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Minxian Lu, Xiaoming Liu, Yu Hao

The Impact of Investment Strategy

of DC Pension Plan on Retirement Age Distribution

Minxian Lu
mlv@uwo.ca
Department of Statistical and Actuarial Sciences
Western University

Dr. Hao Yu
hyu@stats.uwo.ca
Department of Statistical and Actuarial Sciences
Western University

Dr. Xiaoming Liu
xliu@stats.uwo.ca
Department of Statistical and Actuarial Sciences
Western University

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Abstract

Many employer-sponsored pension plans now have shifted from defined benefit (DB) to defined contribution (DC) pension plans. It is notorious that DC plans transfer the risk from employer to employee. We are interested in studying the risk inherent in DC pension plans on an individual and aggregate basis. We adopt a retirement decision model based on replacement ratio exceeding a minimum level. We consider a modified Wilkie's investment model to investigate the impact of various investment strategy on the retirement age distribution. All investment portfolios are made of cash, bond and stock. We investigate the one-asset portfolios, mixed-asset portfolios, and a dynamic investment portfolio which allows a switch from an aggressive portfolio to a conservative one at middle ages. For individuals, portfolio achieving earlier retirement age with lower risk is favoured.

1 Introduction

Define contribution plans have gained increasing popularity especially in the private sector of many countries. Define contribution pension plan (DC) is like an individual saving account in which the employer's contribution is specified. Unlike define benefit pension plan, its future benefit amount is not guaranteed but instead fluctuates on the basis of investment earning. Upon retirement, the wealth accumulated in this individual account is used to provide retirement benefits, for example, an annuity. Nowadays, many insurance companies provide investment options to their DC customers and therefore we are interested in how the selection of investment strategy affects the retirement age distribution. The idea of investigation process is inspired by MacDonald and Cairns (2007). We will use a different investment model. Then we add type switch to the investment portfolio to see whether or not a different conclusion could be drawn. Section 2 talks about the assumptions about the factors associated with the DC pension plan scheme and the structure. We investigate the retirement age distribution of the DC pension plan with single-asset portfolios and mixed-asset portfolios in section 3. At the end of section 3, we pick some efficient portfolios as optimal choices for individual DC pension plan participants. After that in section 4 we assume a dynamic portfolio which allows one switch decision from an aggressive portfolio to a more conservative portfolio at middle ages. The proportion of bond and cash in the conservative portfolio should be bigger than it is in the aggressive one. In section 5, we repeat the investigation process on the investment strategies with one switch decision taking place at middle ages. Section 6 summarizes the results.

2 Investment model and Assumptions About Other Factors

2.1 Factors Associated with DC Scheme

We first adopt an uniform retirement strategy and for the whole population, in other words, the choices of entry age contribution rate, investment portfolio are the same for everyone in this country.

2.1.1 Contribution Rate and Entry Age

The traditional DC pension plan is sponsored by the employers with contribution proportional to one's salary. Like what was done in Bonnie and Andrew (2007), we use a contribution rate of 10% in our investigation. We also assume that people uniformly enters the workforce at age 25.

2.1.2 Wealth Evolution Process

The wealth process advances annually. It is composed of the accumulated wealth one-year ago and the return on the this amount of wealth over the past year. The return is determined by the selected portfolio.

$$Wealth(e,t) = [Wealth(e,t-1) + 0.1 \times salary(e,t-1)] \times (1 + i(e+t)) \quad , \qquad (1)$$

where e is the entry year, t is the number of working years, i.e service year,$i(e+t)$ is the one-year return on the selected portfolio.

2.1.3 Replacement Ratio and Retirement Decision

Most of indexed-annuities are not really indexed to the inflation. Instead the insurance companies let customers choose a fixed index rate. We assume all DC participants' purpose is to buy 2%-indexed annuity. The price of a two-percent-indexed annuity is given by

$$\ddot{\text{a}}(\text{e,t,}e_a) = \sum_{s=0}^{\omega-e_a-t} 1.02^s \times (1 + i_{e+t})^{-s} \cdot {}_s p_{e_a+t} \quad . \qquad (2)$$

Replacement ratio is defined as

$$RR(e,t) = \frac{Wealth(e,t)/\ddot{\text{a}}_{e+t}}{salary(e,t)} \quad , \qquad (3)$$

where e_a is the entry age, $\ddot{\text{a}}_{e+t}$is the price of whole life annuity due paying \$1 every year starting from year e+t, when the participant is at age $e_a + t$. This ratio, measured as a percentage of current salary, is basically the future benefits, one is able to afford with his or her accumulated wealth in the DC pension plan fund. For the minimum replacement ratio (MRR), we first assume it is flat $\frac{2}{3}$ for all ages. Only when the replacement ratio exceeds the minimum replacement ratio (MRR), in other words bigger than $\frac{2}{3}$ that one would and certainly start his or her retirement life. So the earliest retirement age for a particular year is

$$ERA = \min\{\text{e+t} : RR(e,t) \geq \text{MRR(e+t)}\} \quad , \qquad (4)$$

where e is the entry age and t is the number of service year.

2.1.4 Population Structure and Mortality Rates

For mortality rates, the survival probabilities are from the U.S life table 2002. The relative size of one cohort (age x) does not vary with time, and for a particular year it assumed to be given as

$$l_x = \sum_{s=0}^{\omega-x} {}_sp_x \tag{5}$$

with the age limit ω=100.

2.1.5 Dependency Ratio

The dependency ratio is defined as the ratio of the number of retirees to the number of working people.

$$\text{Dependency Ratio}=\frac{\text{Number of retirees}}{\text{Number of working people}}$$

Since the population structure is static, the dependency ratio is inversely proportional to the earliest retirement age.

