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# Ultimate Ruin Probability with Correlation

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**Abstract**

In the Sparre-Andersen model, claim sizes and claim occurrence times are assumed to be independent. The independence assumption was used for simplicity in the literature. In this paper, we introduce correlation and consider two classes of bivariate distributions to model claim sizes and claim occurrence times. We derive exact expressions for the ultimate ruin probability and establish the effect of correlation on ruin probability using the Wiener-Hopf factorization.

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# 1 Introduction

Risk theory refers to a body of techniques used to model and measure risk associated with a portfolio of insurance contracts. It therefore provides an avenue of assessing the solvency of an insurance undertaking. In practice, an insurer commences business with an initial capital $u$, collects premium $c$ and payout claims $X_i$. This results in a net balance called surplus for any fixed time $t > 0$. If the surplus at any time falls below zero, we say the insurer concerned is in a state of insolvency and therefore ruin has occured. In the literature, two risk models have been studied extentively. These models are the Classical Compound Poisson (CCP) risk model and the Continuous Time Renewal process also known as the Sparre Andersen (S-A) risk model.

The risk surplus process $S(t)$ underlying the CCP risk model is given as in:

$$S(t) = u + ct - \sum_{i=1}^{N(t)} X_i \tag{1}$$

where $u \geq 0$ is the initial capital, $N(t)$ the time homogeneous Poisson process with intensity $\lambda$, and the $X_i$'s are independent identically distributed non-negative random variables representing claim sizes.

The risk surplus process $S(t)$ within the framework of S-A risk model is defined as:

$$S(t) = u + \sum_{i=1}^{n}(cT_i - X_i) + c(t - \sum_{i=1}^{n} T_i) \tag{2}$$

where $T_i$'s are independent identically distributed non-negative random variable representing interclaim times with $T_1$ indicating time until the first claim and $t$ is such that $\sum_{i=1}^{n} T_i \leq t \leq \sum_{i=1}^{n+1} T_i$. However, $u$, $c$ and $X_i$ have the same interpretation as in the CCP risk model.

The ultimate ruin probability $\psi(u)$ given an initial capital of $u \geq 0$ is defined as:

$$\psi(u) = P[T < \infty] \tag{3}$$

where $T = min\{t > 0 : S(t) < 0 | U_0 = u\}$ and it represents time of ruin. Ruin probabilities and related quantities have been studied by prominent researchers. Gerber and Shiu (1997b), Dickson (1992, 1993), Dufresne and Gerber (1998), Dickson and Hipp (1998) are among few references.

In the S-A model, $T$ and $X$ are assumed to be independent, however for some real world situations, this assumption is very restrictive. Albrecher and Boxma (2004) and Albrecher and Teugels(2006) obtained approximation for the ruin probability when $T$ and $X$ are related. In the case of Ambagaspitiya (2009), exact expression for the ruin probability was obtained. In this paper, we relax the independence assumption by considering two classes of bivariate distribution for $(T_i, X_i)$. We selected Moran and Downton's bivariate exponential distribution due to its simplicity in terms of its moment generating function. We derive the explicit expressions for the ultimate ruin probability and establish the effect of correlation on ruin probability.

# 2 Wiener-Hopf Factorization Method

The Wiener-Hopf factorization method is used to obtain transforms of quantities related to a random walk. Andersen (1957) used it to obtain infinite time ruin probabilities in the S-A model. Basics of this method is well presented in Prabhu (1980) and Asmussen (2000).

The S-A model can be reduced to the study of random walk. Let

$$S_n = \sum_{i=1}^{n} (X_i - cT_i)$$

$$= \sum_{i=1}^{n} Y_i$$

Note that $S_0 = 0$, $S_n = \{S_1, ..., S_n\}$ for $n \geq 1$ and $E[Y] < 0$. If we define another random variable $M_n$ as $M_n = max(S_0, S_1, ..., S_n)$, then the maximal aggregate loss $L$ of $S(t)$ becomes

$$L = \lim_{n\to\infty} M_n \tag{4}$$

We find the characteristic function (CF) of $L$ from the CF of $Y_i$ as follows:(see Ambagaspitiya (2009)):

1. factorize $1-z\phi_{Y_i}(\omega)$ into two factors $D(z,\omega)$ and $\tilde{D}(z,\omega)$ with the following properties:

a. $D(z,\omega)$ should be analytical and bounded away from zero for $i\omega_2 > 0$,

b. $\tilde{D}(z,\omega)$ should be analytical and bounded away from zero for $i\omega_2 < 0$,

c. $\lim_{i\omega_2\to\infty} D(z,\omega) = 1$.

