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Modelling mortality by cause of death and socio-economic stratification: an analysis of mortality differentials in England

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August 1st, 2013 Temple University Philadelphia

Agenda

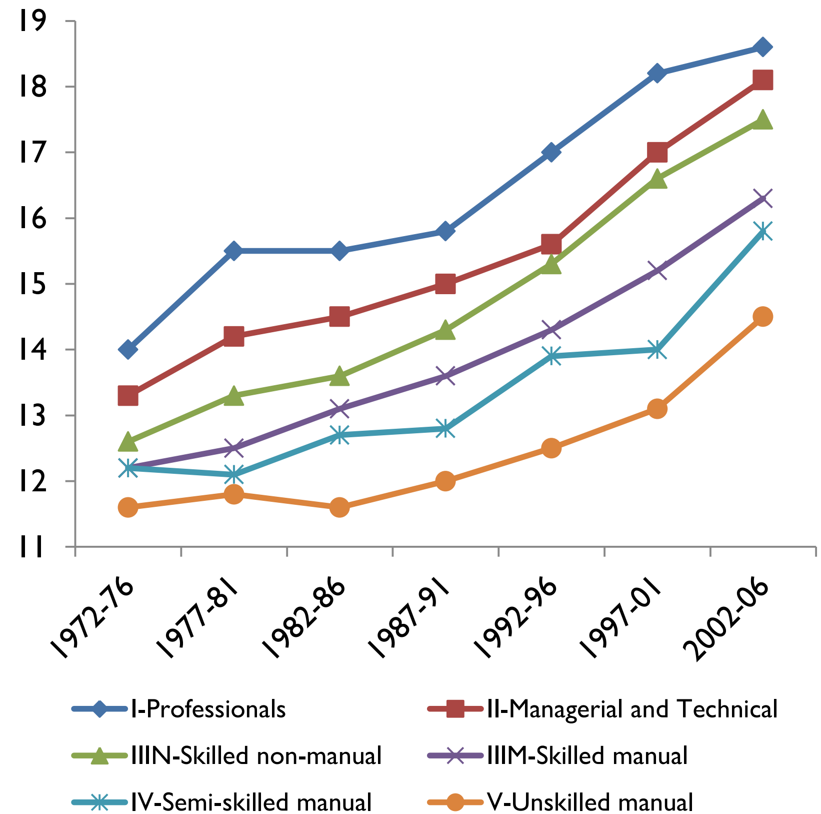
- Motivation
- Modelling mortality by cause of death (CoD)
- Modelling mortality by CoD and socio-economic stratification
- Case study: Mortality by deprivation in England
- Conclusions

Motivation

Socio-economic differences in mortality

- Well-documented relationship between mortality and socioeconomic variables
  - Education
  - Income
  - Occupation
- Important implications on social and financial planning
  - Public policy for tackling inequalities
  - Social security design
  - Annuity reserving and pricing
  - Longevity risk management

Male life expectancy at age 65 by social class -England and Wales



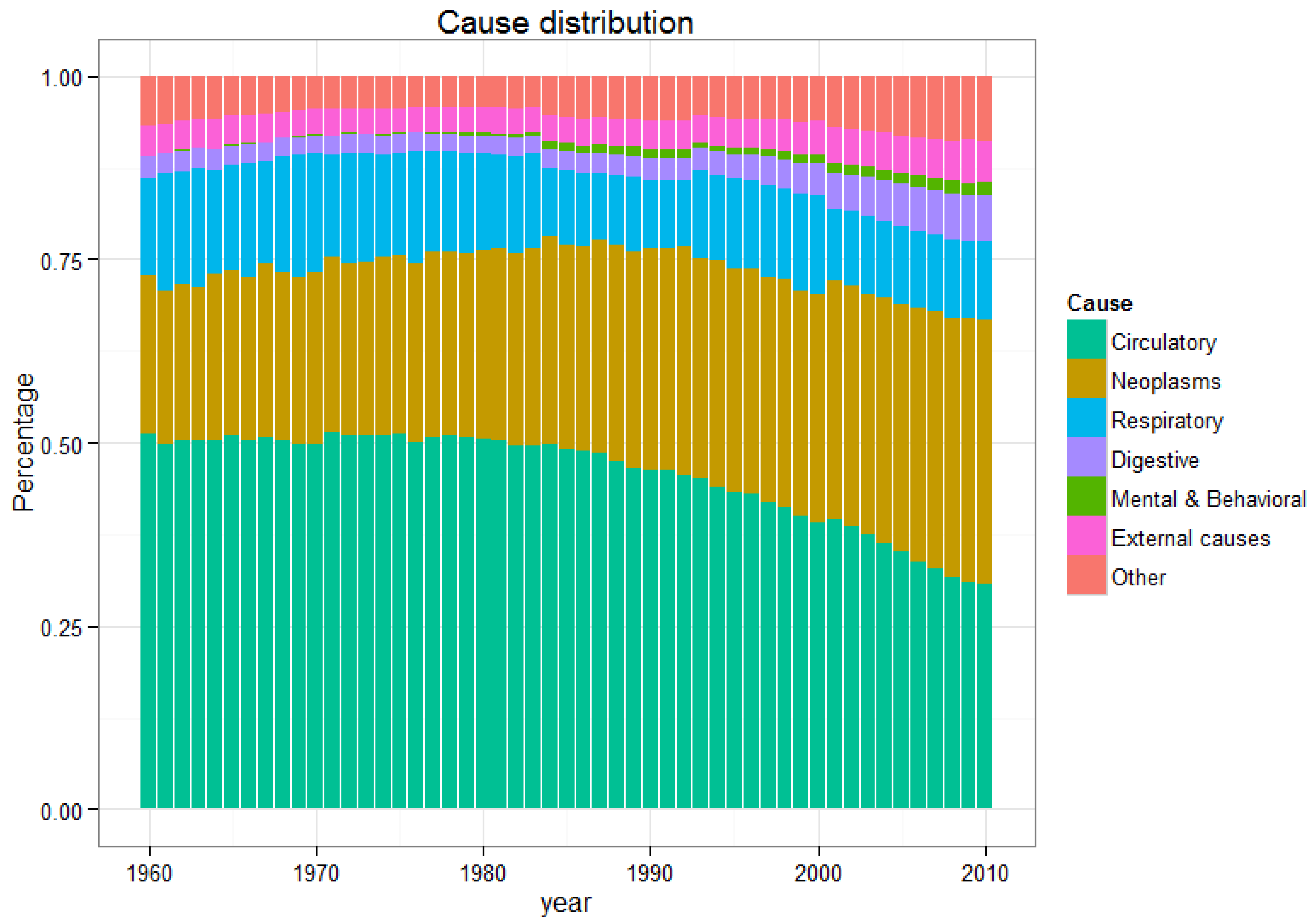
Source: ONS Longitudinal Study

Motivation
Cause-specific mortality

- Forecasts of cause-specific mortality required for many purposes
  - E.g Estimation of health care costs
- Inform the assumptions underlying overall mortality projections
- Shed light on the drivers of
  - Mortality change
  - Mortality differentials

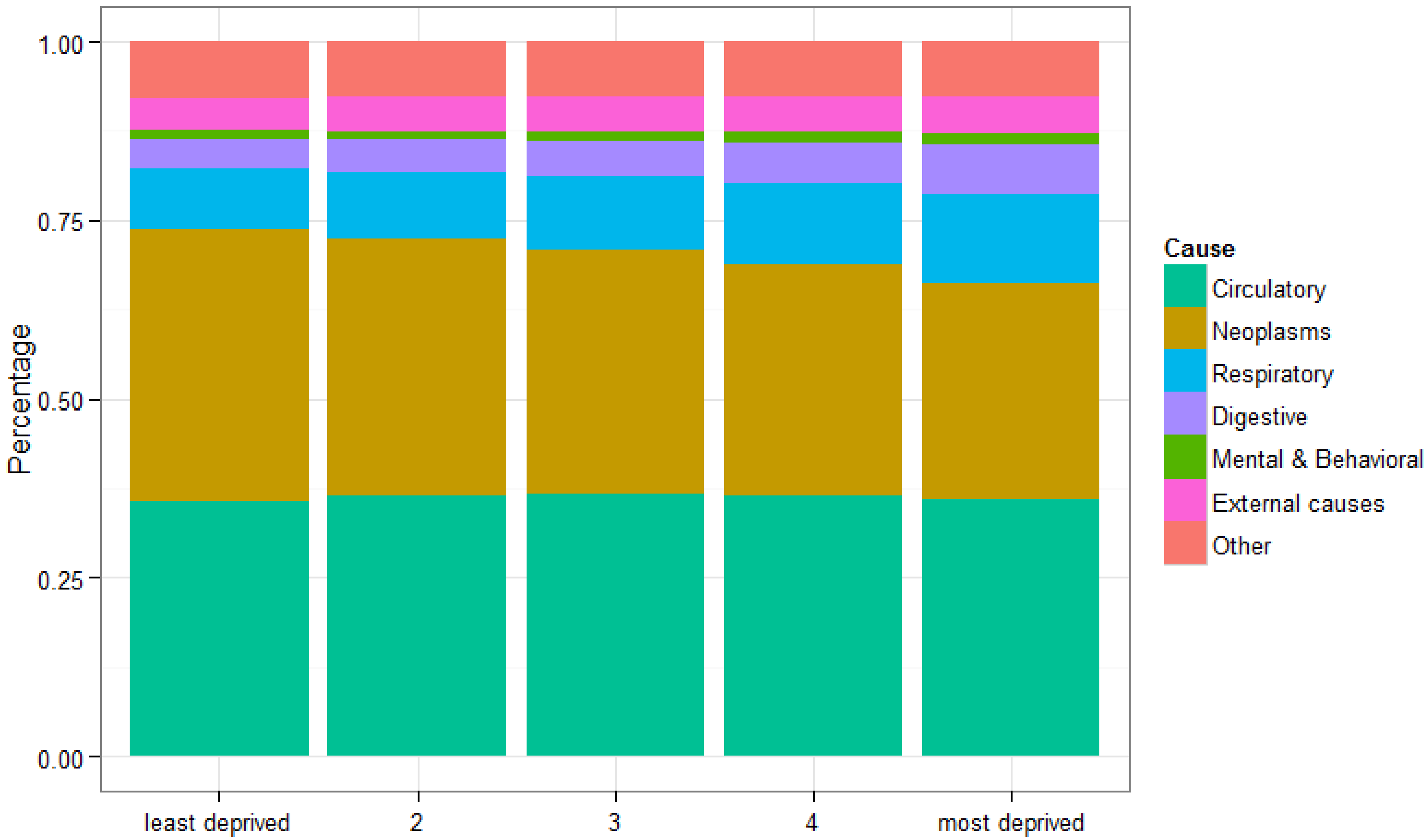
Causes of mortality in England and Wales

Causes distribution in time (ASDR males age 25-84)



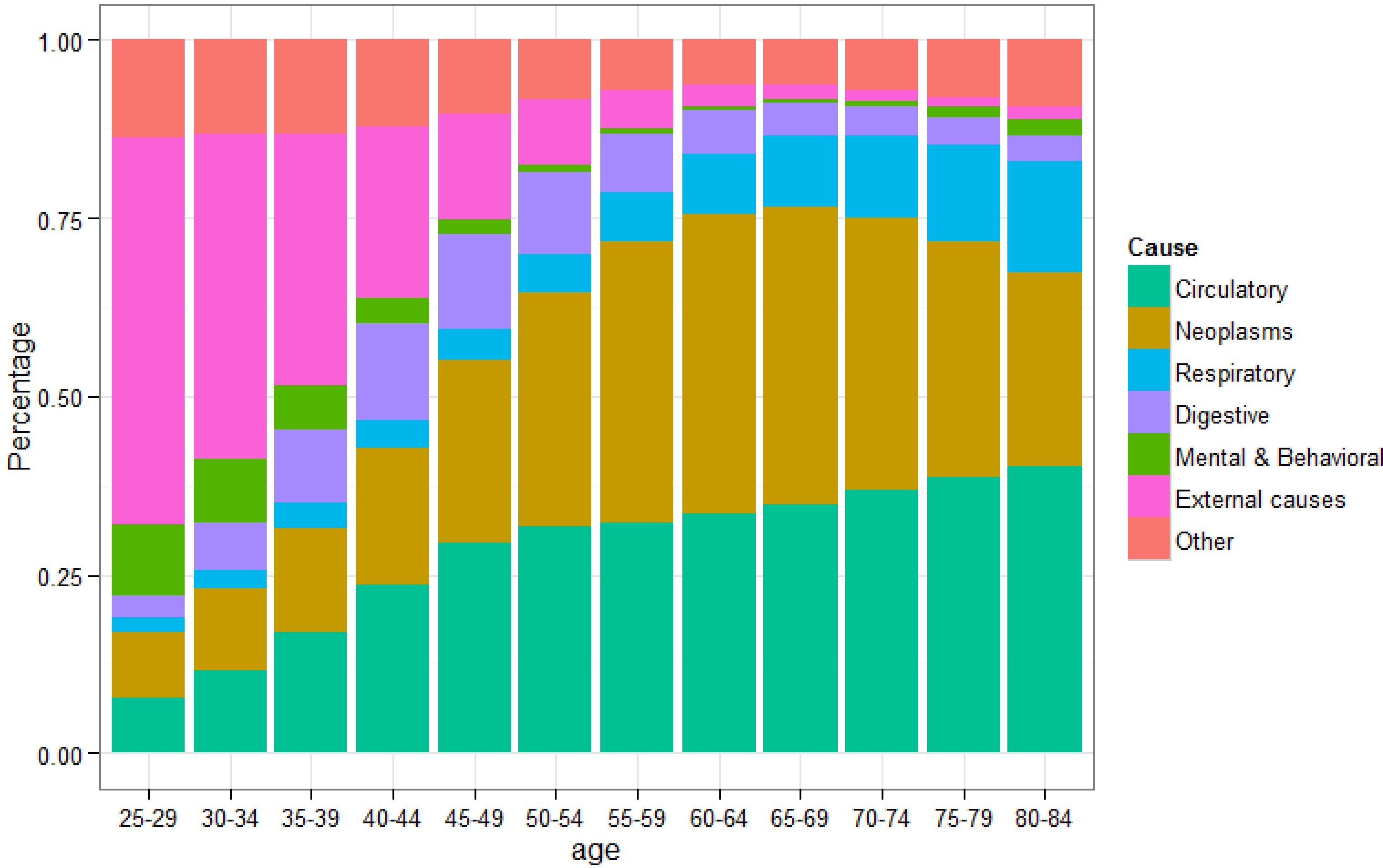
Causes of mortality in England and Wales

Causes distribution by deprivation quintile (males 25-84 2001-2007)