2.2 Investment Model

Bonnie and Andrew (2007) used stochastic differential driven processes to model the investment assets. Specifically, the instantaneous rate was dominated by Vasicek model. Then the term structure was determined by the zero-coupon price under the no-arbitrage framework together with Vasicek model. By doing so, only the concurrent interaction among components were modelled. Therefore we consider a time series model for the investment asset, which allows the influence of historical information of one component itself and other components. We then explore the investment strategy's influence on DC pension plan's retirement age distribution. The structure of modified Wilkie's model given as Figure 1. The key modification we made to the wilkie's investment model structure is that we reverse the direction of influence between long rate and short rate. Reason for this modification is explained in 2.2.2.

Parameter estimate is based on monthly U.S data from 1982 to 2004. For data resource, refer to the Appendix. The inflation is represented by the monthly growth of Consumer Price Index (CPI), and the market performance by the monthly price of S&P500. As for the

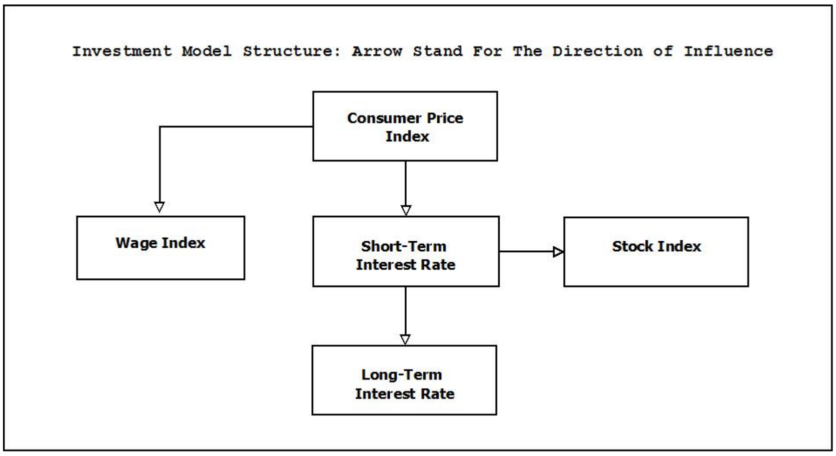


Figure 1: Investment model structure

interest return on cash and bond, they are respectively represented by yield of the 3-month and 10-year Constant Maturity Treasury quoted monthly. Time unit is one month for the model expressions.

2.2.1 CPI and Short-Term Interest Rate

On the January 25th 2012, Mr.Bernanke, chariman of Federal reserve announced a formal inflation target of 2%. Before such an target was set, the members of Federal Open Market Committee[1] regularly states an inflation range around 2%. If the inflation appears to be above the target, the short rate should be increase. Nevertheless, with Mr.Bernake's revolutionary shift to an inflation target, he also stressed that inflation target is not the unique factor affecting decision of Federal Reserve.

> "We are not absolutists...If there is a need to let inflation return a little bit more slowly to target to get a better result on unemployment, then that is something that we would be willing to do."
>
> -Ben S. Bernanke

The reason behind using a state-space model is that we found from the selected period of data, that the influence of CPI on short rate is not obvious, though whose existence is generally believed in. In fact, large proportion of short rate's variation is explained by many things else. Therefore we consider introducing latent variables in state-space model. The latent variable could be regarded as some integrated macroeconomic variable reflecting the current economic condition. The multivariate auto correlation functions indicates that the short rate data is in line with such a policy (See Figure a in Appendix). Along the path of data selection period, short rate seems to respond to the inflation change slowly or does not respond at all. We employ the state-space model with the expression given as follows:

$$\begin{aligned} X_t &= \phi \cdot X_{t-1} + \omega_t & (6) \\ Y(t) &= A \cdot X_t + v_t & (7) \\ \omega_t &\sim N(\mathbf{0}, \mathbf{W}) \qquad v_t \sim N(\mathbf{0}, \mathbf{V}) \end{aligned}$$

$$X_t^T = \begin{pmatrix} S_t & S_{t-1} & \pi_t & \pi_{t-1} \cdots & \pi_{t-11} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 1 & \cdots \end{pmatrix}$$

[1]An organ within the Federal Reserve, which meets on a nearly monthly basis to make key decisions of interest rate and money supply in US.

$$\phi = \begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & SAR_1 \end{pmatrix}$$

$$\mathbf{W} = \begin{pmatrix} \sigma^2_{\omega 1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\omega 2} \end{pmatrix} \qquad \mathbf{V} = \begin{pmatrix} \sigma^2_v & 0 \\ 0 & 0 \end{pmatrix} \quad ,$$

where X_t is latent variable vector of length n, ϕ an 14×14 matrix, A is an 1×14 matrix, Y_t the observation vector of the short rate and log growth of CPI at time t, ω_t an error vector of length n and v_t error term as well. To reduce the number of parameters needed to be estimated, we centered the data before fitting the data to this model. By this setting, the monthly centered log growth of CPI is following an SAR(1) model, when in the same time the latent variable behind the centered short rate evolves with an AR(2) model.