2. Compute $\phi_L(\omega) = \lim_{z\to 1} \frac{1-z}{D(z,\omega)\tilde{D}(z,0)}$.

The survival probability ($\delta(u)$) and $L$ are related by

$$\delta^*(s) = \frac{1}{s} M_L(-s) \tag{5}$$

Therefore, the ultimate ruin probability ($\psi(u)$) becomes

$$\psi(u) = 1 - \delta(u)$$

Note that $\delta(u)$ is obtained by taking the inverse Laplace transform of $\delta^*(s)$. The compliment of $\delta(u)$ gives us the explict expression $\psi(u)$.

# 3 The Ultimate Ruin Probability

**$T$ is Exponential and $X$ is Gamma with Shape 2**

Let $X_1$, $X_2$ form the Moran and Downton's bivariate exponential with correlation coefficient $\rho \geq 0$, then, the associated joint moment generating function (mgf) is given as

$$\begin{aligned} M_{X_1,X_2}(t_1,t_2) &= E[e^{X_1t_1+X_2t_2}] \\ &= \frac{1}{(1-t_1)(1-t_2)-\rho t_1 t_2} \end{aligned}$$

i.e. $X_1$ and $X_2$ each are exponential with mean 1. Let us introduce the following two random variables $T, X$.

$$\begin{aligned} T &= \frac{X_1}{\beta_1} \\ X &= \frac{X_2+Z}{\beta_2}, \end{aligned}$$

where $Z$ is exponential with mean 1 which is independent of $X_1$ and $X_2$. This yields a bivariate distribution with exponential and gamma marginals. The mgf of it can be derived as follows:

$$\begin{aligned} M_{T,X}(t_1,t_2) &= E\left[e^{\frac{X_1t_1}{\beta_1}+\frac{X_2t_2+Zt_2}{\beta_2}}\right] \\ &= M_{X_1,X_2}(\frac{t_1}{\beta_1},\frac{t_2}{\beta_2})M_Z(\frac{t_2}{\beta_2}) \\ &= \frac{1}{\left[\left(1-\frac{t_1}{\beta_1}\right)\left(1-\frac{t_2}{\beta_2}\right)-\rho\frac{t_1t_2}{\beta_1\beta_2}\right]\left(1-\frac{t_2}{\beta_2}\right)} \end{aligned}$$

Assume $T$ represents the inter arrival time between claims and $X$ is the claim size. Then we obtain the mgf of $Y = X - cT$ as follows:

$$\begin{aligned} M_Y(s) &= E[e^{Ys}] \\ &= E[e^{Xs-csT}] \\ &= M_{T,X}(-cs,s) \\ &= \frac{1}{\left[\left(1+\frac{cs}{\beta_1}\right)\left(1-\frac{s}{\beta_2}\right)+\rho\frac{cs^2}{\beta_1\beta_2}\right]\left(1-\frac{s}{\beta_2}\right)} \end{aligned} \tag{6}$$

We let the denomenator of (6) be $g(s)$ as follows:

$$g(s) = \left[\left(1+\frac{cs}{\beta_1}\right)\left(1-\frac{s}{\beta_2}\right)+\rho\frac{cs^2}{\beta_1\beta_2}\right]\left(1-\frac{s}{\beta_2}\right)$$

$$= \left[-\frac{c}{\beta_1\beta_2}(1-\rho)s^2 + \left(\frac{c}{\beta_1} - \frac{1}{\beta_2}\right)s + 1\right]\left(1 - \frac{s}{\beta_2}\right) \tag{7}$$

