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A renewal model for medical malpractice

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Abstract

A renewal model for the aggregate discounted payments and expenses assumed by the insurer is proposed for the "medical malpractice" insurance, where the real interest rates could be stochastic and the dependency is examined through the theory of copulas.

As a first approach to this problem, we present formulas for the first two raw moments and the first joint moment of this aggregate risk process. Examples are given for exponential claims interoccurence times and the dependency is illustrated by an Archimedean copula, in which the autocovariance and the autocorrelation functions are also examined.

Keywords : Aggregate discounted payments; Copulas; Joint and raw moments; Medical malpractice; Renewal process; Stochastic interest rate.

Overview on Medical Malpractice Insurance

Definition :

"Medical malpractice" is generally defined as a professional negligence by act or omission by a health care provider in which the treatment provided falls below the accepted standard of practice in the medical community and eventually causes injury or death to the patient, with most cases involving medical error.

Premiums :

- Medical malpractice insurers take several factors into account when setting premiums, and these are usually charged to individuals, groups of practice, hospitals or governments.

- One of the main factor is the type of work a health-care provider does. Some specialties have a significantly higher rate of claims than others and will thus pay higher premiums, such as in neurosurgery and obstetrics/gynecology.

- Another important factor is the region where a provider practices. Indeed standards and regulations for medical malpractice vary by country and even by jurisdictions within countries, which is particularly apparent in USA.

- Among the other factors that are usually considered by the insurer are : some degree of experience rating, administrative expenses, litigation expenses, future investment income, profit margin sought, insurance business cycle, supply and demand.

- The physician professionals' claims experience is too variable over short time periods but presents more stability for hospitals.

Type of insurance :

- Premiums will also vary depending on the type of insurance coverage choosen for medical malpractice.

- There are essentially two primary types of insurance coverage for medical malpractice : “claims-made” and “occurrence” policies.

- Claims-made insurance, like auto or home insurance, provides coverage for incidents that occur while the policy is in force. However, an important condition is that the claim must also be filed while the policy is in force for the incident to be covered. For this type of insurance, a “tail coverage” is highly recommended to cover incidents that have not been reported to the company during the policy term.

- Occurrence coverage policies differ from the claims-made coverage by the fact that they cover any incident that occurs while the policy is in force, no matter when the claim is filed.

- As generally observed within this insurance market, the first type of insurance is substantially less expensive in the very first years but by the fourth or fifth year it reaches a mature level at about 95% of the cost of an occurrence policy.

- Claims-made policies are what are normally issued by most insurance carriers nowadays. In spite of that, the decision between a claims-made and an occurrence policy will obviously depend of what is best suited for the specific needs of the insured entity.

* In this research, only the claims-made policies will be considered.

Dependency :

- The business line “Medical malpractice” is characterized by a strong degree of uncertainty under many aspects often related.

- Many empirical observations seem to show that there is a positive dependency between the delay from the reception to the settlement of the claim, the final payment of the claim and the amount of expenses allocated to the claim.

- The discount rates used to actualize the payment of the claims and the expenses are not necessarily independent.

- To represent the dependencies mentionned previously, the theory of copulas seems to be most suitable and has been largely applied in the actuarial litterature since the last decade.

A renewal model, with copula and stochastic interest rate

Motivated mainly by the works of Léveillé & Garrido (2001) and Léveillé & Adékambi (2011) on discounted compound renewal sums, we present a stochastic model for the medical malpractice insurance where the counting process is an ordinary renewal process, the discount factors related to the payments and the expenses may be stochastic and dependent, and the dependencies are eventually governed by copulas.

Hence consider the following aggregate discounted payments and expenses process

$$Z(t) = Z_1(t) + Z_2(t) =: \sum_{k=1}^{N(t)} D_1\left(T_k + \tilde{\tau}_k\right) X_k + \sum_{k=1}^{N(t)} D_2\left(T_k + \tilde{\tau}_k\right) Y_k$$

where

- $\{\tau_k, k \in \mathbb{N}\}$ is a sequence of continuous positive independent and identically distributed (i.i.d.) random variables, such that τ_k represents the inter-occurrence time between the $(k-1)$-th and k-th claims.

- $\{T_k, k \in \mathbb{N}\}$ is a sequence of random variables such that $T_k = \sum_{i=1}^{k} \tau_k$, $T_0 = 0$, and then T_k represents the occurrence time of the claims received by the insurer.