2.2.2 Long-Term Bond Yield

Unlike the Wilkie's model, we adopt a model where the short term interest rate influence the long-term rate. This is to reflect the fact that in U.S the short rate is manipulated by the Federal Reserve. However, the yield of the long-term bond is not under its direct control. The initial issue of Treasury bonds are running in the form of an auction. As a consequence, the yield of the long-term yield rate is determined by the market's opinions on many factors such as the future inflation expectation, credit rating and etc. And we observe that the long rate moves in the same direction of short rate. Furthermore, for most of the time the difference of long rate and short rate, which approximates the term premium, behaves in the opposite way of short rate (See Figure b in Appendix). For convenience, we refer term premium to the difference of long rate and short rate. A possible reason for that is whenever the market believes the short rate has been set too low, probably followed by a significant downward movements of the short rate, the market's expectation of future inflation is apt to rise. The time-series expression for long rate is given by,

$$\begin{aligned} TP(t) &= \mu_{TP} + AR_{t1}TP(t-1) + T_1\pi_t + T_2 \triangle S(t) + \\ &\quad (1-\theta_1 B - \theta_2 B^2)a_{3t} \\ L(t) &= TP(t) + S(t) \\ a_{3t} &\sim N(0, \sigma^2_{a_{3t}}) \quad , \end{aligned} \tag{8}$$

where $\triangle S(t)$ is the change of short rate. π_t is the log growth of CPI. where $L(t)$ is the long-term interest rate at time t, $Y(t)$ and π_t stand for the short rate and log growth of inflation as they are in the previous context. The short rate and log growth of CPI enter the model as input variables.

2.2.3 Stock Index

The log growth of the stock index price is modelled with a random walk with a drift term associated with the log growth of the short rate. It is generally believed that the stock and short rate are negatively correlated. This is the reason why we bring log growth of short rate into the expression. The expression is given as

$$\begin{aligned} LH(t) &= \mu_H + H_1 \times LS(t) + a_{4t} \\ H(t) &= H(t-1) \times \exp(LH(t))) \\ a_{4t} &\sim N(0, \sigma^2_{a_{4t}}) \quad , \end{aligned}$$

where LS(t) is the log growth of short term interest rate, H(t) is price of the stock index, LH(t) is the log price of the stock index.

2.2.4 Wage Index

The only data of wage index we could access is annual. We continue to use the settings in Bonnie and Andrew (2007), and generate simulations annually, and simulate the other four components monthly. Their model for wage index is based on the idea that the wage index keeps pace with the CPI in long run (Wilkie 1986). There are two kinds of salary growth, one is the part that is adjusted with the CPI, and the other one is the merit increase. The time series expression for wage index is given as follows:

$$
\begin{aligned}
W(t) &= W(t-12)exp(\sum_{i=t-12}^{t-1} \pi_i + \mu_w + a_{5t}) && (9)\\
\text{Salary level:}R(t) &= \frac{m(t)}{m(0)} \times W(t) && (10)\\
a_{5t} &\sim N(0, \sigma^2_{a_{5t}}) \quad .
\end{aligned}
$$

The merit increase is a function of the number of service years t and takes the form of $m(t)$=1.81 $- e^{-0.1t}$. This is simply saying the maximum value of merit increase is 1.81. It functions with CPI level to determine the salary level.

3 Retirement Age Distribution for Single-Asset Portfolio and Mixed-Asset Portfolio

The pension plan's purpose is to provide participants future benefits upon their retirement. DC participants concerns about the mean growth rate of the strategy and the amount of risk inherent. Because the two factors determine at what age they are going to retire on average and how sure they are able to retire at this mean retirement age. In this section, we first exam the single-asset portfolios, then the mixed ones. All proceeds from the investment is reinvested annually in the same portfolio. Since earliest retirement age is inversely proportional to dependency ratio, searching for the investment strategy that advances the earliest retirement age is equivalent to searching for the one that raising mean dependency ratio. Besides, under the assumption of static population, the mappings between are one-to-one. In U.S., the common normal retirement age is 65, at which the retirees will normally get 70 % of their final salary as retirement benefit. The equivalent dependency ratio is around 0.3619, and we will use this as a benchmark for the retirement age resulted by the DC pension plan and all our assumptions. gives a report of the single-asset portfolios' performance in advancing investor's retirement age.

Figure 2: Simulated CDF of Earliest Retirement Age Distribution for Each Single-Asset Portfolio with Non-Age-Varying MRR

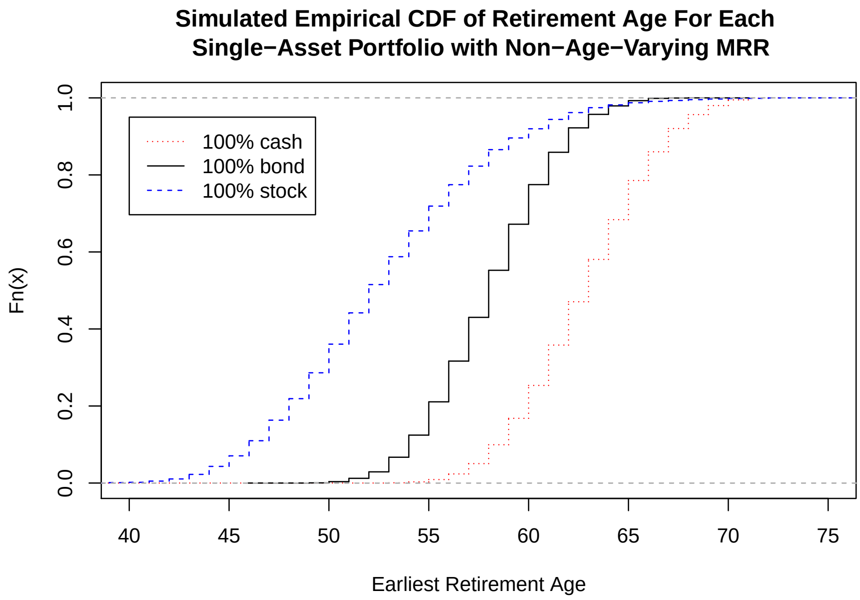


Table 1: The Summary Statistics for The Simulated Dependency Ratio and Eearliest Retirement Age with Single-Asset portfolios