Then, we can re-write $g(s)$ as

$$g(s) = \frac{c(1-\rho)}{\beta_1\beta_2^2}(s+s_1)(s-s_2)(s-s_3) \tag{8}$$

where $s_1 > 0$ is the negative root, $s_2 = \beta_2$ and $s_3$ is the largest root. Now we consider the roots of $g(s) - z$ when $0 < z < 1$. The behaviour of the roots as $z \to 0$ are:

$$\begin{aligned}
\lim_{z\to 0} \quad s_1(z) &\to s_1 \\
\lim_{z\to 0} \quad s_2(z) &\to s_2 = \beta_2 \\
\lim_{z\to 0} \quad s_3(z) &\to s_3
\end{aligned}$$

By the shape of the graph of $g(s)$, relationships for the roots of $g(s) - z$ are:

$$\begin{aligned}
s_1(z) &< s_1 \\
s_2(z) &< \beta_2 \\
s_3(z) &> s_3
\end{aligned}$$

Also, we obtain the following limiting relationships:

$$\begin{aligned}
\lim_{z\to 1} \quad s_1(z) &\to 0 \\
\lim_{z\to 1} \quad s_2(z) &\to s_2(1) < \beta_2 \\
\lim_{z\to 1} \quad s_3(z) &\to s_3(1) > s_3
\end{aligned}$$

Therefore, we can write $g(s) - z$ as:

$$g(s) - z = \frac{c(1-\rho)}{\beta_1\beta_2^2}(s+s_1(z))(s-s_2(z))(s-s_3(z)) \tag{9}$$

Then

$$\frac{g(s)-z}{g(s)} = \frac{(s+s_1(z))(s-s_2(z))(s-s_3(z))}{(s+s_1)(s-s_2)(s-s_3)}$$

We then write the factors of $1-\phi_Y(\omega)$ as follows:

$$1-\phi_Y(\omega)=\frac{(i\omega+s_1(z))(i\omega-s_2(z))(i\omega-s_3(z))}{(i\omega+s_1)(i\omega-s_2)(i\omega-s_3)}$$

where

$$D(z,\omega)=\frac{(i\omega-s_2(z))(i\omega-s_3(z))}{(i\omega-s_2)(i\omega-s_3)}$$

and

$$\tilde{D}(z,\omega)=\frac{(i\omega+s_1(z))}{(i\omega+s_1)}$$

By replacing $\omega_1+i\omega_2$ in $D(z,\omega)$ and $\tilde{D}(z,\omega)$, the following results:

$$D(z,\omega)=\prod_{j=2}^{3}\left[1-\frac{s_j-s_j(z)}{s_j-i\omega_1+\omega_2}\right]$$

and

$$\tilde{D}(z,\omega)=\left[1-\frac{s_1-s_1(z)}{s_1+i\omega_1-\omega_2}\right]$$

At this point, it is obvious that

1. $\tilde{D}(z,\omega)$ is a bounded function, bounded away from zero for $\omega_2<0$.
2. $D(z,\omega)$ is a bounded function, bounded away from zero for $\omega_2>0$.
3. $\lim_{\omega_2\to\infty} D(z,\omega)=1$.

Therefore $\tilde{D}(z,\omega)$ and $D(z,\omega)$ are the required factors. Quite importantly in arriving at the ultimate ruin probability, they are conditions precedent. Substituting $s=0$ in (7) and (8). Since $g(0)=1$ we have

$$\frac{c(1-\rho)}{\beta_1\beta_2^2}s_1s_2s_3=1 \tag{10}$$

and

$$\frac{c(1-\rho)}{\beta_1\beta_2^2}s_1(z)s_2(z)s_3(z)=1-z \tag{11}$$

hence

$$\frac{1-z}{s_1(z)} = \frac{s_2(z)s_3(z)}{s_1 s_2 s_3}$$

Therefore

$$\begin{aligned}
\phi_L(\omega) &= \frac{1}{D(1,\omega)} \lim_{z\to 1} \frac{1-z}{\tilde{D}(z,0)} \\
&= \frac{(i\omega - s_2)(i\omega - s_3)}{(i\omega - s_2(1))(i\omega - s_3(1))} s_1 \frac{s_2(1)s_3(1)}{s_1 s_2 s_3} \\
&= \frac{(i\omega - s_2)(i\omega - s_3)}{(i\omega - s_2(1))(i\omega - s_3(1))} \frac{s_2(1)s_3(1)}{s_2 s_3}
\end{aligned}$$