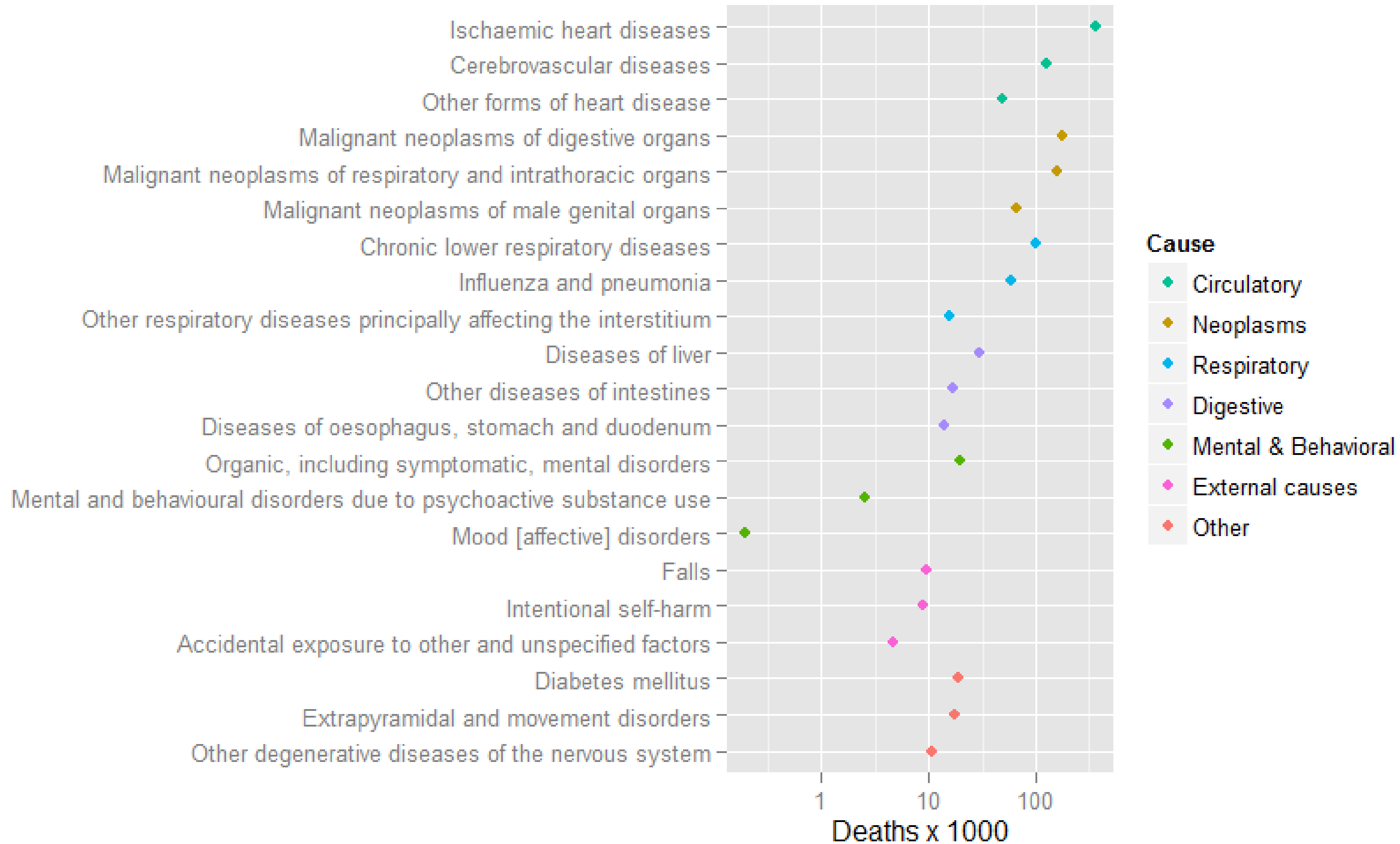
Causes of mortality in England and Wales

Causes distribution by age (males 2001-2010)



Causes of mortality in England and Wales

Main causes for males aged 50-84 (2001-2010)



Causes of mortality in England and Wales Main causes for males aged 25-49 (2001-2010)

Modelling mortality by cause of death

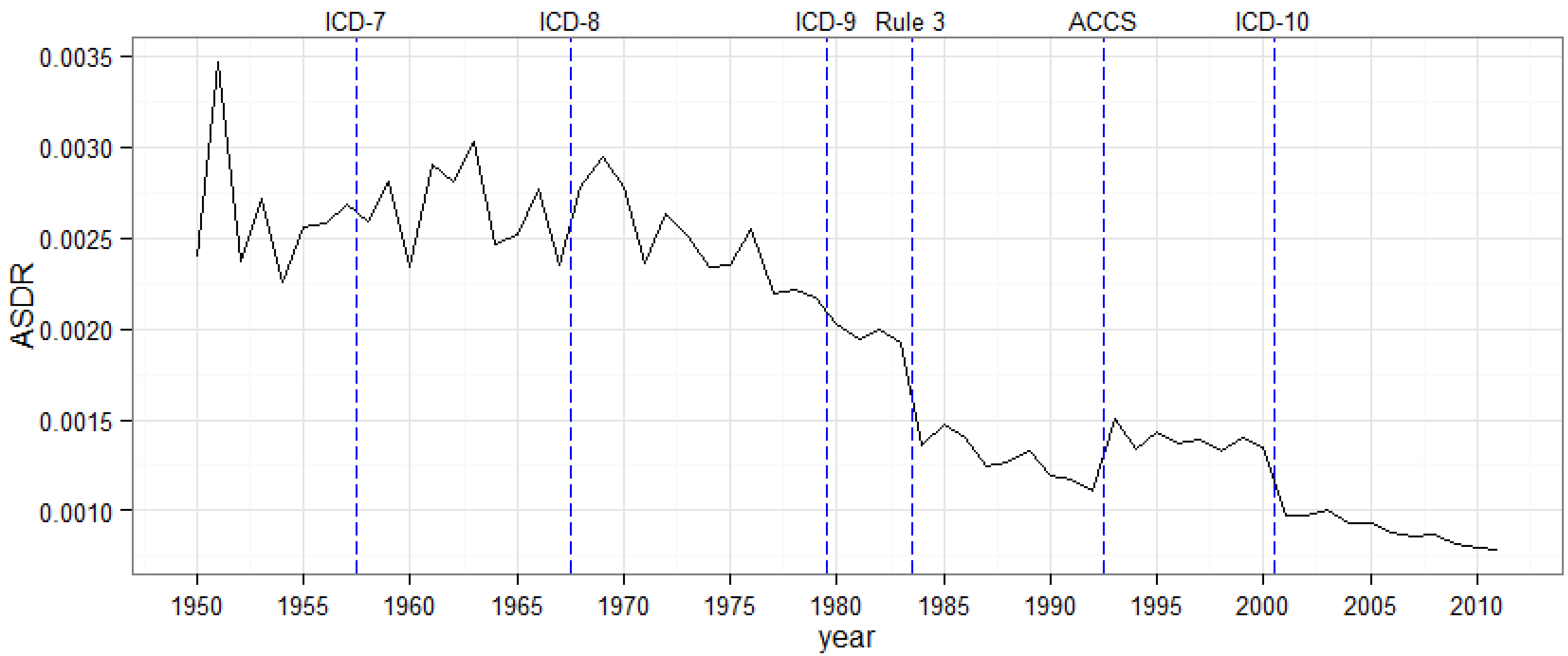
Challenges

- Correlation between causes
  - Same risk factor can affect several causes (e.g. smoking and some cancers and heart diseases)
  - Reduction in the relative importance of one cause can lead to further improvements on other causes
- Increase in dimensionality induced by the disaggregation
  - The same modelling methods might not be appropriate for all causes
  - Major empirical exercise
- Changes in classification of causes of death difficult the analysis of trends

Modelling mortality by cause of death

Cause of death coding changes

Age-standardised mortality rate for respiratory diseases
(Male age 25-84 – England and Wales)



Modelling mortality by cause of death

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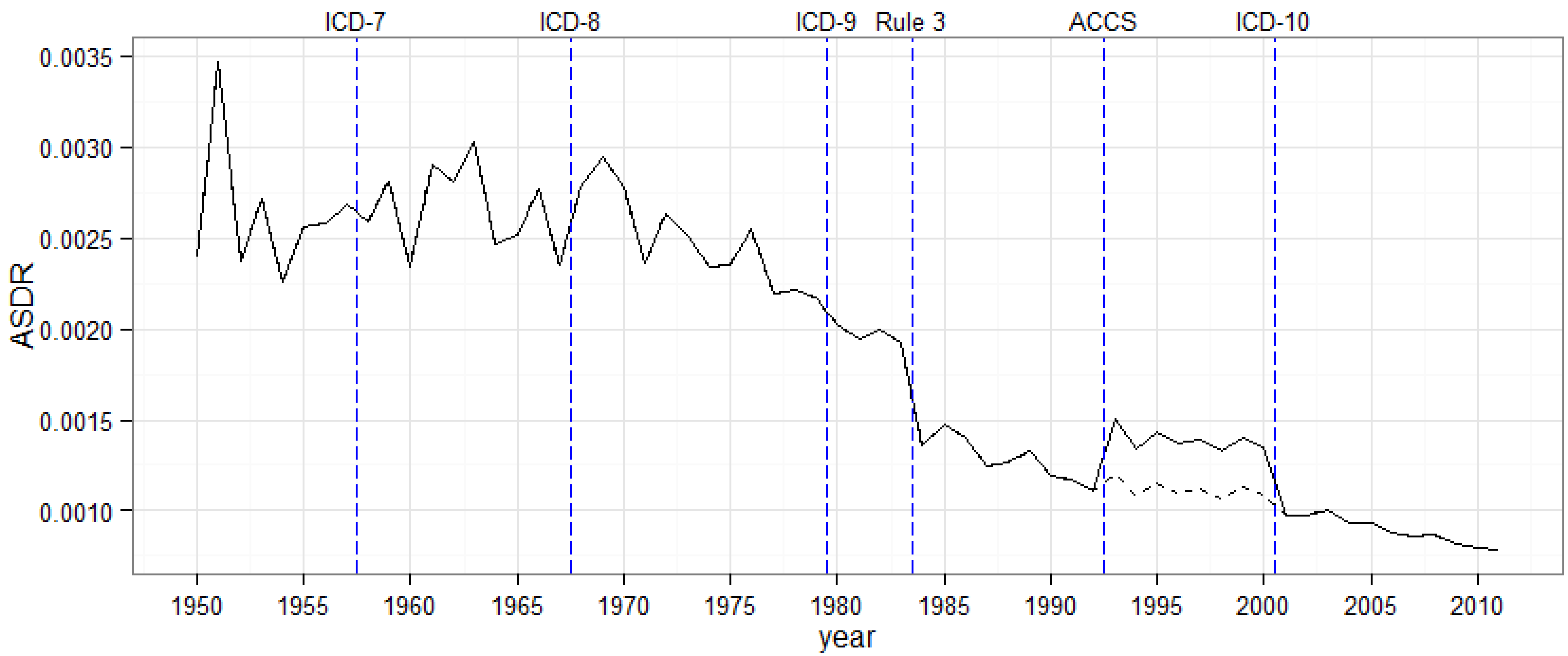


- Adjustment methods
  - Bridge coding and comparability ratios (e.g. ONS for ICD-9 to ICD10)
  - Statistical correction methods (e.g. Rey et al (2009), Park et al (2006))

Modelling mortality by cause of death

Lee-Carter model with coding changes

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t$$

Modelling mortality by cause of death

Lee-Carter model with coding changes

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t$$

Age-specific mortality pattern

Age-modulating parameters

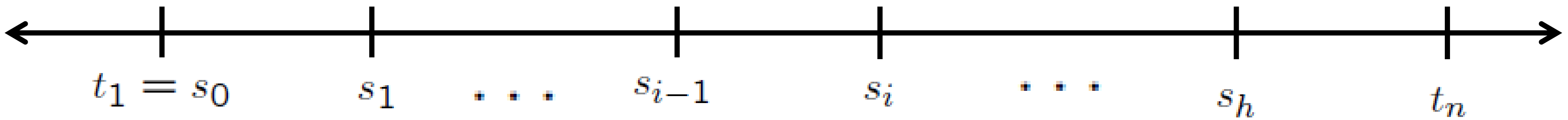
Overall time trend of mortality

Modelling mortality by cause of death

Lee-Carter model with coding changes

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t$$

- α_x: Age-specific mortality pattern
- β_x: Age-modulating parameters
- κ_t: Overall time trend of mortality



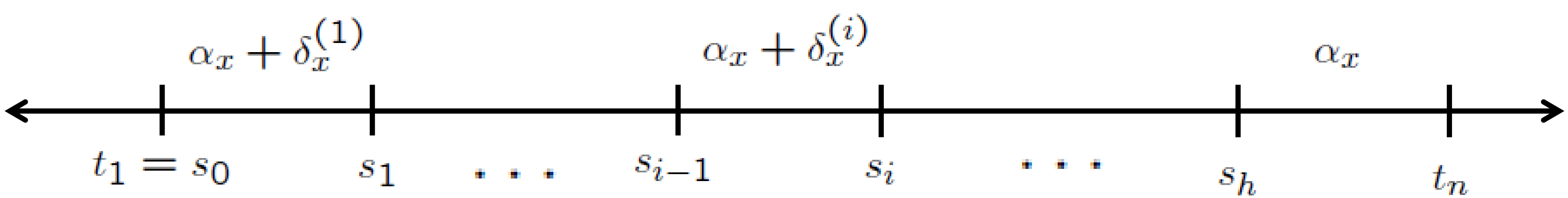
Modelling mortality by cause of death

Lee-Carter model with coding changes

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t + \sum_{i=1}^{h} \delta_x^{(i)} f^{(i)}(t)$$

- α_x: Age-specific mortality pattern
- β_x: Age-modulating parameters
- κ_t: Overall time trend of mortality
- $\sum_{i=1}^{h} \delta_x^{(i)} f^{(i)}(t)$: Adjustment for coding changes

$$f^{(i)}(t) = \mathcal{I}_{\{s_{i-1} \le t < s_i\}}$$



Modelling mortality by cause of death

Lee-Carter model with coding changes – Invariant transformations

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t + \sum_{i=1}^{h} \delta_x^{(i)} f^{(i)}(t)$$

This specification is invariant to the following parameter transformations

Standard Lee-Carter transformations

$$\{\tilde{\alpha}_x, \tilde{\kappa}_t\} = \{\alpha_x + b_1 \beta_x, \kappa_t - b_1\}$$

$$\left\{\tilde{\beta}_x, \tilde{\kappa}_t\right\} = \left\{\frac{1}{b_2}\beta_x, b_2 \kappa_t\right\}$$

Modelling mortality by cause of death

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New transformations

$$\left\{\tilde{\delta}_x^{(i)}, \tilde{\kappa}_t\right\} = \left\{\delta_x^{(i)} + a_i\beta_x, \kappa_t - a_i f^{(i)}(t)\right\} \quad i = 1, \dots, h$$

Modelling mortality by cause of death

Lee-Carter model with coding changes – Identifiability constraints

Standard Lee-Carter

Make the last year in the data the reference

$$\{\tilde{\alpha}_x, \tilde{\kappa}_t\} = \{\alpha_x + b_1\beta_x, \kappa_t - b_1\} \longrightarrow \kappa_{t_n} = 0$$

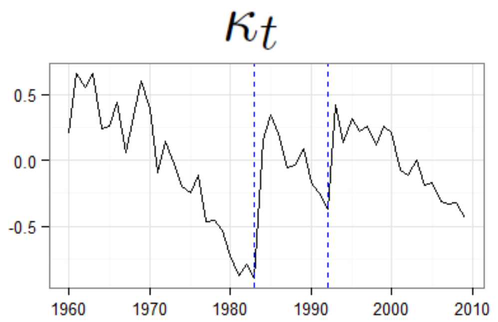
Normalise the age gradient

$$\left\{\tilde{\beta}_x, \tilde{\kappa}_t\right\} = \left\{\frac{1}{b_2}\beta_x, b_2\kappa_t\right\} \longrightarrow \sum_x \beta_x = 1$$

Modelling mortality by cause of death

Lee-Carter model with coding changes – Identifiability constraints

$$\left\{\tilde{\delta}_x^{(i)}, \tilde{\kappa}_t\right\} = \left\{\delta_x^{(i)} + a_i\beta_x, \kappa_t - a_i f^{(i)}(t)\right\}$$