	Mean	Standard Deviation	Associated Mean Earliest Retirement Age	Efficiency
100% Cash	0.433005	0.100937	62.8028	0.40688152
100% Bond	0.595729	0.122406	58.098	1.66489
100% Stock	0.879635	0.289479	52.686	1.6847488

From the summary in Table 1, the 100 % stock portfolio has the highest mean dependency ratio therefore the lowest mean earliest retirement age, at the cost of the lowest stability. Bond takes the second place in terms of the highest mean dependency ratio, with a considerably smaller standard deviation. Cash further lower the mean dependency ratio without obviously improve the stability. What happen if combining the assets to make mixed-asset portfolios? We first fix the exposure to stock every time, and let proportions of the other two assets in a portfolio varies 10% every time to make a different combination. We group the portfolios by their stock exposure. Five thousand simulations are done to each investment strategy to find the efficient portfolio for each investment strategy. Refer to figure 3 for the efficient portfolios and tangency portfolio.

The efficient portfolios which situate along the efficient frontier must contain no cash. For portfolios that have a non-zero fixed stock component, the more it positions in cash the less efficient it is. There seems to be no way for portfolios that have no stock components to lie at the efficient frontier. They are all less efficient than portfolios with 10% stock. Tangency portfolio, or the optimal investment strategy for individual participants, turns out to be composed of 30% stock and 70% long-term bond.

To identify each asset's role in advancing the earliest retirement age, we plot the mean retirement ages against their standard deviations in figure 4. The red hollow dots represent the means and the vertical segments are one-standard-deviation intervals around the corresponding means. We first fix stock exposure to be 40%. As we increase the position size in cash, we see that the mean retirement age is declining, while the width of the one-standard-deviation interval virtually remain the same. For other portfolios with different fixed stock position size, the plots of mean retirement age and the one-standard-deviation interval look similar. Therefore we conclude that cash is not a good choice as pension investment option. Now that it is proved to be useless to the DC participants, on the right figure we only concentrate on portfolios which are made of long-term bonds and stock only. We observe that increasing position in stock help lower the mean retirement age at the cost of rising risk.

Figure 3: Efficient Frontier and Tangency Portfolio for Simulated Dependency Ratios: The percentage beside each cluster of dots is stock exposure

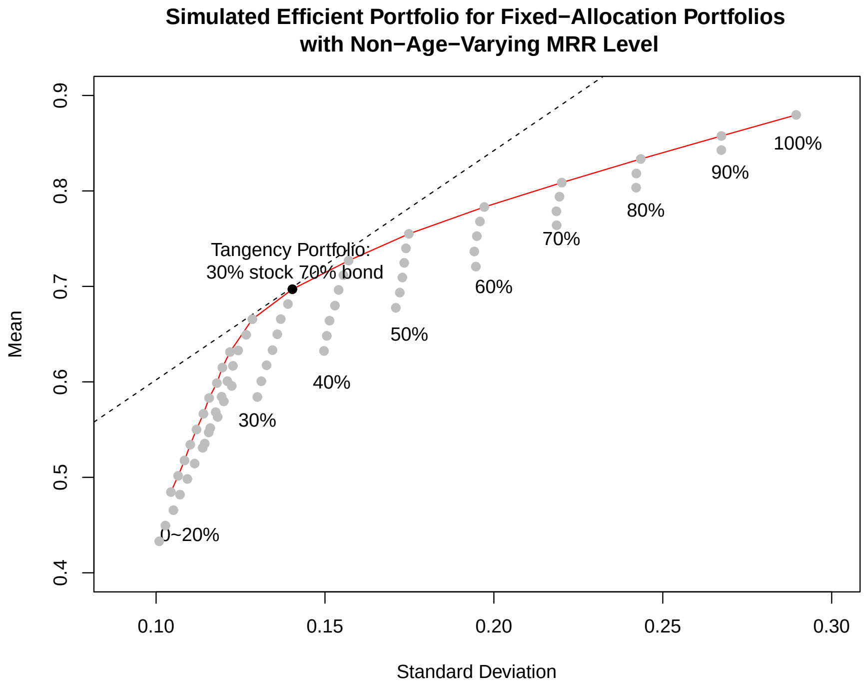
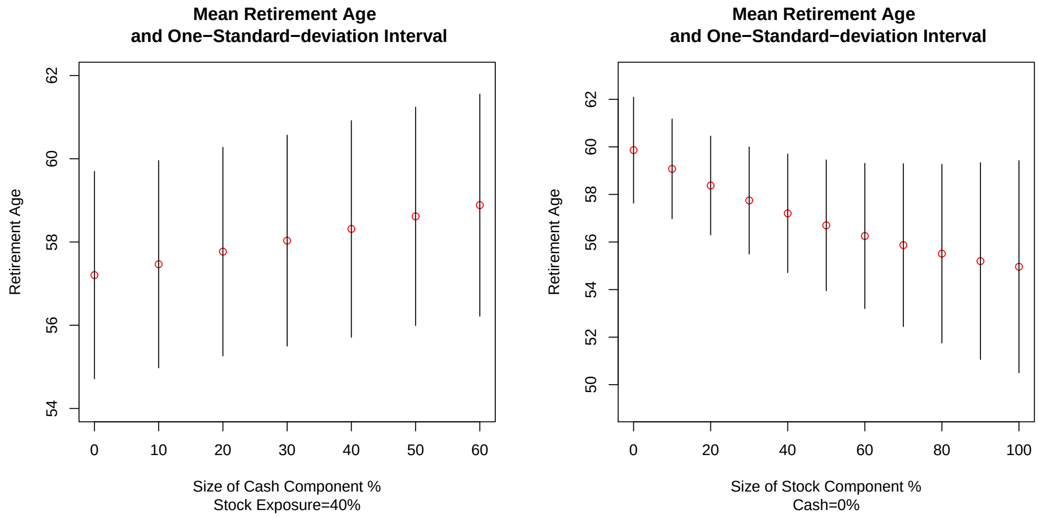