Now

$$\delta^*(s) = \frac{1}{s} \frac{(s+s_2)(s+s_3)}{(s+s_2(1))(s+s_3(1))} \frac{s_2(1)s_3(1)}{s_2 s_3}$$

we break $\delta^*(s)$ into partial fractions as follows:

$$\delta^*(s) = \frac{A}{s} + \frac{B}{s+s_2(1)} + \frac{C}{s+s_3(1)}$$

A, B and C are respectively given as $A = 1$, $B = -\frac{\left(1-\frac{s_2(1)}{s_2}\right)\left(1-\frac{s_2(1)}{s_3}\right)}{1-\frac{s_2(1)}{s_3(1)}}$ and $C = -\frac{\left(1-\frac{s_3(1)}{s_2}\right)\left(1-\frac{s_3(1)}{s_3}\right)}{1-\frac{s_3(1)}{s_2(1)}}$.

Therefore the survival probability $\delta(u)$ is obtained by inverting $\delta^*(s)$

$$\delta(u) = 1 - \left[ \frac{\left(1-\frac{s_2(1)}{s_2}\right)\left(1-\frac{s_2(1)}{s_3}\right)}{1-\frac{s_2(1)}{s_3(1)}} e^{-s_2(1)u} + \frac{\left(1-\frac{s_3(1)}{s_2}\right)\left(1-\frac{s_3(1)}{s_3}\right)}{1-\frac{s_3(1)}{s_2(1)}} e^{-s_3(1)u} \right] \tag{12}$$

**$T$ is Exponential and $X$ is Gamma with Shape $m$**

Let us consider another form of the random variable $X$.

$$
\begin{aligned}
T &= \frac{X_1}{\beta_1} \\
X &= \frac{X_2 + Z}{\beta_2},
\end{aligned}
$$

where $Z$ is gamma with shape $m-1$ which is independent of $X_1$ and $X_2$. With this notation $T$ and $X$ are correlated random variables. The mgf of it can be obtained as follows:

$$
\begin{aligned}
M_{T,X}(t_1, t_2) &= E\left[e^{\frac{X_1 t_1}{\beta_1} + \frac{X_2 t_2 + Z t_2}{\beta_2}}\right] \\
&= M_{X_1,X_2}(\frac{t_1}{\beta_1}, \frac{t_2}{\beta_2}) M_Z(\frac{t_2}{\beta_2}) \\
&= \frac{1}{\left[\left(1 - \frac{t_1}{\beta_1}\right)\left(1 - \frac{t_2}{\beta_2}\right) - \rho \frac{t_1 t_2}{\beta_1 \beta_2}\right]\left(1 - \frac{t_2}{\beta_2}\right)^{m-1}}
\end{aligned}
$$

Now, we need the moment generating function of $Y = X - cT$

$$
\begin{aligned}
M_Y(s) &= E[e^{Ys}] \\
&= E[e^{Xs - csT}] \\
&= M_{T,X}(-cs, s) \\
&= \frac{1}{\left[\left(1 + \frac{cs}{\beta_1}\right)\left(1 - \frac{s}{\beta_2}\right) + \rho \frac{cs^2}{\beta_1 \beta_2}\right]\left(1 - \frac{s}{\beta_2}\right)^{m-1}}
\end{aligned}
$$

writing

$$
\begin{aligned}
g(s) &= \left[\left(1 + \frac{cs}{\beta_1}\right)\left(1 - \frac{s}{\beta_2}\right) + \rho \frac{cs^2}{\beta_1 \beta_2}\right]\left(1 - \frac{s}{\beta_2}\right)^{m-1} \\
&= \left[-\frac{c}{\beta_1 \beta_2}(1-\rho)s^2 + \left(\frac{c}{\beta_1} - \frac{1}{\beta_2}\right)s + 1\right]\left(1 - \frac{s}{\beta_2}\right)^{m-1}
\end{aligned}
$$

Here too, writing $g(s)$ and $g(s)-z$ as before

$$g(s) \quad = \quad (-1)^m \frac{c(1-\rho)}{\beta_1 \beta_2^m}(s+s_1)(s-s_3)(s-s_2)^{m-1} \tag{13}$$

and

$$g(s)-z \quad = \quad (-1)^m \frac{c(1-\rho)}{\beta_1 \beta_2^m}(s+s_1(z))(s-s_3(z))(s-s_2(z))^{m-1} \tag{14}$$