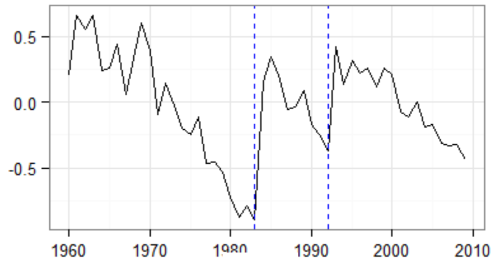


Modelling mortality by cause of death

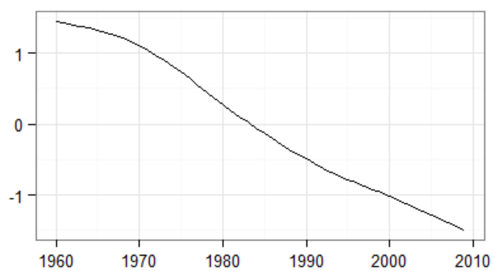
Lee-Carter model with coding changes – Identifiability constraints

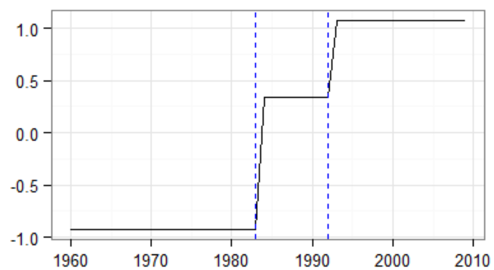
$$\left\{\tilde{\delta}_x^{(i)}, \tilde{\kappa}_t\right\} = \left\{\delta_x^{(i)} + a_i\beta_x, \kappa_t - a_i f^{(i)}(t)\right\}$$

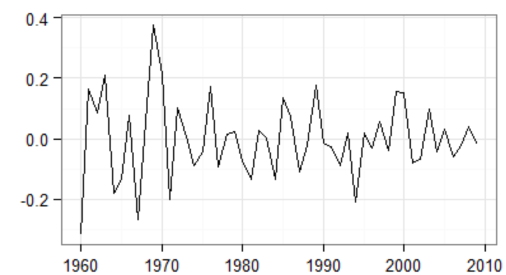
$$\kappa_t$$



$$= \quad g(t) + \sum_{i=1}^{h} a_i f^{(i)}(t) + \epsilon_t$$





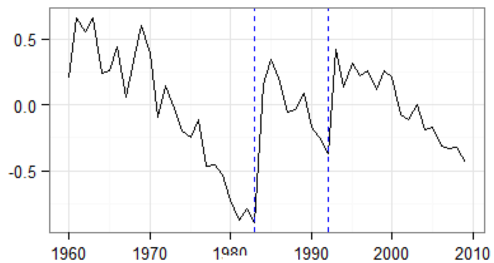


Modelling mortality by cause of death

Lee-Carter model with coding changes – Identifiability constraints

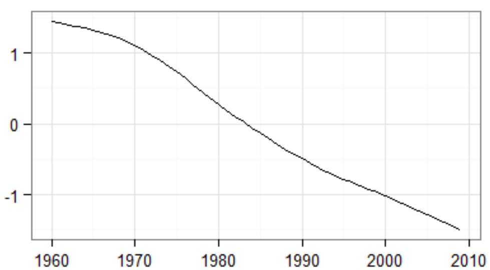
$$\left\{\tilde{\delta}_x^{(i)}, \tilde{\kappa}_t\right\} = \left\{\delta_x^{(i)} + a_i\beta_x, \kappa_t - a_i f^{(i)}(t)\right\}$$

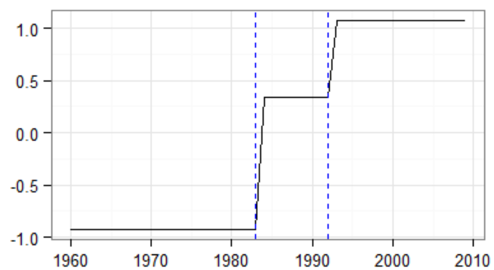
$$\kappa_t$$

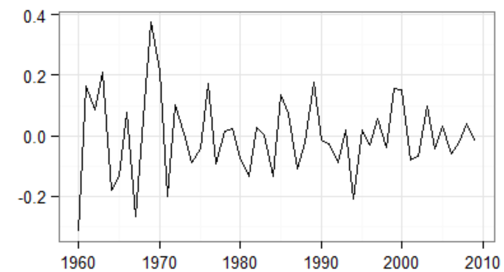


$$=$$

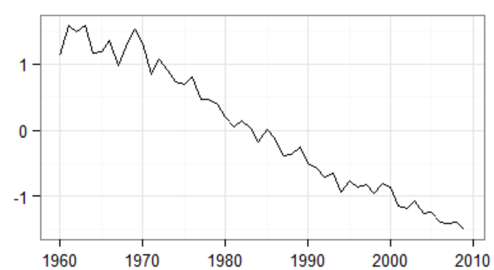
$$g(t) \quad + \quad \sum_{i=1}^{h} a_i f^{(i)}(t) \quad + \quad \epsilon_t$$





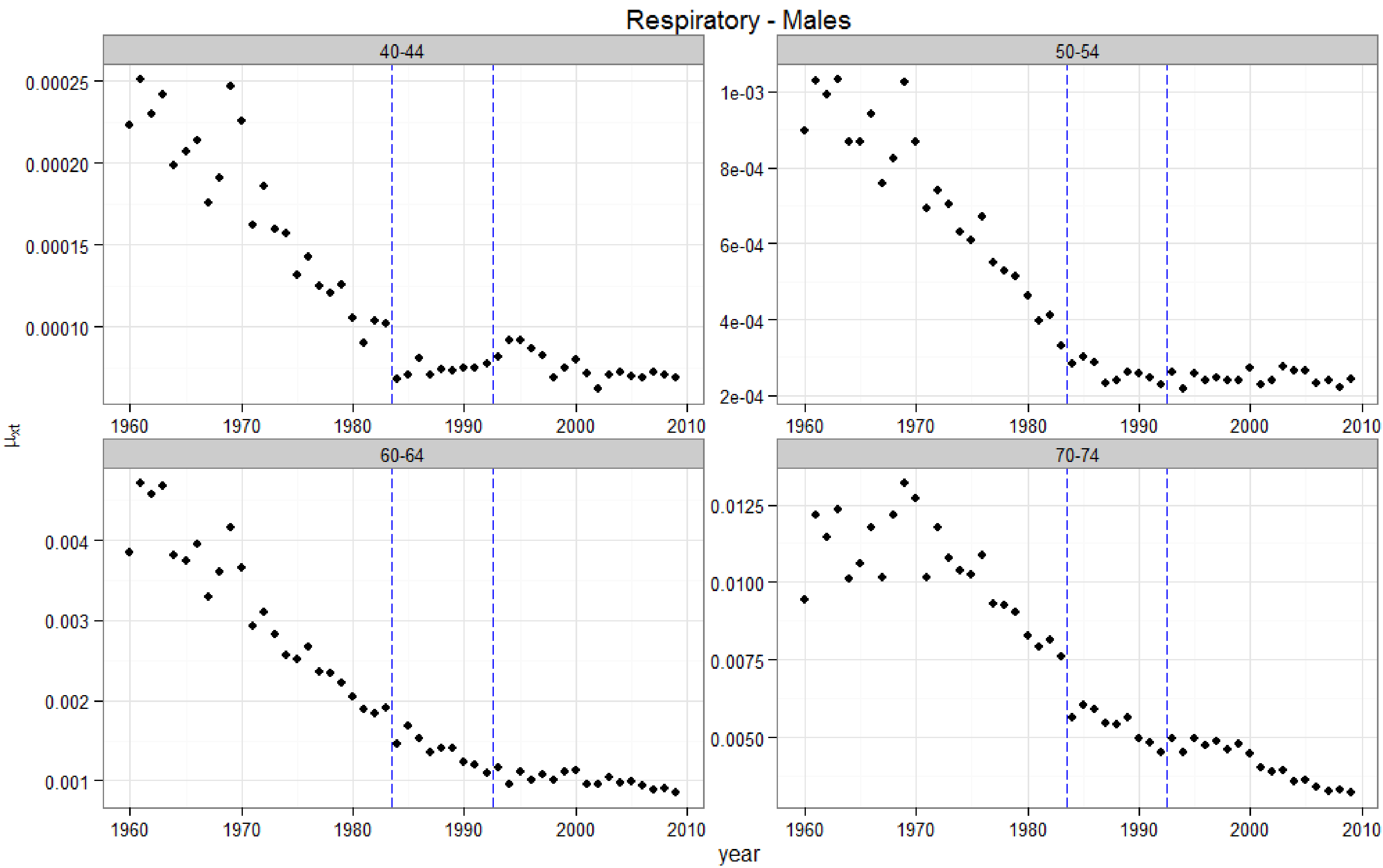


$$g(t) + \epsilon_t = \tilde{\kappa}_t = \kappa_t - \sum_{i=1}^{h} a_i f^{(i)}(t)$$



Modelling mortality by cause of death

Lee-Carter model with coding changes – Example



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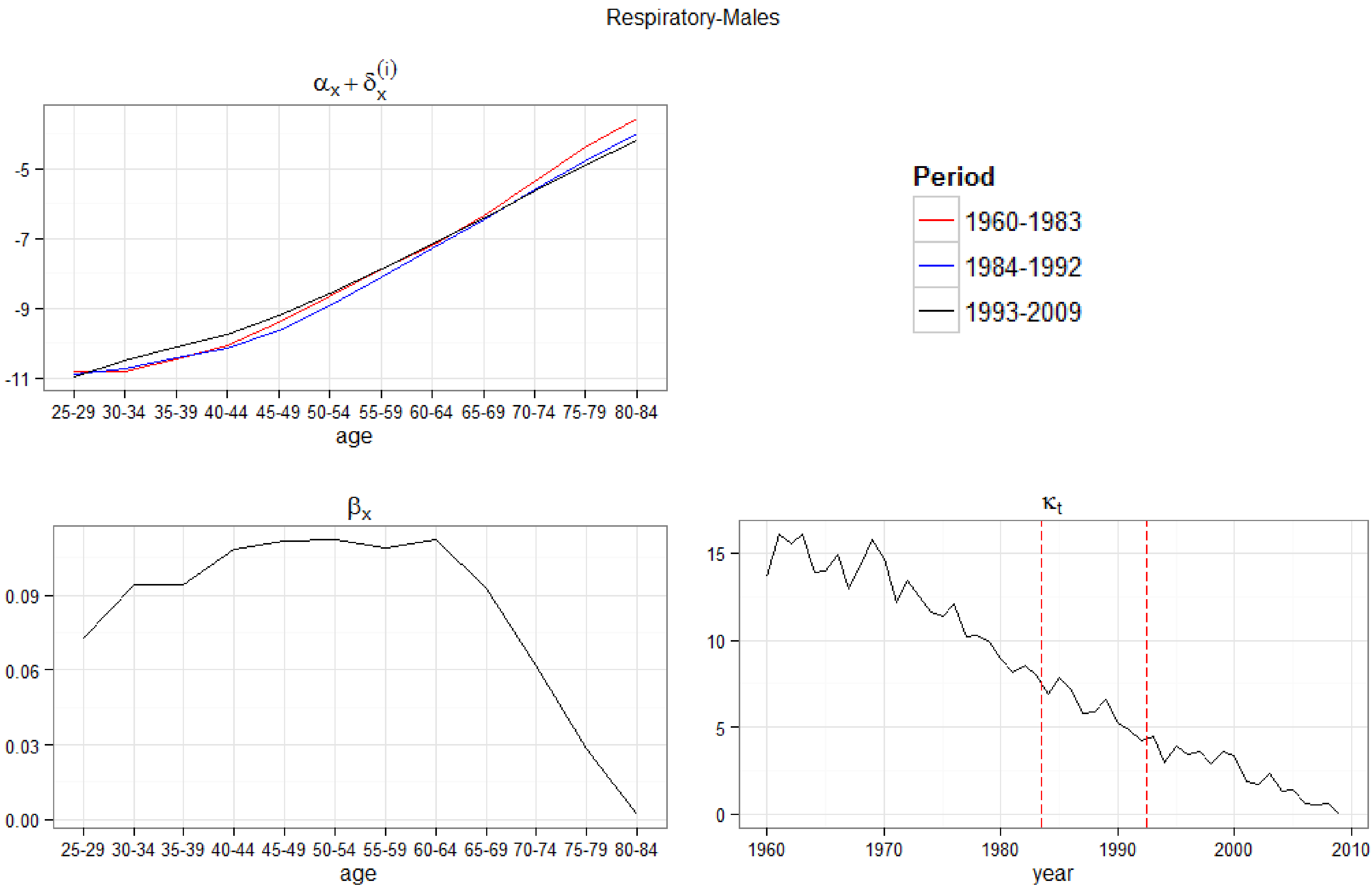

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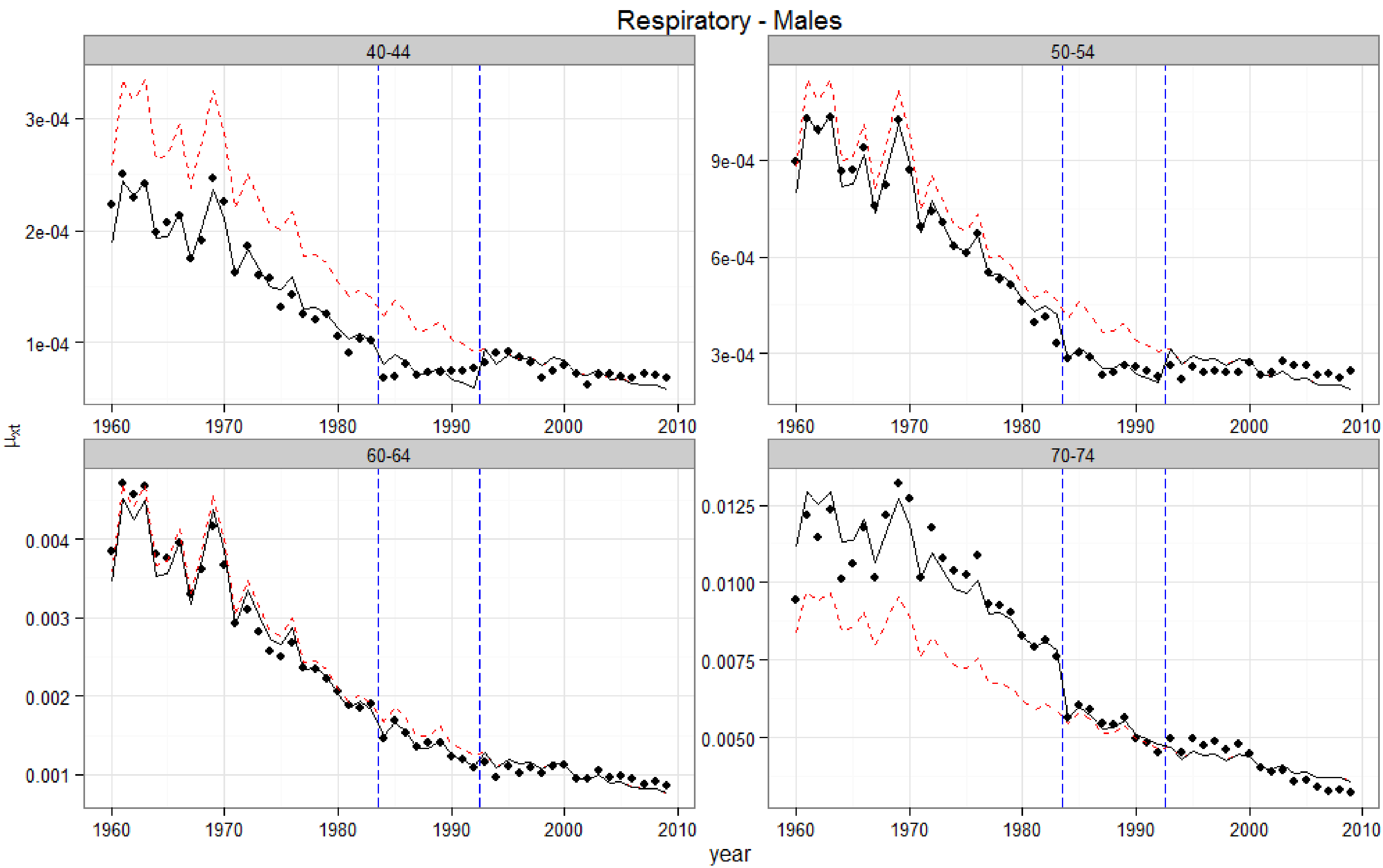