Figure 4: Each Asset's Role in Resulting in the Earliest Retirement Age



(a) Effect of Cash (b) Bond and Stock

4 Switch Decision at Middle Ages

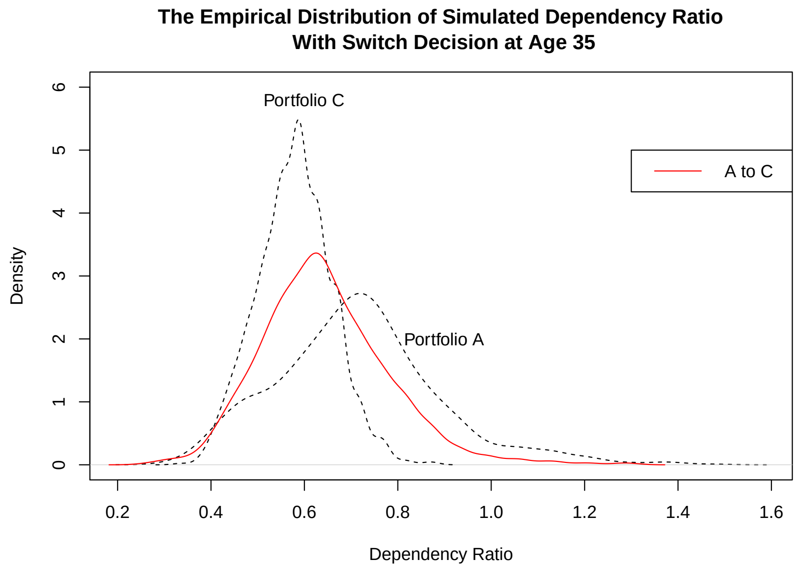
In this section, we examine the effect of switch at middle ages on the retirement age distribution. The reason why we would like to consider switch decision is that people's attitude towards risk tends to change as they age. This kind of switch decision within our consideration has nothing to do within people's anticipation on the assets. For this purpose, we picked 3 portfolios from the set of efficient portfolios in the previous section.

- Portfolio A: 80 % Stock, 20 % Bond
- Portfolio B: 50 % Stock, 50 % Bond
- Portfolio C: 20% Stock, 80% Bond

As people tend to get less aggressive in investment strategy at old ages, we only allow switch that reduce the stock exposure in order to lower the uncertainty about retirement age. Refer to figure 5 to see the effect of switch decision from portfolio A to C at age 35, 10 years after entry.

We observe that the retirement age distribution sits somewhere between the distributions resulted from holding portfolios it switch from and to. While on average the dependency ratio is lower than that of portfolio C, the instability has not been improved apparently. For more insights into the effect of switch decision, we examined the effect of switch timing every

Figure 5: The Distribution of Retirement Age with One Switch Decision From A to C at Age 35

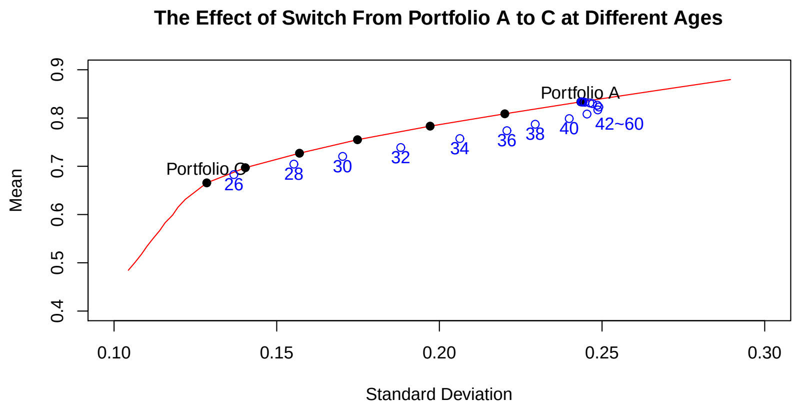


other year from entry (age 25) to age 60. The set of Figure 6 includes the efficient mixed-asset portfolios stated in previous section, represented by black solid dots, of which equity exposure ranges from the investment strategy it switch from and to. The efficient portfolios are connected by the efficient frontier. The blue hollow dots represent the investment strategy with switch decision occurs at the age given by the number attached below. As the decision is made later, the switch does not help to distinguish this investment strategy from the portfolio it switch from, and vice versa. Surprisingly, we also notice that none of these switch decision make an efficient portfolio, as they cannot break the efficient frontier created by the mixed-asset portfolios in section 3. Portfolios with switch decision look like mixed-asset portfolios that contains cash component and was hold for the whole course.