With a little bit of complex analysis and some derivations, we obtain the ultimate ruin probability as

$$\psi(u) \quad = \quad \sum_{j=1}^{m-1} B_j e^{-s_{2,j}(1)u} + Ce^{-s_3(1)u} \tag{15}$$

where

$$-B_j \quad = \quad \frac{s_3(1)(s_2 - s_{2,j}(1))^{m-1}(s_3 - s_{2,j}(1)) \prod_{k=1}^{m-1} s_{2,k}(1)}{s_{2,j}(1)(s_3(1) - s_{2,j}(1)) \prod_{k=1, k\neq j}^{m-1} \left[s_{2,k}(1) - s_{2,j}(1)\right]}$$

and

$$-C \quad = \quad \frac{(s_2 - s_3(1))^{m-1}(s_3 - s_3(1)) \prod_{k=1}^{m-1} s_{2,k}(1)}{\prod_{k=1}^{m-1}(s_{2,k}(1) - s_3(1))}$$

**The Effect of Correlation on Ruin Probability**

Let $s_3 \to f_3(\rho)$ and $s_3(1) \to f_3^*(\rho)$

where

$$f_3(\rho) \quad = \quad \frac{\left(\frac{c}{\beta_1} - \frac{1}{\beta_2}\right) + \sqrt{\left(\frac{c}{\beta_1} - \frac{1}{\beta_2}\right)^2 + \frac{4(1-\rho)c}{\beta_1 \beta_2}}}{2(1-\rho)\frac{c}{\beta_1 \beta_2}}$$

but

$$\begin{aligned}
\lim_{\rho\to 0} \quad f_3(\rho) \quad &\to \beta_2 \\
\lim_{\rho\to 1} \quad f_3(\rho) \quad &\to +\infty \\
\lim_{\rho\to 0} \quad f_3^*(\rho) \quad &> \lim_{\rho\to 0} f_3(\rho) \to \beta_2 \\
\lim_{\rho\to 1} \quad f_3^*(\rho) \quad &> \lim_{\rho\to 1} f_3(\rho) \to +\infty
\end{aligned}$$

As $\rho \to [0,1]$, $s_3 \to [\beta_2, +\infty)$ and $s_3\,(1) \to (\beta_2, +\infty)$. Note that,

1. $s_3$ is an increasing function of $\rho$;

2. $s_3\,(1)$ is an increasing function of $\rho$;

3. $s_2\,(1) < \beta_2$ is neither an increasing nor a decreasing function of $\rho$;

4. $\frac{\left(1-\frac{s_2(1)}{s_2}\right)\left(1-\frac{s_2(1)}{s_3}\right)}{1-\frac{s_2(1)}{s_3(1)}}$ is a decreasing function of $\rho$;

5. $\frac{\left(1-\frac{s_3(1)}{s_2}\right)\left(1-\frac{s_3(1)}{s_3}\right)}{1-\frac{s_3(1)}{s_2(1)}}$ is a decreasing function of $\rho$.

Therefore as $\rho$ increases $\psi\,(u)$ decreases.

# 4 Numerical Examples

## 4.1 Example 1

Let $c = 3$, $\beta_1 = 2$, $\beta_2 = 4$, $\rho = \frac{2}{5}$ and $m = 2$. By equating (7) to zero and solving, we get:
$s_1 = s_1(0) = 0.7093$, $s_2 = s_2(0) = \beta_2 = 4$ and $s_3 = s_3(0) = 6.2650$.
By equating (9) to zero and letting $z = 1$, we get:
$s_1(1) = 0$, $s_2(1) = 2.5309$ and $s_3(1) = 7.0247$.
Putting all the above into (12) and subracting from 1, we arrive at:
$\psi(u) = 0.3422e^{s_2(1)u} - 0.0516e^{s_3(1)u}$.

