An out-of-sample analysis of investment guarantees for equity-linked products: Lessons from the financial crisis of the late-2000s

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Introduction

- Insurance companies have been selling investment guarantees in many insurance products
  - Segregated funds in Canada, variable annuities in the US
  - Universal and participating policies
  - Other equity-linked insurance and annuities
- Life insurance AND protection against market downturns (and crashes)
- An investment guarantee is a long-term put option that is not typically traded on financial markets

Introduction

- Risk management
  - Model the cost of the guarantee: stochastic models
  - Actuarial approach and dynamic hedging approach
- Actuarial approach
  - Project the value of the guarantee using multiple scenarios of the underlying asset
  - Calculate a reserve based upon tail risk measures
- Dynamic hedging approach
  - Replicate the payoff of the guarantee with stocks and bonds or other available assets: Financial engineering
  - Calculate a reserve for hedging errors

Agenda

- Introduction
- Overview of data and models
- Actuarial approach – Left tail analysis
- Dynamic hedging approach
- Conclusion

Overview of data and models

- Data: log-returns on the S&P 500 Total Return Index
  - Period: February 1956 – December 2010
  - Frequency: monthly
- Classes of models
  - Independent
  - GARCH and extensions
    - Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)
    - Asymmetric power ARCH (APARCH)
    - Exponential GARCH (EGARCH)
  - Regime-switching (RS) and mixtures
  - RS-GARCH models (Gray (1996), Klaassen (2002) and Haas et al. (2004)) and extensions

Overview of data and models

- Distribution of the error term in models
  - Normal (NORM)
  - Student (STD)
  - Normal Inverse Gaussian (NIG)
  - Generalized Error (GED) (a.k.a. Exponential Power)
  - Skewed versions of these distributions were also considered
- In total, 78 models are considered
- Analysis of fit
  - Global fit: log-likelihood, AIC and BIC
  - Normality of residuals: Jarque-Bera and Shapiro-Wilk
  - Heteroskedasticity: ARCH-LM and Ljung-Box

Overview of data and models

Model	Params	BIC	Heteroskedasticity	Normality
RS-EGARCH (SNORM)	10	2,355	PASS	PARTIAL
EGARCH (SSTD)	7	2,351	PARTIAL	PASS
APARCH (SNIG)	8	2,340	PASS	PASS
MIX-APARCH (SNORM)	9	2,338	PASS	PASS
MIX-GARCH (NORM)	7	2,338	PASS	PARTIAL
RS-GARCH-Klaassen (NORM)	8	2,333	PASS	PARTIAL
RS-GARCH-Gray (NORM)	8	2,325	PASS	PARTIAL
RS (SGED)	9	2,311	PASS	PASS
RS (NORM)	6	2,310	FAIL	PASS
SNIG	4	2,302	FAIL	PASS
GARCH (NORM)	4	2,301	PASS	FAIL
NORM	2	2,264	FAIL	FAIL

Overview of data and models

- Summary
  - Recent econometric models can offer an improved fit over the RS (NORM) model
  - However, there is no model that performs best overall
  - When APARCH models are combined with RS, it is generally the mixture version of these models that is preferred. This entails that the role of RS is mainly to provide a possibility for the volatility to jump and that persistence in volatility may be better explained by GARCH-type dynamics than solely by regime persistence
  - Good global fit is interesting but in the context of investment guarantees, the fit in the left tail is most important

Actuarial approach – Left tail analysis

- Objectives
  - Were the capital requirements generated by the models sufficient to cover an insurer's loss on investment guarantees during the financial crisis?
  - Are models capable of generating low cumulative returns over long periods of time? This is essential if an investment guarantee is to mature in-the-money
- The worst cumulative returns on the S&P 500 (TR) on a horizon of 10 years or less generally occur for periods ending in February 2009 (month-end)
  - Two stock market crashes between 1999 and 2009

Actuarial approach – Left tail analysis

- Cumulative returns on the S&P 500 (TR) for periods ending in February 2009
  - 3 years: -39%
  - 5 years: -29%
  - 7 years: -24%
  - 10 years: -30%
- **Out-of-sample exercise**: check whether the risk measures generated by the models were close to these kinds of cumulative returns

Actuarial approach – Left tail analysis

- Guaranteed Minimum Maturity Benefit (GMMB)
  - Initial investment: 100$
  - Product fees: decreasing with maturity but 0.5% MER
  - Guarantee: return of capital on maturity
  - No mortality
  - No lapses
  - *n*-year maturity ending February 2009
  - Models estimated using data from the beginning of the sample (February 1956) to February 2009 minus *n*