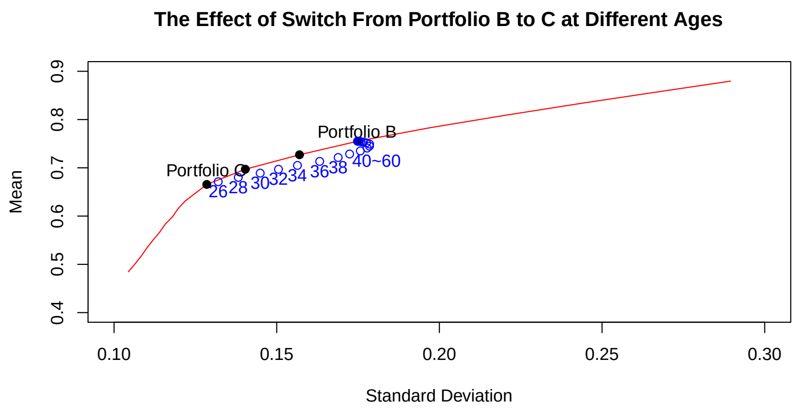
5 Age-Varying Minimum Replacement Ratio Level

In this section, we assume an age-varying minimum replacement ratio (see Figure 7(a)). Particularly, we set it about 0.7 at age 65. It is not very likely for young workers (before attaining age 55) to start live on retirement benefit that is only a proportion of the current salary, especially when they still have children to raise. As ageing, people's demand for high replacement ratio is relaxed and drop to around 70% at age 65. So if the actual Replacement Ratio looks like the red line in Figure 7(b), the earliest retirement age for this year is around

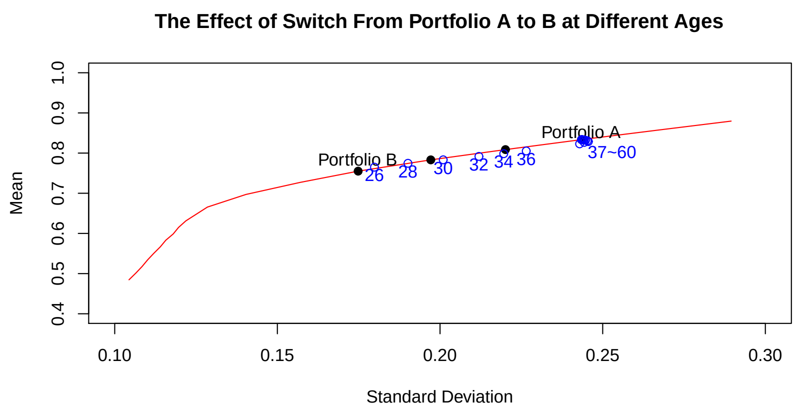
Figure 6: The Effect of Switch Decision



(a) A to C



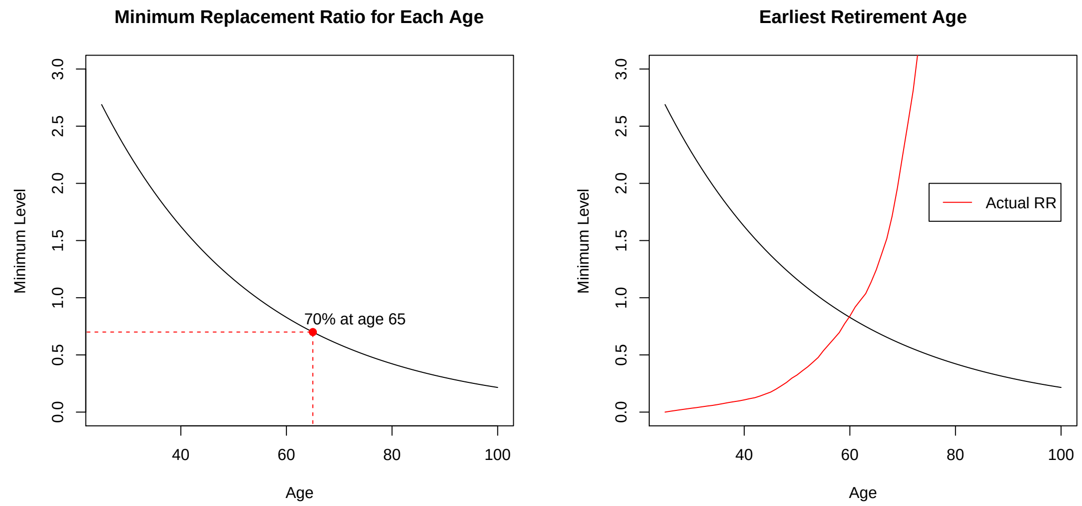
(b) B to C



(c) A to B

60.

Figure 7: Age-Varying Minimum Replacement Ratio Assumption



(a) Plot of the Age-Varying Minimum Replacement Ratio

(b) Search for the Earliest Retirement Age

The expression for this age-varying minimum ratio level is given as equation 11, an exponentially decreasing function.

$$MRR(e_a + t) \quad = \quad 6.2363 \times e^{-0.03365 \times (e_a + t)} \tag{11}$$

Under this age-varying minimum ratio assumption, the cumulative distribution functions of earliest retirement age with single-asset portfolios are shown in figure 8. Compared to that under the flat minimum ratio assumption in figure 2, the starting point on the x-axis of CDF line with pure-bond or pure-cash investment strategy is moved to the right. As expected, fixed assets' (bond and cash) ability of advancing retirement age is impaired. The spread of their CDFs are narrowed, that is to say earliest retirement age with pure-bond or pure-cash portfolios is not as volatile as it is in section 3 and 4. From the CDF of 100% stock portfolio, its shape and location of starting point are not obviously changed. It remains almost the same as it is under the original assumption, the most powerful also the riskest investment strategy.

