed versions of these distributions were also considered

• In total, 78 models are considered

• Analysis of fit
  – Global fit: log-likelihood, AIC and BIC
  – Normality of residuals: Jarque-Bera and Shapiro-Wilk
  – Heteroskedasticity: ARCH-LM and Ljung-Box
## Overview of data and models

<table>
<thead>
<tr>
<th>Model</th>
<th>Params</th>
<th>BIC</th>
<th>Heteroskedasticity</th>
<th>Normality</th>
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</thead>
<tbody>
<tr>
<td>RS-EGARCH (SNORM)</td>
<td>10</td>
<td>2,355</td>
<td>PASS</td>
<td>PARTIAL</td>
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<td>EGARCH (SSTD)</td>
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<tr>
<td>SNIG</td>
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<td>NORM</td>
<td>2</td>
<td>2,264</td>
<td>FAIL</td>
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Overview of data and models

• Summary
  – Recent econometric models can offer an improved fit over the RS (NORM) model
  – However, there is no model that performs best overall
  – When APARCH models are combined with RS, it is generally the mixture version of these models that is preferred. This entails that the role of RS is mainly to provide a possibility for the volatility to jump and that persistence in volatility may be better explained by GARCH-type dynamics than solely by regime persistence
  – Good global fit is interesting but in the context of investment guarantees, the fit in the left tail is most important
Actuarial approach – Left tail analysis

• Objectives
  – Were the capital requirements generated by the models sufficient to cover an insurer’s loss on investment guarantees during the financial crisis?
  – Are models capable of generating low cumulative returns over long periods of time? This is essential if an investment guarantee is to mature in-the-money

• The worst cumulative returns on the S&P 500 (TR) on a horizon of 10 years or less generally occur for periods ending in February 2009 (month-end)
  – Two stock market crashes between 1999 and 2009
Actuarial approach – Left tail analysis

• Cumulative returns on the S&P 500 (TR) for periods ending in February 2009
  – 3 years: -39%
  – 5 years: -29%
  – 7 years: -24%
  – 10 years: -30%

• Out-of-sample exercise: check whether the risk measures generated by the models were close to these kinds of cumulative returns
Actuarial approach – Left tail analysis

- Guaranteed Minimum Maturity Benefit (GMMB)
  - Initial investment: 100$
  - Product fees: decreasing with maturity but 0.5% MER
  - Guarantee: return of capital on maturity
  - No mortality
  - No lapses
  - $n$-year maturity ending February 2009
  - Models estimated using data from the beginning of the sample (February 1956) to February 2009 minus $n$
<table>
<thead>
<tr>
<th>Model</th>
<th>3-Year</th>
<th>10-Year</th>
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<tbody>
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<td>99% VaR</td>
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Actuarial approach – Left tail analysis

• For a period of 3 years, risk measures generated by models with a good fit were comparable; for a period of 10 years there was much more variability.

• This last statement implies that long-term investment risk may be hard to quantify for long-term periods, i.e., there is a lot of uncertainty around the determination of reserves for investment guarantees with a long-term maturity.

• Hence, it is important to take into account model risk when quantifying long-term investment risk.
Dynamic hedging approach

- More and more companies are now dynamically hedging their investment guarantees.
- Within the Black-Scholes (B-S) framework, an option can be perfectly replicated under these conditions:
  - The market model is a geometric Brownian motion (GBM) and its parameters are known with certainty.
  - Trading can occur in continuous time.
  - It is possible to borrow and lend cash at a known constant risk-free rate.
  - There are no market frictions (no transaction costs and no constraints on trading).
Dynamic hedging approach

- These conditions are clearly not satisfied in the real world
- **Question:** How robust is the B-S delta hedge when its assumptions are violated?
- How can we evaluate the robustness of the B-S delta hedge?
  - Generate returns under the real-world probability measure using each of the models estimated previously
  - Apply the B-S delta hedge with a monthly rebalancing
  - Calculate the present value of hedging errors (PVHE)
Dynamic hedging approach

- Assumptions used in the Black-Scholes delta hedge
  - Volatility: empirical in-sample volatility (volatility of the log-returns prior to the inception of the contract)
    - This corresponds to between 14% and 15% depending on the maturity of the contract
  - Risk-free rate: set to 3%
    - This corresponds roughly to the average 1-month Treasury Constant Maturity rate prior to the financial crisis
  - Transaction costs: 0.2% of the change in the market value of the stock position that is used for hedging

- We remain in an out-of-sample context and assume that the investment guarantee matures in February 2009
## Dynamic hedging approach

| Model                         | 3-Year | PV of Hedging Errors | 10-Year |  |  |
|-------------------------------|--------|-----------------------|---------|  |  |
|                              | 95% CTE| 99% VaR               | 95% CTE | 99% VaR |  |
| EGARCH (SSTD)                 | 6.15   | 7.54                  | 8.22    | 9.76     |  |
| APARCH (SNIG)                 | 5.77   | 7.05                  | 8.24    | 9.81     |  |
| RS-EGARCH (SNORM)             | 5.61   | 6.92                  | 7.87    | 9.27     |  |
| MIX-APARCH (SNORM)            | 6.45   | 7.87                  | 7.78    | 9.18     |  |
| GARCH (NORM)                  | 4.96   | 6.13                  | 6.68    | 7.84     |  |
| MIX-GARCH (NORM)              | 6.16   | 7.27                  | 5.90    | 6.89     |  |
| RS-GARCH-Klaassen (NORM)      | 5.92   | 7.05                  | 5.84    | 6.82     |  |
| RS-GARCH-Gray (NORM)          | 5.47   | 6.51                  | 4.93    | 5.84     |  |
| RS (NORM)                     | 5.46   | 6.49                  | 4.24    | 5.05     |  |
| RS (SGED)                     | 4.26   | 5.11                  | 3.15    | 3.69     |  |
| SNIG                          | 4.61   | 5.43                  | 3.14    | 3.71     |  |
| NORM                          | 3.57   | 4.11                  | 2.23    | 2.56     |  |
Dynamic hedging approach

• The B-S delta hedge is very sensitive to its assumption of a GBM

  Model risk is very important

• For a 10-year maturity, the distribution of the PVHE is not only highly variable but it is also very uncertain

• The effectiveness of the B-S delta hedge is highly dependent on the underlying market model which implies that it is not a robust hedging strategy
Dynamic hedging approach

• How can we improve the B-S delta hedge?
  – Volatility input: we may use an inference on volatility based on a RS-GARCH model, for example
  – Greeks: options must be traded in the replicating portfolio; there is no guarantee that using Greeks in the B-S framework will lead to an improvement!
  – Rebalancing more frequently: by rebalancing more frequently, we may expose ourselves to increased model risk as the market model deviates more significantly from a GBM on higher frequencies
  – True replicating portfolio under more complex models: market is incomplete; reliance on a risk premium parameter or process
Conclusion

- It is important to take into account model risk when evaluating long-term investment risk or implementing a hedging strategy.
- Rantala (2006): “In the face of model risk, rather than to base decisions on a single selected ‘best’ model, the modeller can base his inference on an entire set of models by using model averaging.”
Thank You!

Questions?