- $\{\tilde{\tau}_k, k \in \mathbb{N}\}$ is a sequence of continuous positive i.i.d. random variables, independent of the τ_k, such that $\tilde{\tau}_k$ is the time from T_k taken by the insurer to pay the k-th claim.

- $\{X_k, k \in \mathbb{N}\}$ is a sequence of positive i.i.d. random variables, independent of the T_k, such that X_k represents the deflated amount of the claim effectively paid by the insurer.

- $\{Y_k, k \in \mathbb{N}\}$ is a sequence of positive i.i.d. random variables, independent of the T_k, such that Y_k represents the deflated amount of the expenses incurred by the insurer to fix the payment corresponding to the k-th claim.

- $\{N(t), t \geq 0\}$ is an ordinary renewal process generated by the inter-occurrence times $\{\tau_k, k \in \mathbb{N}\}$, which represents the number of claims received by the insurer in $[0,t]$.

• The random variables X_k, Y_k and $\tilde{\tau}_k$ are eventually dependent and this dependency relation is generated by a copula $C(u_1,u_2,u_3)$, where $(u_1,u_2,u_3)\in[0,1]^3$, which has positive measures of dependence and concordance.

• $D_i(t)=\exp\left\{-\int_0^t \delta(u)du\right\}$, $i=1,2$, is the discount factor at $t=0$ corresponding to $Z_i(t)$ and $\delta_i(t)$ is the force of net interest which could be deterministic or stochastic. Moreover, we will assume that $\{\delta_1(t), t\geq 0\}$ and $\{\delta_2(t), t\geq 0\}$ could be dependent but are independent of the processes $\{N(t), t\geq 0\}$, $\{\tilde{\tau}_k, k\in\mathbb{N}\}$, $\{X_k, k\in\mathbb{N}\}$ and $\{Y_k, k\in\mathbb{N}\}$.

* Here, we make the choice of not representing the possible dependency between the discount factors by another copula in order not to weigh down our model.

First and second raw moments of $Z(t)$

The following theorem gives an integral expression for the first moment of $Z(t)$, in agreement with our hypotheses.

Theorem 1 : Consider the discounted aggregate payments and expenses process, such as assumed previously. Then, for stochastic forces of interest $\delta_1(t)$ and $\delta_2(t)$, the first moment of $Z(t)$ is given by :

$$E\left[Z(t)\right] = \int_0^\infty E\left[X \middle| \tilde{\tau} = v\right] \left\{ \int_0^t E\left[D_1(u+v)\right] dm(u) \right\} dF_{\tilde{\tau}}(v)$$

$$+ \int_0^\infty E\left[Y \middle| \tilde{\tau} = v\right] \left\{ \int_0^t E\left[D_2(u+v)\right] dm(u) \right\} dF_{\tilde{\tau}}(v) \ ,$$

where $m(t)$ is the renewal function. □

Corollary 1 : For positive constant forces of interest δ_1 and δ_2 Theorem 1 yields

$$E\left[Z(t)\right] = E\left[e^{-\delta_1 \tilde{\tau}} X\right] \int_0^t e^{-\delta_1 v}\, dm(v) + E\left[e^{-\delta_2 \tilde{\tau}} Y\right] \int_0^t e^{-\delta_2 v}\, dm(v) \ .$$

Example 1: Assume that the deflated amounts X_k and Y_k have respectively, for $x > 0$, Pareto distributions

$$F_X(x) = 1 - \left[\frac{\beta_1}{\beta_1 + x}\right]^{\alpha_1} \quad , \quad F_Y(x) = 1 - \left[\frac{\beta_2}{\beta_2 + x}\right]^{\alpha_2} ,$$

where $\beta_1 > 0$, $\beta_2 > 0$, $\alpha_1 > 2$ and $\alpha_2 > 2$,

and that the interoccurrence times of the claims τ_k and the delays $\tilde{\tau}_k$ have respectively, for $t>0$, exponential distributions

$$F_\tau(t)=1-e^{-\lambda t} \quad , \quad F_{\tilde{\tau}}(t)=1-e^{-\tilde{\lambda} t} \ ,$$

where $\lambda>0$, $\tilde{\lambda}>0$.

Furthermore assume that the dependency relation between X_k, Y_k and $\tilde{\tau}_k$ is generated by the Archimedian copula

$$C(u_1,u_2,u_3)=1-\left[1-\prod_{i=1}^{3}\left[1-(1-u_i)^{\gamma}\right]\right]^{\frac{1}{\gamma}}=:1-f^{\frac{1}{\gamma}}(u_1,u_2,u_3,\gamma) \ ,$$

where $u_1=F_X(x)$, $u_2=F_Y(y)$, $u_3=F_{\tilde{\tau}}(t)$ and $\gamma\geq 1$.