Actuarial approach – Left tail analysis

Model	3-Year		10-Year	
	95% CTE	99% VaR	95% CTE	99% VaR
Out-of-sample	47.5		42.4	
RS-EGARCH (SNORM)	34.0	41.2	20.3	35.1
RS-GARCH-Gray (NORM)	40.3	46.7	19.3	32.5
RS-GARCH-Klaassen (NORM)	36.8	42.6	19.0	31.9
MIX-GARCH (NORM)	36.2	42.0	18.9	31.3
MIX-APARCH (SNORM)	35.3	42.3	18.8	33.1
APARCH (SNIG)	34.6	41.6	18.6	33.3
EGARCH (SSTD)	34.1	41.1	17.8	32.2
RS (NORM)	36.8	43.3	10.2	20.8
GARCH (NORM)	27.1	32.2	11.4	22.4
RS (SGED)	27.8	33.6	5.9	12.0
SNIG	30.9	35.9	5.3	10.3
NORM	30.0	34.7	4.9	9.3

Actuarial approach – Left tail analysis

- For a period of 3 years, risk measures generated by models with a good fit were comparable; for a period of 10 years there was much more variability
- This last statement implies that long-term investment risk may be hard to quantify for long-term periods, i.e., there is a lot of uncertainty around the determination of reserves for investment guarantees with a long-term maturity
- Hence, it is important to take into account model risk when quantifying long-term investment risk

Dynamic hedging approach

- More and more companies are now dynamically hedging their investment guarantees
- Within the Black-Scholes (B-S) framework, an option can be perfectly replicated **under these conditions**
  - The market model is a geometric Brownian motion (GBM) and its parameters are known with certainty
  - Trading can occur in continuous time
  - It is possible to borrow and lend cash at a known constant risk-free rate
  - There are no market frictions (no transaction costs and no constraints on trading)

Dynamic hedging approach

- These conditions are clearly not satisfied in the real world
- **Question:** How robust is the B-S delta hedge when its assumptions are violated?
- How can we evaluate the robustness of the B-S delta hedge?
  - Generate returns under the real-world probability measure using each of the models estimated previously
  - Apply the B-S delta hedge with a monthly rebalancing
  - Calculate the present value of hedging errors (PVHE)

Dynamic hedging approach

- Assumptions used in the Black-Scholes delta hedge
  - Volatility: empirical in-sample volatility (volatility of the log-returns prior to the inception of the contract)
    - This corresponds to between 14% and 15% depending on the maturity of the contract
  - Risk-free rate: set to 3%
    - This corresponds roughly to the average 1-month Treasury Constant Maturity rate prior to the financial crisis
  - Transaction costs: 0.2% of the change in the market value of the stock position that is used for hedging
- We remain in an out-of-sample context and assume that the investment guarantee matures in February 2009

Dynamic hedging approach

Model	PV of Hedging Errors			
	3-Year		10-Year	
	95% CTE	99% VaR	95% CTE	99% VaR
EGARCH (SSTD)	6.15	7.54	8.22	9.76
APARCH (SNIG)	5.77	7.05	8.24	9.81
RS-EGARCH (SNORM)	5.61	6.92	7.87	9.27
MIX-APARCH (SNORM)	6.45	7.87	7.78	9.18
GARCH (NORM)	4.96	6.13	6.68	7.84
MIX-GARCH (NORM)	6.16	7.27	5.90	6.89
RS-GARCH-Klaassen (NORM)	5.92	7.05	5.84	6.82
RS-GARCH-Gray (NORM)	5.47	6.51	4.93	5.84
RS (NORM)	5.46	6.49	4.24	5.05
RS (SGED)	4.26	5.11	3.15	3.69
SNIG	4.61	5.43	3.14	3.71
NORM	3.57	4.11	2.23	2.56

Dynamic hedging approach

- The B-S delta hedge is very sensitive to its assumption of a GBM

→ **Model risk is very important**

- For a 10-year maturity, the distribution of the PVHE is not only highly variable but it is also very uncertain
- The effectiveness of the B-S delta hedge is highly dependent on the underlying market model which implies that it is not a robust hedging strategy

Dynamic hedging approach

- How can we improve the B-S delta hedge?
  - **Volatility input**: we may use an inference on volatility based on a RS-GARCH model, for example
  - **Greeks**: options must be traded in the replicating portfolio; there is no guarantee that using Greeks in the B-S framework will lead to an improvement!
  - **Rebalancing more frequently**: by rebalancing more frequently, we may expose ourselves to increased model risk as the market model deviates more significantly from a GBM on higher frequencies
  - ***True* replicating portfolio under more complex models**: market is incomplete; reliance on a risk premium parameter or process

Conclusion

- It is important to take into account model risk when evaluating long-term investment risk or implementing a hedging strategy
- Rantala (2006): “In the face of model risk, rather than to base decisions on a single selected ‘best’ model, the modeller can base his inference on an entire set of models by using model averaging.”

Thank You!

Questions?