Figure 8: Simulated Empirical CDF of Retirement Age For Each Single-Asset Portfolio with Age-Varying MRR

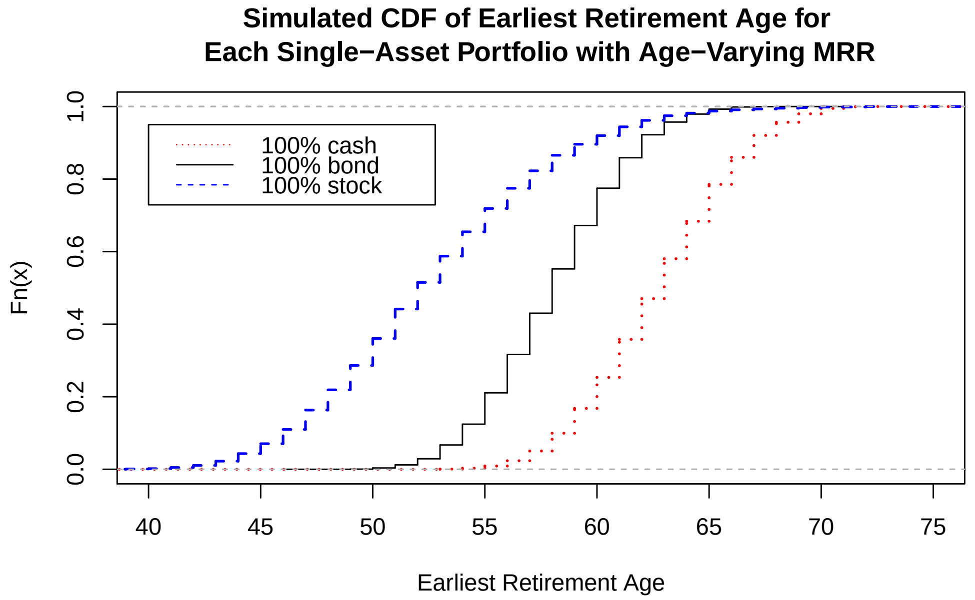
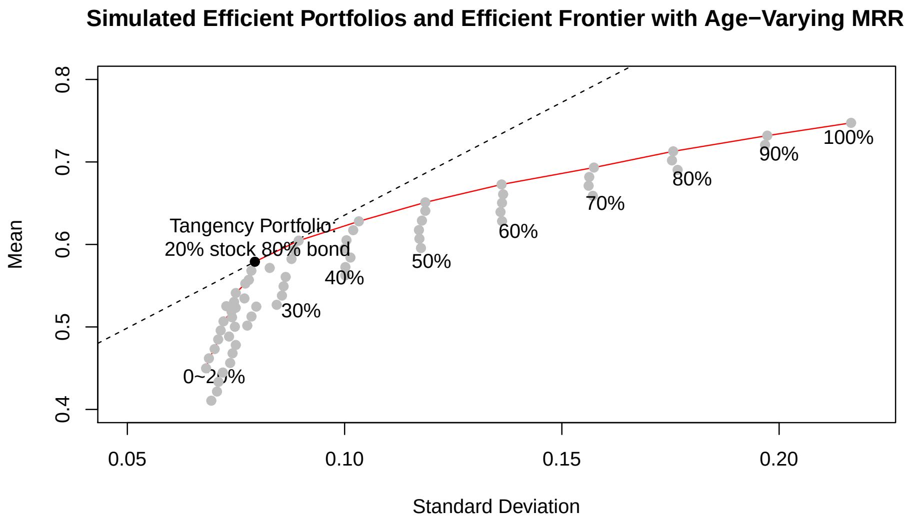
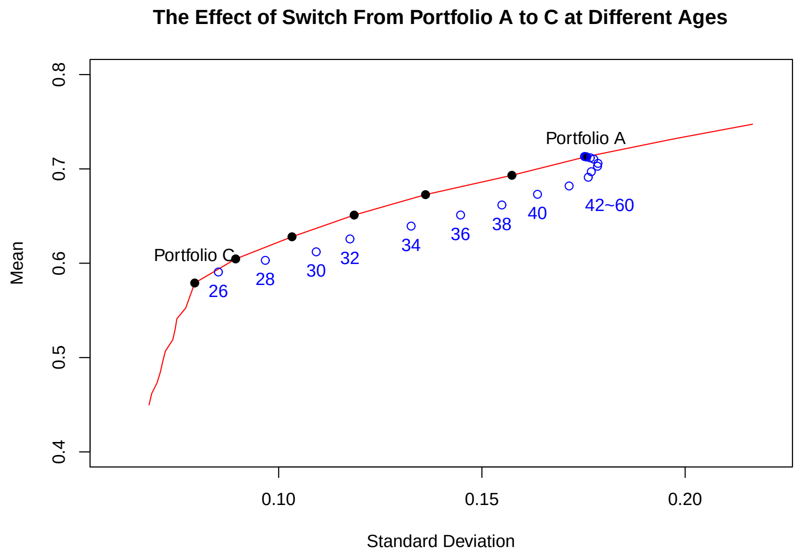


Figure 9: The Efficient Portfolios and Efficient Frontier with Age-Varying MRR



Yet the influence of an age-varying MRR assumption can hardly be detected from the CDF of retirement age with pure-stock portfolio, it is still there. Observe the efficient portfolios and frontier in figure 9 ,stock exposure of tangency portfolio moved from 30% to 20 %. As adopting higher minimum replacement ratio level for younger people, it lowers the possibilities of retiring early before age 55. Under the new assumption, the portfolios with one switch decision are no as efficient as holding a certain mixed-asset portfolio homogeneously over the time. See the efficiency of investment strategy with switch decision from A to C in Figure 10.