Applying Corollary 1, and with the help of a software such as "Maple", the preceding identity for $E[Z(t)]$ can be calculated numerically.

Hence, if we consider the particular case where $\delta_1 = 0.02$, $\delta_2 = 0.03$, $\alpha_1 = 3$, $\alpha_2 = 5$, $\beta_1 = \beta_2 = 1$, $\lambda = 1$, $\tilde{\lambda} = 2$ and $\gamma = 2$, then we get from Corollary 1 the following function for the first raw moment of $Z(t)$,

$$E[Z(t)] \approx (24.48)\left[1 - e^{-0.02t}\right] + (8.1)\left[1 - e^{-0.03t}\right] .$$

* The next theorem gives an integral expression for the second moment of $Z(t)$ where the assumed dependencies of the model are also present in each term of this expression.

Theorem 2 : Consider the discounted aggregate payments and expenses process, such as assumed previously. Then, for stochastic forces of interest $\delta_1(t)$ and $\delta_2(t)$, the second moment of $Z(t)$ is given by:

$$E\left[Z^2(t)\right] = \int_0^\infty E\left[X^2 \middle| \tilde{\tau} = w\right] \left\{ \int_0^t E\left[D_1^2(v+w)\right] dm(v) \right\} dF_{\tilde{\tau}}(w)$$

$$+ \int_0^\infty E\left[Y^2 \middle| \tilde{\tau} = w\right] \left\{ \int_0^t E\left[D_2^2(v+w)\right] dm(v) \right\} dF_{\tilde{\tau}}(w)$$

$$+2 \int_0^\infty E\left[XY \middle| \tilde{\tau} = w\right] \left\{ \int_0^t E\left[D_1(v+w) D_2(v+w)\right] dm(v) \right\} dF_{\tilde{\tau}}(w)$$

$$+2 \int_0^\infty \int_0^\infty E\left[X \middle| \tilde{\tau} = w\right] E\left[X \middle| \tilde{\tau} = w'\right]$$

$$\times \left\{ \int_0^t \int_0^{t-v} E\left[D_1(u+w) D_1(u+v+w')\right] dm(u) dm(v) \right\} dF_{\tilde{\tau}}(w) dF_{\tilde{\tau}}(w')$$

$$+2\int_0^{\infty}\int_0^{\infty} E\left[Y\middle|\tilde{\tau}=w\right]E\left[Y\middle|\tilde{\tau}=w'\right]$$

$$\times\left\{\int_0^{t}\int_0^{t-v} E\left[D_2\left(v+w\right)D_2\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

$$+2\int_0^{\infty}\int_0^{\infty} E\left[X\middle|\tilde{\tau}=w\right]E\left[Y\middle|\tilde{\tau}=w'\right]$$

$$\times\left\{\int_0^{t}\int_0^{t-v} E\left[D_1\left(v+w\right)D_2\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

$$+2\int_0^{\infty}\int_0^{\infty} E\left[X\middle|\tilde{\tau}=w'\right]E\left[Y\middle|\tilde{\tau}=w\right]$$

$$\times\left\{\int_0^{t}\int_0^{t-v} E\left[D_2\left(v+w\right)D_1\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

□

Corollary 2 : For positive constant forces of real interest δ_1 and δ_2, Theorem 2 yields

$$E\left[Z^2(t)\right] = E\left[e^{-2\delta_1\tilde{\tau}}X^2\right]\int_0^t e^{-2\delta_1 v}\,dm(v) + E\left[e^{-2\delta_2\tilde{\tau}}Y^2\right]\int_0^t e^{-2\delta_2 v}\,dm(v)$$

$$+2E\left[e^{-(\delta_1+\delta_2)\tilde{\tau}}XY\right]\int_0^t e^{-(\delta_1+\delta_2)v}\,dm(v)$$

$$+2\left\{E^2\left[e^{-\delta_1\tilde{\tau}}X\right]\int_0^t\int_0^{t-v} e^{-\delta_1(u+2v)}\,dm(u)\,dm(v) + E^2\left[e^{-\delta_2\tilde{\tau}}Y\right]\int_0^t\int_0^{t-v} e^{-\delta_2(u+2v)}\,dm(u)\,dm(v)\right.$$

$$\left.+E\left[e^{-\delta_1\tilde{\tau}}X\right]E\left[e^{-\delta_2\tilde{\tau}}Y\right]\int_0^t\int_0^{t-v} e^{-(\delta_1+\delta_2)v}\left[e^{-\delta_1 u}+e^{-\delta_2 u}\right]dm(u)\,dm(v)\right\}.$$