Modelling by CoD and socio-economic stratification

Three-way Lee-Carter model (Russolillo et al, 2011)

$$\log \mu_{xtg}^{c} = \alpha_x^c + \beta_x^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

Modelling by CoD and socio-economic stratification

Three-way Lee-Carter model (Russolillo et al, 2011)

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \quad \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

Level differentials

Modelling by CoD and socio-economic stratification

Three-way Lee-Carter model (Russolillo et al, 2011)

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

Level differentials

Improvement differentials

Modelling by CoD and socio-economic stratification

Three-way Lee-Carter model

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

- Estimate the model parameters using a two stage estimation procedure with a reference population
  - National population data available for longer periods of time than socio-economic disaggregated data
  - More precise estimation of the long-run mortality trend
  - Coherency with the national mortality trend
- Stage I:
  - Estimate $\alpha_x^c, \beta_x^c, \kappa_t^c, \delta_x^{c,(i)}$ using the reference population data
- Stage II:
  - Estimate $\alpha_{xg}^c, \lambda_g^c$ conditional on $\alpha_x^c, \beta_x^c, \kappa_t^c, \delta_x^{c,(i)}$

Case study: Mortality by deprivation in England

Application data

Subpopulation data

- England population disaggregated by deprivation quintile using the 2007 version of the English Index of Multiple Deprivation (IMD 2007)
- Ages: 25-29,30-34,…,80-84
- Period: 1981-2007

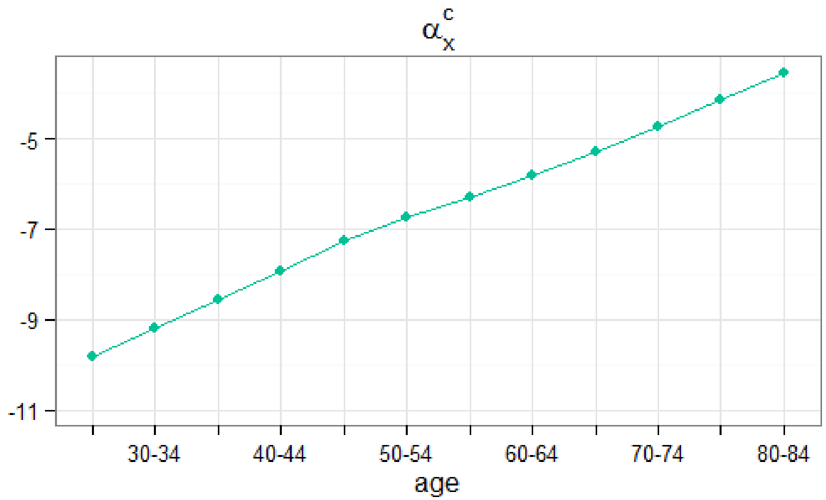
Reference population data

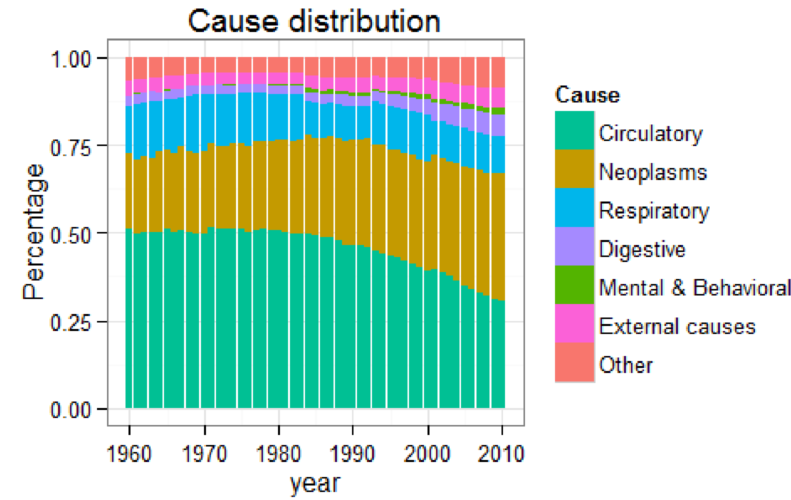
- England and Wales population
- Ages: 25-29,30-34,…,80-84
- Period: 1960-2009

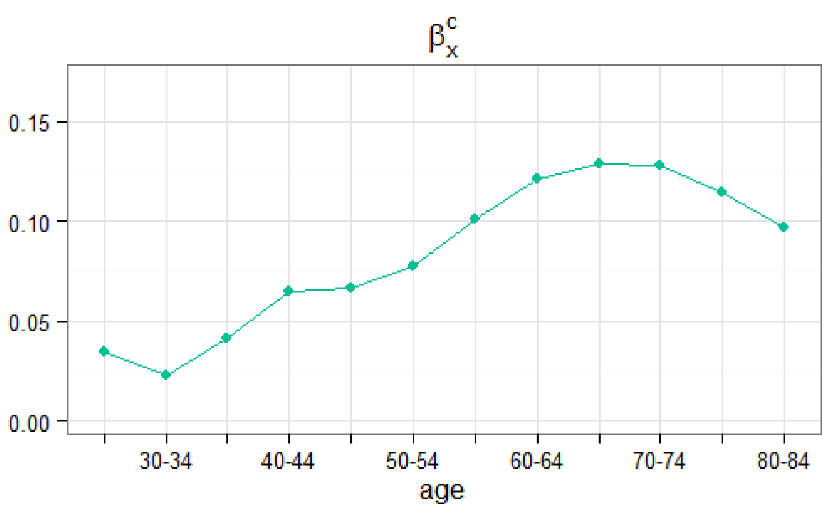
Case study: Mortality by deprivation in England

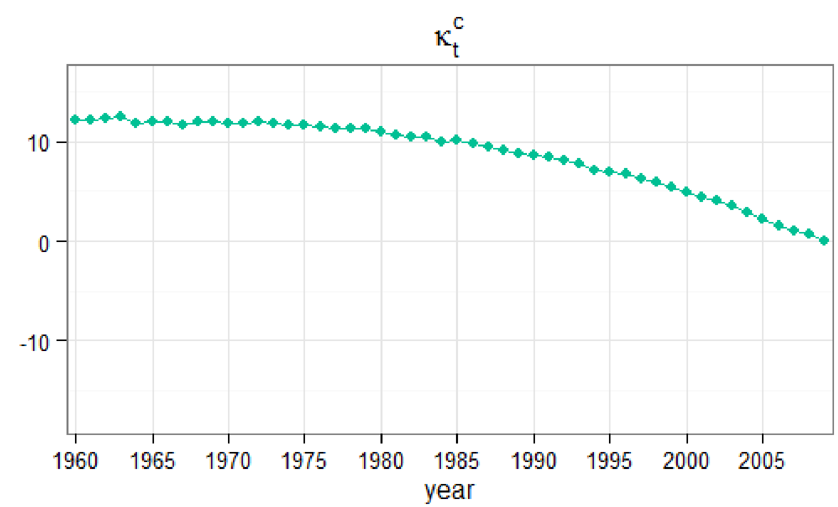
England and Wales Male population parameters

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$









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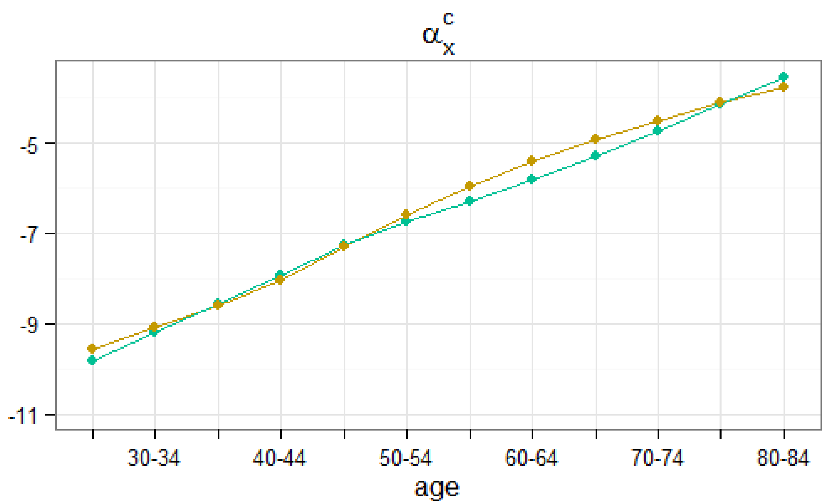

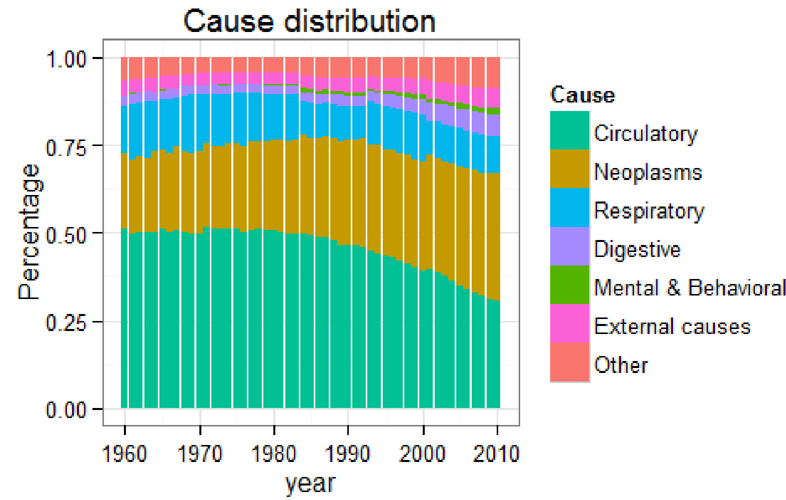

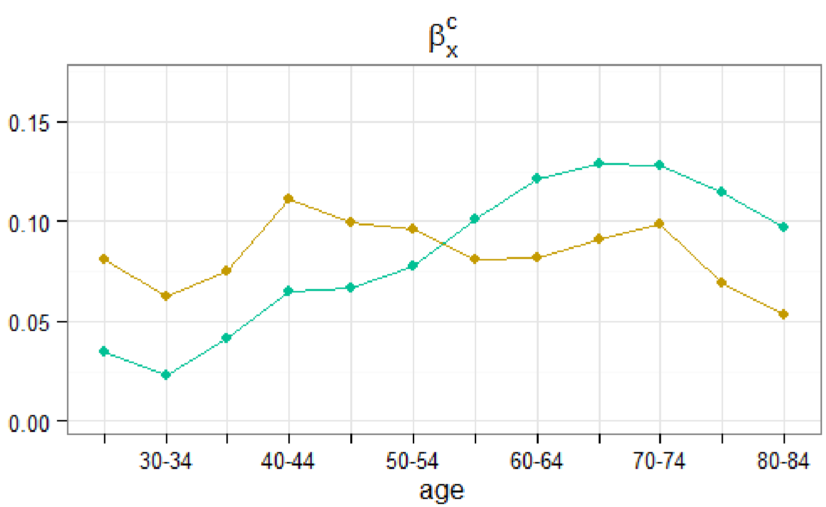

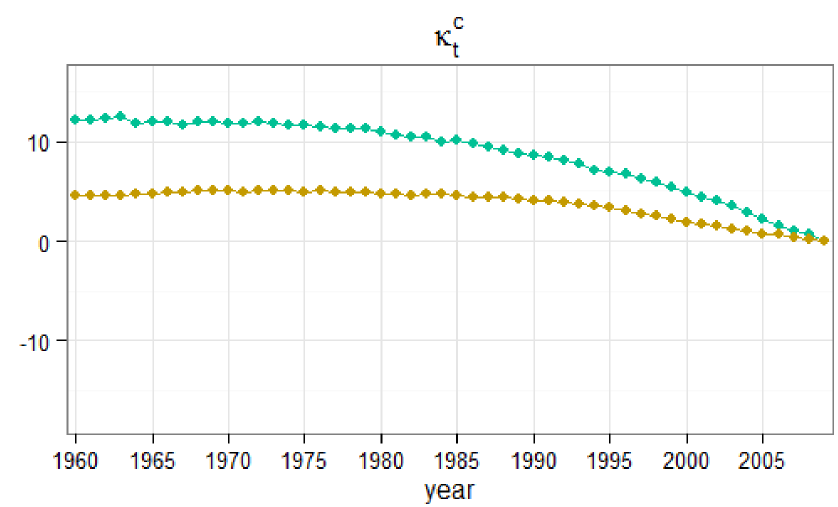

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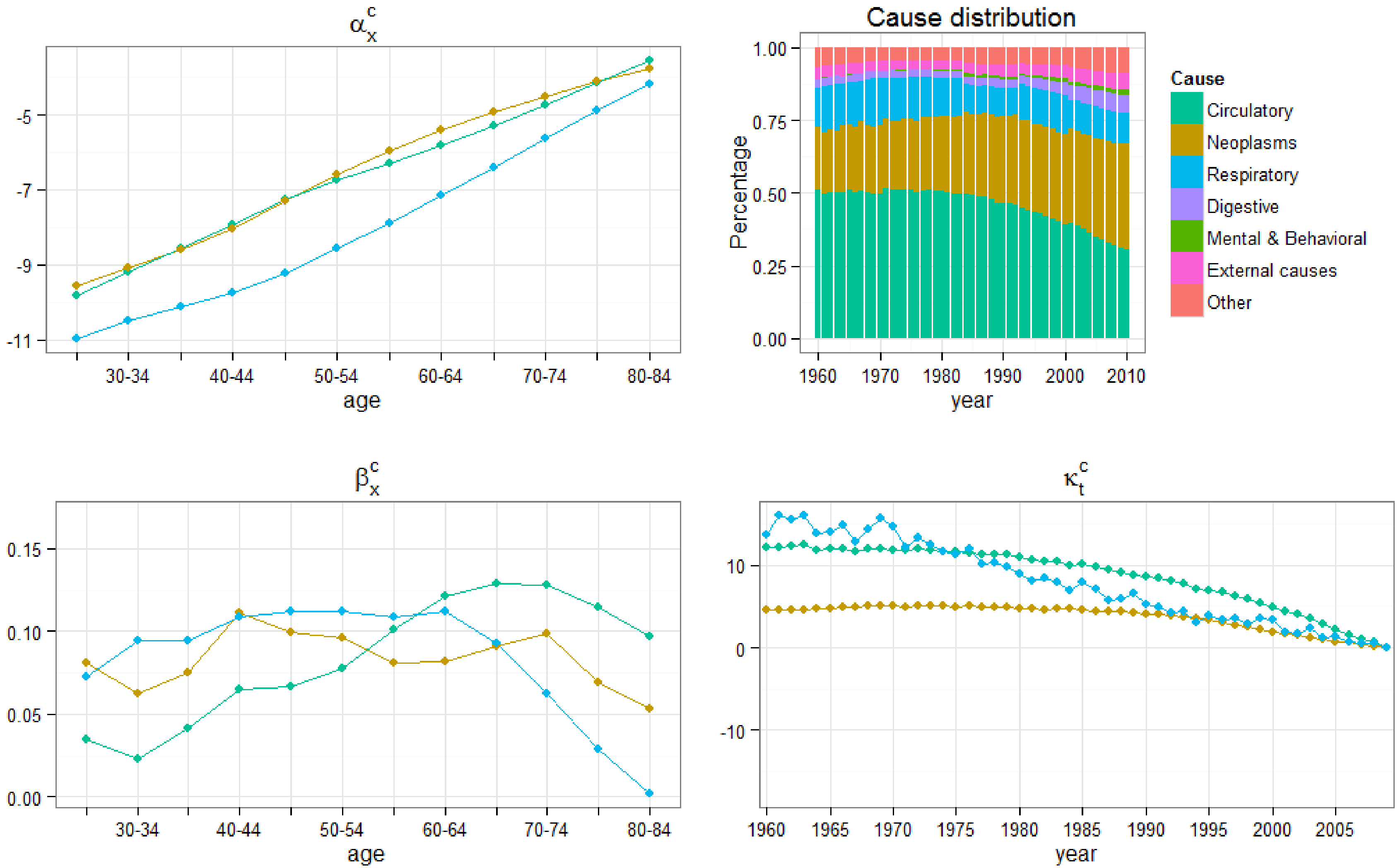