Figure 10: The Efficiency of Switch Decison from A to C with Age-Varying MRR



6 Conclusion

Being selected in the pension investment portfolio, stock plays the role in enabling people to have chances to retire early. Yet its ability of doing so is not guaranteed, we mingle it with long-term bonds to improve the instability. Cash is proved to be of little help as a long-term pension investment option. The switch decision based on risk preference which is changing with age does not make a better portfolio than holding one certain mixed-asset portfolio homogeneously over the time. The influence of assumption of an age-varying MRR on 100 % stock is not as obvious as it is on the pure-fixed-asset portfolios. 100% stock portfolio still gives its investors chances to retire at very young ages, though this possibilities is reduced under the age-varying MRR assumption. The plots that may indicate the strong ability of DC pension plan in advancing retirement age is a little exaggerated, as now we haven't take tax rate, management fees and other related factors into account.

Appendix

Data and Parameter Estimates

The Investment model is fitted to U.S data from 1982 to 2004. For the long rate, we let it be represented by the 10-year Constant Maturity Treasury Bond's yield rate, and short rate by the 3-month Constant Maturity Treasury Bond's yield rate. The historical yield rates of the bonds are available on the website of Federal Reserve. The national average wage index data is recorded annually and available at the official website of the U.S Social Securities Administration. Data source is given as follow.

- CPI data: http://www.winflationdata.com/
- Bond yield rates: http://www.federalreserve.gov/releases/h15/data.htm
- Stock Index:http://ca.finance.yahoo.com/
- Wage index:http://www.ssa.gov/oact/cola/AWI.html

Table B: Parameter Estimate

ϕ_1=1.0238047658	ϕ_2=-0.04866567	SAR_1=0.2370525	σ_{ω_1}=0.004169	σ_{ω_2}=0.0024986789
σ_v=0.0006059602	u_{TP}=0.0177	AR_{t1}=0.9351	T_1=0.0574	T_2=-0.1747
θ_1=0.5847	θ_2=0.0255	$\sigma_{a_{3t}}$=0.002484552	u_H=0.0083	H_1=-0.0531
$\sigma_{a_{4t}}$=0.044271887	u_w=0.01011	$\sigma_{a_{5t}}$=0.01913		

Figures

Figure k: Multivariate Autocorrelation Plot of Log Growth of Short Rate and Log Growth of CPI

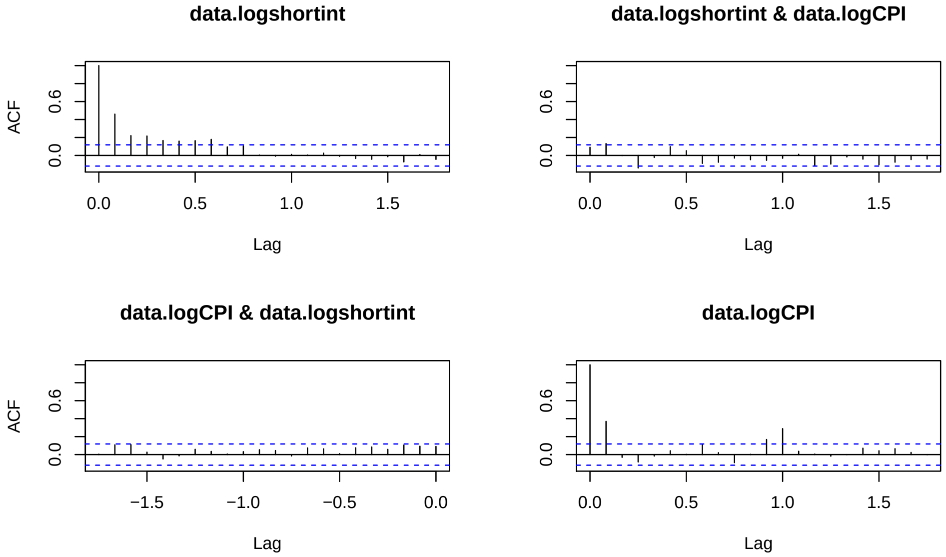
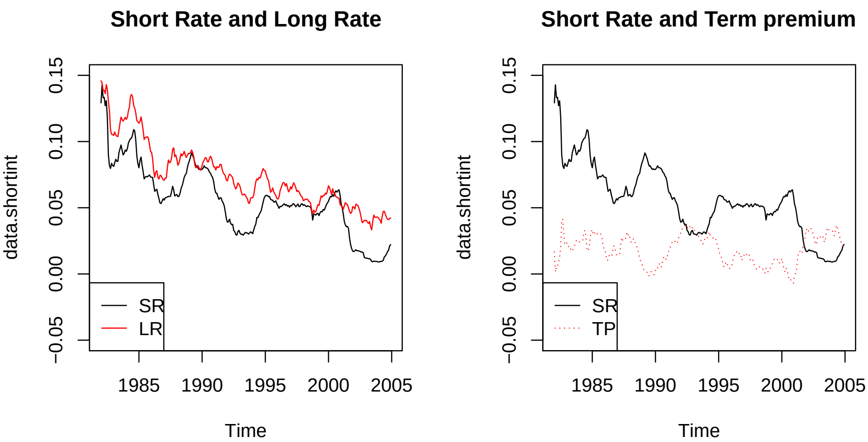


Figure l: Historical Data of Short Rate and Long Rate



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