### 4.2 Example 2

Let $c = 3$, $\beta_1 = 2$, $\beta_2 = 4$, $\rho = \frac{2}{5}$ and $m = 5$. By equating (13) to zero and solving, we get:
$s_1 = s_1(0) = 0.7093$, $s_2 = s_2(0) = \beta_2 = 4$ (4 repeated roots) and $s_3 = s_3(0) = 6.2650$.
By equating (14) to zero and letting $z = 1$, we get:
$s_1(1) = 0$, $s_{2,1}(1) = 3.7993 - 2.7569i$, $s_{2,2}(1) = 3.7993 + 2.7569i$, $s_{2,3}(1) = 6.8463 - 1.3592i$, $s_{2,4}(1) = 6.8463 + 1.3592i$ and $s_3(1) = 0.2650$.
Putting all the above into (15), we arrive at:
$\psi(u) = 0.86139e^{-0.2649u} - 0.011022sin(1.3692u) - 0.021099cos(1.3693u)e^{-6.8468u} - 0.034101sin(2.7551u) - 0.017639cos(2.7551u)e^{-3.7985u}$.

# 5 Conclusions

From the above results, we conclude that, it is possible to derive analytically the expressions for the ultimate ruin probability when $T$ and $X$ are correlated. Particularly in our case where both $T$ and $X$ are exponential and gamma (sum of exponential) respectively. Again in our case, we have succeeded in establishing the effect of correlation on ulitmate ruin probability. Thus ultimate ruin probability tends to decrease as correlation increases.

# References


[1] Albrecher, Hansjorg, Hans U. Gerber, and Hailiang Yang. (2010). A direct approach to the discounted penalty function. North American Actuarial Journal 14(4): 420- 447.

[2] Albrecher, Hansjorg and Onno J. Boxma. (2004). A ruin model with dependence between claim sizes and claim intervals. Insurance: Mathematics and Economics 35(2): 245-254.

[3] Albrecher, Hansjorg and Jef L. Teugels. (2006). Exponential behavior in the presence of dependence in risk theory. Journal of Applied Probability 43(1): 257-273.

[4] Albrecher, Hansjorg and Jef L. Teugels. (2006). Exponential behavior in the presence of dependence in risk theory. Journal of Applied Probability 43(1): 257-273.

[5] Andersen, E.S. (1957). On the collective theory of risk in the case of contagion between the claims.In: Transactions of XVth International Congress of Actuaries, New York 2, pp. 219-229.

[6] Asmussen, Soren. (2000). Ruin Probabilities. Singapore: World Scientific.

[7] Dickson, D.C.M. (1992). On the distribution of the surplus prior to ruin, Insurance: Mathematics and Economics, 11,191-207.

[8] Dickson, D.C.M. (1993). On the distribution of the surplus prior to ruin, Insurance: Mathematics and Economics, 12, 143-154.

[9] Dickson, David C.M. and Christian Hipp. (1997). Ruin probabilities for Erlang(2) risk processes. Insurance Mathematics Economics 22, 251-262.

[10] Dickson, David C.M. and Christian Hipp. (1998). On a class of renewal risk process. North American Actuarial Journal 2 (3), 60-68.

[11] Dickson, David C.M. and Christian Hipp. (2001). On the time to ruin for Erlang(2) risk processes. Insurance Mathematics Economics 29, 333-344.

[12] Dufresne, F., Gerber, H.U., (1988). The surpluses immediately before ruin and at ruin, and the amount of the claim causing ruin, Insurance: Mathematics and Economics, vol. 7, 193-199.

[13] Gerber, Hans U. and Elias S.W. Shiu. (1997b). The joint distribution of the time of ruin, the surplus immediately before ruin, and the de cit at ruin. Insurance Mathematics Economics 21(2): 129-137.

[14] Gerber, Hans U. and Elias S.W. Shiu. (1998). On the time value of ruin. North American Actuarial Journal 2(1): 48-78.

[15] Prabhu, N.U. (1980). Stochastic Storage Processes: Queues, Insurance Risk and Dams. Springer-Verlag, New York.

[16] Ramsay, C. M. (2003). A solution to the ruin problem for Pareto distributions. Insurance: Mathematics and Economics., 33: 109116.

[17] Ambagaspitiya, R.S. (2009). Ultimate ruin probability in the Sparre Andersen model with dependent claim sizes and claim occurrence times. Insurance: Mathematics and Economics., 44(3): 464472.