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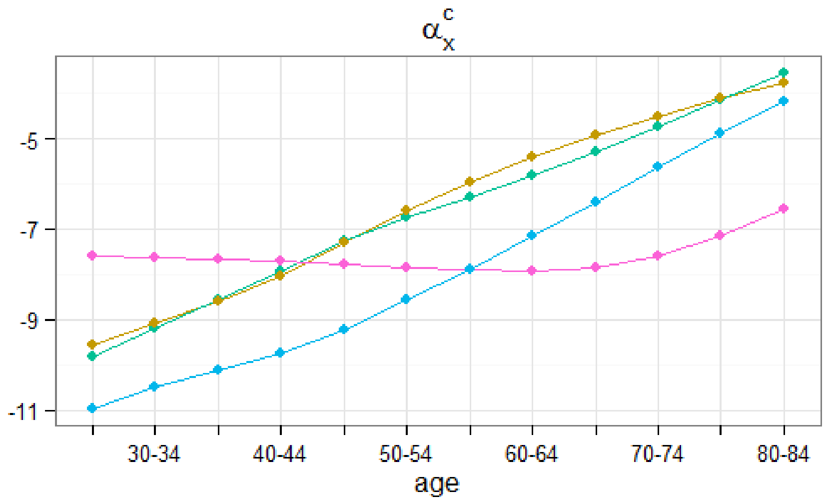

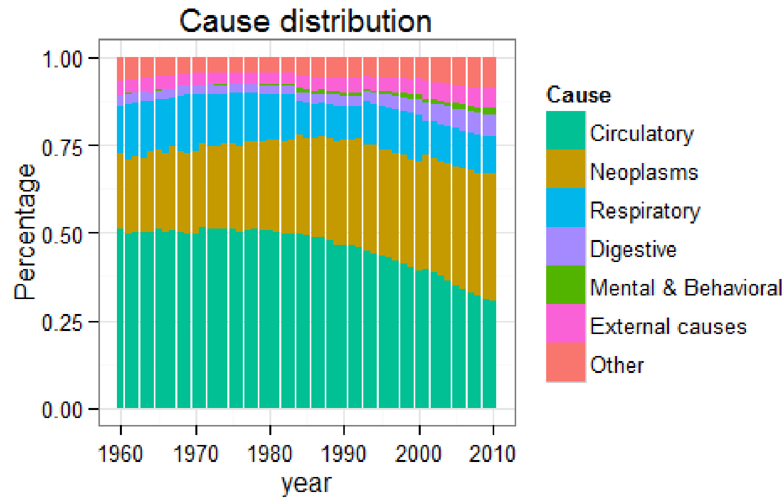

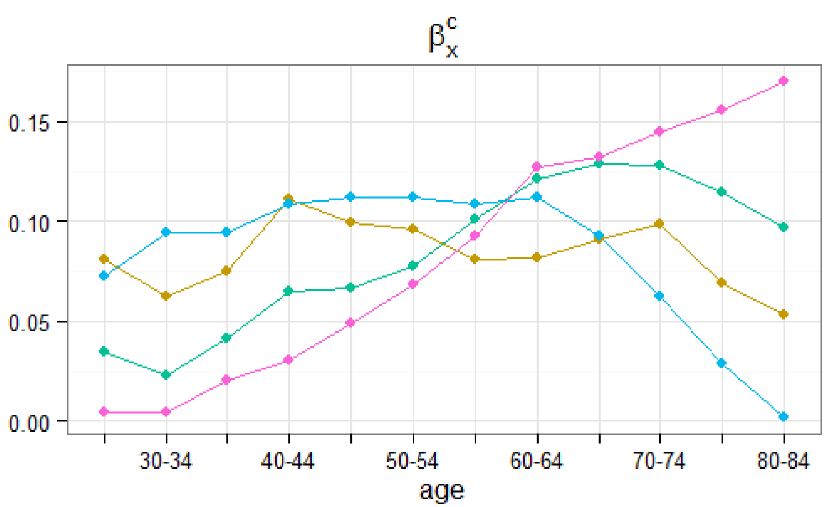

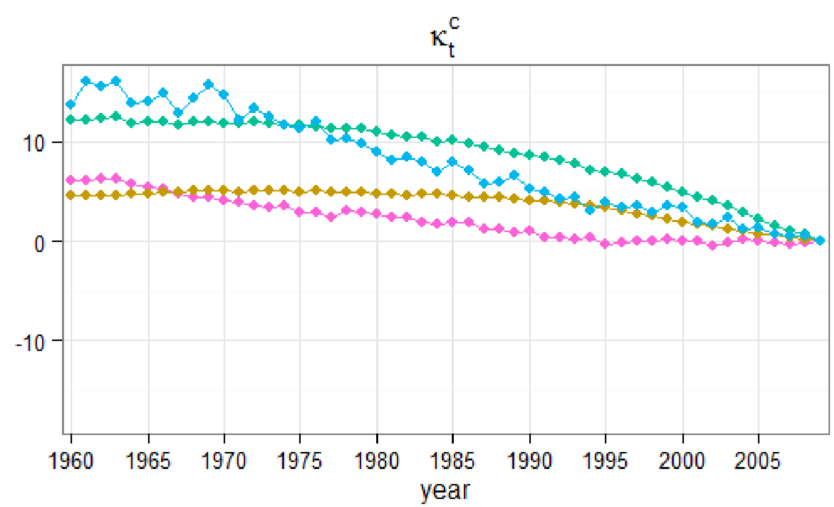

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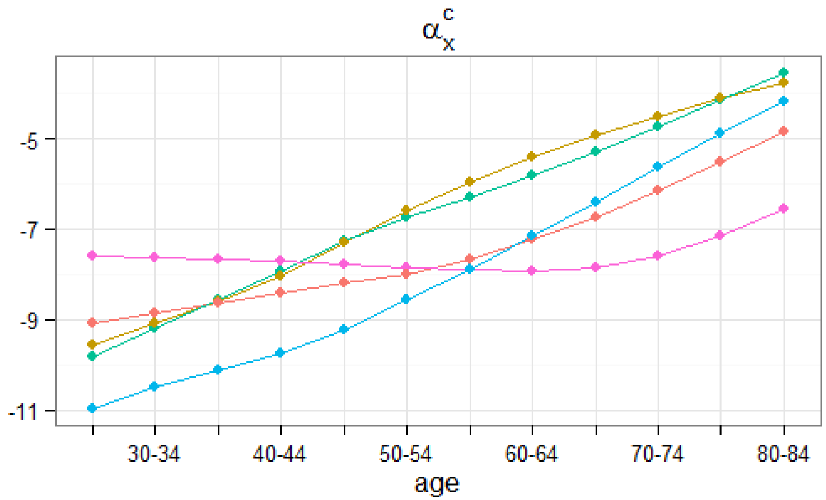

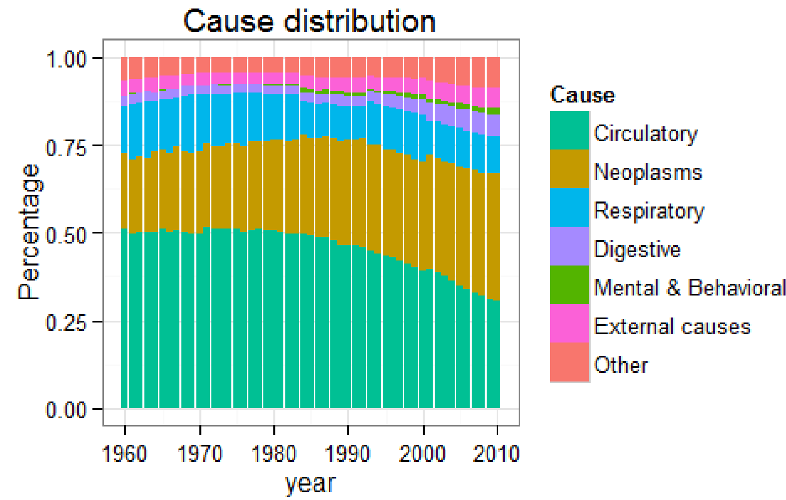

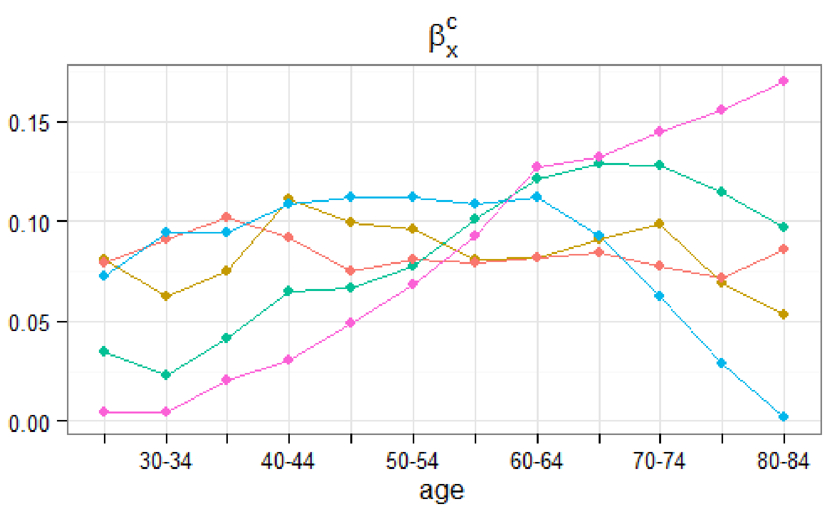

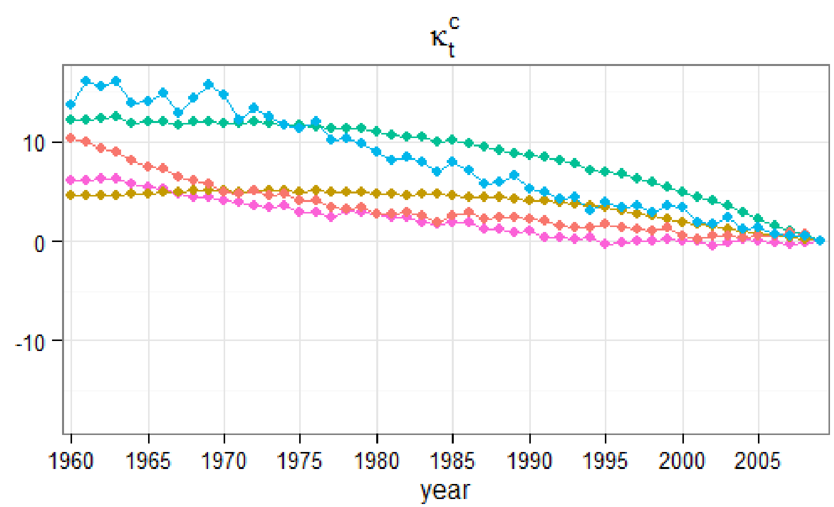

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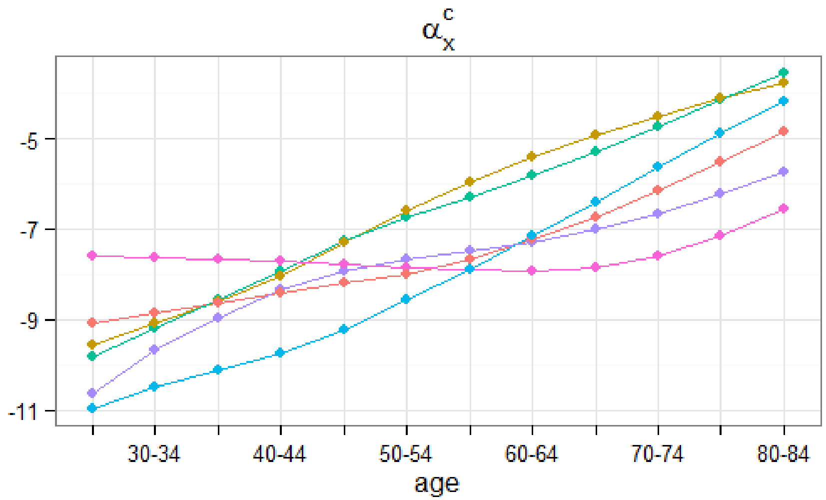