Example 2 : Consider the same distributions, copula and parameters such as given in Example 1, then by combining the results of Example 1 and Corollary 2 we get

$$E\left[Z^2(t)\right] \approx (22.75)\left[1-e^{-0.04t}\right]+(2.5)\left[1-e^{-0.06t}\right]+(11.6)\left[1-e^{-0.05t}\right]$$
$$+2\left\{(773,762.5)\left[1-2e^{-0.02t}+e^{-0.04t}\right]+(36,450.29)\left[1-2e^{-0.03t}+e^{-0.06t}\right]\right.$$
$$\left.+(330,480.66)\left[1-e^{-0.02t}-e^{-0.03t}+e^{-0.05t}\right]\right\}.$$

From Examples 1 and 2, we get the following table for the expectation and standard deviation of $Z(t)$, and for a premium based on the standard deviation principle.

Table 1: $E[Z(t)]$, $\sigma[Z(t)]$, $\Pi[Z(t)] = E[Z(t)] + \sigma[Z(t)]$.

t	1	2	3	4	5	6	7
$E[Z(t)]$	0.72	1.43	2.12	2.80	3.46	4.10	4.73
$\sigma[Z(t)]$	32.53	64.32	95.40	125.80	155.52	184.59	213.00
$\Pi[Z(t)]$	33.25	65.75	97.52	128.60	158.98	188.69	217.73

If time t is measured in units of year and the paid amounts and the expenses are both measured in units of $10,000 , then we note that the premium charged by the insurer is very expensive and increases substantially with the insurance coverage period.

Remark : The choice of the Archimedian copula used in the previous examples is arbitrary but presents positive measures of concordance and dependence as our model requires it.

To verify that, hereafter we compute (numerically) three classical measures corresponding to the copula used in our examples, precisely

... the trivariate Kendall's tau τ_3 defined by

$$\tau_3 = \frac{1}{3}\left\{8\int_{[0,1]^3} C(u_1,u_2,u_3)\,dC(u_1,u_2,u_3) - 1\right\} \;\Rightarrow\; \tau_3 \approx 0.354 \;,$$

... the trivariate Spearman's rho ρ_3 defined by,

$$\rho_3 = 8\int_{[0,1]^3} C(u_1,u_2,u_3)\,du_1\,du_2\,du_3 - 1 \;\Rightarrow\; \rho_3 \approx 0.44 \;,$$

... the multivariate upper tail dependence coefficient $\lambda_U^{1..h|h+1..3}$ defined by

$$\lambda_U^{1..h|h+1..3} = \lim_{u\to 1-} \frac{\sum_{i=1}^{n}\left\{\binom{n}{n-i}(-1)^i\left[\varphi^{-1}\left(i\varphi(u)\right)\right]\right\}}{\sum_{i=0}^{n-h}\left\{\binom{n-h}{n-h-i}(-1)^i\left[\varphi^{-1}\left(i\varphi(u)\right)\right]\right\}} \quad , \quad h=1,2 \ ,$$

where $\varphi(u)$ is the generator of this Archimedian copula

$$\varphi(u) = -\ln\left[1-(1-u)^{\gamma}\right] \ \Rightarrow \ \varphi^{-1}(u) = 1-\left[1-e^{-u}\right]^{\gamma^{-1}} ,$$

which implies that

$$\lambda_U^{1|2,3} \approx 0.84 \quad , \ \lambda_U^{1,2|3} \approx 0.49 \ .$$

First joint moment between $Z(t)$ and $Z(t+h)$

In this section, we present an integral expression for the covariance between $Z(t)$ and $Z(t+h)$, where the terms depending on h are also highly "affected" by the dependencies of the model.