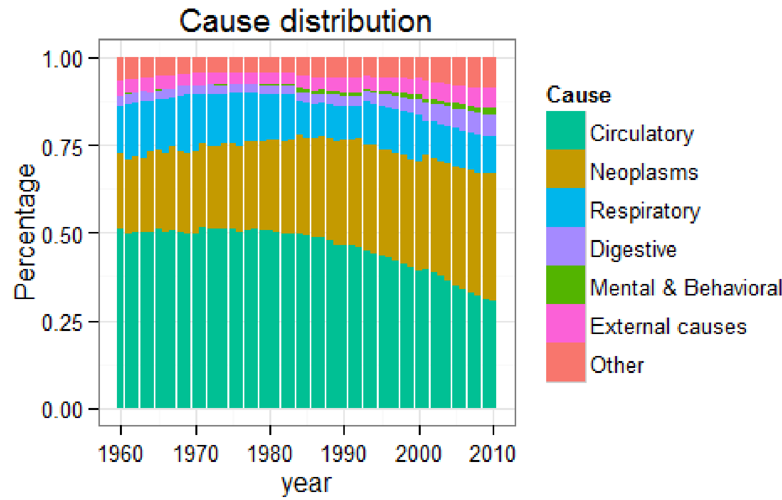

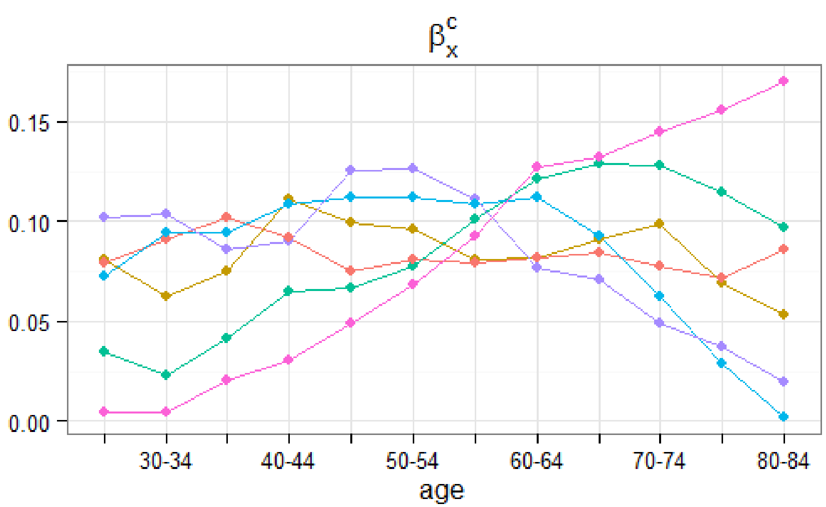

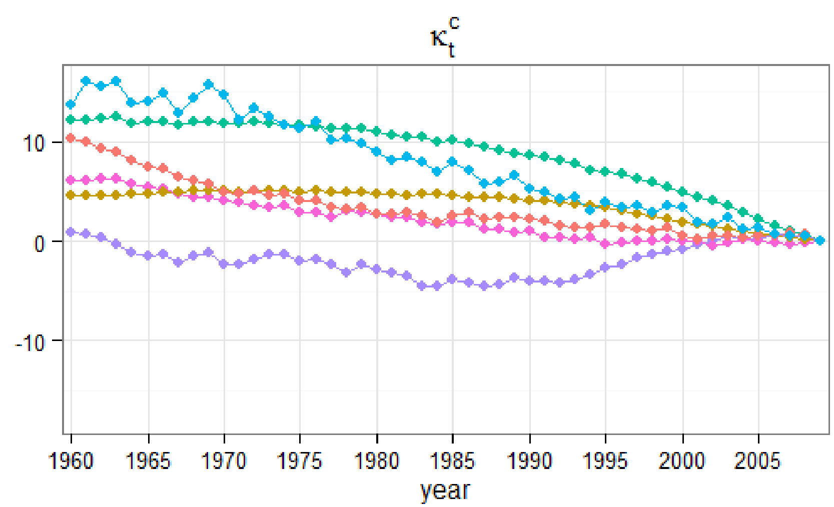

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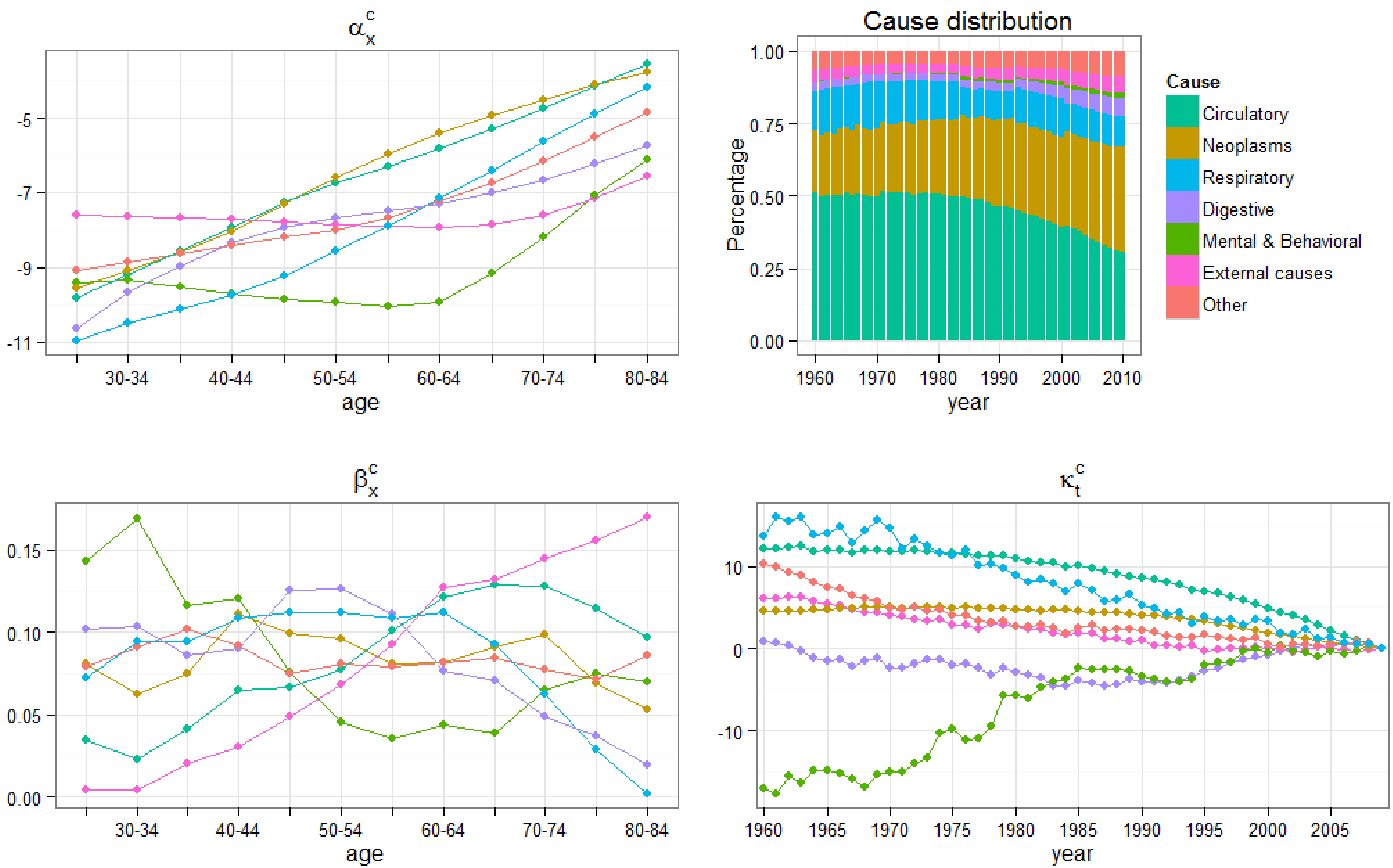
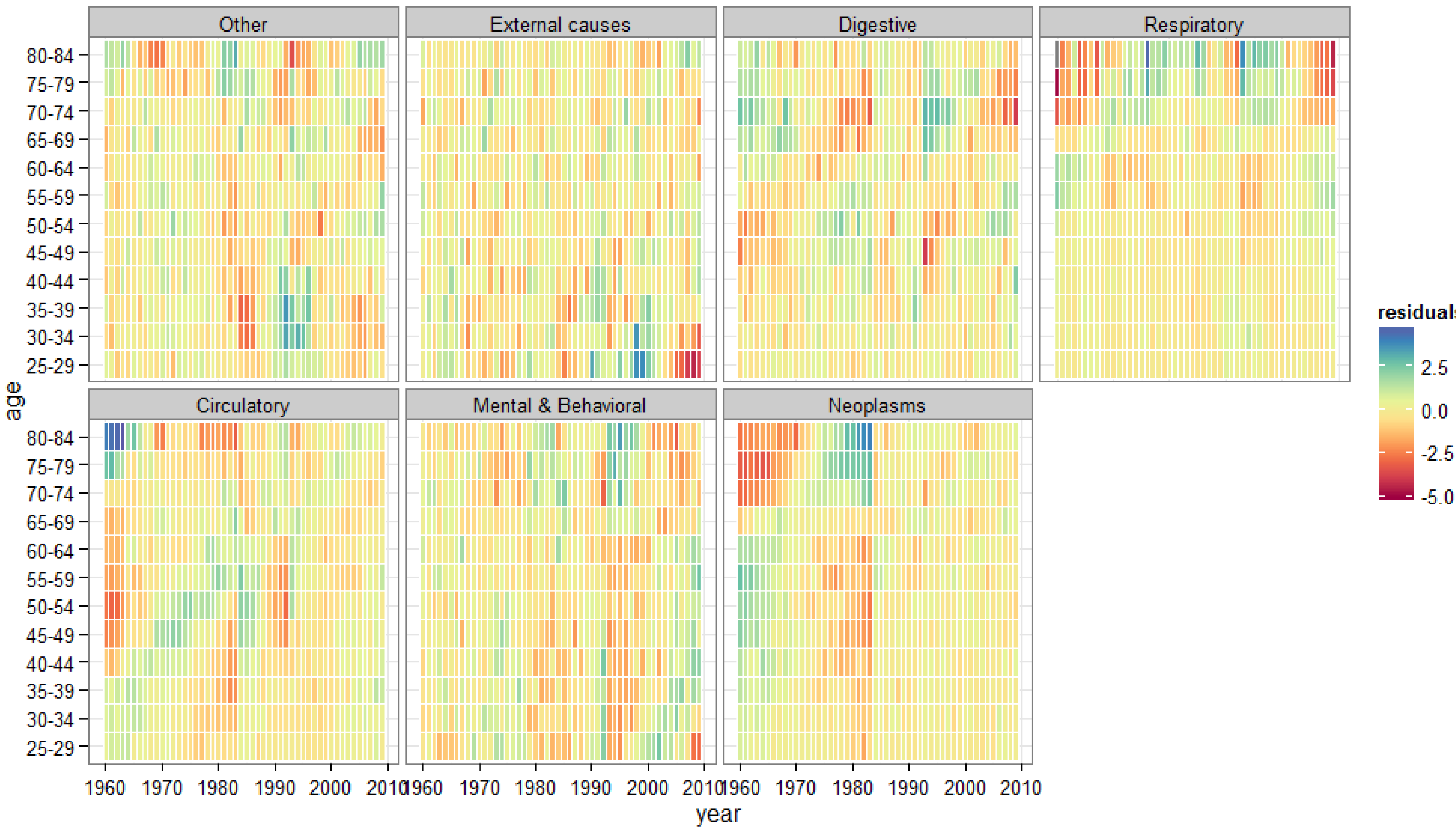


Case study: Mortality by deprivation in England

England and Wales Male population - Residuals

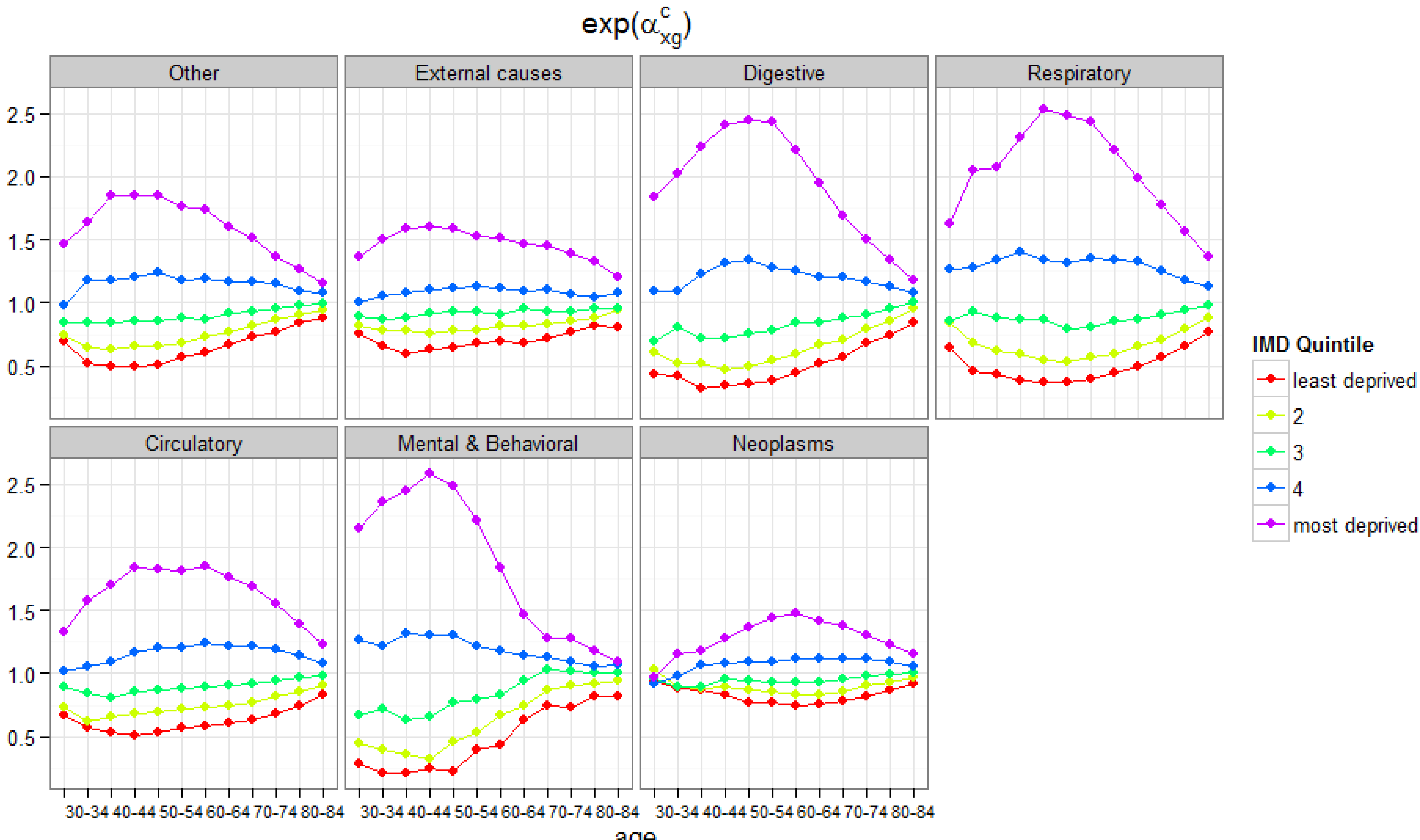
$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$



Case study: Mortality by deprivation in England

Level differences by deprivation quintile

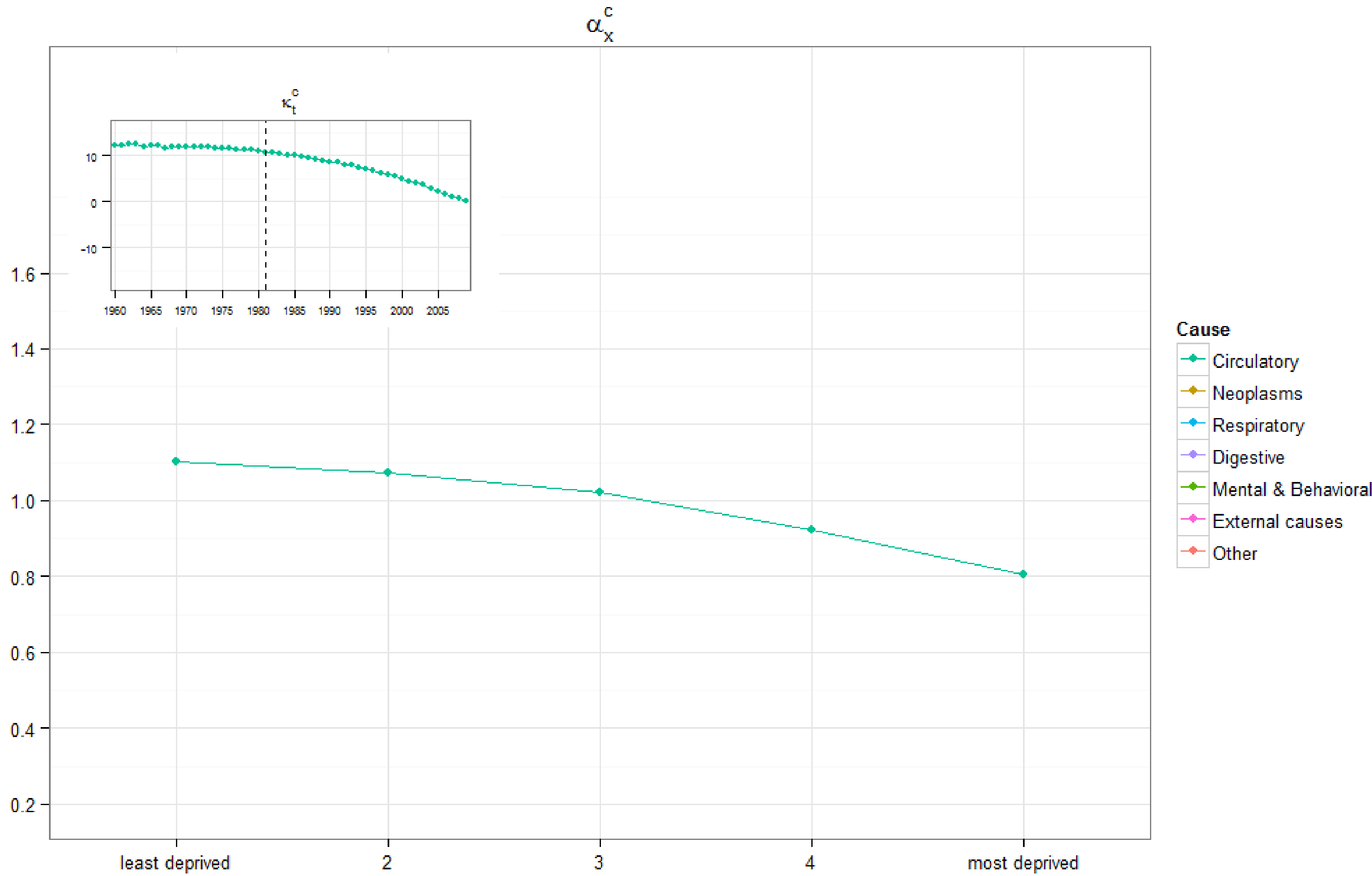
$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$



Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$



Case study: Mortality by deprivation in England

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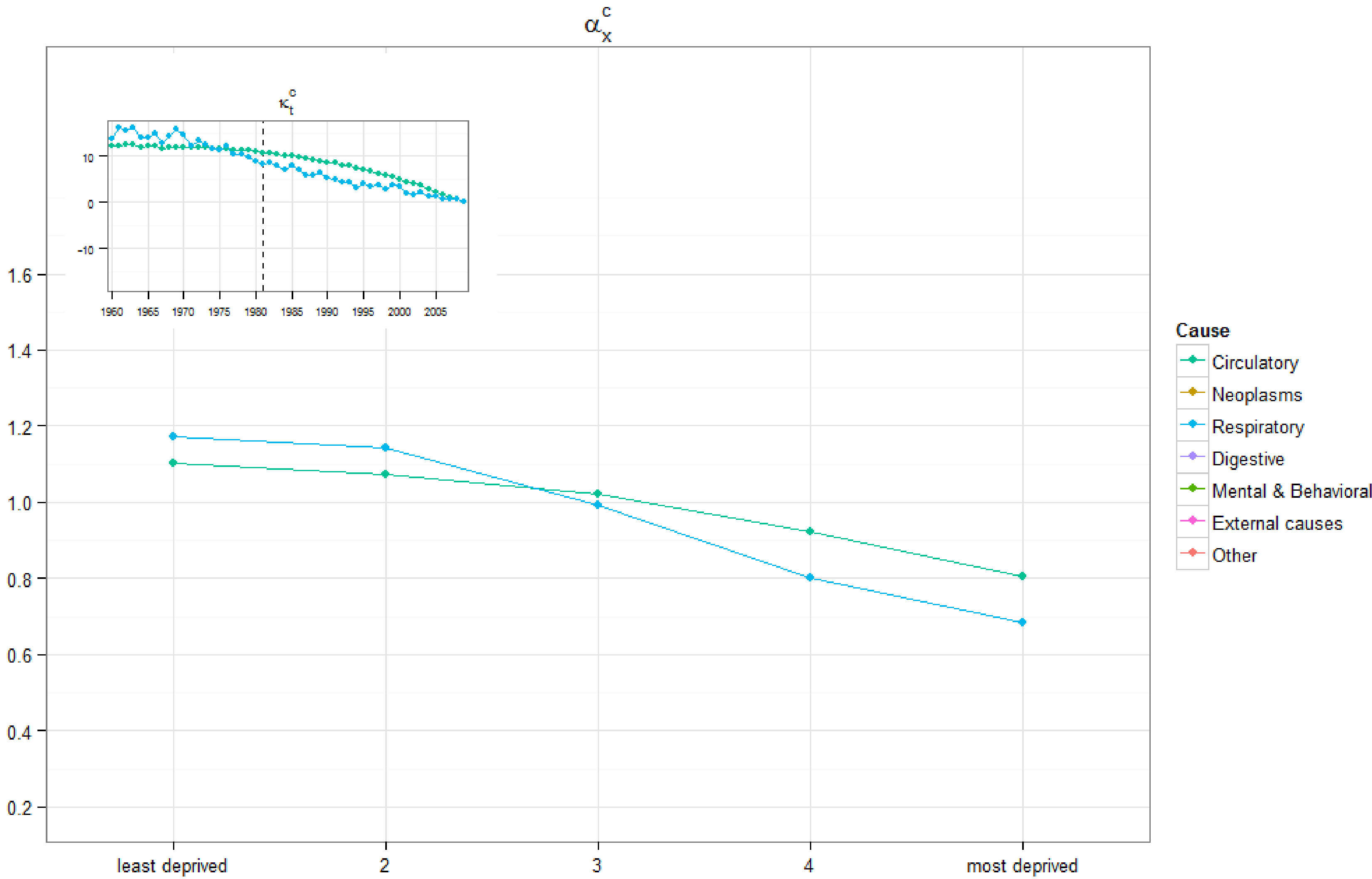

Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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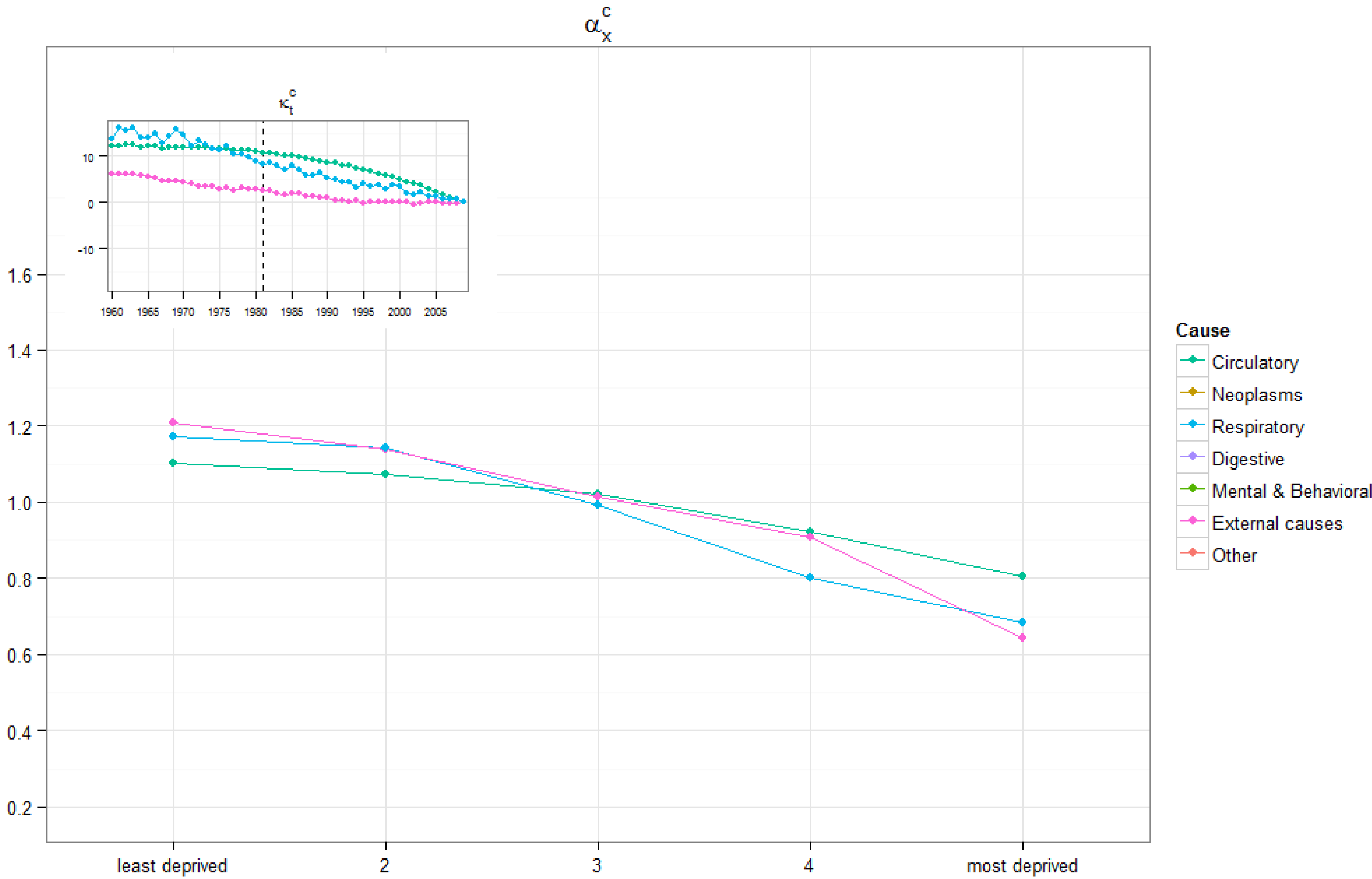

Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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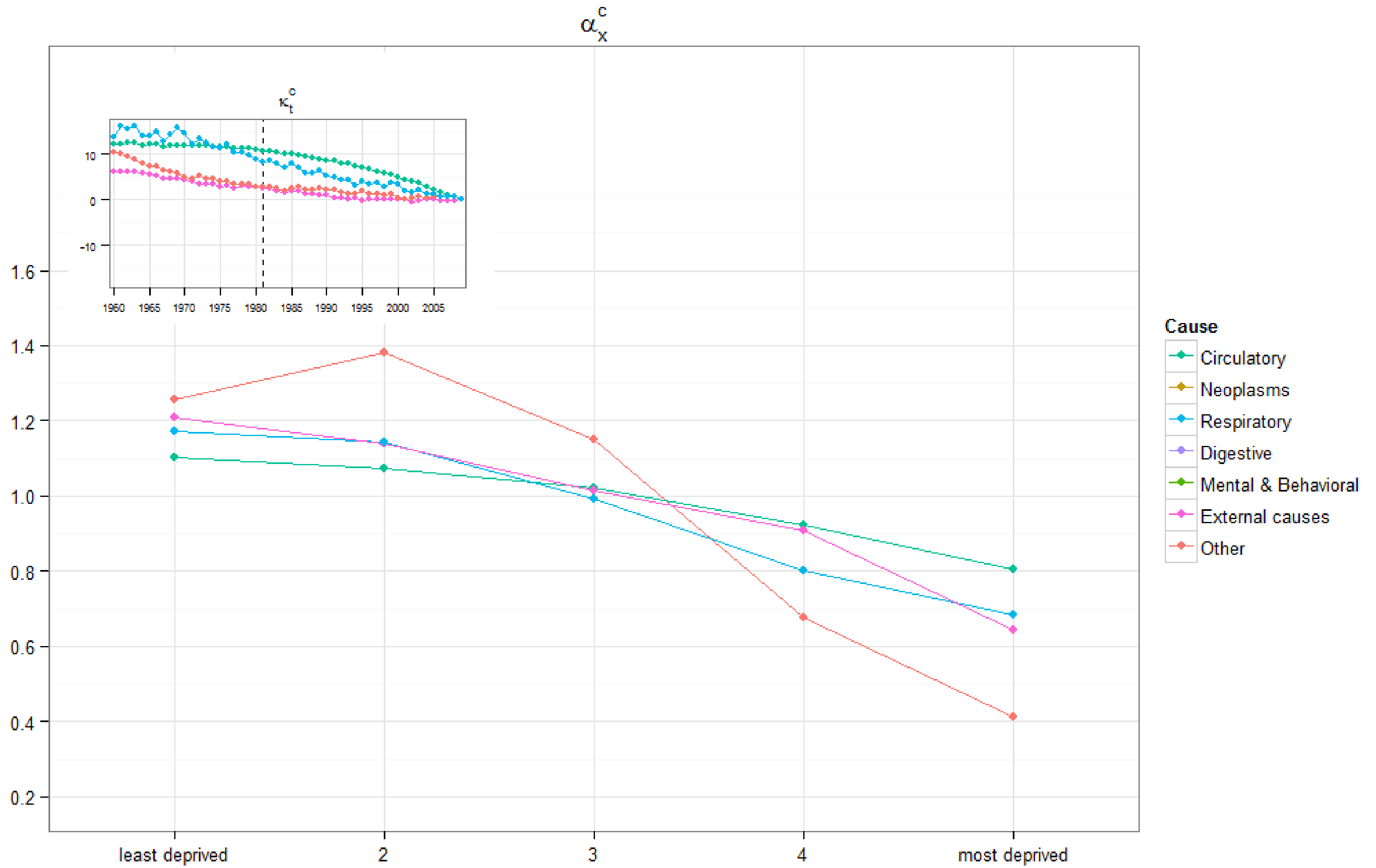
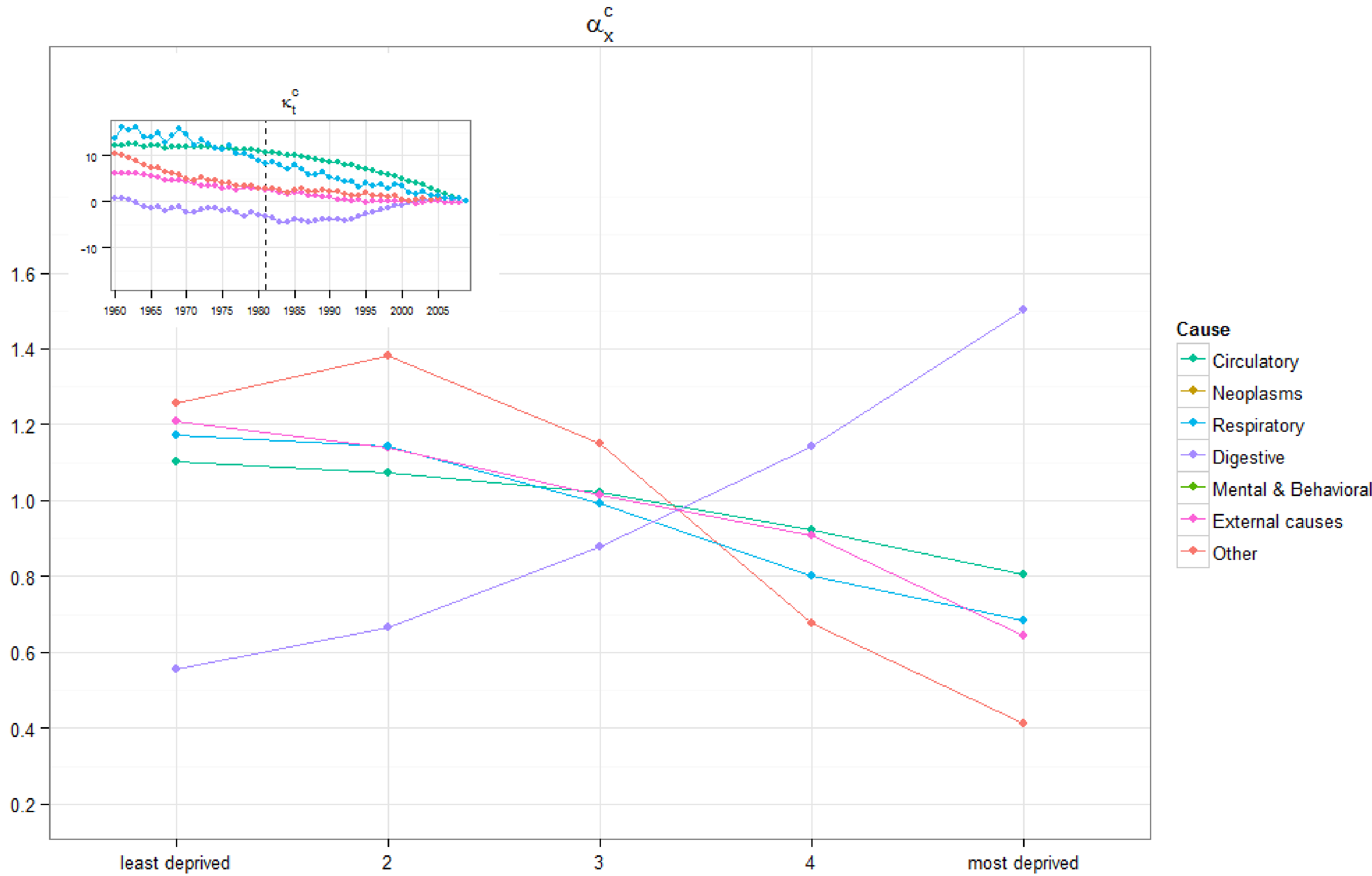


Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

$$\log \mu^{c}_{xtg} = \alpha^{c}_{x} + \alpha^{c}_{xg} + \beta^{c}_{x}\lambda^{c}_{g}\kappa^{c}_{t} + \sum_{i=1}^{h} \delta^{c,(i)}_{x} f^{(i)}(t)$$



Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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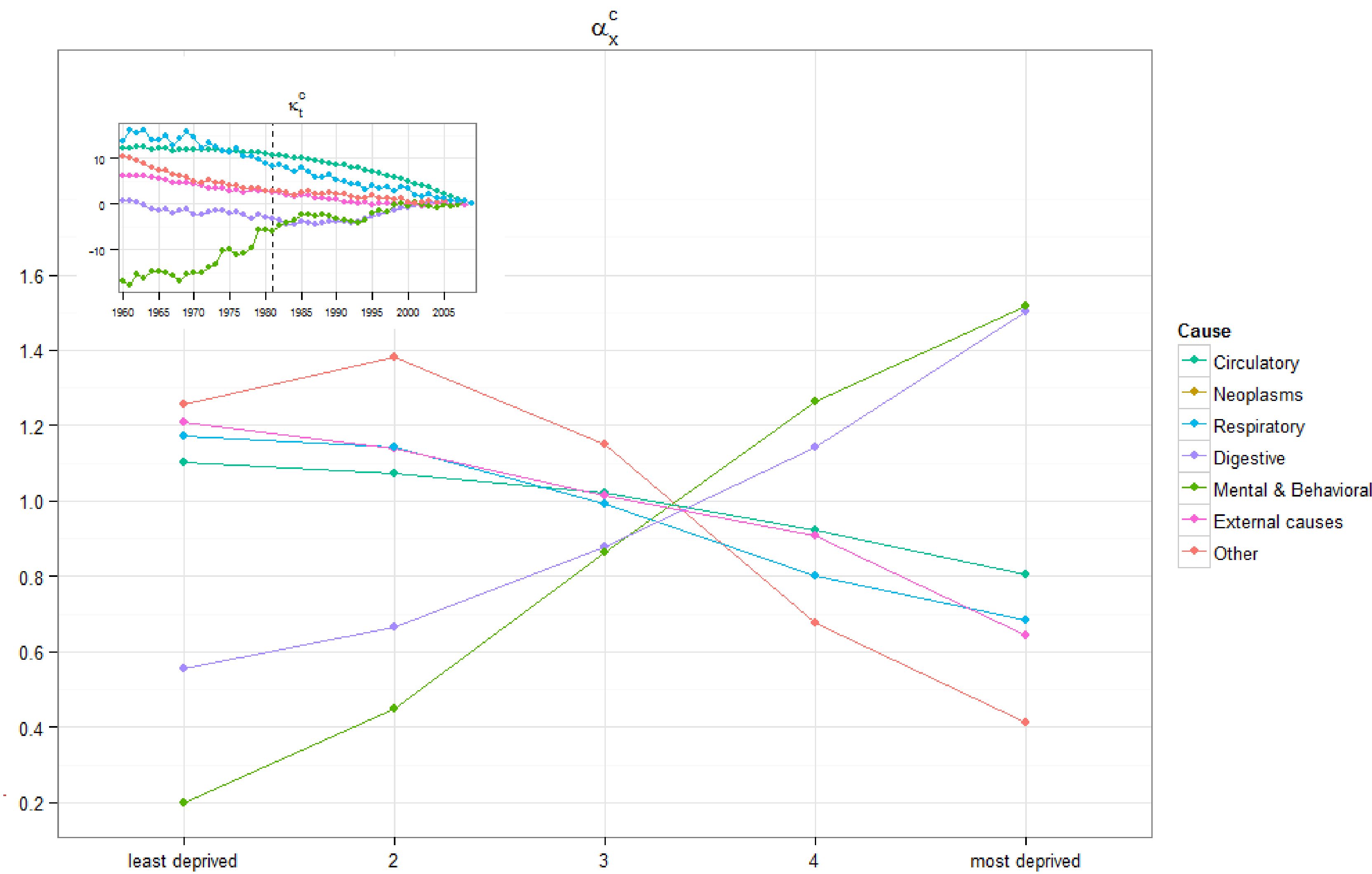

Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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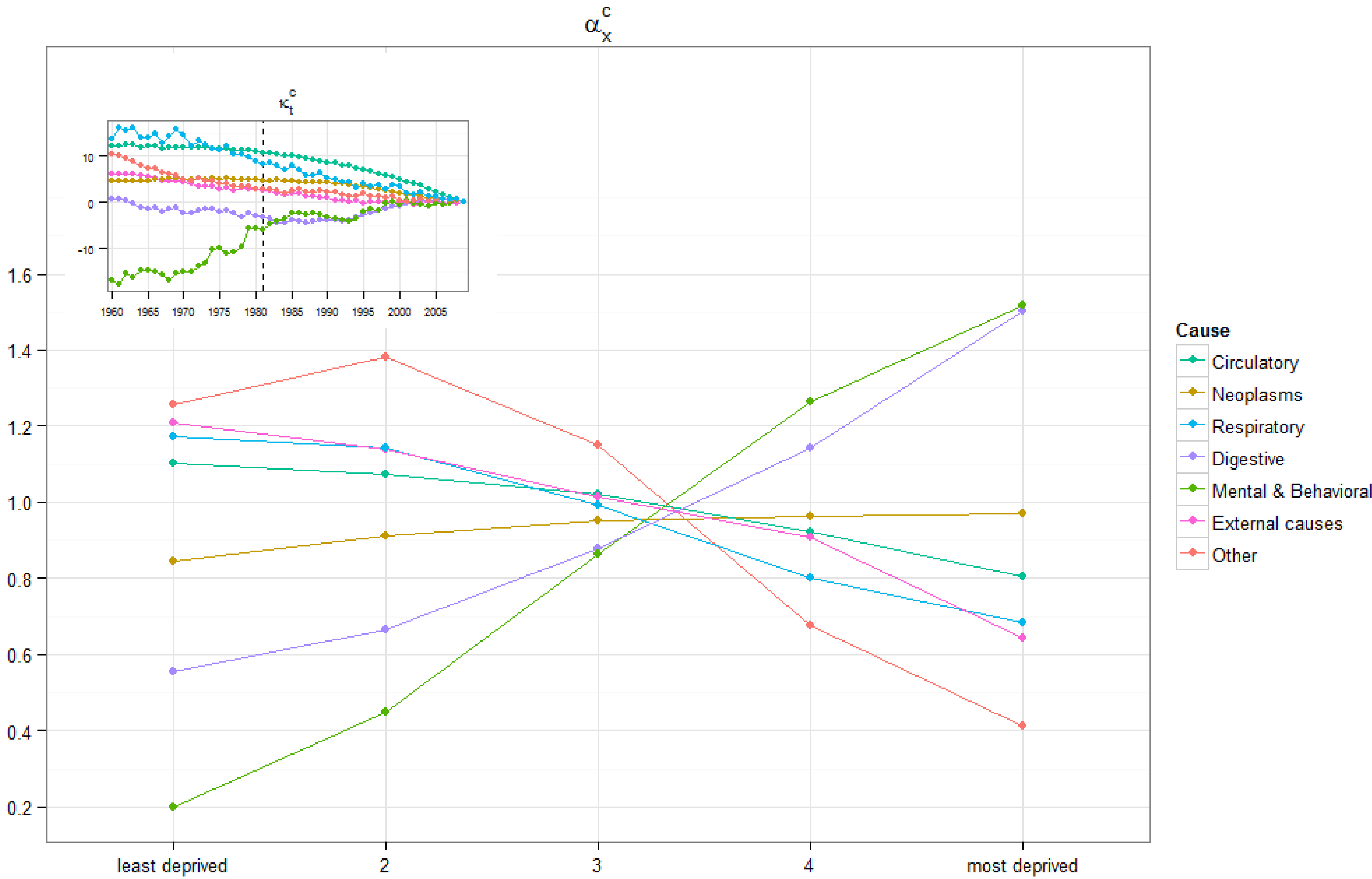

Conclusions

- Introduce an extension of the Lee-Carter model to deal with production changes in cause-specific mortality
- Embed this model in a multipopulation framework to assess socio-economic differences in cause of death
- Application in the analysis of the extent of mortality differentials across deprivation subgroups in England for the period 1981- 2007
  - Clear inverse relationship between area deprivation and mortality for all causes
  - Reduction of differentials in cancer mortality
  - Offset of this reduction by marked differentials in digestive, respiratory and mental and behavioural diseases

Thank you!

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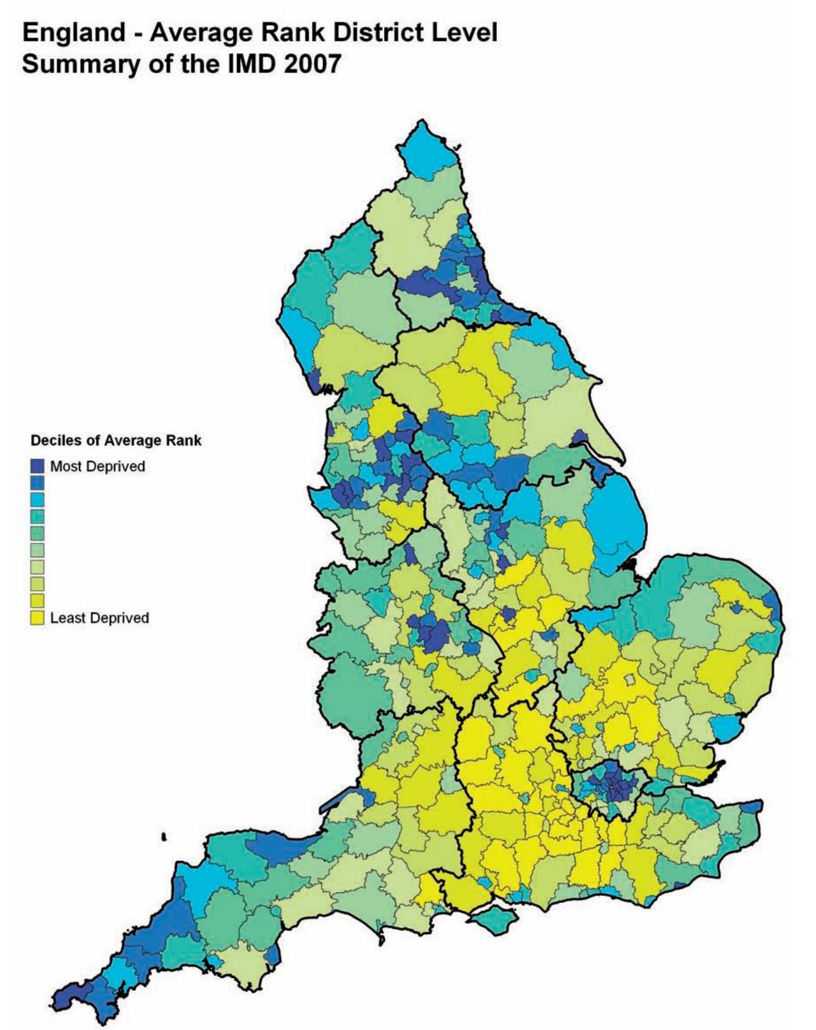
Reserve Slides

Case study: Mortality by deprivation in England

Application data - IMD 2007

- Socio-economic classification of the population obtained using the Index of Multiple Deprivation 2007 (IMD 2007)
- IMD 2007 combines indicators across 7 deprivation domains into a single deprivation score for each geographically defined Lower Layer Super Output Area (LSOA)
  - Income, employment, health, education, housing and services, crime, and living environment
- 32,482 LSOA in England with approximately 1,500 people each
- LSOAs ranked from 1 to 32.482 by their IMD 2007 score and grouped into quintiles
  - Q1: Least deprived quintile
  - Q5: Most deprived quintile

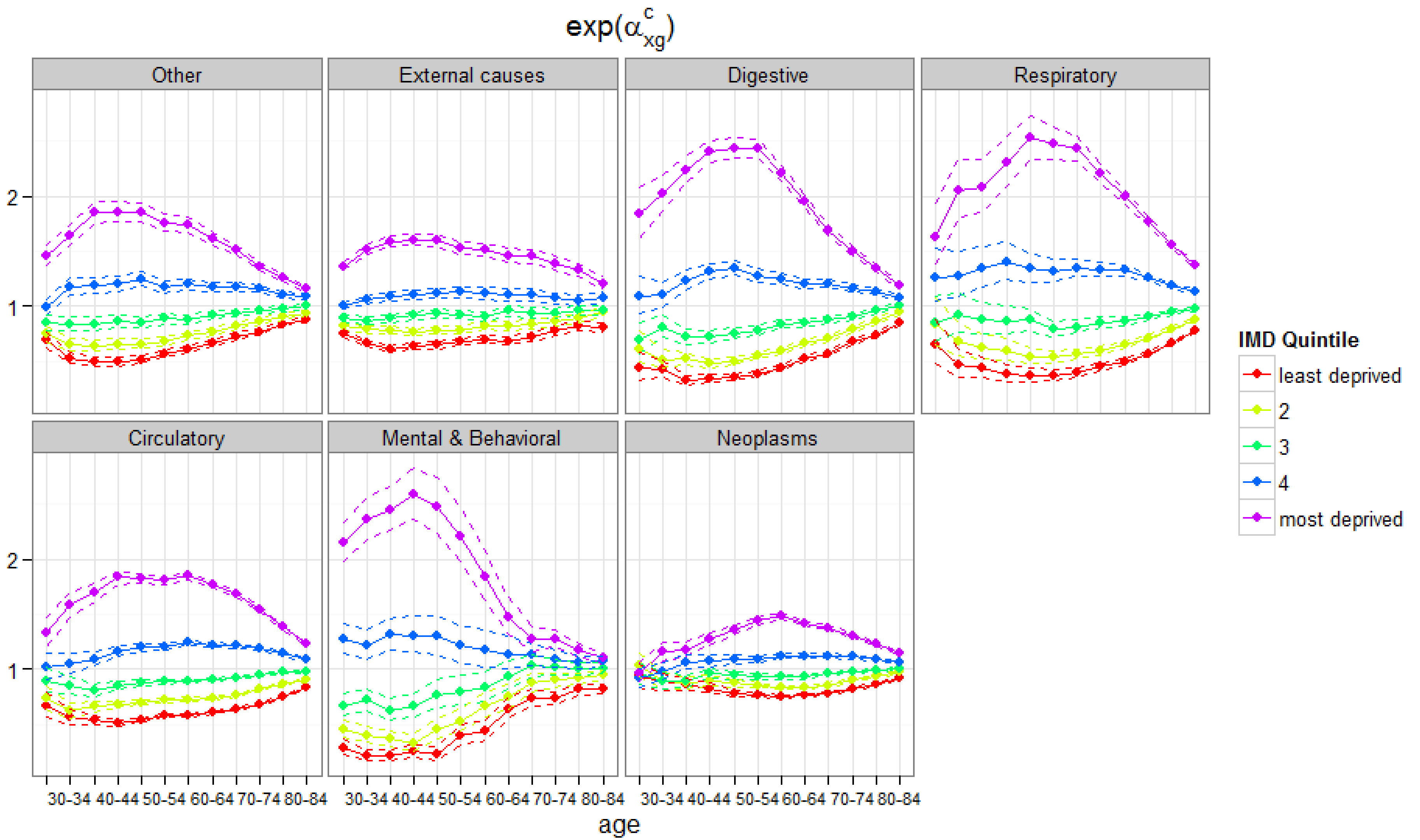


Source: Noble et al (2007)

Case study: Mortality by deprivation in England

Level differences by deprivation quintile

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c$$



Case study: Mortality by deprivation in England

Trend differences by deprivation quintile