Theorem 3 : Consider the discounted aggregate payments and expenses process, such as assumed previously. Then, for stochastic forces of interest $\delta_1(t)$ and $\delta_2(t)$, the first joint moment between $Z(t)$ and $Z(t+h)$ is given by :

$$E\left[Z(t)Z(t+h)\right] = E\left[Z^2(t)\right]$$
$$+\int_0^\infty\int_0^\infty E\left[X|\tilde{\tau}=w\right]E\left[X|\tilde{\tau}=w'\right]$$
$$\times\left\{\int_0^t\int_{t-v}^{t+h-v} E\left[D_1(v+w)D_1(u+v+w')\right]dm(u)dm(v)\right\}dF_{\tilde{\tau}}(w)dF_{\tilde{\tau}}(w')$$

$$+\int_0^\infty\int_0^\infty E\left[Y\middle|\tilde{\tau}=w\right]E\left[Y\middle|\tilde{\tau}=w'\right]$$

$$\times\left\{\int_0^t\int_{t-v}^{t+h-v} E\left[D_2\left(v+w\right)D_2\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

$$+\int_0^\infty\int_0^\infty E\left[X\middle|\tilde{\tau}=w\right]E\left[Y\middle|\tilde{\tau}=w'\right]$$

$$\times\left\{\int_0^t\int_{t-v}^{t+h-v} E\left[D_1\left(v+w\right)D_2\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

$$+\int_0^\infty\int_0^\infty E\left[Y\middle|\tilde{\tau}=w\right]E\left[X\middle|\tilde{\tau}=w'\right]$$

$$\times\left\{\int_0^t\int_{t-v}^{t+h-v} E\left[D_2\left(v+w\right)D_1\left(u+v+w'\right)\right]dm\left(u\right)dm\left(v\right)\right\}dF_{\tilde{\tau}}\left(w\right)dF_{\tilde{\tau}}\left(w'\right)$$

□

Corollary 3 : For positive constant forces of interest δ_1 and δ_2, Theorem 3 yields

$$E\left[Z(t)Z(t+h)\right] = E\left[Z^2(t)\right] + E^2\left[e^{-\delta_1\tilde{\tau}}X\right]\int_0^t \int_{t-v}^{t+h-v} e^{-\delta_1(u+2v)}\,dm(u)\,dm(v)$$

$$+ E^2\left[e^{-\delta_2\tilde{\tau}}Y\right]\int_0^t \int_{t-v}^{t+h-v} e^{-\delta_2(u+2v)}\,dm(u)\,dm(v)$$

$$+ E\left[e^{-\delta_1\tilde{\tau}}X\right]E\left[e^{-\delta_2\tilde{\tau}}Y\right]\int_0^t \int_{t-v}^{t+h-v} e^{-(\delta_1+\delta_2)v}\left[e^{-\delta_1 u} + e^{-\delta_2 u}\right]dm(u)\,dm(v).$$

Example 3 : Again, using the same assumptions and the results obtained in Examples 1 and 2, we get

$$E\left[Z(t)Z(t+h)\right] \approx E\left[Z^2(t)\right] + (1{,}498{,}176)\, e^{-0.02t}\left[1-e^{-0.02t}\right]\left[1-e^{-0.02h}\right]$$
$$+ (72{,}899.93)\, e^{-0.03t}\left[1-e^{-0.03t}\right]\left[1-e^{-0.03h}\right]$$
$$+ (330{,}480.66)\left\{e^{-0.02t}\left[1-e^{-0.02h}\right] + e^{-0.03t}\left[1-e^{-0.03h}\right] - e^{-0.05t}\left[2-e^{-0.02h}-e^{-0.03h}\right]\right\},$$

where $E\left[Z^2(t)\right]$ is given in Example 2.

By using the data of Table 1, we obtain the following table for the autocovariance and autocorrelation functions of $Z(t)$, where we define

$$C(t,h) = Cov\left(Z(t), Z(t+h)\right) \quad , \quad \rho(t,h) = \frac{C(t,h)}{\sigma\left[Z(t)\right]\sigma\left[Z(t+h)\right]} \quad .$$

Table 2 : $C(t,h)$, $\rho(t,h)$.

h	1	2	3	4	5	6	7
$C(1,h)$	2072.7	3064.5	4034.4	4982.7	5910.0	6816.8	7703.6
$\rho(1,h)$	0.9905	0.9874	0.9858	0.9848	0.9842	0.9837	0.9833

This table corroborates the strong linear correlation observed between the values of $E\left[Z(t)\right]$ in Table 1. Obviously, this last function is concave and tends (approximatively) to the value 32.58 as $t \to \infty$.

Conclusion

A renewal model for medical malpractice insurance has been proposed. This model incorporates stochastic interest rates and a copula to establish the dependence between the payment of the claim, the expenses and the delay between the receipt and the settlement of the claim.

Integral formulas has been given for the first two raw moments and the first joint moment of our risk process. The autocorrelation function has also been examined, as well as the incidence of our model on the premium.

Several important challenges arise now from this model, such as the calibration of the copulas that will characterize adequately the dependency relations within our problem and the choice of the discount rates that will best represent the yields expected by the insurer, to mention only those.

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